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SIMPLIFIED KINEMATIC WAVES AT A DIVERGE

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Abstract: Queuing at a diverge is an interesting but sophisticated phenomenon where one stream of traffic is split into two or more. It is generally not a problem when downstream supplies are sufficient to accommodate upstream demand. If, however, the demand does exceed the supplies, congestion will back onto upstream link and constrain traffic there. This paper, based on analyzing diverging behavior, reviews the existing models and proposes a contribution-based weighted splitting (CBWS) diverge model that takes into consideration queuing from different diverging branches. Based on this, Newell's simplified theory of kinematic waves is extended to incorporate diverges.

Key Words: Traffic simulation, macroscopic traffic model, kinematic waves, simplified theory, queuing, diverge.

INTRODUCTION

At the core of ITS (Intelligent Transportation Systems) are Advanced Traffic Management System (ATMS), which is intended to improve operational control and reduce congestion, and Advanced Traveler Information System (ATIS), which helps driving easier and more efficient. In both systems, traffic engineers rely heavily on traffic models to develop traffic control strategies, compare incident recovery alternatives, devise ramp metering rates, and publish traveler-related information. Macroscopic traffic models are well-suited to serve these purposes due to its efficiency in describing traffic evolution in a large roadway network. The so-called LWR model (Lighthill and Whitham, 1955; Richards 1956) is such a

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model based on vehicle conservation in time-space domain and some functional relationship between traffic flow and density. Because the LWR model is typically difficult to solve, Newell further simplified the kinematic model based on some assumptions and the resulting simplified theory of kinematic waves (Newell 1993a, b, c) is easy to handle either graphically or numerically. Unfortunately, the simplified theory deals only with freeway mainline, which greatly limits its application, especially when the performance of a regional network is of interest. As part of the effort to extend Newell's simplified theory, kinematic waves at a diverge is the focus of this paper.

Several diverge models are identified and analyzed in this paper, but no intention is made to criticize any of these models since they work fine under this or that circumstances. The purpose of this paper is to propose a sound and efficient diverge model based on analysis of diverging behavior that is suited to extend the simplified theory of kinematic waves.

REVIEW OF EXISTING WORK ON DIVERGES

A diverge comes into play because it links two or more downstream routes that are possibly interact at their common upstream link. Very often, one is faced with splitting traffic between the diverging branches and a demand-supply framework is typically used to explain queuing at a diverge.

In his simplified theory, Newell (1993a, b, c) assumed unlimited supply on exits, so there is essentially no congestion from off-ramps. If, however, vehicles are prevented from departing at a diverge either because of insufficient capacity at the diverge point or congestion from downstream mainline, Newell assumed that all vehicles upstream will be affected regardless of their destinations, i.e., traffic state (e.g., speed, flow, density, etc.) is uniform over all lanes at the upstream link. This diverge model serves as one of the underlying assumptions based on which the simplified theory of kinematic waves is formulated.

In one of his later papers as an attempt to estimate delays caused by an off-ramp queue, Newell (1999) slightly relaxed the above assumption and tended to believe that queues from different diverging branches need to be treated separately. This implies that traffic states can be different for traffic bound for different destinations. However, to be consistent with his original theory and avoid going into the detail of lane-by-lane difference, Newell limited the differences in the vicinity of the diverge by assuming vertical queues. If, however, one feels that a vertical queue might over-simplify things and a physical queue makes more sense, there will be a problem of dealing with more than one queue and hence more than one traffic state on a link, which is typically not captured in most macroscopic traffic models.

Daganzo (1995) proposes a diverge model similar to Newell's first model but with finite supply at an off-ramp. The model assumes that, if either diverging branch is blocked, all upstream traffic will be restricted regardless of destination. In his another paper regarding generalized theory of kinematic waves (Daganzo 1997), Daganzo proposes a second diverge model to deal with freeways with special lanes. This model assumes two vehicle types and three traffic regimes such that, upstream of a diverge, there are "two pipes" carrying two sets of fluids with different speeds and, further upstream, there is "one pipe" carrying the mixed fluid with uniform speed. This is a closer approximation of the reality at the cost of allowing more than one traffic state on a link concurrently.

Lebacque (1996) proposes two diverge models. The first one determines the upstream demand and then divides it among the diverging branches based on turning proportions. This mode is basically the same as Daganzo's first model, i.e., if one of the diverging branches is unable to provide any supply, no vehicle can depart at the upstream link. The second model determines the supplies at the diverging branches and sums them up to get the upstream departure. This model implies that, if there is congestion from one of the diverging branches, a hypothetical storage is needed to store the vehicles that are unable to depart while traffic on other lanes are not affected. It also implies that traffic state on different lanes may not be the same, though the lane-by-lane difference is not explicitly modeled.

In summary, there are generally two modeling strategies at a diverge: one that allows multiple concurrent traffic states on the upstream link and the other that doesn't. The former treats queues from different diverging branches separately and a queue only affects its corresponding part of upstream traffic. However, this strategy may involve lane-by-lane difference and is hard to capture at a macroscopic level. This is probably part of the reason why Newell assumes vertical queue and Lebacque assumes hypothetical storage. The latter spreads downstream congestion over all lanes of the upstream link and all traffic there will be affected regardless of destination. This is relatively cheap to model but at some cost of realism. In this paper, we are going to extend the simplified theory of kinematic waves to model queuing at a diverge, which takes advantages of the above seemingly competing strategies such that the former is used to split flow and move traffic forward at a diverge and the latter is used to update traffic state for each link.

ANALYSIS OF DIVERGING BEHAVIOR

Figure 1 shows a sketch of a diverge. There are two types of vehicles, type 1 vehicles which are destined for the left branch (branch 1) of the diverge and type 2 vehicles which are destined for the right branch (branch 2) of the diverge. Our discussion here addresses a generic case that incorporates or can be extended to off-ramps, diverging highways or freeways, as well as intersections.

Before any queue backs up from downstream of the diverge, let's assume that traffic upstream is operating in the free-flow regime and traffic state is pretty uniform over all lanes. Though lane variation exists, this is neither the main focus of nor typically captured at a macroscopic level. Under this assumption, there are some 1-vehicles traveling on the right lanes and some 2 vehicles traveling on the left lanes and these drivers have the piece in mind that they are able to change to their desired lanes whenever needed. When a queue backs up from downstream, it can originate from branch 1 or 2 or both. Let's assume it comes from branch 2 and base our following discussion on this assumption. The same discussion also applies to branch 2. Several pictures are proposed to model queuing at the diverge.

One picture is illustrated in Figure 2 where the 2-vehicle queue dictates the overall traffic condition upstream. This is the diverge model assumed in Newell (1993a, b, c) and in Daganzo (1995) and also the first diverge model proposed by Lebacque (1996). As the 2-vehicle queue backs up, all vehicles upstream will be affected and delay is experienced by all vehicles regardless of their destinations. This model is more appropriate if one is willing to model traffic at a higher level and achieves efficiency at cost of some realism. This model also makes more sense if the upstream link has only one lane or the majority of the road users are tourists often characterized as alert, courteous, and curious. However, this model might not be realistic when the upstream link has multiple lanes, especially when the road users are predominated daily commuters, a case that is often seen in urban freeways.

Another picture is proposed by Newell (1996) where the queue is assumed vertical and confined somewhere near the diverge at the side of branch 2. Upstream of the 2-vehicle queue, there could be another queue mainly formed by 1-vehicles if their arrival rate is higher than the capacity of the left lanes. Further upstream of the 1-vehicle queue, traffic states tend to be uniform over all lanes. This model is proposed primarily for evaluating delays caused by congestion at an off-ramp and the model applies to isolated exits without interaction with further upstream and downstream links, features that render the model unsuitable for extending the simplified theory of kinematic waves to corridor/network application. Despite of this, Newell did imply that queues from different branches need to be treated differently, which is the idea that this paper is in favor of.

