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Delay and Energy Tradeoff in Multi-state Wireless Sensor Networks

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***Abstract*—This paper discusses a first attempt to investigate, using analytic means, the transmission delay and energy characteristics of a multi-state wireless sensor network. For such a network with n nodes, where each node can be active, resting or sleeping, a model that describes the transition of a node from one state to another and the probability associated with each state is proposed. Asymptotic analyses of transmission delay and energy are presented. We report the presence of a threshold for the arrival rate of data packets that decides which energy component dominates. The transmission delay-energy tradeoff is presented for the case where transmission energy varies directly with distance raised to a power and when the reported threshold is exceeded.**

I. Introduction

A. Preamble and Motivation

Advances in the power management of wireless sensor networks have given rise to the possibility of multi-state networks. Multi-state networks refer to those which contain nodes that can function in more than two operating modes or conditions, each with a different level of power consumption. In this paper, we discuss wireless sensor networks with nodes that can operate in three different states. These states are the fully functional active state which has the highest power consumption, the passive listening or resting state with limitations in functions so as to conserve power and the sleep state where the node is not functioning and requires the least amount of power. It is assumed that resting nodes can be "awakened" by other active nodes to become fully functional active nodes. We investigate the delay and energy characteristics and tradeoffs of such networks.

The presence of resting nodes in a sensor network intuitively suggests a saving in power consumption and hence, an extension of node and network lifespan. This suggestion, however, needs further verification. In addition, the effect of resting nodes on other performance parameters such as delay requires further investigation. As such, the need to better understand the performance of multi-state networks motivates this line of research.

B. Related Work

The interest in wireless networks has motivated studies and generated papers addressing different aspects of this subject. In [1], a detailed discussion on the opportunities and techniques for reducing energy use by wireless network at each level of the protocol stack and in the network architecture is presented. In [2], the authors investigate the energy-delay tradeoff for a wireless sensor network by varying the transmission range. Topology management as a means to trade latency for energy savings is discussed in [3].

Several important papers [4-9] relate to the scaling and tradeoff of important network performance metrics. They, however, do not consider multi-state networks. In particular, [8] discusses the latency of wireless sensor networks under a two-state uncoordinated power saving scheme.

Paging systems, such as the Motorola FLEX [10], are early adopters of power-saving protocols that involve signalling a node prior to data transmission. An early work on multi-state nodes with regards to dynamic power management in wireless sensor networks is found in [11]. The notion of resting sensor nodes that can be awakened is discussed in [12-17]. In particular, power save mechanisms where nodes go to sleep but can be awakened using a wakeup radio are discussed in [16] and [17]. A sentry-based approach to power management, where sentries wake up non-sentries when necessary, in a two-state network is discussed in [18].

C. Contributions and Outline of Paper

This paper contributes to the understanding of multi-state sensor network by providing the following:

- transmission delay and energy characteristics of deterministically-deployed one- and two-dimensional multi-state sensor networks
- arrival rate thresholds that decide the dominant energy component
- the delay and energy tradeoff where applicable, and
- insights on arrival rate and power consumption to aid the design of multi-state sensor networks.

The following section describes the state transition and network models adopted in this discussion. This is followed by sections on the delay and energy characteristics as well as the tradeoff between the two where applicable. The concluding remarks suggest directions for future research.

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II. Models

A. State Transition Model

In this paper, we consider the case where the sensor nodes in a large-scale sensor network can be in three states: *A*ctive, *R*esting and *S*leeping. The methodology developed here is applicable and extensible to more complex models and scenarios. The time each node spends in each state is denoted by T_X where $X = A, R$ or S and p_X represents the probability that a node is in state X. Figure 1 depicts the state transition diagram, where the dashed arrow represents the waking up of a resting node to become active.

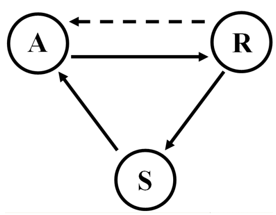

Fig. 1. State Transition for Sensor Nodes.

For a sleeping node i, we define the random variable (r.v.) X_i as the time the node wakes up to enter the active state, where $0 \leq X_i \leq T_S$. The r.v. X_i is assumed to be uniformly distributed between 0 and T_S, i.e. $X_i \sim U(0, T_S)$.

