A Kinematic Wave Model For Merge Queuing

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ABSTRACT
Queuing at a merge is of particular interest because it is often related to highway congestion. Kinematic waves models such as simplified theory of kinematic waves are welcomed by traffic engineers because these models are simple and efficient yet still provide sufficient measures of effectiveness (MOEs) without going into the low level details of individual vehicle behavior. However, the original simplified theory deals only with freeway mainline and it would be of great interest to both theory and practice if this simple and elegant algorithm can be extended to address network traffic. As part of such effort, this paper deals with simplified kinematic waves at a merge. Several merge models are reviewed and a capacity-based weighted fair queuing (CBWFQ) merge model is proposed. This merge model exhibits several attractive features which make it ideal for modeling merge queuing in terms of simplified theory of kinematic waves.

Keywords: Simulation and Modeling, Traffic flow, Simplified theory of kinematic waves, Queuing, Merge.

REVIEW OF EXISTING MERGE MODELS

To facilitate future discussion, a merge is sketched below which has two upstream links/branches and one downstream link. A merge can be a junction where an on-ramp joins a freeway, two freeways or highways come into one, or even a multi-leg intersection if properly defined. Suppose at time \( t \), link 1 wants to send \( S_1 \) vehicles, link 2 wants to send \( S_2 \) vehicles, and the downstream link can receive \( R \) vehicles. Denote \( d_1 \) the out flow (i.e., departure count) of link 1, \( d_2 \) the out flow of link 2, and \( d \) the inflow of the downstream link, where \( d = d_1 + d_2 \). Denote also \( p_1 \) the priority factor or splitting coefficient of link 1 and \( p_2 \) that of link 2. They are non-negative fractions. See the sketch of a merge in Figure 1.

Lebacque’s Lane-Based Merge Model
This model [1] is based on user optimal strategy, i.e., the model tries to maximize outflows at the merging branches and sum them up to obtain the downstream inflow. The model works as follows:

\[
\begin{align*}
    d_1 &= \min\{S_1, p_1 R\} \\
    d_2 &= \min\{S_2, p_2 R\} \\
    d &= d_1 + d_2
\end{align*}
\]

The splitting coefficients, \( p_1 \) and \( p_2 \), defined as the ratios of maximum densities of the upstream links to that of the downstream link, often translate to the ratios of their respective number of lanes. There is no guarantee that the splitting coefficients sum up to 1.

Solution space of Lebacque’s model is illustrated by the shaded area in Figure 2. Notice that, in addition to the upsloping solid line through the origin, the dashed lines indicate other possibilities of splitting coefficients. If \( R \) is fixed and \( S_1 + S_2 \leq R \), the solution lies on bold line S2-1-2-3-4-S1. If otherwise \( S_1 + S_2 > R \), the solution lies on bold line S2-1-6-4-S1. The above cases hold when \( p_1 + p_2 \leq 1 \). If, however, if \( p_1 + p_2 > 1 \), a solution found may not be realistic because the sum of upstream out flows can be greater than downstream supply, and something has to be done to make it feasible. Nevertheless, this model is comprehensive and yields the largest solution space.

Daganzo’s Priority-Based Merge Model
Unlike Lebacque’s model, this model [2] is based on system optimal strategy, i.e., the model tries to maximize the downstream inflow while keeping upstream outflows feasible. The model works as follows:

\[
\begin{align*}
    d_1 &= \min\{S_1, p_1 R\} \\
    d_2 &= \min\{S_2, p_2 R\} \\
    d &= d_1 + d_2
\end{align*}
\]

The splitting coefficients, \( p_1 \) and \( p_2 \), defined as the ratios of maximum densities of the upstream links to that of the downstream link, often translate to the ratios of their respective number of lanes. There is no guarantee that the splitting coefficients sum up to 1.
\[
\begin{align*}
\begin{cases}
d_1 = S_1 & \text{if } R \geq S_1 + S_2 \\
d_2 = S_2 & \text{if } R < S_1 + S_2 \\
d_1 = \text{mid} \{S_1, R - S_2, p_2 R\} & \text{if } R < S_1 + S_2 \\
d_2 = \text{mid} \{S_2, R - S_1, p_2 R\} & \text{if } R < S_1 + S_2
\end{cases}
\end{align*}
\]

where operator mid means the middle point of the three points intersected by line \( R = d_1 + d_2 \).