Figure 3 probably gives a more realistic representation of queuing at a diverge. As 2-vehicle queue backs onto the upstream link, traffic at branch 2 is dictated by the queue and, hence, exhibits a high density. Slightly upstream of the queue, i.e., the area on the right lanes near the diverge, traffic state is almost the same as that of branch 2 because the former is a nature extension of the latter. Also the difference of traffic states between the right lanes and the left lanes becomes sharper as one gets closer to the diverge. This is especially true when origin-destination flow is predetermined and route choice is absent, as most macroscopic model does. Several reasons help to maintain the 2-vehicle queue on the right lanes. First, most 1-vehicles, noticing that the 2-vheicle queue is building up, tend to change to the left lanes because they are not intended to exit via branch 2 and because traffic is traveling at higher speeds on the left lanes. Second, those who are bound to exit via branch 2 have to stay in the queue even though the adjacent lanes exhibit higher speeds. Third, given the short distance and the high speed difference, queued vehicles who are close to the diverge may not be able to change to the left lanes even though they want to divert. However, the difference in traffic states between the left lanes and the right lanes diminishes as one goes further upstream due to a transition from congested to uncongested condition on the right lanes. Traffic density in the transition area is lower then that of the congested area, but higher than the uncongested area. Still in this area, downstream congestion becomes foreseeable and vehicles bound for different destinations begin to consider changing lanes with 2-vehicles changing to or staying at the right lanes even though they can travel faster at adjacent lanes. Though there are some 2-vehicles who bypass the queue and squeeze in at the head of the queue at the expense of others' delay, these vehicles are relatively few and the relatively high speed difference near the diverge may not be favorable for them to sneak in. This and other phenomena such 1-vehicle's squeezing out from right lanes can be modeled by a friction at the interface between the left and right lanes. This friction, in turn, acts as a variable constraint on effective capacity based on current traffic state (e.g., density). Upstream of the transition area, traffic is free-flowing (or nearly so), and downstream congestion has not been perceived by travelers here, so traffic tends to be distributed uniformly over all lanes.

Technically, it is difficult to model transition of traffic state at a macroscopic level, and this is where a shock wave is introduced to describe the discontinuity of traffic states. Figure 4 can be approximated by the Figure 5, in which there is an abrupt change of traffic state between branch 2 and the right lanes. This happens to be the second diverge model proposed by Lebacque (1996).

Considering that queue tail might be located somewhere on the right lanes, the right lanes might contain a congested section and a uncongested section. On the other hand, there might be vehicles exchanging between the left and the right lanes and this acts as a friction at the interface. Given these, a closer approximation of Figure 4 can be the one illustrated in Figure 6 where there is a discrete changeover from congested to uncongested condition and a traffic state-based variable capacity near the diverge. If there is a concurrent queue from branch 1, the same treatment applies and there is another discrete changeover on the left lanes, and this picture happens to be the second diverge model proposed by Daganzo, i.e., the one with two vehicle types and three traffic states.

Figure 6 depicts the contribution-based weighted splitting (CBWS) diverge model based on which the simplified theory of kinematic waves is going to be extended. As stated before, we are trying to mix the two modeling strategies and taking the best out of them. More specifically, we are going to split flow and move traffic forward based on the strategy of multiple traffic states, while we update link traffic state based on the strategy of single traffic state. The simplified kinematic waves at a diverge works as follows. First, the departure counts of the two diverging branches are evaluated individually. Second, their sum is used as one of the constraints to evaluate the aggregate departure count to the left of the diverge. Also, this is the place where the friction comes into play. A traffic state-based friction factor is applied to the capacity which is also one of the constraints to the aggregate departure count. The resulting aggregate departure count may not exactly be the sum of the departure counts of the two branches determined earlier, so a splitting scheme based on downstream contributions is used to determine the actually downstream departure counts. Queues from different branches are treated separately such that travel times of vehicles in these queues are evaluated individually and the travel times are then used to advance multiple-destination flows.

Of course, one would arguably say that treating different queues separately violates the FIFO (first-in-first-out) assumption of a queuing system. This is not necessarily the case because FIFO still holds if the two queues from the two diverging branches are evaluated individually. On the other hand, vehicles for different destinations will operate independently once they have past the diverge and FIFO lost its meaning for them.

SIMPLIFIED THEORY OF KINEMATIC WAVES AT A DIVERGE

We consider here a diverge as shown in Figure 7. The upstream link is (x_i, x_l) and the two diverging branches are (x_l, x_n) and (x_l, x_m) . x_r denotes any downstream destination of x_l via x_n and x_s denotes any downstream destination of x_l via x_m . $A_{i,n}(x_l^-, t)$ denotes the cumulative number of vehicles waiting somewhere to the left of x_l at time t originated from node x_i destined for node x_n and beyond at time t . $D_{i,n}(x_l^+, t)$ denotes the cumulative number of vehicles past the right of x_l at time t originated from node x_i destined for node x_n and beyond at time t . $Q_{il}, k_{il}, v_{il}, n_{il}, l_{il}, u_{il}$ denote the capacity, jam density, forward wave speed, number of lanes, length, and backward wave speed of link (x_i, x_l) . The meaning of the above notations applies to similar symbols.

The simplified theory of kinematic waves starts from boundary conditions and works on lattice points in a time-location domain such that, at each time step, all nodes are evaluated from the first to the last and then time advances to the next step. Suppose, from boundary conditions and previous time steps, we know the cumulative departure curves to the right of x_i originated from x_i destined for all destinations x_z ($z = l, m, n, r, s, etc.$) up to time t , $D_{i,z}(x_i^+, t)$. Suppose also that geometry data and traffic characteristics data are well defined for each link. Our goal here is to determine the cumulative departure curves past x_l destined for all destinations. This can be done by a 5-step procedure based on Newell's simplified theory of kinematic waves.

A. Departure to the right

There are two links to the right of x_l, (x_l, x_n) and (x_l, x_m), so cumulative departure curves $D_{l,n}(x_l^+, t)$ and $D_{l,m}(x_l^+, t)$ are evaluated individually. According to Newell, the cumulative departure curve to the right of x_l originated from x_l destined for x_n and beyond, $D_{l,n}(x_l^+, t)$, is constrained by the following:

a. Upstream arrival

$$A_{l,n}(x_l^+, t) = A_{i,n}(x_l^-, t) = D_{i,n}(x_i^+, t - l_{il} / v_{il})$$

b. Right capacity

$$D_{l,n}(x_l^+, t - \tau) + \tau \times Q_{\ln}$$

c. Downstream queue

$$D_{l,n}(x_n^-, t - l_{\ln} / u_{\ln}) + l_{\ln} \times k_{\ln}$$

d. Left capacity

There is a problem here. Usually the capacity to the left of x_l is enough to handle traffic destined for x_n and beyond. However, this capacity is, at the same time, shared by traffic destined for x_m and beyond. The question is, how much of the capacity can be utilized by the former? It is hard to answer at this point and let's leave it for a second. For now, $D_{l,n}(x_l^+, t)$ is simply the minimum of a, b, and c, i.e.,

$$D_{l,n}(x_l^+, t) = \min\left\{A_{l,n}(x_l^+, t), D_{l,n}(x_l^+, t - \tau) + \tau \times Q_{\ln}, D_{l,n}(x_n^-, t - l_{\ln} / u_{\ln}) + l_{\ln} \times k_{\ln}\right\}$$

Similarly, we can obtain the cumulative departure curve to the right of x_l originated from x_l destined for x_m and beyond, $D_{l,m}(x_l^+, t)$:

$$D_{l,m}(x_l^+, t) = \min\left\{A_{l,m}(x_l^+, t), D_{l,m}(x_l^+, t - \tau) + \tau \times Q_{lm}, D_{l,m}(x_m^-, t - l_{lm} / u_{lm}) + l_{lm} \times k_{lm}\right\}$$

