Direct Methods of Determining Traffic Stream Characteristics by Definition

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Abstract

There are generally two approaches to derive traffic stream characteristics (i.e. flow, space mean speed, and density): indirect approach which derives traffic stream characteristics by estimation from correlated variable(s) and direct approach which determines traffic stream characteristics by definition from field observations. A method of the former approach inevitably involves estimation error while a method of the latter provides “the ground truth”. This paper examines and contrasts three direct methods, namely the HCM method, the x-t method, and the n-t method. The HCM method determines traffic stream characteristics based on their definition adopted by Highway Capacity Manual (HCM). The x-t method determines traffic stream characteristics based on their definition proposed by Edie in generalized terms. The x-t method is so named because it works on vehicle trajectories in space-time (x-t) domain. The n-t method also determines traffic stream characteristics based on Edie’s definition, but the method works on cumulative vehicle counts in cumulative number-time (n-t) domain. In terms of accuracy, the x-t method and the n-t method are almost equivalent and both are more accurate than the HCM method. In terms of the fundamental relationship among flow, speed, and density, the x-t method and the n-t method guarantee the relationship while the HCM method does not. In terms of the working mode, the HCM method and the n-t method are capable of working on-line/real-time while the x-t method typically works off-line. In terms of data sources, the n-t method uses as input point sensor data while the x-t method and the HCM method require aerial photograph data or a combination with others. In summary, the n-t method provides a promising means for traffic management centers (TMC) to accurately determine traffic stream characteristics by definition on-line/real-time from data collected by ITS traffic surveillance systems that predominantly consist of point sensors.
INTRODUCTION

Accurate information of traffic stream characteristics (i.e. flow, density, and space mean speed (SMS)) is very important for many traffic engineering studies. In addition, many Intelligent Transportation Systems (ITS) applications require on-line/real-time provision of such information. There are generally two approaches to derive traffic stream characteristics: direct and indirect approaches. A direct approach determines traffic stream characteristics by definition from field observations, while an indirect approach estimates traffic stream characteristics from correlated variable(s) such as travel time.

A widely accepted definition of traffic stream characteristics is that adopted by the Highway Capacity Manual (HCM 1985 and 2000). This simple definition defines flow as the number of vehicles passed a fixed location during unit time, density as the number of vehicles on a roadway segment over unit space, and SMS as the harmonic mean of (spot) speeds over a length of roadway. However, this definition sometimes gives misleading results in that the fundamental relationship among flow, speed, and density (referred to as “the fundamental relationship” thereafter) may not hold (1, 2). A method that strictly follows the HCM definition in determining traffic stream characteristics will be called “the HCM method” thereafter. Another definition of traffic stream characteristics was proposed by Edie in generalized terms (3, 4). This definition defines flow, density, and SMS based on a common ground, i.e. a space-time region, so the fundamental relationship is guaranteed. The traditional means to determine traffic stream characteristics based on Edie’s definition is to construct vehicle trajectories in space-time (x-t) domain and calculate total distance and total time traveled by all vehicles in a predetermined space-time region. This method will be called “the x-t method”. Considering that the x-t method requires vehicle trajectory information which is typically obtained by means of aerial photographs, an alternative method is desirable if it is able to make use of point sensor data which are more readily available on-line/real-time from most ITS systems. Our research demonstrates that this is possible, i.e. one can determine traffic stream characteristics on-line/real-time by definition from vehicle counts at fixed locations. To do so, one only needs to construct curves of cumulative number of vehicles in cumulative number-time (n-t) domain and follow Edie’s definition. We will call this method “the n-t” method.

The above paragraph identifies three methods that follow a direct approach. Because they determine traffic stream characteristics by definition, their results are “the ground truth”. There are some other methods which follow an indirect approach. These methods estimate traffic stream characteristics from correlated variables that are easier to measure and/or data of which are more readily available. It is because these methods involve estimation they are inherently prone to error. On the other hand, a method of the indirect approach may require calibration to customize the method to local traffic, which can be a costly process. Due to the above limitations of the indirect approach, this paper focuses only on the direct approach.

Considering the nature and needs of ITS systems, it is desirable to have a method which meets the following requirements: (i) it is able to determine traffic stream characteristics by definition rather than by estimation, (ii) it is theoretically sound and preserves the fundamental relationship, (iii) it is able to work on-line/real-time, (iv) it is applicable to point sensor data which are typically provided by most ITS systems, (v) it does not require additional investment in existing ITS systems. The objective of this paper is to make a close examination of the aforementioned three direct methods and identify which is better suited to meet the above requirements. Emphasis will be placed on studying the performance of the three methods and their sensitivity to varying conditions. To serve such a purpose, we designed a small-sized numerical example which is manageable by manual calculation and is reproducible for independent verification. We also presented some empirical examples using vehicle trajectory data provided by the Next Generation Simulation (NGSIM) program. This set of data was collected with an unprecedented level of detail that it allows a closer study of traffic flow.

SURVEY OF PRIOR APPROACHES

Generally, traffic stream characteristics can be determined in two ways: direct determination by definition and indirect determination by estimation.