Again, solution space of this model is shown by the shaded area in Figure 3 and the dashed lines indicate the many possibilities of the priority factors. The solution under certain supply and demand conditions can be multiple and all the points in the shaded are feasible, though they might not be equally likely. To find a unique solution, some additional constraints must be provided. For example, the priority constraint assumes that flow on link 1, \( d_1 \), has higher priority than flow on link 2, \( d_2 \), i.e., \( p_1 > p_2 \). This eliminates virtually half of the solution space. If \( p_1 \) and \( p_2 \) are fixed, the solution space reduces to the bold line 1-2-3-4. Depending on the values of sending flows and receiving flow, unique solution can be found at point 1 if both merging branches are constrained by backward waves, or at point 3 if link 1 is constrained by forward wave and link 2 is constrained by backward wave, or at point 4 if both branches are dictated by forward waves.

**FIGURE 3 Daganzo's priority-based merge model**

**Jin and Zhang’s Simplest Distribution Scheme**

In an attempt to develop a special and simplest case of Daganzo’s priority constraint, Jin and Zhang [3] proposed a distribution scheme based on contributions of upstream demands, i.e.,

\[
\begin{align*}
\begin{cases}
d_1 = S_1 & \text{if } S_1 + S_2 < R \\
d_2 = S_2 & \text{if } S_1 + S_2 \geq R \\
d_1 = \frac{R \times S_1}{S_1 + S_2} & \text{if } S_1 + S_2 < R \\
d_2 = \frac{R \times S_2}{S_1 + S_2} & \text{if } S_1 + S_2 \geq R
\end{cases}
\end{align*}
\]

Based on this assumption the solution space of Daganzo’s reduces to the bold line in Figure 4.

**FIGURE 4 Jin and Zhang’s demand-based merge model**

Unfortunately, this model may yield unrealistic results under certain conditions. For example, suppose that upstream mainline demand is \( S_1 = 2000 \), on-ramp demand is \( S_2 = 200 \), and downstream supply is \( R = 2000 \). This distribution scheme suggests a solution of \( d_1 = 1818 \) and \( d_2 = 182 \). This implies that the on-ramp can depart only one vehicle at the departure of every 10 vehicles at the upstream mainline, a scenario that is very rare in real life.

**Newell’s Simplified Merge Model**

As one of his underlying assumptions of the simplified theory of kinematic waves [4][5][6], Newell assumed that ramp-entering vehicles can always bypass a queue, if any, and experience no delay. Unlike Daganzo’s model, here the full priority is given to on-ramp traffic, i.e., \( p_1 = 1 \) and \( p_2 = 0 \). The model works as follows:

\[
\begin{align*}
\begin{cases}
d_2 = S_2 & \text{if } S_1 + S_2 \leq R \\
d_1 = S_1 & \text{if } S_1 + S_2 > R \\
d_2 = R - S_2 & \text{if } S_1 + S_2 > R
\end{cases}
\end{align*}
\]

With this, the solution space is reduced to the bold line indicated in Figure 5. The solution is at point 1 if link 1 is dictated by backward waves or at point 2 if link 1 is dictated by forward waves. In either case, link 2 is always dictated by forward waves.

**FIGURE 5 Newell’s simplified merge model**

**Banks’ Ramp-Metering Merge Model**

As part of his effort to analyze the effect of ramp-metering in reducing traffic delay, Banks extends a little bit Newell’s...
assumption on a merge such that ramp out flow is constrained by its demand $S_2$, capacity $Q_2$, and metering rate $M_2$, i.e.,

$$d_i = \min\{S_2, Q_2, M_2\}$.