B. Departure to the left

The cumulative departure curve to the left of x_l originated from x_i destined for x_l and beyond, $D_{i,l}(x_l^-, t)$, is simply the minimum of:

a. Upstream arrival

$$A_{i,l}(x_l^-, t) = D_{i,l}(x_i^+, t - l_{il} / v_{il})$$

b. Left capacity

$$D_{i,l}(x_l^-, t - \tau) + \tau \times Q_{il}$$

c. Downstream departure

$$D_{l,n}(x_l^+, t) + D_{l,m}(x_l^+, t)$$

Notice that the destination of $D_{i,l}(x_l^-, t)$ is x_l, not x_n or x_m. We are considering the aggregate flow at link (x_i, x_l). As mentioned before, the use of the lanes near the diverge may not be balanced, i.e., 2-vehicles may stay at the right lanes and 1-vehicles may use all lanes though left lanes are usually preferred. Also, there might be vehicle exchange between left and right lanes and this is modeled as a friction which is a function of traffic state (e.g. traffic density). The effect of the friction on traffic operation can be reflected by reducing capacity accordingly. Let $f_{il}(\phi, t)$ denotes the friction factor of link (x_i, x_l) at time t and ϕ is the current traffic state. $f_{il}(\phi, t)$ is usually not known and has to be treated as a design parameter. The effective capacity is then $Q_{il}' = Q_{il}(1 - f_{il}(\phi, t))$, and the capacity constraint becomes $D_{i,l}(x_l^-, t - \tau) + \tau \times Q_{il}'$. Therefore,

$$D_{i,l}(x_l^-, t) = \min\left\{A_{i,l}(x_l^-, t), D_{i,l}(x_l^-, t - \tau) + \tau \times Q_{il}', D_{l,n}(x_l^+, t) + D_{l,m}(x_l^+, t)\right\}$$

Notice that, as the friction factor approaches 0, our proposed technique reduces to Lebacque's second model. In response to the problem of left capacity in A, this step guarantees that the cumulative departure destined for x_l (i.e., the sum of those destined for x_n and x_m) won't exceed the capacity to the left of x_l.

Now, a new problem arises. Of the amount $D_{i,l}(x_l^-, t)$ determined above, how much is destined for x_n, i.e. $D_{i,n}(x_l^-, t)$, and how much is destined for x_m, i.e., $D_{i,m}(x_l^-, t)$? They might be the same as $D_{l,n}(x_l^+, t)$ and $D_{l,m}(x_l^+, t)$, respectively, if $D_{i,l}(x_l^-, t)$ is determined by downstream departures. However, when $D_{i,l}(x_l^-, t)$ is determined by upstream arrival or left capacity, $D_{i,n}(x_l^-, t)$ and $D_{i,m}(x_l^-, t)$ may not be the same as $D_{l,n}(x_l^+, t)$ and $D_{l,m}(x_l^+, t)$, respectively. In either case, $D_{i,l}(x_l^-, t)$ is split between $D_{i,n}(x_l^-, t)$ and $D_{i,m}(x_l^-, t)$ based on their respective downstream contributions, which explains how the proposed contribution-based weighted splitting (CBWS) diverge model gains its name. Let:

$$d_{l,n}(x_l^+, t) = D_{l,n}(x_l^+, t) - D_{l,n}(x_l^+, t-\tau)$$

$$d_{l,m}(x_l^+, t) = D_{l,m}(x_l^+, t) - D_{l,m}(x_l^+, t-\tau)$$

$$d_{i,l}(x_l^-, t) = D_{i,l}(x_l^-, t) - D_{i,l}(x_l^-, t-\tau)$$

Then

$$d_{i,n}(x_l^-, t) = d_{i,l}(x_l^-, t) \times \frac{d_{l,n}(x_l^+, t)}{d_{l,n}(x_l^+, t) + d_{l,m}(x_l^+, t)}$$

$$D_{i,n}(x_l^-, t) = D_{i,n}(x_l^-, t-\tau) + d_{i,n}(x_l^-, t)$$

$$d_{i,m}(x_l^-, t) = d_{i,l}(x_l^-, t) \times \frac{d_{l,m}(x_l^+, t)}{d_{l,n}(x_l^+, t) + d_{l,m}(x_l^+, t)}$$

$$D_{i,m}(x_l^-, t) = D_{i,m}(x_l^-, t-\tau) + d_{i,m}(x_l^-, t)$$

If $d_{l,n}(x_l^+, t) + d_{l,m}(x_l^+, t) = 0$, no traffic discharges for either downstream link, i.e., $d_{i,n}(x_l^-, t) = 0$ and $d_{i,m}(x_l^-, t) = 0$.

C. Link travel time

At pervious step, we determined the cumulative departure curves based on aggregate flow for the upstream link. At this step, we evaluate the queues for the two diverging branches individually. This is done by computing link travel time for vehicles destined for each diverging branch.

Travel time for 1-vehicles at link (x_i, x_l) can be obtained by comparing curve pair $D_{i,n}(x_i^+, t)$ vs. $D_{i,n}(x_l^-, t)$ as follows. We trace $D_{i,n}(x_i^+, t)$ backwards until some prior time t' such that $D_{i,n}(x_i^+, t')$ is equal to $D_{i,n}(x_l^-, t)$. Then the link travel time for 1-vehicles is $T_{il}^n(t) = t - t'$, where the superscript denotes the branch having x_n.

Similarly, travel time for 2-vehices at link (x_i, x_l), $T_{il}^m(t)$, can be obtained by curve pair $D_{i,m}(x_i^+, t)$ vs. $D_{i,m}(x_l^-, t)$.

D. Departure to the left – multi-destinations

With link travel time $T_{il}^n(t)$ obtained above, the cumulative departure curve to the left of x_l originated from x_i destined for x_r and beyond, $D_{i,r}(x_l^-, t)$, is determined as

$$D_{i,r}(x_l^-, t) = D_{i,r}(x_i^+, t - T_{il}^n(t))$$

Similarly, the cumulative departure curve to the left of x_l originated from x_i destined for x_s and beyond, $D_{i,s}(x_l^-, t)$ is determined as

$$D_{i,s}(x_l^-, t) = D_{i,s}(x_i^+, t - T_{il}^m(t))$$

E. Departure to the right – multi-destinations

Since this is a diverging scenario, no traffic enters from any on-ramp. The cumulative departure curve to the right of x_l originated from x_i destined for x_n and beyond, $D_{i,n}(x_l^+, t)$ is the same as $D_{i,n}(x_l^-, t)$, and the cumulative departure curve to the right of x_l originated from x_i destined for x_m and beyond, $D_{i,m}(x_l^+,t)$ is the same as $D_{i,m}(x_l^-,t)$, i.e.,

$$D_{i,n}(x_l^+,t) = D_{i,n}(x_l^-,t)$$

$$D_{i,m}(x_l^+,t) = D_{i,m}(x_l^-,t)$$

Notice that, at step A, we have preliminarily determined $D_{l,n}(x_l^+,t)$ and $D_{l,m}(x_l^+,t)$, which are the equivalent of $D_{i,n}(x_l^+,t)$ and $D_{i,m}(x_l^+,t)$, respectively. As the procedure goes on, those preliminary values are fine-tuned and updated.

Similarly, for other destinations x_r x_s, we have:

$$D_{i,r}(x_l^+,t) = D_{i,r}(x_l^-,t)$$

$$D_{i,s}(x_l^+,t) = D_{i,s}(x_l^-,t)$$

So far we have extended Newell's simplified theory of kinematic waves to accommodate queuing at a diverge. With the cumulative departure curves at both ends of a link (e.g., $D_{i,l}(x_i^+,t)$ and $D_{i,l}(x_l^-,t)$ for link (x_i, x_l)), traffic state such as speed, flow, and density and other measures of effectiveness (MOEs) can be computed. This treatment results in an aggregated traffic state over all lanes of the link. Of course, one can work on partial curves (e.g., $D_{i,m}(x_i^+,t)$ and $D_{i,m}(x_l^-,t)$) based on partial lanes (e.g., the right lanes) to compute partial traffic states if one is really interested, though this would be much involved.