The definition of traffic stream characteristics has appeared in many forms, some of which may not be equivalent. The simple definition adopted by the Highway Capacity Manual (HCM 1985 and 2000) has been widely accepted and applied in the profession. The definition is quite straightforward which defines flow as “the number of
vehicles passing a point of roadway in unit time (typically one hour), density as “the number of vehicles on a roadway of unit length (typically one mile),” and SMS as “the average speed based on the average travel time of vehicles to traverse a segment of roadway.” However, there obviously lacks a common ground based on which these traffic stream characteristics are defined. This contributes to the fact that the fundamental relationship, which is assumed to be an identity, does not hold in some cases (1, 2). Another definition was presented by Edie (3, 4) in generalized terms. The common ground for the three traffic stream characteristics is a space-time region. Flow is defined as the total distance traveled by all vehicles in the region divided by the “area” of the region, density is defined as the total time traveled by all vehicles in the region divided by the “area” of the region, and SMS is defined as the total distance traveled by all vehicles divided by the total time spent by all vehicles in the region. Because of the common ground, this definition guarantees the fundamental relationship.

Traditionally, one would have to rely on aerial photographs if one were to determine traffic stream characteristics by Edie’s definition. As a useful means of studying traffic stream characteristics, the aerial photographs technique was initially explored by Greenshields (5-7). Makigami et al (8) gave a complete discussion on traffic flow chrematistics and presented them in a 3D model. In 1985, Makigami et al (9) reported a study which used aerial photographs to construct vehicle trajectories and used the trajectories to determine traffic stream characteristics by Edie’s definition. Makigami et al (8) pointed out the interchangeability between the a space-time graph and a cumulative number-time graph, which opened a door to alternative means of determining traffic stream characteristics using point sensor data. Unfortunately, this idea did not seem to have been explored until we present this paper. Several other methods were also identified which determine traffic stream characteristics based on the HCM definition. In 1993, Newell (10) demonstrated some of the properties of curves of cumulative vehicle counts, one of which deals with determining traffic density from the curves. Later some researchers (11, 12) used the idea to extract traffic data from simulation results. Coifman (13) presented a vehicle re-identification technique to determine traffic density by finding the number of vehicles passed a sensor.

While the above paragraph identifies some direct methods, one can also determine traffic stream characteristics by means of indirect methods. An indirect method determines traffic stream characteristics by estimation from correlated variables that are easier to measure and/or whose data are more readily available. One of the techniques that was explored most was the Kalman filtering technique. A series of papers in the early 1970s discussed a Kalman filtering procedure (14-18) to estimate traffic density. This procedure records the flow and speeds of vehicles that cross the input and the output of a roadway section for each time interval. The speeds are used to estimate the travel time over the section. Vehicle storage in this section is then estimated based on the travel time and the flow data. The vehicle storage is then fine-tuned using a Kalman filter and traffic density can be computed from the vehicle storage. Chang and Gazis (19) extended the Kalman filtering procedure by considering explicitly lane-changing on a multilane freeway and identified potential opportunity to reduce estimation error. The reduction of error is greater when the roadway section is longer and hence the number of lane-changes greater. Nahi and Trivedi (18, 20) proposed a different Kalman filtering density estimator by considering vehicle conservation. As an effort to validate the Kalman filtering procedure for estimating traffic density, Gazis and Szeto (21) used aerial photography data as a benchmark to evaluate the effectiveness of the procedure. They reported two important findings. First, density estimation yields more accurate result when multiple lanes are treated as combined than as separated. This is because in the combined case lane-changing becomes an internal issue and is self-contained, while in the separated case lane-changing is an external issue for each lane and this resulted in potential observation error. Second, sensitivity tests showed that estimation accuracy remained low and quite constant over a detector spacing range between 1,000 and 3,000 ft. Before and after this range, estimation accuracy deteriorated. In 1980, Kurkjian et al (22) revisited the Kalman filtering approach by using flow and occupancy data to estimate the density on a link. They formulated a scalar Kalman filter and achieved simplicity by compensating the filter with a scalar generalized likelihood ratio event detection algorithm. Payne et al (23) used steady-state Kalman filters to estimate traffic density and SMS from presence detector data. Bhouri et al (24) applied a scalar Kalman filter to estimate traffic density based on presence detector data. Estimation results were then compared to real traffic density measurements extracted from a film. Cremer and Putensen (25) contributed yet another Kalman filter-based approach for estimating traffic density which took a model-based estimation scheme using both loop detectors and vehicle trip data. Davis and Kang (26) applied Szeto and Gazis’s approach (16) to track the density of a freeway section broken down by destination and tested the filter using simulated data and actual freeway data obtained from Interstate 35W. Gazis and Liu (27) extended Szeto and Gazis’s approach (16) to estimate vehicle counts for two roadway sections which were treated jointly. Improvements were achieved over those which treated the two sections separately. This is because counting error, if any, will propagate from the upstream section to the downstream section. Sun et al (28)
developed a mixture Kalman filter to estimate vehicle densities from loop detector data. The mixture Kalman filter algorithm, based on a sequential Monte Carlo method, is used to approximately solve the difficult problem of inference on a switching state-space model with an unobserved discrete state.