For M independent sleeping nodes, we represent the time for any of the nodes to wake up by the r.v. *X*. $\Pr(X \geq \tau) = \Pr(X_1 \geq \tau, ..., X_M \geq \tau) = \prod_{i=1}^{M}(1 - F_{X_i}(\tau)) = (1 - \frac{\tau}{T_S})^M$. Other measures of interest are the conditional CDF of X given $X \leq T$, which is given by $F_{X|X \leq T}(\tau) = \frac{1-(1-F_{X_i}(\tau))^M}{1-(1-F_{X_i}(T))^M}$ and the conditional PDF $f_{X|X \leq T}(\tau) = \frac{M(1-\frac{\tau}{T_S})^{M-1}}{T_S[1-(1-\frac{T}{T_S})^M]}$. The expected value of X given $X \leq T$ can be found using $\mathbf{E}(X|X \leq T) = \int_0^T \tau f_{X|X \leq T}(\tau) d\tau$.

B. Network and Transmission Model

Each node is independently and randomly in state X with probability p_X, where $X = A$, S or R. The networks considered are deterministically-deployed ones where nodes are equally-spaced apart in a line network or in a square grid for a two-dimensional (2D) network, similar to those discussed in [19] and [20]. Results for networks under the random deployment model can be obtained using a similar approach.

Nodes can only communicate with their neighbors i.e. nodes within the radius of communication r and are assumed to have knowledge of their neighbors' states. Delay due only to transmission, and not congestion, is investigated. The r.v. D denotes the delay or time taken to send a message from one end of the network to the other. The network is assumed to have two channels- one for data transmission and the other, presumably with a narrower bandwidth, for the purpose of sending wake-up beacons and node status updates.

It is assumed that time is slotted, similar to [21], and a single-hop transmission requires t time units and an additional slot of t time units is required for a resting node to be awakened by a neighbor to be ready and partake in the communication process. A node in state X will expend power at the rate of E_X energy units per unit time, where $E_A > E_R > E_S$. A node expends E_t units of energy in transmitting a data packet and E_{tw} units of energy if it needs to wake up a neighbor before transmitting the packet. E_t and E_{tw} are given by βr^α and γr^α respectively, where $\alpha > 1$ and $\beta > \gamma$. Alternatively, E_t and E_{tw} can also be constants independent of r if a simpler model is preferred. We further differentiate between static energy E_{static} and dynamic energy $E_{dynamic}$ of a network. Static energy of a network, which refers to the operating energy required regardless of transmission activity, is given by $E_{static} = n(p_A E_A + p_R E_R + p_S E_S)T_0$, where T_0 is the total deployment time of the network. Dynamic energy refers to energy that is expended only in the process of transmission and requires separate formulation for different cases. The total energy expended is given by $E = E_{static} + E_{dynamic}$.

Subsequent analysis and derivations are based on a general case, of which both the supercritical or connected and subcritical or disconnected phases [8] of the network are special cases of. Connectivity of a network in this case is decided by the radius of communication r [22].

III. Deterministic Line Networks

A. Formulation

Consider a line network of unit length with n sensors spaced equi-distant apart, as depicted in Figure 2, where a data packet from the node on the left end (dot) needs to reach the node on the right end (diamond). Each node can be in state X with probability p_X where $X = A, S$ or R. We divide the line into $\lceil \frac{1}{l} \rceil$ segments each of length l, where $\lceil y \rceil$ denotes the smallest integer larger than or equal to y. For simplicity, we ignore edge effects and assume that all segments are of length l.

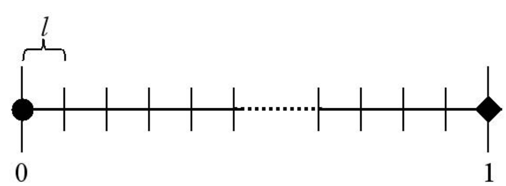

Fig. 2. Line network subdivided into segments of length l with source (dot) and destination (diamond) nodes.

The r.v.'s $D_P(l)$ and $E_P(l)$ represent the time and dynamic energy respectively needed to propagate the data packet from source to sink. By propagate, we refer to the condition where a non-sleeping node is present in a segment to receive the data packet and forward it to the next segment. There is no need to consider whether the communicating nodes are within range for the purpose of finding upper or lower bounds. In addition, T and E are r.v.'s that represent the time taken and energy

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needed respectively to propagate the data packet across one segment. Due to space constraints, the PDF's of T and E are presented in Tables I and II without proof since they can be easily derived.