As depicted in Figure 6, the right lanes can have two traffic states, congested and uncongested, if a queue tail is located on the right lanes and one can determine the queue tail if interested. Locating a queue tail is based on the presence of a shock wave which separates different traffic states as well as the fact that a shock wave always coincides with a queue tail under the assumption of triangular flow-density relationship. Son (1996) has given an excellent description on how to locate a queue tail, which is not repeated here. The basic idea behind is that, suppose at time t, the cumulative number of vehicles vs. location curve can be obtained either from forward moving wave, say $A_{i,m}(x,t)$, or from backward moving wave, say $D_{i,m}(x,t)$, where $x_i < x < x_l$ and also suppose the queue on the right lanes is of interest. At the location of the shock wave/queue tail, x', the two curves intersect, i.e., $A_{i,m}(x',t) = D_{i,m}(x',t)$. A little search routine can be devised to find such location x' on link (x_i, x_l) such that the sign of $A_{i,m}(x',t)$ - $D_{i,m}(x',t)$ flips before and after this location. Otherwise, the queue tail has past this link or hasn't yet reached this link. Once the queue tail has been located, cumulative departure curve at this location can be evaluated by interpolation, and traffic states before and after it can then be computed separately.

It should be pointed out that, though the proposed model evaluates queues originated from different diverging branches separately and provides the potential to differentiate traffic states even in certain lane groups, the limited goal of simulation at a macroscopic level might not warrant keeping track of such level of detail. Actually, traffic engineers might be more interested in knowing the aggregate state of a link rather than the lane-by-lane difference and this is especially true as the network size gets big.

EMPRIRCAL TESTS

Empirical tests of the proposed model are based on data collected from GA-400 by Georgia NAVIGATOR system, an automatic traffic surveillance system covering the greater Atlanta metropolitan area. Link traffic states are reported and compared in an aggregate manner in these tests.

The Test Site

The test site is a diverge on the northbound of GA-400, as illustrated in Figure 8. The test site consists of an entrance link (4000043-4000046) followed by an upstream link (4000046-6006). There are two downstream links (6006-4000048 and 6006-4006006) after the diverging point. All mainline links have 4 lanes with approximately the same capacity. The diverging point, node 6006, might be a bottleneck because queues can back up from either of the downstream links. The geometry data and traffic characteristic data of the test site are summarized in Table 1.

Empirical Test 1

Data for test 1 was collected on Thursday, September 12, 2002. Comparison of model prediction and field observation is based on qualitative as well as quantitative measures.

Figure 9 shows the density vs. time curves of each link in test 1. The solid line is observed density and the dashed line is predicted density. The X axis is time of day and the Y axis is density in veh/km/ln. The test runs from 00:01 to 23:51, almost a whole day. There are two peaks during this day. The morning peak originates from the downstream of node 4000048 probably caused by insufficient capacity, while the afternoon peak backs up from the downstream of node 4006006 due primarily to high exit volume.

Generally, it is preferable to have congestion contained in the bounding box of the time-space diagram. However, congestion backing up from downstream is also acceptable. If congestion backs up past the upstream end, there is a problem because traffic is now operating at the congested side of the underlying flow-density curve and the arrival flow no longer represents the true demand. If this is the case, one generally goes further upstream, trying to find a node where congestion never reaches. In this way, the congestion is contained in the new bounding box.

Unfortunately, the afternoon peak in our case passes the upstream end, but we are unable to find an uncongested node further upstream because, otherwise, we would have to incorporate another freeway junction which is unacceptable in this test. Therefore, limited confidence should be given to the portion where the congestion exceeds the upstream boundary.

Nevertheless, this test is a good example to demonstrate the validity of the proposed model – queues can build up from either branch of a diverge and constrain upstream traffic accordingly. In general, the contours show a good agreement between the prediction and the observation.

In Figure 10, it is clearer that the afternoon queue does exceed the upstream end. Fortunately, this queue lasts less than 30 minutes and its impact on the rest part of the figure is limited. More specifically, we are confident with the full process of the morning queue, and we are also sure about the formation of the afternoon queue up to the upstream end, but only limited confidence can be given to the portion beyond this point and the queue dissipation.

In Figure 11, the highest bar represents 846 samples and the second highest represents 249 samples. Therefore, prediction error within ±5 makes up 96% of the total samples.

A simultaneous statistical test, based on batch means technique (Goldsman and Tokol 2000), is performed with two hypothesizes: the mean of prediction error is not statistically different than 0 (i.e., the model is unbiased) and the variance of prediction error is sufficiently small. The results show that the first hypothesis holds at 95% confidence level, and the second hypothesis translates to a 95% confidence interval of (-0.024786, 0.031377) $\times$100% for percentage error if the sample variance is taken as small enough.

Empirical Test 2

Data for empirical test 2 of diverging scenario was collected on Monday, December 9, 2002 from 00:00 to 23:45, almost a whole day.

Figure 12 shows density vs. time curves at each link of the site. There are two peaks during this day, a morning peak and an afternoon peak. The morning peak, originated from the downstream of node 4000048, is caused by insufficient downstream capacity. What makes this example interesting is that the afternoon peak is caused by congestion at both downstream links (6006-4000048 and 6006-4006006). At approximately 16:40, a queue backs onto link 6006-4000048 and continues to move towards upstream. At approximately 17:51, another queue pops in at link 6006-4006006 and also keeps moving backwards. Both of the two queues propagate past node 6006 and continue to build up on the upstream link. The off-ramp queue clears out around 18:15, and the mainline queue dissipates at approximately 18:00.

The figure generally shows a good fit between the prediction and the observation with a little over-prediction at both peaks on link 6006-4000048 and a little under prediction in the middle.

Figure 13 shows a good agreement between the prediction and the observation as far as congested region is concerned. The morning peak is smaller in scale, while the afternoon peak starts with a tiny queue and, barely after this queue clears out, another larger queue returns and reaches somewhere between nodes 4000042 and 4000046.

The highest bar in Figure 14 represents 722 samples, and the second highest 335 points, so prediction error within ±5 veh/km/ln roughly accounts for more than 92% of the total samples.

Simultaneous statistical test shows that there is also a lack of evidence that the mean of prediction error is statistically different than 0 at 95% level of confidence. The test suggests a 95% confidence interval of (-0.043319, 0.040738) $\times$100% for percentage prediction error.

CONCLUSION

This paper proposes a contribution-based weighted splitting (CBWS) diverge model that evaluates queues from different diverging branches separately and considers friction between right and left lanes. Based on this, the simplified theory of kinematic waves is extended to incorporate diverges and the proposed diverge model is used to split and advance traffic. Though traffic state at a link is modeled in an aggregate manner as most macroscopic simulation models do, the treatment of queuing from different diverging branches enhances the realism and accuracy of traffic simulation at the diverge, as supported by empirical tests.

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TABLE 1 Geometry and traffic characteristics data for the test site

Link	Up node	Down node	Length (km)	# of lanes	Type	FFS * (km/h)	Capacity (veh/h/ln)	Jam density (veh/km/ln)
1	4000043	4000046	1.504	4	Mainline	108.8	2200	112.5
2	4000046	6006	0.368	4	Mainline	108.8	2200	112.5
3	6006	4000048	0.832	4	Mainline	96	2200	112.5
4	6006	4006006	0.8	1	Off-ramp	96	2000	112.5