A few indirect methods other than Kalman filter were reported. Lopez-Lopez and Houpt (29) provided an alternative approach to estimating SMS and density based on point processing techniques. This approach treated vehicle arrival at a given location as a point or counting Poisson process whose rate is a function of the state of the traffic at every instant of time. The traffic state is modeled as a finite-state Markov chain. A sequential point process filter, optimum in the mean-squared error sense, was designed to estimate the state from observations of the vehicle arrival-time sequence. Sheu (30) proposed a stochastic system modeling approach for dynamic prediction of section-wide lane traffic characteristics on freeways. This was a recursive estimation algorithm using an extended Kalman filter, truncation and normalization procedures, and a density-updating procedure. This method provided information of inter- and intra-lane traffic variables, though the procedure was very complicated. Wang and Nihan (31) addressed estimation of traffic density and SMS from single-loop data. They found that the assumed uniform vehicle length used to convert occupancy to density is not a constant and varies with the fraction of long vehicles such as trucks in the fleet. They estimated the percentage of long vehicles based on occupancy variance and computed mean vehicle length from a log-linear regression model based only on single-loop outputs. The estimated mean vehicle length could then be used to estimate speed and density. Angel et al (32) applied automated vehicle identification to estimate traffic flow variables including traffic density, but the report did not provide further detail about estimation methodology. Junghans et al (33) used wide area video surveillance for measuring wide area traffic flow parameters such as traffic density. Occlusion robust measurement of traffic density was based on motion measurements and the evaluation of the continuity equation. Astarita et al (34) proposed a method to estimate traffic flow characteristics from a sample of traffic population by using instrumented vehicle counts.

THREE DIRECT METHODS TO BE EXAMINED

In this section, we highlight three direct methods which determine traffic stream characteristics by definition. More specifically, the three methods are (i) the HCM method which determines traffic stream characteristics based on their definition adopted by HCM, (ii) the x-t method which determines traffic stream characteristics based on Edie’s definition and works on vehicle trajectories in x-t domain, and (iii) the n-t method which determines traffic stream characteristics based on Edie’s definition and works on cumulative vehicle numbers in n-t domain.

The HCM Method

According to HCM (35), traffic stream characteristics are defined as follows:

**Flow Rate**

The equivalent hourly rate at which vehicles, bicycles, or persons pass a point on a lane, roadway, or other trafficway. It is computed as the number of vehicles, bicycles, or persons passing the point, divided by the time interval (usually less than 1 h) in which they pass, and, expressed as vehicles, bicycles, or persons per hour.

\[ q = \frac{N}{T} \]  
(Eq. 1)

where \( q \) is flow rate, \( N \) is the number of vehicles, and \( T \) is the elapsed time.

**Density**

The number of vehicles on a roadway segment averaged over space, usually expressed as vehicles per kilometer or vehicles per kilometer per lane.

\[ k = \frac{N}{L} \]  
(Eq. 2)

where \( k \) is density, \( N \) is the number of vehicles, and \( L \) is the length of the segment.
SMS

HCM has two definitions for SMS: (i) the harmonic mean of (spot) speeds over a length of roadway, (ii) an average speed based on the average travel time of vehicles to traverse a segment of roadway, in kilometers per hour.

Note that the above two definitions are somehow equivalent because the harmonic mean is calculated by converting the individual spot speeds to individual travel time rates, then calculating the average travel time rate, and finally inverting the average travel time rate to obtain an average speed. This is shown in the following equation:

\[ \bar{v} = \frac{L}{\frac{1}{N} \sum \limits_{i} t_i} = \frac{L}{\frac{1}{N} \sum \limits_{i} \frac{1}{u_i}} = \frac{1}{\frac{1}{N} \sum \limits_{i} \frac{1}{u_i}} \]  

(Eq. 3)

where \( \bar{v} \) is SMS, \( t_i \) is the time traveled by the \( i \)-th vehicle, and \( u_i \) is the spot speed of the \( i \)-th vehicle.

This use of spot speeds to calculate SMS was agreed by many researchers (4, 36, 37). There is a third and widely accepted definition for SMS: (iii) the arithmetic mean of speeds over a given segment at a given instant observed on an aerial photograph, i.e.

\[ \bar{v} = \frac{1}{N} \sum \limits_{i} v_i \]  

(Eq. 4)

where \( v_i \) is the speed of the \( i \)-th vehicle observed on an instant photograph.

The x-t Method

This method is based on the Edie’s definition (3, 4). To determine traffic stream characteristics, one constructs vehicle trajectories on a space-time (x-t) domain, as in the left graph of Figure 1. The vertical axis (x) of this graph represents distance from some arbitrary starting point along the road in the direction of travel and the horizontal axis (t) represents elapsed time from some arbitrary starting time. A x-t region \( A_n \) is defined as the shaded rectangle bounded by \( x_{lo} \), \( x_{hi} \), \( t_{lo} \), and \( t_{hi} \) : \( A_n : (x, t) \in A_n, x \in (x_{lo}, x_{hi}), t \in (t_{lo}, t_{hi}) \). Next, one determines the following three basic quantities:

The three basic quantities

(i) The total time spent by all vehicles in region \( A_n \), \( t(A_n) \):

\[ t(A_n) = \sum \limits_{i=n_{lo}}^{n_{hi}} \left( \min(t^{(i)}(x_{hi}), t_{hi}) - \max(t^{(i)}(x_{lo}), t_{lo}) \right) \]  

(Eq. 5)

where subscripts “lo” means a “low” number and “hi” means a “high” number.

\( x_{hi}, x_{lo} \): the upper and lower boundaries of the region in x domain, respectively.

\( t_{hi}, t_{lo} \): the upper and lower boundaries of the region in t domain, respectively.

\( n_{lo}, n_{hi} \): the first vehicle and the last vehicle crossing the region, respectively.