This model is basically the same as Newell's, i.e., full priority is still given to link 2 and backward waves never reach this branch. If we combine constraints $S_2, Q_2, and M_2$ into a new demand $S'_2$, Bank's model yields exactly the same solution space as Newell's.

As can be seen from above analysis, Lebacque's and Daganzo's models are comprehensive but require calibration which can be expensive. In addition, not all solutions are feasible, nor are feasible solutions practical. The other three models are very simple and easy to implement and require no calibration. However, these models are subject to over-simplification and may yield unrealistic results under certain conditions. The CBWFQ merge model proposed below preserves the advantages of the above models while addressing their unattractive features such that: (1) it deals with both forward and backward waves, so it is ideal for solving simplified kinematic waves problem; (2) it always yields unique solution, so it is well-formulated; (3) the solutions are physically meaningful and highly likely, so the model eliminates lots of unnecessary possibilities that may come at costs; (4) the model is simple and easy to understand and implement, so it makes a lot of sense to traffic engineers; (5) it takes into consideration as many factors as possible, such as demand, supply, road geometry, capacity, ramp-metering strategies, etc. at no extra cost, so it has lots of free stuff. (6) it has a good extensibility, so it has the potential to deal with a merge with multiple merging branches.

**THE CBWFQ MERGE MODEL**

As part of the effort to extend Newell's simplified theory of kinematic waves, a model is desired to deal with kinematic waves at a merge. Let:

- $a_1$: Traffic demand, i.e., number of vehicles, waiting somewhere upstream the merge at the side of link 1, that want to use link 3. $a_1$ can also be viewed as the arrival waiting to be served by link 1. $a_1$ is referred to as the upstream arrival thereafter. Here, we are refrained from using capital A since it is reserved for cumulative arrival count.
- $Q_1$: Capacity of link 1. Capacity is affected by factors such as number of lanes, per lane capacity, and various traffic control strategies such as ramp-metering if applicable.
- $S_1$: Number of vehicles on link 1 waiting to be served by link 3. $S_1$ is referred to as upstream demand thereafter.
- $\Delta_1$: Downstream supply shared by link 1 proportionally to its capacity.
- $d_1$: Number of vehicles that can actually move onto link 3 from link 1, i.e., the out flow or departure count thereafter. Similar to $a$ and $A$, capital $D$ is reserved for cumulative departure count.

All the above symbols may be functions of time which is omitted to keep the notation simple. The same set of symbols applies to link 2.

- $R$: Number of vehicles that can be received by link 3, i.e., the downstream supply thereafter.
- $Q_2$: Capacity of link 3.
- $d_2$: Number of vehicles that can actually move onto link 3.

Since upstream demand is constrained by upstream arrival and capacity (including traffic control strategies such as ramp metering if any), we have:

$$S_1 = \min\{a_1, Q_1\}$$

for link 1, and

$$S_2 = \min\{a_1, Q_1\}$$

for link 2.

To formulate the merge model, let's assume that traffic is greedy and wants to pass as many vehicles as possible. We also assume a capacity-based weighted fair queuing at a merge, which means that, for those merging branches dictated by backward waves, their demands are satisfied, or downstream supply is distributed, proportionally to their respective capacities.

Suppose the downstream supply is $R$, let's define:

$$\Delta_1 = R \times \frac{Q_1}{Q_1 + Q_2}$$

as the fair share of downstream supply for link 1, and

$$\Delta_2 = R \times \frac{Q_2}{Q_1 + Q_2}$$

as the fair share for link 2.

Suppose the upstream demands are $S_1$ and $S_2$. There are 4 possible cases:

Case 1: For each upstream link, its demand is less than or equal to its fair share, i.e., $S_1 \leq \Delta_1$ and $S_2 \leq \Delta_2$. In this case, each upstream link is dictated by forward wave and every one gets chance to depart without delay, i.e., $d_1 = S_1$ and $d_2 = S_2$.