We introduce a binary constant $c = 0$ or 1 to capture the notion of connectivity where $c = 1$ corresponds to the connected case and $c = 0$ otherwise. The notation $\delta(\cdot)$ represents the Kronecker delta function. The network is connected if $r \geq \frac{(1+\epsilon)\ln(n)}{2n\ln(\frac{1}{p_S})} = \frac{k\ln(n)}{n}$, $k = \frac{(1+\epsilon)}{2\ln(\frac{1}{p_S})}$ and $\epsilon > 0$.

TABLE I
PDF OF T

$\hat{\tau}$	$\Pr(T = \hat{\tau})$
t	$1 - e^{-np_A r}$
$2t$	$(e^{-np_A r} - \delta(c)p_S^{nr})e^{-\frac{tnrp_S}{T_S(p_S+p_R)}}$
$t + \frac{(Mt+T_S)(1-\frac{t}{T_S})^M - T_S}{(1+M)(-1+(1-\frac{t}{T_S})^M)}$	$(e^{-np_A r} - \delta(c)p_S^{nr})\times(1 - e^{-\frac{tnrp_S}{T_S(p_S+p_R)}})$
$t + \frac{T_S}{M+1}$	$\delta(c)p_S^{nr}$

TABLE II
PDF OF E

ε	$\Pr(E = \varepsilon)$
E_t	$1 - e^{-np_A r}$ + $\delta(c)p_S^{nr}$ + $(e^{-np_A r} - \delta(c)p_S^{nr})e^{-\frac{tnrp_S}{T_S(p_S+p_R)}}$
E_{tw}	$(e^{-np_A r} - \delta(c)p_S^{nr})(1 - e^{-\frac{tnrp_S}{T_S(p_S+p_R)}})$

With reference to Table I, the first case from the top corresponds to the case where an active node is present in a segment and t units of time are needed to propagate the data packet to the next segment. The second and third cases arise when active nodes are absent and a resting node has to be awakened to complete the propagation. The second case corresponds to the situation where the propagation is completed by the awakened resting node while the third case occurs when a sleeping node awakens by itself and completes the propagation during the t units of time the resting node is being awakened. The fourth case is possible only in disconnected networks where a segment has only sleeping nodes and the propagation is delayed until one of the sleeping nodes wakes up on its own to become active. The PDF for E consists of only two cases-one which involves the waking up of resting nodes and the other without.

Lemma 1: The expected delay in propagating a data packet from source to sink is $\mathbf{E}(D_P(l)) = \lceil\frac{1}{l}\rceil[(1 - e^{-np_A l})t + 2(e^{-np_A l} - \delta(c)p_S^{nl})e^{\frac{-tnlp_S}{T_S(p_S+p_R)}}t + (e^{-np_A l} - \delta(c)p_S^{nl})(1 - e^{\frac{-tnlp_S}{T_S(p_S+p_R)}})(\frac{(T_S - tnl)e^{-\frac{tnl}{T_S}} + T_S}{(1+nl)(1-e^{-\frac{tnl}{T_S}})} + t) + \delta(c)p_S^{nl}(\frac{T_S}{(1+nl)} + t)]$.

Lemma 2: The expected energy expended in propagating a data packet from source to sink is $\mathbf{E}(E_P(l)) = \lceil\frac{1}{l}\rceil\{(1 - e^{-np_A l} + \delta(c)p_S^{nl})E_t + (e^{-np_A l} - \delta(c)p_S^{nl})(1 + e^{\frac{-tnlp_S}{T_S(p_S+p_R)}})E_{tw}\}$.

Proof: Since there are $\lceil\frac{1}{l}\rceil$ segments, the expected delay $\mathbf{E}(D_P(l))$ and energy expended $\mathbf{E}(E_P(l))$ are given by $\lceil\frac{1}{l}\rceil\mathbf{E}(T)$ and $\lceil\frac{1}{l}\rceil\mathbf{E}(E)$ respectively. ■

B. Asymptotic Analysis of Delay and Energy

Theorem 1: If $r = \frac{k\ln(n)}{n}$, then

$$\lim_{n\to\infty}\mathbf{E}(D) = \theta(\frac{n}{\ln(n)}).$$

Proof: Observe that $\mathbf{E}(D) \geq \mathbf{E}(D_P(r))$. By Lemma 1, $\lim_{n\to\infty}\mathbf{E}(D) \geq \lim_{n\to\infty}\lceil\frac{n}{k\ln(n)}\rceil[(1 - e^{-p_A k\ln(n)})t + 2(e^{-p_A k\ln(n)} - \delta(c)p_S^{k\ln(n)})e^{\frac{-tp_S k\ln(n)}{T_S(p_S+p_R)}}t + (e^{-p_A k\ln(n)} - \delta(c)p_S^{k\ln(n)})(1 - e^{\frac{-tp_S k\ln(n)}{T_S(p_S+p_R)}})(\frac{(T_S - tk\ln(n))e^{-\frac{tk\ln(n)}{T_S}} + T_S}{(1+k\ln(n))(1-e^{-\frac{tk\ln(n)}{T_S}})} + t) + \delta(c)p_S^{k\ln(n)}(\frac{T_S}{(1+k\ln(n))} + t)] = \frac{nt}{k\ln(n)}$.