\( t^{(i)}(x_{hi}) \) is the time when the \( i \)-th vehicle passes \( x_{hi} \) and similar definition applies to \( t^{(i)}(x_{lo}) \).

(ii) The total distance traveled by all vehicles in region \( A_n \), \( d(A_n) \):

\[ d(A_n) = \sum \limits_{i=n_{lo}}^{n_{hi}} \left( \min(x^{(i)}(t_{hi}), x_{hi}) - \max(x^{(i)}(t_{lo}), x_{lo}) \right) \]  

(Eq. 6)

where \( x^{(i)}(t_{hi}) \) is the location of the \( i \)-th vehicle at time \( t_{hi} \) and similar definition applies to \( x^{(i)}(t_{lo}) \).
(iii) The area of the x-t region $A_{n}$, $|A_{n}|$:

$$|A_{n}| = (x_{hi} - x_{lo})(t_{hi} - t_{lo})$$

(Eq. 7)

$|A_{n}|$ has a unit miles*hours.

With the above preparation, traffic stream characteristics are determined as follows:

**Traffic stream characteristics**

(i) Flow $q(A_{n})$: the total distance traveled by all vehicles in $A_{n}$, $d(A_{n})$, divided by the “area” of $A_{n}$, $|A_{n}|$:

$$q(A_{n}) = \frac{d(A_{n})}{|A_{n}|}$$

(Eq. 8)

(ii) Density $k(A_{n})$: the total time spent by all vehicles in $A_{n}$, $t(A_{n})$, divided by the “area” of $A_{n}$, $|A_{n}|$:

$$k(A_{n}) = \frac{t(A_{n})}{|A_{n}|}$$

(Eq. 9)

(iii) SMS $\bar{V}(A_{n})$: the total distance traveled by all vehicles in $A_{n}$, $d(A_{n})$, divided by the total time spent by all vehicles in $A_{n}$, $t(A_{n})$:

$$\bar{V}(A_{n}) = \frac{d(A_{n})}{t(A_{n})}$$

(Eq. 10)

**FIGURE 1** illustration of the x-t and the n-t methods.
The n-t Method

Considering that the x-t method relies on vehicle trajectories in x-t domain, data collection of which is typically expensive and is carried out on a non-regular basis. It is of great interest if this method can somehow be transformed such that the resulted method is able to utilize point sensor data such as those collected by most ITS Systems. To serve such a purpose, we developed a n-t method (38) (Ni and Leonard, unpublished data) which is also based on Edie’s definition. Instead of working on vehicle trajectories as the x-t method does, the n-t method works on cumulative number of vehicles in cumulative number-time (n-t) domain. Basically, if one sets two point sensors at locations \( x_{lo} \) and \( x_{hi} \) and count vehicles passing these locations cumulatively, one obtains a pair of cumulative curves as shown in the right graph of Figure 1 where the vertical axis (n) represents cumulative number of vehicles referenced from some arbitrary vehicle and the horizontal axis (t) represents elapsed time from some arbitrary starting time. The n-t region \( A_n \) which is equivalent to the x-t region \( A_x \) is shown as the shaded area in the right graph. \( A_n \) is bounded by the two cumulative curves \( n(x_{hi}, t) \) and \( n(x_{lo}, t) \) and the two time instants \( t_{hi} \) and \( t_{lo} \).

Following the same procedure of the x-t method, one determines the three basic quantities:

**The three basic quantities**

(i) The total time spent by all vehicles in region \( A_x \), \( t(A_x) \) is:

\[
t(A_x) = \sum_{i=n_{lo}}^{n_{hi}} t^{(i)}(A_x) = \sum_{i=n_{lo}}^{n_{hi}} \left\{ \min(t_{x=x_{hi}}(n_i), t_{hi}) - \max(t_{x=x_{lo}}(n_i), t_{lo}) \right\}
\]

(Eq. 11)

where:

\( t^{(i)}(A_x) \) is the travel time that the i-th vehicle spends in region \( A_x \),

\( t_{x=x_{hi}}(n_i) \) is the time when the i-th vehicle passes \( x_{hi} \) and this time can be determined by finding the intersection of line \( n = n_i \) and curve \( n(x_{hi}, t) \). Similarly,

\( t_{x=x_{lo}}(n_i) \) is the time when the i-th vehicle passes \( x_{lo} \) and this time can be determined by finding the intersection of line \( n = n_i \) and curve \( n(x_{lo}, t) \).

(ii) The total distance traveled by all vehicles in region \( A_x \), \( d(A_x) \) is:

\[
d(A_x) = \sum_{i=n_{lo}}^{n_{hi}} d^{(i)}(A_x) = \sum_{i=n_{lo}}^{n_{hi}} t^{(i)}(A_x) \frac{x_{hi} - x_{lo}}{t_{x=x_{hi}}(n_i) - t_{x=x_{lo}}(n_i)}
\]

(Eq. 12)

where \( d^{(i)}(A_x) \) is the distance traveled by the i-th vehicle in region \( A_x \). Since in the n-t domain, location information (x) becomes discrete, distance traveled by the i-th vehicle in region \( A_x \) is determined as proportional to the time it spends in the same region, i.e. an assumption of constant speed is implicit here.