Case 2: Demand of link 1 is less than or equal to its fair share, but demand of link 2 is greater than its fair share, i.e., $S_1 \leq \Delta_1$ and $S_2 > \Delta_2$. In this case, traffic on link 1 deserves its chance and can depart without delay, while traffic on link 2 can depart depending on the remainder of the downstream supply, i.e.,

$$d_1 = S_1$$

and

$$d_2 = \min\{S_2, R - d_1\}$$.

Case 3: the situation of case 2 reverses, i.e., $S_1 > \Delta_1$ and $S_2 \leq \Delta_2$. In this case, traffic on link 2 can depart without delay, while traffic on link 1 make use of the remainder of downstream supply, i.e.,

$$d_1 = \Delta_1$$

and $d_2 = \Delta_2$.

The solution space is illustrated by the shaded areas in Figure 6 which works as follows. A downsloping line at an angle of $135^\circ$ passing point B4 denotes supply line where the coordinates of every point on it sum up to downstream supply. $\Delta_1$ and $\Delta_2$ are link 1 and link 2's fair shares of downstream supply, respectively. Lines $\Delta_2$B4 and $\Delta_1$B4 divide the first quadrant into four areas numbered in circles. Let C denotes a demand point whose coordinates are upstream demands $(S_1, S_2)$ and B denotes a solution point whose coordinates are upstream out flows $(d_1, d_2)$. Area 1 corresponds to case 1 where demand point C1 coincides with solution point B1. Area 2 corresponds to case 2 and is further divided into two subsections by the supply line. In the lower left (shaded) subsection, demand point C2 coincides with solution point B2, while, in the upper right subsection, solution point B2’ is found at the intersection of the supply line and a vertical line through demand point C2’. Similar situation occurs in area 3 which corresponds to case 3, and the solution can be found as illustrated. Area 4 corresponds to case 4 where the solution is at B4 regardless of where the demand point C4 is.
The goal of this section is to extend the simplified theory of kinematic waves to incorporate a merging scenario. Figure 7 shows a sketch of the merge. There are two upstream links, \((x_i, x_j)\) and \((x_j, x_l)\), and one downstream link, \((x_l, x_n)\). Let \(x_i\) denote any further destination of \(x_i\) via \(x_n\). \(A_{i,n}(x_i, t)\) denotes the cumulative arrival curve (i.e., cumulative number of vehicles waiting somewhere) to the left of \(x_i\) at time \(t\) originated from node \(x_i\) destined for node \(x_n\) and beyond at time \(t\). \(D_{i,n}(x_i, t)\) denotes the cumulative departure curve (i.e., cumulative number of vehicles past) to the right of \(x_i\) at time \(t\) originated from node \(x_i\) destined for node \(x_n\) and beyond at time \(t\). \(x_i\) is the length of link \(x_i\). Capacity reflects number of lanes, per lane capacity, and traffic control strategies such as ramp-metering if any. The meaning of the above notations applies to similar symbols.

The simplified theory of kinematic waves starts from boundary conditions and proceeds in an iterative manner such that, at each time step (typically no less than the time that a vehicle traverses the shortest link at free flow speed), all nodes, from the beginning to the end, are evaluated one by one, and then time tick advances and the above process repeats. Suppose, from boundary conditions and previous time steps, we know the cumulative departure curves to the right of \(x_i\) destined from \(x_i\) destined for all destinations \(x_j\) (\(z = l, n, r, etc.\)) up to time \(t\). \(D_{i,z}(x_j, t)\) and the cumulative departure curves to the right of \(x_j\) destined from \(x_j\) destined for all destinations \(x_i\) (\(z = l, n, r, etc.\)) up to time \(t\). \(D_{i,z}(x_i, t)\). Suppose also that geometry data and traffic characteristics data are well defined for each link. Our goal here is to determine the cumulative departure curve past \(x_i\) destined for all destinations. This can be done by a 5-step procedure based on Newell’s simplified theory of kinematic waves.