* FFS – Free Flow Speed

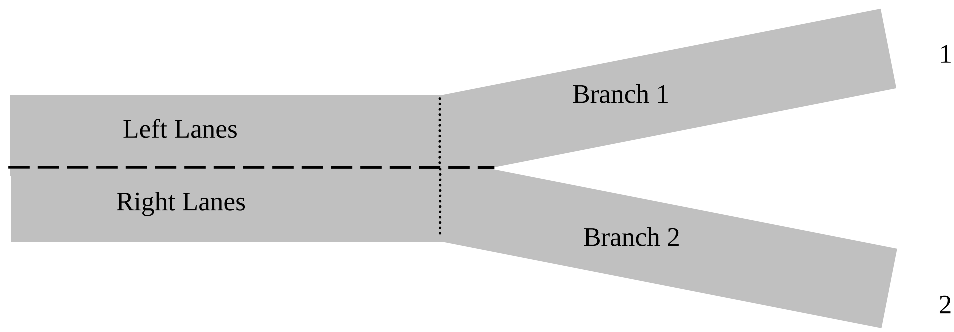


FIGURE 1 Sketch of a diverge

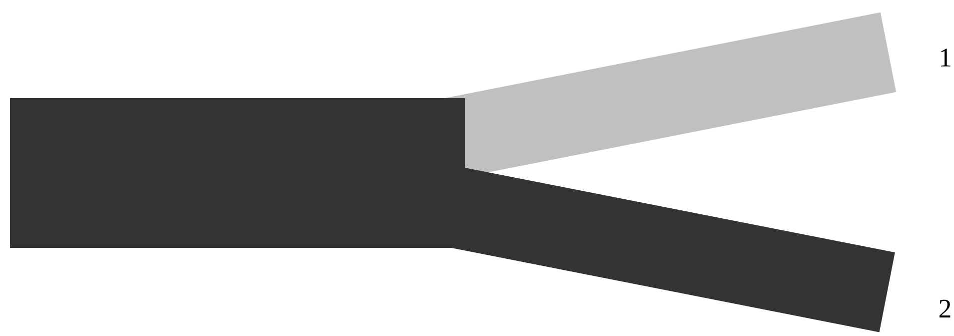


FIGURE 2 Diverge model 1 – Newell's 1st model, Daganzo's model, and Lebacque's 1st model

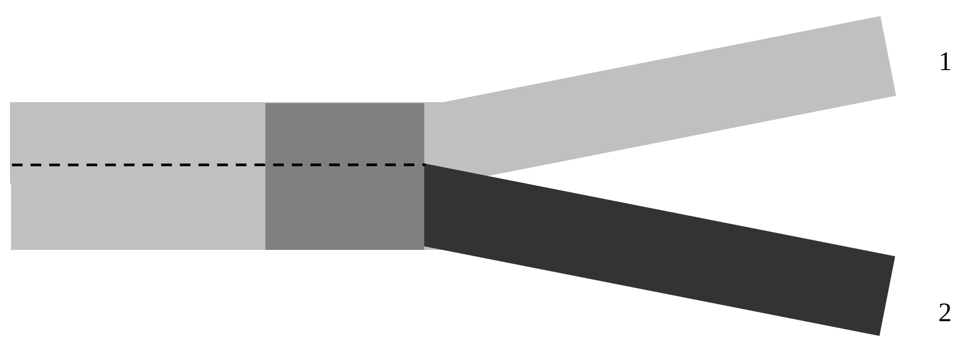


FIGURE 3 Diverge model 2 - Newell's 2nd model

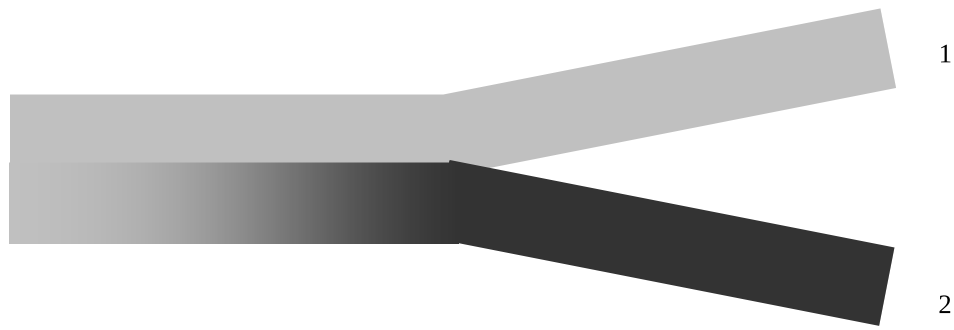


FIGURE 4 Diverge model 3 – the realistic model

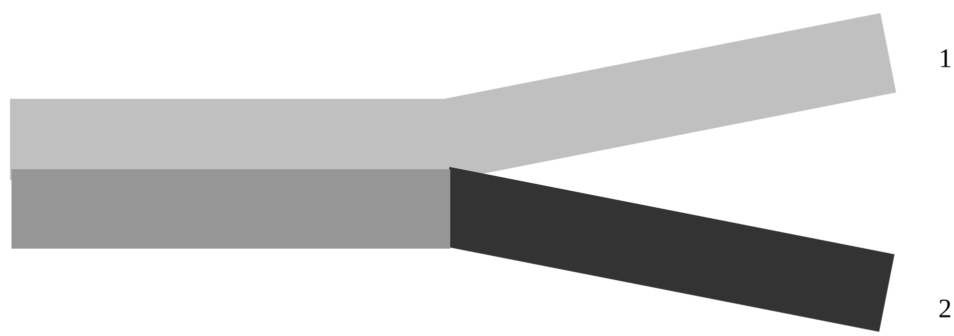


FIGURE 5 Diverge model 4 - Lebacque's 2nd model

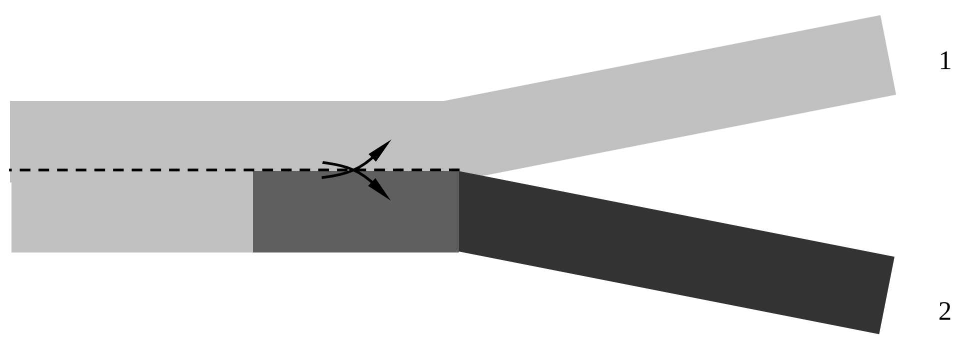


FIGURE 6 Diverge model 5 – the contribution-based weighted splitting (CBWS) diverge model