(iii) The area of the n-t region \( A_x \), \( |A_x| \):

Though region \( A_x \) appears in a different shape than \( A_x \), their areas are the same, i.e.

\[
|A_x| = |A_x| = (x_{hi} - x_{lo})(t_{hi} - t_{lo})
\]

(Eq. 13)

Traffic stream characteristics

Therefore, traffic stream characteristics can be determined in the n-t domain as follows:

\[
q(A_x) = \frac{d(A_x)}{|A_x|}
\]

(Eq. 14)
EXAMPLES AND COMPARISONS

This section presents some numerical and empirical examples to contrast the three direct methods. We designed a small-sized numerical example which is manageable by manual calculation and is reproducible for independent verification. We also conducted empirical comparison using vehicle trajectory data provided by the Next Generation Simulation (NGSIM) program. This set of data was collected with such a detail that vehicle movement was recorded every one fifteenth second.

Overall Approach

In the following paragraphs, we contrast and compare the performance of the three methods in terms of determining traffic stream characteristics by definition. More specifically, for the x-t method, we find x-t regions based on $x_{hi}, x_{lo}, t_{hi}, t_{lo}$ in the x-t domain and determine flow, density and SMS using equations 8, 9, and 10 respectively. For the n-t method, we find n-t regions that are equivalent to the x-t regions and determine flow, density and SMS using equations 14, 15, and 16 respectively. For the HCM method, we determine flow using Equation 1 by counting vehicles at location $(x_{hi} + x_{lo})/2$; we determine density using Equation 2 by counting vehicles at instant $(t_{hi} + t_{lo})/2$; we determine SMS based on Equation 3 by using vehicles speeds observed at location $(x_{hi} + x_{lo})/2$ and this SMS will be called “harmonic SMS”; we compute an additional SMS (called “arithmetic SMS”) based on Equation 4 using vehicle speeds observed on an instant photograph taken at time $(t_{hi} + t_{lo})/2$.

Numerical Examples

A numerical example was designed and shown in Figure 2. Part A of the figure shows a x-t domain with x axis representing distance referenced from some arbitrary starting point and t axis representing elapsed time from some arbitrary starting time. To facilitate subsequent discussion, we assume that space increment is 1 mile (i.e. $x_{i+1}-x_i = 1$, $i=1, 2, \ldots$). Time increment is 1 minute (i.e., $t_{i+1}-t_i = 1$, $i=1, 2, \ldots$). The trajectory of the first vehicle (V1) is generated by the following equation:

$$x = f(t) = 0.020833t^3 - 0.3125t^2 + 2.0417t$$

(Eq. 17)

The trajectory of the n-th vehicle (Vn) is generated by translating the first vehicle trajectory to the right by (n-1) time increments.

![Figure 2: A numerical example.](image)
The x-t region of interest is the one defined by the shaded area \(\{A_n: (x, t) \in A_n, x \in (2, 4) \text{ and } t \in (5, 10), x \text{ in miles and } t \text{ in minutes thereafter}\}\). Seven vehicles pass this region, i.e. V3 ~ V9. Results based on the x-t method are listed in Test 1 of Table 1. The results show that total time traveled, \(t(A_n)\), is 10 minutes, total distance traveled, \(d(A_n)\), is 10 miles, and the area of the region \(A_n\) is 10 miles*minutes. These convert to a flow \(q(A_n)\) of 60 vph (vehicles per hour), a SMS \(v(A_n)\) of 60 mph (miles per hour), and a density \(k(A_n)\) of 1 vpm (vehicles per mile), as listed in column 2 of Test 1 of Table 1. Next, we do it again using the n-t method. The vehicle trajectories in the x-t region of Part A correspond to the solid lines in the n-t region (shaded) of Part B. Using Equations 14 through 16, we obtain exactly same results as in column 3 of Test 1 of Table 1. For the HCM method, we find flow by observing traffic at location x=3 miles. 5 vehicles (i.e. V4 through V8) pass this location between t=5 and t=10 minutes and their speeds are the same, i.e. 52.5 mph. We find density by taking an instant photograph at time t=7.5 minutes. There are two vehicles (i.e. V6 and V7) on the photograph and their speeds are 75 and 52.5 mph respectively. Based on the above, the harmonic SMS is determined as 52.5 mph and the arithmetic SMS is 63.75 mph. the results are listed in column 4 and 5.

Test 1 shows that the n-t method is equivalent to the x-t method. However, the HCM method underestimates SMS which corresponds to an error of 12.5%. The instant photograph technique overestimates SMS with an error of 6.25%. The fundamental relationship holds for the x-t and the n-t methods but does not hold for the HCM method.

<table>
<thead>
<tr>
<th>TABLE 1 Comparison of Numerical Tests</th>
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<tr>
<td>Test 1</td>
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<td>x-t method</td>
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<tr>
<td>Flow (vph)</td>
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<tr>
<td>Density (vpm)</td>
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<td>SMS (mph)</td>
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<th>Test 2</th>
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<tr>
<td>x-t method</td>
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<td>Flow (vph)</td>
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<tr>
<td>Density (vpm)</td>
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<td>SMS (mph)</td>
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Next, we repeat the above process by examining another x-t region \(\{A_n: x \in (1, 6) \text{ and } t \in (4, 10)\}\) and list the results in Test 2 of Table 1. At location x=3.5 miles, 6 vehicles (i.e. V3 through V8) are observed and their speeds are the same, i.e. 52.49 mph. On an instant photograph taken at t=7 minutes, 6 vehicles are observed (i.e. V2 through V7) and their speeds are 75, 52.49, 37.5, 30, 30, and 37.5 mph respectively. This converts to an arithmetic SMS of 43.75 mph. Test 2 shows that the x-t method and the n-t method yield the same density. The flow and SMS determined using the n-t method are 2% less than those by the x-t method and this small difference is due to the constant speed assumption in the n-t method. The fundamental relationship holds for the x-t and the n-t methods. Flow, density and SMS determined using the HCM method differ from those by the x-t method by 5.3%, 3.5%, and 6.7% respectively. The fundamental relationship does not hold for the HCM method. The arithmetic SMS differs from that of the x-t method by 11.4%.