A. Departure to the right
As proposed by Newell, the cumulative departure curve to the right of \(x_i\) originated from \(x_i\) destined for \(x_n\) and beyond, \(D_{i,n}(x_i, t)\) is constrained by the following:

a. Upstream arrival
According to the forward wave propagation rule of the simplified theory, the cumulative arrival curve to the left of \(x_i\) originated from \(x_i\) destined for \(x_n\) and beyond, \(A_{i,n}(x_i, t)\), can be obtained by translating the cumulative departure curve to the right of \(x_i\) originated form \(x_i\) destined for \(x_n\) and beyond, \(D_{i,n}(x_i, s)\), by a free link travel time \(T_{sf} = l_i/v_i\), i.e.,

\[A_{i,n}(x_i, t) = D_{i,n}(x_i, t - l_i/v_i)\]

Similarly, \(A_{i,n}(x_i, t)\) can be obtained from \(D_{i,n}(x_i, t)\), i.e.,

\[A_{i,n}(x_i, t) = D_{i,n}(x_i, t - l_i/v_i)\]

The arrival to the right of \(x_i\). \(A_{i,n}(x_i, t)\), is then the sum of \(A_{i,n}(x_i, t)\) and \(A_{j,n}(x_i, t)\), i.e.,

\[A_{i,n}(x_i, t) = A_{j,n}(x_i, t) + A_{i,n}(x_i, t) = D_{i,n}(x_i, t - l_i/v_i) + D_{j,n}(x_i, t - l_i/v_i)\]

b. Right capacity
d. Left capacity
The maximum number of vehicles that are allowed to depart from link \((x_i, x_j)\) at current time step is \(\tau \times Q_{ij}\) and the maximum number that are allowed to depart from link \((x_j, x_l)\) is \(\tau \times Q_{ij}\). So, the left capacity constraint can be:

\[D_{i,n}(x_i, t - \tau) + \tau Q_{ij} = \tau Q_{ij}\]

Therefore, Cumulative departure to the right of \(x_i\),

\[D_{i,n}(x_i, t)\]

is the minimum of the above four, i.e.,

\[D_{i,n}(x_i, t) = \min\{A_{i,n}(x_i, t), D_{i,n}(x_i, t - \tau) + \tau Q_{ij}, B_{i,n}(x_i, t - \tau) + \tau Q_{ij}, \}\]

B. Departure to the left

Now, we are interested in knowing, of $D_{x_n} (x'_i, t)$, how much is contributed by link $(x_i, x_l)$ and how much by $(x_l, x_j)$. This is the place where our proposed CBWFQ merge model plugs in. Let:

$a_{il}(x'_i, t)$ be the current arrival to the left of $x_i$ destined for $x_l$ and beyond. Notice that $a$ still follows the meaning of traffic demand as before, but now it is qualified by more variables. The same applies to $d$ and is not repeated.

$d_{il}(x'_i, t)$ be the current departure to the left of $x_i$ destined for $x_l$ and beyond,

$d_{jl}(x'_j, t)$ be the current departure to the left of $x_j$ destined for $x_l$ and beyond,

$s^\beta$ be the demand of link $(x_i, x_l)$, and $s^\rho$ be the demand of link $(x_l, x_j)$.

$R_{in}$ be the supply of link $(x_i, x_n)$.

$\Delta_{il}$ be link $(x_i, x_l)$’s fair share of downstream supply, and $\Delta_{jl}$ be link $(x_l, x_j)$’s fair share downstream supply.

Obviously, we have:

$R_{in} = D_{il} (x'_i, t) - D_{il} (x'_i, t - \tau)\quad \Delta_{il} = R_{in} \cdot \frac{Q_\beta}{Q_\beta + Q_\rho} \quad \Delta_{jl} = R_{in} \cdot \frac{Q_\rho}{Q_\beta + Q_\rho}$

$a_{il}(x'_i, t) = A_{il} (x'_i, t) - D_{il} (x'_i, t - \tau)\quad a_{jl}(x'_i, t) = A_{jl} (x'_i, t) - D_{jl} (x'_i, t - \tau)$