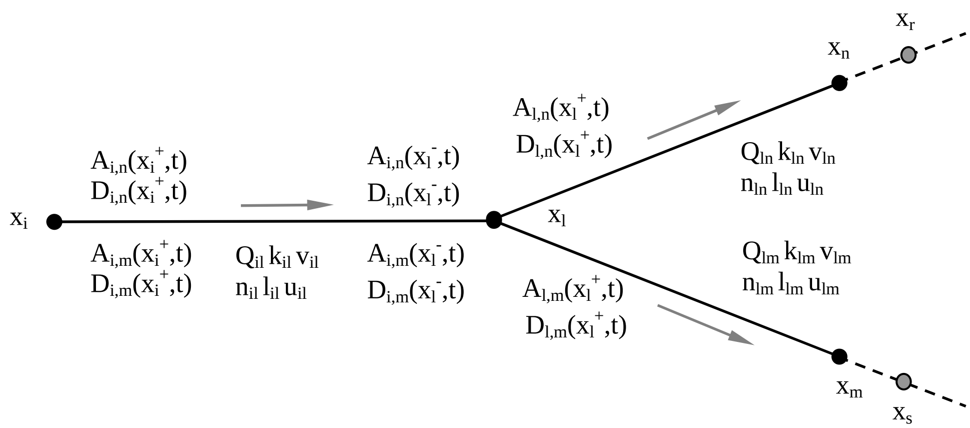


FIGURE 7 Data of simplified kinematic waves at a diverge

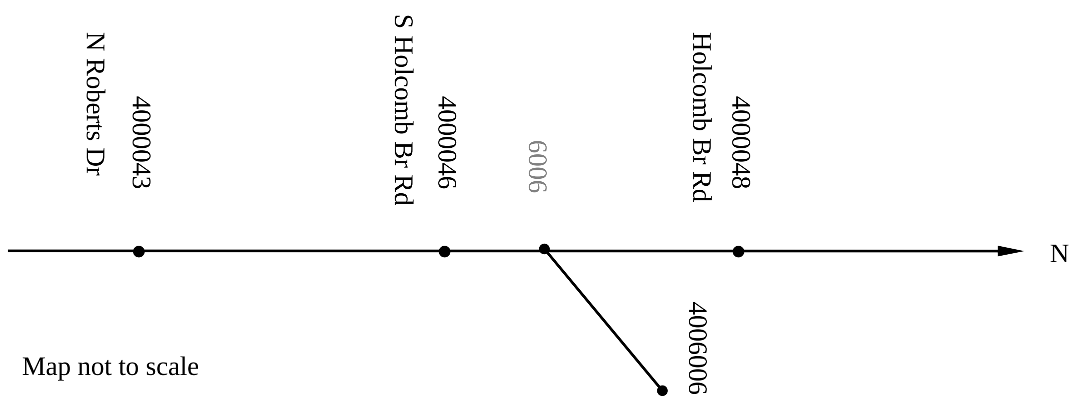
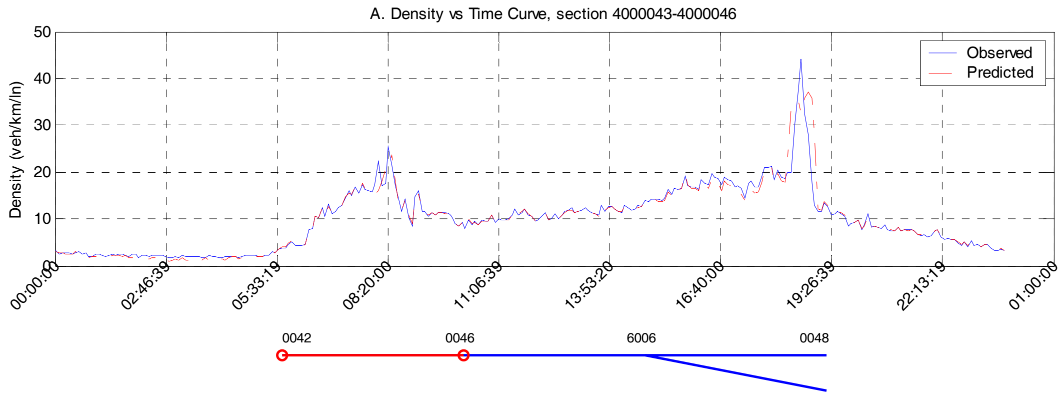
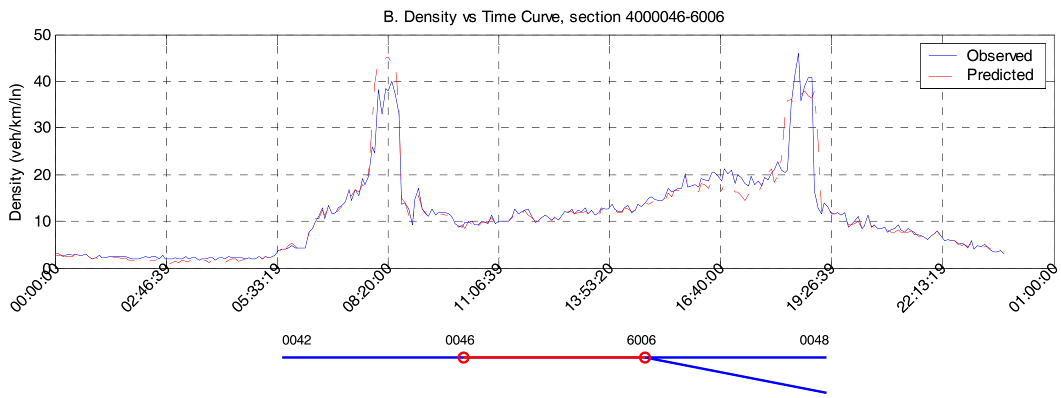


FIGURE 8 The test site of simplified kinematic waves at a diverge


A. Density vs Time Curve, section 4000043-4000046
Density (veh/km/ln)
Observed
Predicted
00:00:00
02:46:39
05:33:19
08:20:00
11:06:39
13:53:20
16:40:00
19:26:39
22:13:19
01:00:00
0042
0046
6006
0048


B. Density vs Time Curve, section 4000046-6006
Density (veh/km/ln)
Observed
Predicted
00:00:00
02:46:39
05:33:19
08:20:00
11:06:39
13:53:20
16:40:00
19:26:39
22:13:19
01:00:00
0042
0046
6006
0048