Further tests by choosing different x-t regions indicate that:

1. The n-t method and the x-t method yield the same density and this result strictly holds under any circumstance.
2. There might be some error in SMS and flow in the n-t method due to the loss of resolution in space domain (which necessitates a constant speed assumption). However, the error is very small and the error affects flow and SMS in the same way.
3. Significant error may be resulted in the HCM method. The arithmetic SMS determined using the instant photograph technique is not reliable either. It is interesting to note that, contrary to what was commonly
Ni and Leonard believed, minimizing the time interval (i.e. \( t_{hi} - t_{lo} \rightarrow 0 \)) of the x-t region may not eliminate the error in the HCM method as well as in the instant photograph technique.

**Empirical Examples**

To further verify the above observations, we examined the three methods using empirical data provided by the Next Generation Simulation (NGSIM) program. The data contains detailed vehicle trajectories collected on eastbound Interstate 80 for a period of approximately half an hour between 2:35 and 3:05 PM on Wednesday, December 3, 2003. Figure 3 shows the sketch of the data collection site. Six video cameras (positions are not shown in the figure) monitored the whole site which is 2590 ft long. The site consists of six lanes as indicated in circles and an on-ramp at 755 ft and an off-ramp at 2360 ft measured from entry, i.e. the left boundary. A vehicle, once entered the site, was attached a vehicle ID and the vehicle was traced all the way until it exited the site. Complete vehicle trajectories were transcribed at a resolution of 15 frames per second. For each vehicle in each frame, a total of 16 data columns are recorded, including vehicle ID, local X and Y positions which are referenced from the top left corner of the site, and global time referenced from January 1, 1970. For more detailed description of the data structure, please refer to NGSIM website (http://ngsim.camsys.com). The data we are interested in are vehicle ID, local Y, and global time. All the lanes are treated as combined to achieve better result as suggested by (21).

**FIGURE 3 Sketch of the NGSIM data collection site.**

*The Effect of Extremely Brief Presence of Vehicle in x-t Region*

In principle, the x-t method is the benchmark method which should yield the most reliable result. However, in computerized implementation, this method sometimes performs less satisfactorily than the n-t method in terms of computing density. The example in Figure 4 defines a x-t region \( A_x: (x, t) \in A_x, x \in (1000, 1100) \) and \( t \in (20, 21) \), x in feet and t in seconds thereafter. Six vehicles pass this region as shown in the left graph. A computerized algorithm searches trajectory data and determines when a vehicle crosses the left or the bottom of the x-t region (labeled as “tlo” in Table 2) and when the vehicle crosses the top or right of the x-t region (labeled as “thi” in Table 2). The time the vehicle spent in the region is the difference of thi and tlo. Computation results based on the x-t method and the n-t method are listed in Table 2. The two methods yield almost the same result except that the x-t method fails to count the first vehicle (V1) (see the short bold line at the top left corner of the x-t region). A closer look at the trajectory data reveals that this is because the presence of this vehicle in the x-t region is so brief that it is less than the resolution of the trajectory data (1/15 second in this case). However, the n-t method successfully detects this brief presence and gives more accurate density result. This test shows that the n-t method outperforms the x-t method when there is extremely brief presence (i.e. less than time resolution) of vehicle in the x-t region.
Trajectories in region 1000<x<1100 and 20<t<21

Cumulative curve for the same x-t region

FIGURE 4 An example with extremely brief presence of vehicle in x-t region.

TABLE 2 Comparison of the x-t and the n-t Methods in Case of Extremely Brief Presence of Vehicle

<table>
<thead>
<tr>
<th>Results based on the x-t method</th>
<th>Veh2</th>
<th>Veh3</th>
<th>Veh4</th>
<th>Veh5</th>
<th>Veh6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{lo} )</td>
<td>20</td>
<td>20</td>
<td>20.434</td>
<td>20.811</td>
<td>20.848</td>
</tr>
<tr>
<td>( t_{hi} )</td>
<td>20.264</td>
<td>20.899</td>
<td>21</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>( t_{hi} - t_{lo} )</td>
<td>0.264</td>
<td>0.899</td>
<td>0.566</td>
<td>0.189</td>
<td>0.152</td>
</tr>
<tr>
<td>( t(A_n) = 2.07 ) sec, ( A_n = 100 ) sec*ft, ( k(A_n) = 18.216 ) vpmpl</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results based on the n-t method</th>
<th>Veh ID</th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
<th>V6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{lo} )</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20.4345</td>
<td>20.812</td>
<td>20.848</td>
<td></td>
</tr>
<tr>
<td>( t_{hi} )</td>
<td>20.035</td>
<td>20.2647</td>
<td>20.8993</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>( t_{hi} - t_{lo} )</td>
<td>0.035</td>
<td>0.2647</td>
<td>0.8993</td>
<td>0.5655</td>
<td>0.188</td>
<td>0.152</td>
<td></td>
</tr>
<tr>
<td>( t(A_x) = 2.1045 ) sec, ( A_x = 100 ) sec*ft, ( k(A_x) = 18.5196 ) vpmpl</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Effect of Detector Spacing