$s^\beta = \min \{a_{il}(x'_i, t) - QR_{il}, \Delta_{il} \} \quad s^\rho = \min \{a_{jl}(x'_i, t) - QR_{jl}, \Delta_{jl} \}$

As discussed before, there are 4 possible cases:

a. Case 1: $S^\beta \leq \Delta_{il}$ and $S^\rho \leq \Delta_{jl}$. The solution is:

$d_{il}(x'_i, t) = s^\beta$ and $d_{jl}(x'_i, t) = s^\rho$

b. Case 2: $S^\beta \leq \Delta_{il}$ and $S^\rho > \Delta_{jl}$. The solution is:

$d_{il}(x'_i, t) = S^\beta$ and $d_{jl}(x'_i, t) = \min \{S^\beta, R_{in} - d_{il}(x'_i, t) \}$

c. Case 3: $S^\beta > \Delta_{il}$ and $S^\rho \leq \Delta_{jl}$. The solution is:

$d_{il}(x'_i, t) = \min \{S^\beta, R_{in} - d_{il}(x'_i, t) \}$ and $d_{jl}(x'_i, t) = S^\rho$

d. Case 4: $S^\beta > \Delta_{il}$ and $S^\rho > \Delta_{jl}$. The solution is:

$d_{il}(x'_i, t) = \Delta_{il}$ and $d_{jl}(x'_i, t) = \Delta_{jl}$

Based on the above rules, the cumulative departure curve to the left of $x_i$ originated from $x_l$ destined for $x_n$ and beyond, $D_{x_n} (x'_i, t)$, and the cumulative departure curve to the left of $x_j$ originated from $x_j$ destined for $x_n$ and beyond, $D_{x_n} (x'_j, t)$, are determined as follows:

$D_{x_n} (x'_i, t) = D_{x_n} (x'_i, t - \tau) + d_{il}(x'_i, t)$

$D_{x_n} (x'_j, t) = D_{x_n} (x'_j, t - \tau) + d_{jl}(x'_j, t)$

Since this is a merge, no traffic exists. The cumulative departure curve to the left of $x_i$ originated from $x_j$ and beyond, $D_{x_n} (x'_i, t)$, is the same as $D_{x_n} (x'_j, t)$, and $D_{x_n} (x'_j, t)$ is the same as $D_{x_n} (x'_i, t)$, i.e.,

$D_{x_n} (x'_i, t) = D_{x_n} (x'_j, t)$

$D_{x_n} (x'_j, t) = D_{x_n} (x'_i, t)$

C. Link travel time

According to Newell, link travel time is obtained by comparing the cumulative departure curves at both ends of a link. Therefore, travel time at link $(x_i, x_j)$, $T_{x_i} (t)$, can be found by comparing the following pair of cumulative departure curves:

$D_{x_i} (x'_i, t) \quad \text{vs.} \quad D_{x_i} (x'_j, t)$

If we trace $D_{x_i} (x'_i, t)$ backwards until some prior time $t'$ such that $D_{x_i} (x'_i, t')$ is equal to $D_{x_i} (x'_j, t)$, then $T_{x_i} (t) = t - t'$ is the link travel time for the vehicle bearing the “number” $D_{x_i} (x'_i, t)$ at this link, and the link travel times for other vehicles at the same link are assumed to be the same regardless of their destinations. In a similar fashion, travel time on link $(x_j, x_l)$, $T_{x_j} (t)$, can be found by comparing curve pair $D_{x_j} (x'_j, t) \quad \text{vs.} \quad D_{x_j} (x'_j, t)$.