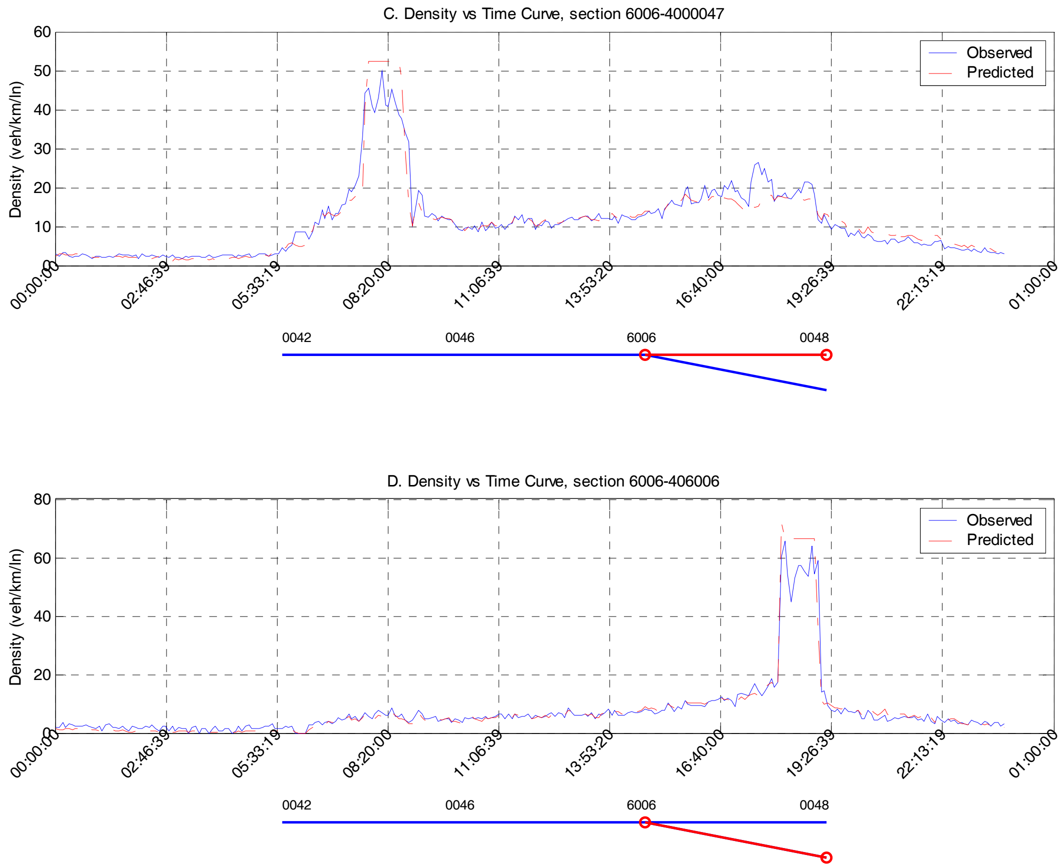


FIGURE 9 Density vs. time curves of test 1

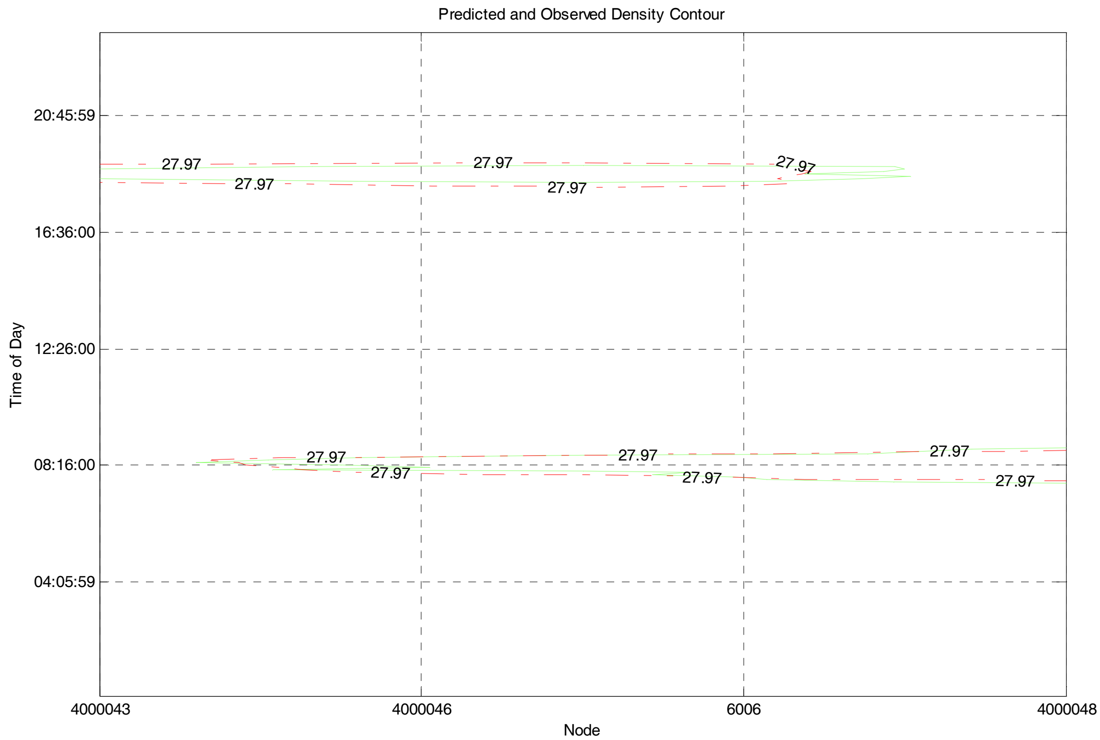


FIGURE 10 Density contours of test 1

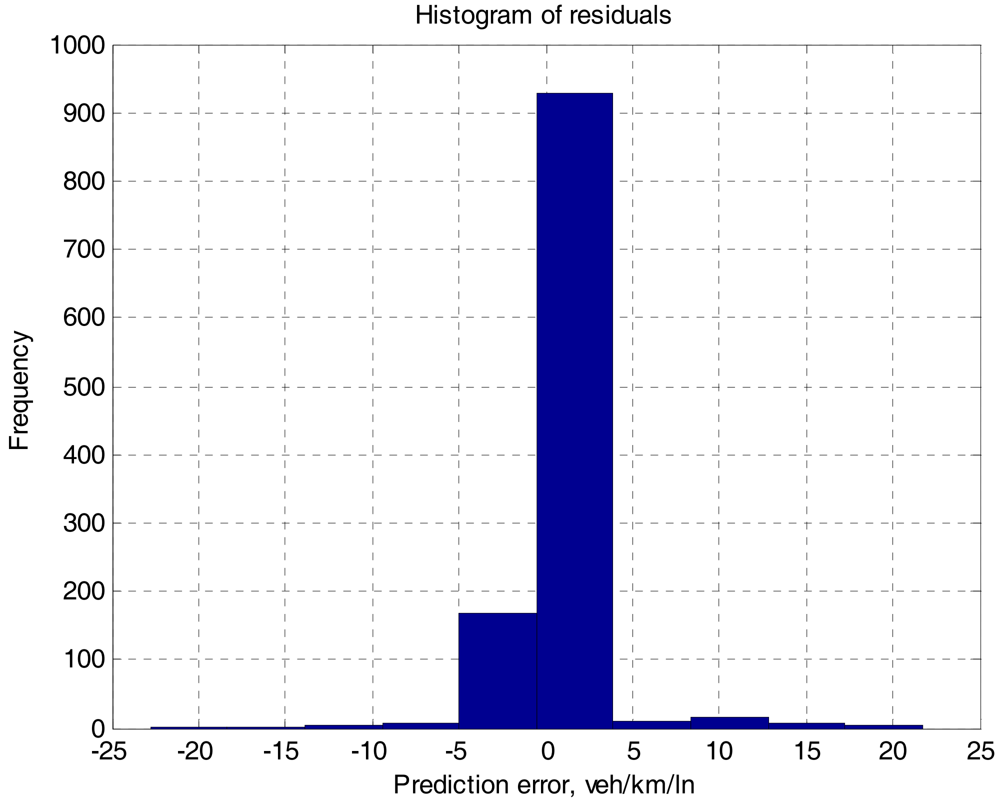
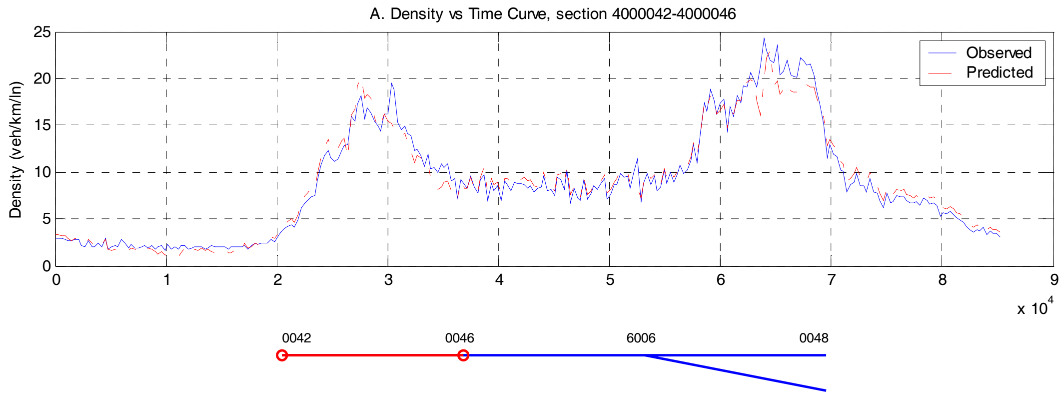
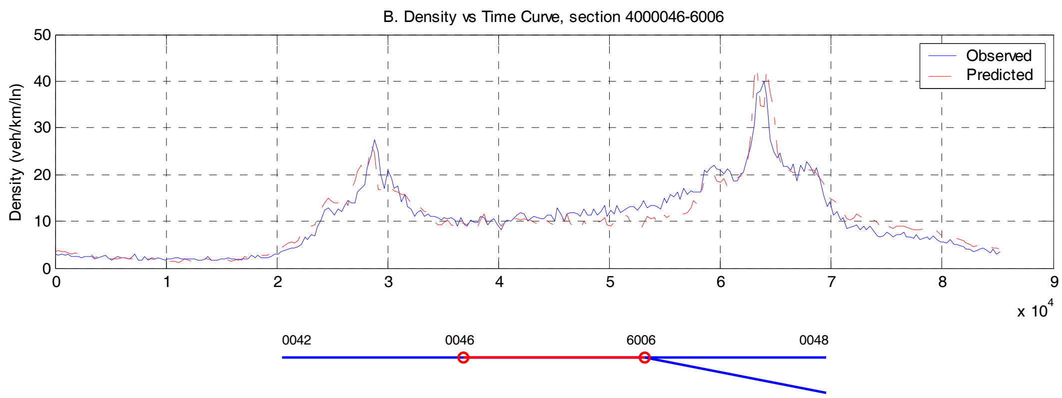


FIGURE 11 Histogram of prediction error of test 1 (1144 Samples)