To investigate the sensitivity of the three methods to detector spacing, we start with a x-t region \( \{A_w: x \in (755, 2255) \} \) which covers a roadway section of 1500 feet. We set imaginary detectors at both ends of the section. We then progressively insert imaginary detectors to evenly divide the section into two parts, three parts, and so on. In each case, the average flow, density, and SMS determined using the x-t method remain almost constant at 1680 vphpl (vehicles per hour per lane), 28 vpmpl (vehicles per mile per lane), and 60 mph respectively. The flow, density, and SMS determined using the n-t method and the HCM method are compared against those of the x-t method. Mean absolute percentage error (MAPE) is used in the comparison:

\[
MAPE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{y_i - y_0}{y_0} \right|
\]

where \( N \) is the number of samples, \( y \) is a traffic stream characteristic (flow, density, or SMS) determined using the n-t method or the HCM method, and \( y_0 \) is the corresponding traffic stream characteristic determined using the x-t method.
Comparison results are shown in Figure 5. The top graph shows the MAPE of flow where the MAPE of the n-t method (the solid line) increases slowly as detector spacing increases, but the MAPE is negligible (below 0.15%). The error of the HCM method (the dotted line) is much larger than that of the n-t method with a sudden drop when detector spacing approaches 1500 feet (no evidence shows that this sudden drop remains as detector spacing becomes even larger). The middle graph shows the MAPE of density where the MAPE of the n-t method (the solid line) remains zero regardless of how detector spacing varies. This confirms our previous observation. The MAPE of the HCM method (the dotted line) generally decreases as detector spacing becomes large. Notice that the error of the HCM method is significant with most of the line above 10%. The bottom graph shows the MAPE of SMS. There are three lines here. The solid line, which is almost flat at zero level, represents the error of the n-t method. The dotted line, which runs mostly between 2~4%, represents the error of the HCM method (harmonic SMS). The dash-dotted line, which gradually increases from 1% to nearly 6%, represents the error of the instant photograph technique (the arithmetic SMS). The comparison confirms that the n-t method is almost equivalent to the x-t method, while the HCM method yields large error. The instant photograph technique is not a good way to determine SMS.

FIGURE 5 The effect of detector spacing.

The Effect of Detector Headway

To investigate the sensitivity of the three methods to detector headway (detector spacing in time domain), we start with a x-t region \( \{A_x; x \in (1000, 1100) \text{ and } t \in (40, 1540)\} \) which spans a duration of 1500 seconds. We set imaginary detectors at both ends of the duration. We then progressively insert imaginary detectors to evenly divide the duration into two parts, three parts, and so on. In each case, the average flow, density, and SMS determined using the x-t method remain almost constant at 1644.4 vphpl, 25.6 vpmpl, and 64.3 mph respectively. Comparison treatment is similar to the detector spacing case and Figure 6 shows comparison results. In the top, middle, and bottom graphs, the lines corresponding to the error of the n-t method are almost flat at zero level, suggesting that the
The n-t method is not sensitive to the change of detector headway and the n-t method is nearly equivalent to the x-t method. The error of the HCM method is much larger than that of the n-t method. The error of the instant photograph technique (the dash-dotted line in the bottom graph) is constant (2%) regardless of how detector headway varies.

The Effect of a Partial Trip in x-t Region

In some cases, the trajectory of a vehicle may terminate within a x-t region. This might be because the detection system fails to keep track of the vehicle before it reaches the downstream boundary of the region or because the vehicle stops on the roadway or because of any other reason. An example of such a scenario in the NGSIM data is illustrated in Figure 7 where the x-t region \( \{A_t : x \in (2129,2130), t \in (807,808)\} \) is shaded in the left graph. The third trajectory terminates within the region (a partial trip in the region). In the right graph, we show cumulative number of vehicles in the n-t domain. Notice that there is an obvious jump in the lower curve due to the partial trip.

FIGURE 6 The effect of detector headway.
FIGURE 7 An example where a trip terminates within a x-t region (a partial trip).

Manual calculation based on the above example (though a 1 foot detector spacing is not realistic) using the x-t method yields the following results: $t(A_n)=0.035$ sec, $d(A_n)=3.5$ ft, $A_n=1$ ft*sec. This converts to flow $q(A_n)=2100$ vphpl, SMS $v(A_n)=68.2$ mph, and density $k(A_n)=30.8$ vpmpl. To investigate the effect of partial trip on the performance of the three methods, we vary detector spacing, headway, and region area by doing the following: (1) starting with $A_n$, we progressively increase detector spacing by setting the upstream space detector back; (2) starting with $A_n$, we progressively increase detector headway by setting the earlier time detector back; (3) starting with $A_n$, we progressively increase the area of x-t region by setting upstream space detector and earlier time detector back simultaneously. For each iteration in each case, we determined traffic flow characteristics based on the three methods and compare the results of the n-t method and the HCM method against those of the x-t method. Table 3 shows density computed using the x-t method. Column 1 and 2 show how density varies with detector spacing. Column 3 and 4 show how density varies with detector headway. Column 5 and 6 show how density varies with the area of x-t region. Using the results in Table 3 as a benchmark, we compare against it the n-t method and the HCM method by computing their corresponding MAPEs. Comparison results are shown in Figure 8.