D. Departure to the left - multi-destinations

Based on Newell’s assumption that vehicles on the same link experience the same link travel time regardless of their destinations, the cumulative departure curves to the left of $x_i$ originated from $x_l$ destined for other destinations $x_n$, $D_{x_l} (x'_i, t)$, can be obtained by simply translating $D_{x_l} (x'_i, t)$ to the right by a link travel time, $T_{x_l} (t)$, and the same applies to other multi-destination cumulative departure curves $D_{x_l} (x'_i, t)$, i.e.,

$D_{x_l} (x'_i, t) = D_{x_l} (x'_i, t - T_{x_l} (t))$

$D_{x_l} (x'_i, t) = D_{x_l} (x'_i, t - T_{x_l} (t))$

E. Departure to the right - multi-destinations

The cumulative departure curve past the right of $x_i$ originated from $x_l$ destined for other destination $x_n$, $D_{x_l} (x'_i, t)$, is simply:

$D_{x_l} (x'_i, t) = D_{x_l} (x'_i, t) + D_{x_l} (x'_i, t)$

So far we have extended Newell’s simplified theory of kinematic waves to accommodate queuing at a merge.

Empirical Test

Empirical test of the proposed model is based on data collected from GA-400 by Georgia NAVIGATOR system, an
automatic traffic surveillance system covering the greater Atlanta metropolitan area.

The Test Site

The test site is a merge on the northbound of GA-400, as illustrated in Figure 8:

![Map of the test site](image)

FIGURE 8 The test site

This site consists of three mainline links, and an on-ramp link. The merge might be a bottleneck because the capacity downstream is less than the sum of that of upstream links. The geometry data and traffic characteristic data of the test site are summarized in Table 1.

### Table 1: Geometry and traffic characteristics data

<table>
<thead>
<tr>
<th>Link</th>
<th>Up Node</th>
<th>Down Node</th>
<th>Length (km)</th>
<th>Lanes</th>
<th>Type</th>
<th>FFS* (km/h)</th>
<th>Capacity (veh/h/ln)</th>
<th>Jam Density (veh/km/ln)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4000053</td>
<td>5008</td>
<td>0.16</td>
<td>3</td>
<td>Mainline</td>
<td>4400</td>
<td>4200</td>
<td>400</td>
</tr>
<tr>
<td>2</td>
<td>4000053</td>
<td>5008</td>
<td>0.45</td>
<td>3</td>
<td>Mainline</td>
<td>4400</td>
<td>4200</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>5008</td>
<td>4000055</td>
<td>0.05</td>
<td>3</td>
<td>Mainline</td>
<td>3000</td>
<td>3200</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>5008</td>
<td>4000055</td>
<td>0.50</td>
<td>1</td>
<td>On-ramp</td>
<td>2000</td>
<td>1800</td>
<td>180</td>
</tr>
</tbody>
</table>

* FFS – Free Flow Speed

Empirical Test

Data for the test was collected on Thursday, October 14, 2002 from 5:52 to 19:57. Comparison of model performance is based on traffic density. Figure 9 shows contours of observed (solid line) and predicted (dashed line) density at a level of 45 veh/mi/ln which is used in this study to delineate the boundary of congested region in time-space domain. The X axis represents location (nodes along the freeway mainline) and the Y axis represents time of day. There are two congested regions backing up from downstream node 4000055 towards upstream node 4000051. The figure shows a very good fit between the prediction and the observation.

![Density contours of the test](image)

FIGURE 9 Density contours of the test

Figure 10 illustrates the frequency of prediction error. As expected, residuals are densely concentrated around zero with the rest balanced at both sides – a bell-shaped distribution as expected. There are 676 samples in total. The highest bar represents 466 samples, accounting for 69% of the total. The number between ±10 makes up approximately 90% of the total samples.

![Histogram of residuals](image)

FIGURE 10 Error Histogram of the test (676 Samples)

CONCLUSION

Among the many possibilities, a CBWFQ model is proposed to split flow at a merge. Capacity seems to be a better basis to determine the merging branches’ shares of downstream supply because capacity reflects a number of factors including number of lanes, per lane capacity, and various traffic control measures, and the above conjecture is supported by field observations. The major goal of this paper is to explore whether Newell’s simplified theory can be extended to a merge where two streams of traffic come into one, a feature that is of critical interest but somehow is missing from the original theory. By plugging in the proposed CBWFQ model, simplified kinematic waves at a merge is formulated in a 5-step procedure. Empirical tests support the validity of the extension and show satisfactory modeling accuracy.

REFERENCE