A. Density vs Time Curve, section 4000042-4000046
Density (veh/km/ln)
Observed
Predicted
x 10^4
0042
0046
6006
0048


B. Density vs Time Curve, section 4000046-6006
Density (veh/km/ln)
Observed
Predicted
x 10^4
0042
0046
6006
0048

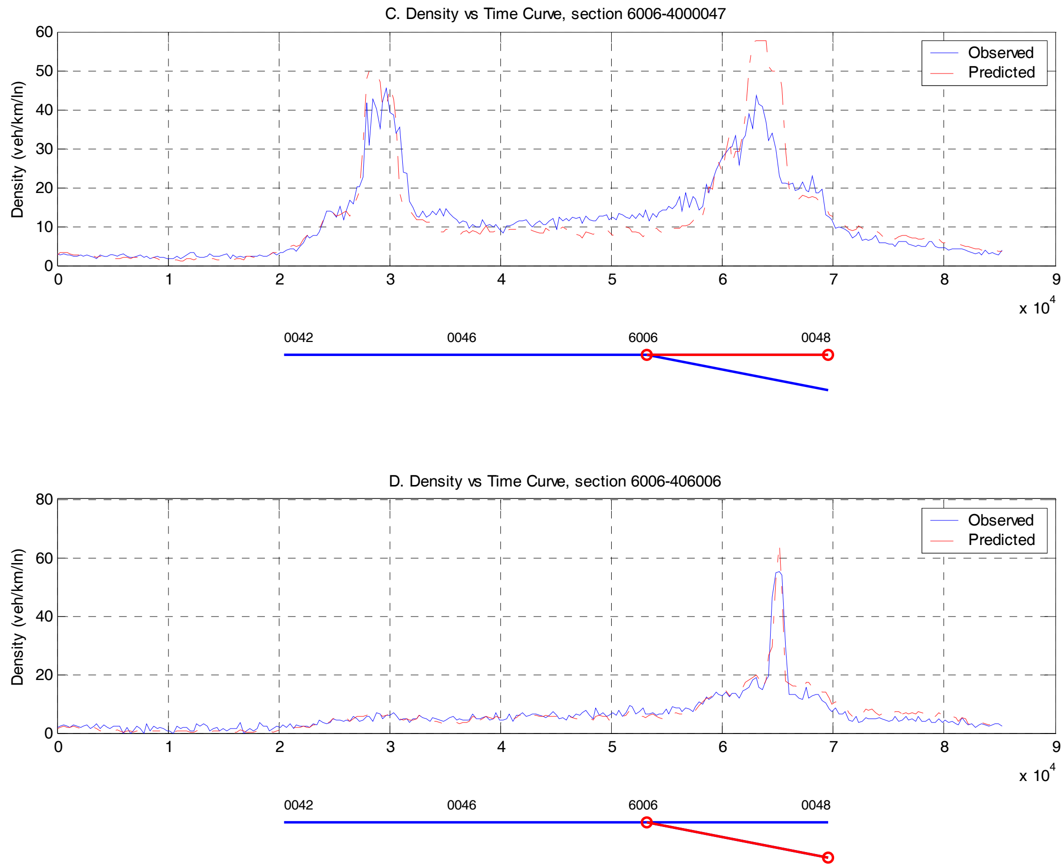


FIGURE 12 Density curves of test 2

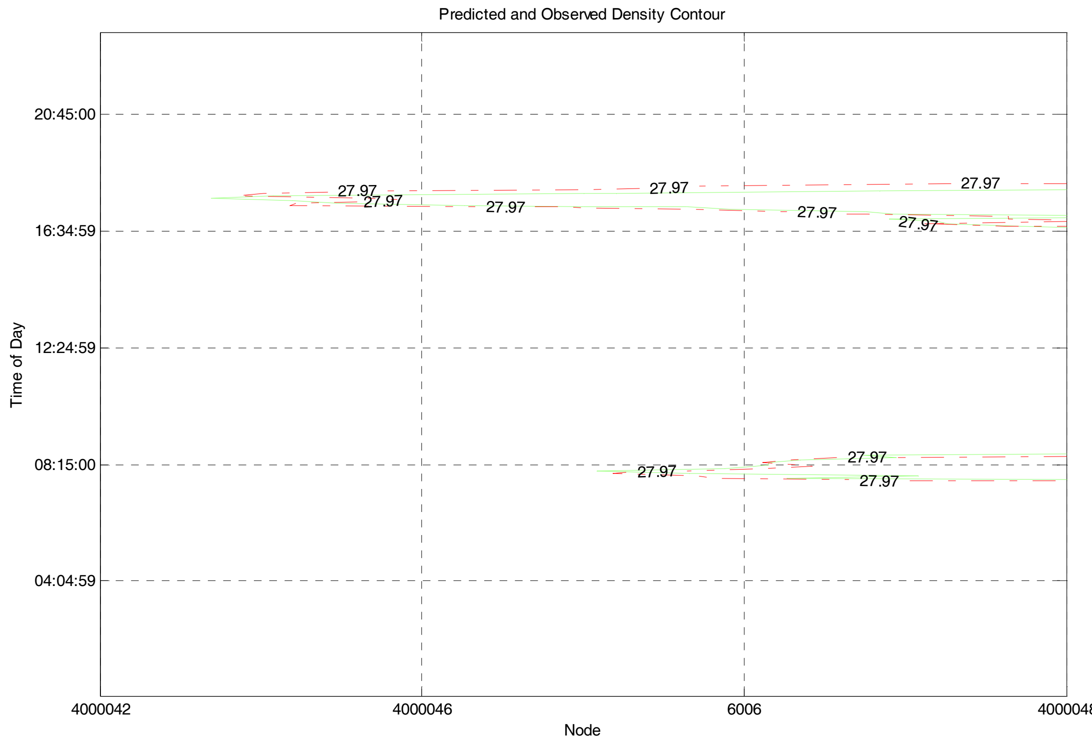


FIGURE 13 Density contours of test 2

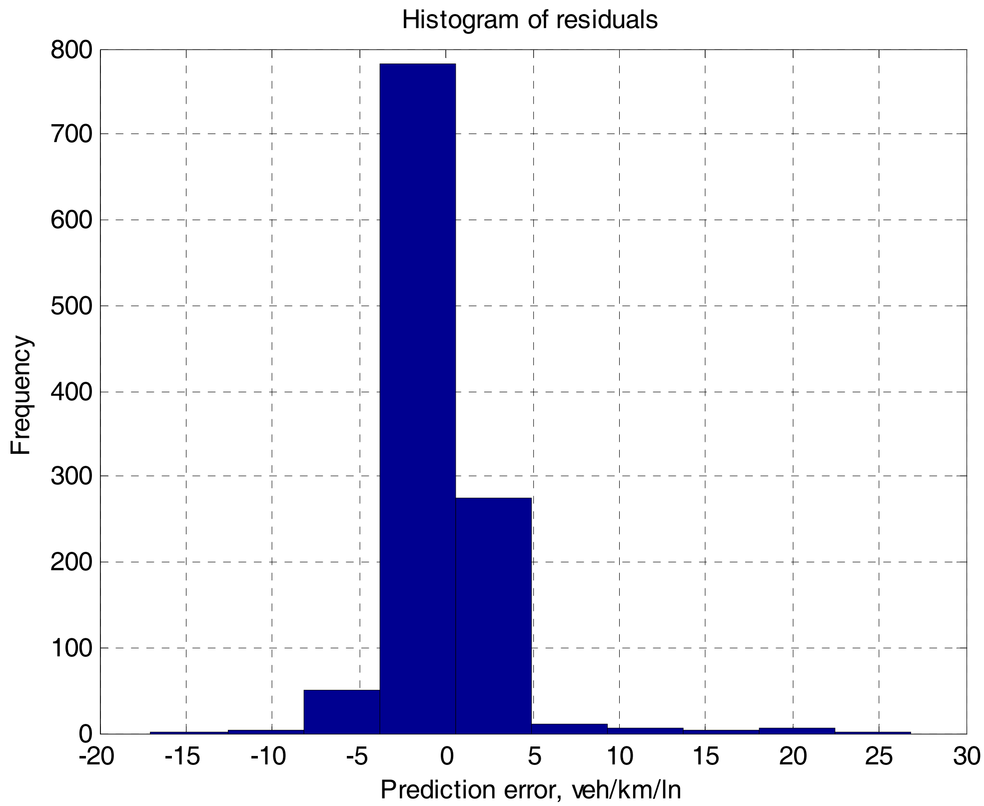


FIGURE 14 Histogram of prediction error of test 2 (1144 Samples)