### TABLE 3 Density Based on the x-t Method for the Partial Trip Example

<table>
<thead>
<tr>
<th>Spacing ft</th>
<th>Density vpmpl</th>
<th>Headway sec</th>
<th>Density vpmpl</th>
<th>Area ft*sec</th>
<th>Density Vpmpl</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>40.200</td>
<td>1</td>
<td>39.687</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>27.318</td>
<td>10</td>
<td>37.552</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>38.055</td>
<td>50</td>
<td>26.935</td>
<td>50*50</td>
<td>25.929</td>
</tr>
<tr>
<td>500</td>
<td>37.262</td>
<td>100</td>
<td>27.685</td>
<td>100*100</td>
<td>25.925</td>
</tr>
<tr>
<td>750</td>
<td>38.596</td>
<td>250</td>
<td>29.09</td>
<td>250*250</td>
<td>27.482</td>
</tr>
<tr>
<td>1000</td>
<td>34.855</td>
<td>500</td>
<td>29.803</td>
<td>500*500</td>
<td>28.880</td>
</tr>
<tr>
<td>1300</td>
<td>30.209</td>
<td>750</td>
<td>29.379</td>
<td>750*750</td>
<td>29.089</td>
</tr>
</tbody>
</table>

The top graph of Figure 8 indicates comparable performance between the n-t method and the HCM method in that both methods yield large MAPE when detector spacing is small, i.e. less than 50 feet. However, the MAPE drops and approaches zero when detector spacing becomes large. The middle graph shows 100% MAPE for the HCM method regardless how detector headway varies. The MAPE for the n-t method is large when detector headway is small (i.e. less than 100 seconds), but the MAPE constantly decreases and approaches zero as detector headway becomes large. The bottom graph shows that the MAPE for the HCM method is always large (mostly above 20%) as the x-t region area varies, while the MAPE for the n-t method is nearly zero. Figure 8 indicates that, for the n-t method, the error due to the partial trip diminishes as detector spacing, headway, or region area becomes large. This suggests that too small detector spacing or headway may render the n-t method sensitive to partial trip error. Of course, partial trip presents a detection problem and will inevitably affect the performance of any method.
Therefore, it is recommended that partial trip problems be fixed whenever possible, or, if the problems can not be fixed, partial trips be excluded from consideration.

![Graphs showing the effect of detector spacing, headway, and region area on partial trip error.](image)

**FIGURE 8** The performance of the n-t method and the HCM method in case of partial trip.

**SUMMARY AND CONCLUSIONS**

The study is motivated by a desire to find a direct method to determine traffic stream characteristics by definition. Meanwhile, the method is expected to be able to work on-line/real-time and utilize point sensor data provide by most ITS systems. Prior approaches such as the x-t method and the HCM method do not meet these requirements. Though accurate, the x-t method relies on vehicle trajectory data which requires expensive equipment and is not supported by most ITS systems. The HCM method, on the other hand, is not reliable because it does not define flow, density, and SMS on the same basis. We developed a n-t method which is a transformed presentation of the x-t method. In addition to its accuracy, the n-t method works on cumulative number of vehicles observed at fixed locations, so the n-t method is better suited to meet the aforementioned requirements.

In this paper, we examined the three methods (i.e., the x-t method, the n-t method, and the HCM method) with an emphasis on their performance of determining traffic stream characteristics. The detailed vehicle trajectory data provided by the NGSIM program enabled us to examine these methods empirically with a very fine resolution. Tests of both numerical and empirical examples suggested the following:

1. Generally, the n-t method is almost equivalent to the x-t method which is widely accepted as a benchmark of determining traffic stream characteristics. More specifically, the n-t method and the x-t method are strictly equivalent in terms of determining density. Though there might be some error in the n-t method when determining flow and SMS due to the loss of resolution in space domain, the error is negligible and
the error affects flow and SMS in the same way. On the other hand, the fundamental relationship holds in both the n-t method and the x-t method.

2. The HCM method is generally not reliable and the error can be as high as 100%. The HCM method may not preserve the fundamental relationship because the HCM defines flow and density on different basis. Meanwhile, using the instant photograph technique to determine SMS is not reliable either.

3. Increasing detector spacing tends to increase the error of the n-t method in terms of flow and SMS. However, the error, if any, is negligible. Detector spacing has a mixed effect on the HCM method, but no evidence shows that increasing detector spacing helps improve the performance of the HCM method.

4. The n-t method is not sensitive to detector headway. Again, detector headway has a mixed effect on the HCM method, but no evidence shows that increasing detector headway helps improve the performance of the HCM method.

5. Minimizing detector spacing or headway won’t make the HCM method approximate the x-t method or the n-t method.

6. In computerized implementation, the n-t method outperforms the x-t method when there is extremely brief presence (i.e. less than time resolution) of vehicle in the x-t region.

7. With the presence of partial trip in a x-t region, the difference between the n-t method and the x-t method diminishes as the x-t region becomes larger, but the difference between the HCM method and the x-t method does not. However, partial trip is itself a detection problem and should be avoided whenever possible.

ACKNOWLEDGEMENT

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