Similarly, observe that $\mathbf{E}(D) \leq \mathbf{E}(D_P(\frac{r}{2}))$ and by Lemma 1, $\lim_{n\to\infty}\mathbf{E}(D) \leq \frac{2nt}{k\ln(n)}$. Hence we conclude $\lim_{n\to\infty}\mathbf{E}(D) = \theta(\frac{n}{\ln(n)})$. ■

Theorem 2: If $r = \frac{k\ln(n)}{n}$ and $E_t = c_1 \in \Re$, then

$$\lim_{n\to\infty}\mathbf{E}(E_{dynamic}) = \theta(\frac{n\lambda(n)}{\ln(n)}),$$

where $\lambda(n)$ is the arrival rate of the data packets.

Proof: Observe that $\mathbf{E}(E_{dynamic}) \geq \mathbf{E}(E_P(r))\lambda(n)T_0$. By Lemma 2, $\lim_{n\to\infty}\mathbf{E}(E_{dynamic}) \geq \lim_{n\to\infty}\lceil\frac{n}{k\ln(n)}\rceil\{(1 - e^{-p_A k\ln(n)} + \delta(c)p_S^{k\ln(n)})E_t + (e^{-p_A k\ln(n)} - \delta(c)p_S^{k\ln(n)})(1 + e^{\frac{-tp_S k\ln(n)}{T_S(p_S+p_R)}})E_{tw}\}\lambda(n)T_0 = \frac{E_t T_0 n\lambda(n)}{k\ln(n)}$. Since $\mathbf{E}(E_{dynamic}) \leq \mathbf{E}(E_P(\frac{r}{2}))\lambda(n)T_0$, $\lim_{n\to\infty}\mathbf{E}(E_{dynamic}) \leq \frac{2T_0 E_t\lambda(n)n}{k\ln(n)}$. This implies $\mathbf{E}(E_{dynamic}) = \theta(\frac{n\lambda(n)}{\ln(n)})$. ■

Theorem 3: If $r = \frac{k\ln(n)}{n}$ and $E_t = \beta r^\alpha$, then

$$E_{dynamic}(n) = \theta(\lambda(n)[\frac{1}{D(n)}]^{\alpha-1})$$

where $\lambda(n)$ is the arrival rate of the data packets.

Proof: Proof follows from the proof to Theorem 2 and substituting $E_t = \beta r^\alpha$, which gives $\lim_{n\to\infty}\mathbf{E}(E_{dynamic}) = \theta(\lambda(n)[\frac{\ln(n)}{n}]^{\alpha-1})$. ■

IV. DETERMINISTIC TWO-DIMENSIONAL LINE NETWORKS

A. Formulation

The ensuing formulation is for the purpose of deriving the upper bounds for delay and energy expended for the 2D network. Consider a unit square covered by $\sqrt{n}\times\sqrt{n}$ sensors arranged in a grid. Subsequent analysis assumes the scenario where a message is sent from the node in the bottom left-hand corner to the destination node, say a base station, at the top right-hand corner.

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The analysis is based on Algorithm 1, which is briefly described by the following. The sending node, denoted by the asterisk in Figure 3, will attempt to forward the message first to an active node in region 1. In the absence of any active node in region 1, the sending node will try to forward the message to an active node in region 2. If this is not possible, the sending node will wake up a resting node in region 1. Finally, if this is not possible, it will wake up a resting node in region 2 to complete the message passing. In the process of waking up a resting node, however, should a sleeping node become active, the message will be sent to the active node that has just become active. Figure 4 depicts a flowchart illustrating Algorithm 1. The process continues until the data packet has covered at least an effective distance of $\sqrt{2}$ along the diagonal of the unit square. The efficiency of this algorithm is not of importance here as it is proposed for the purpose of deriving the upper bound for delay and dynamic energy.

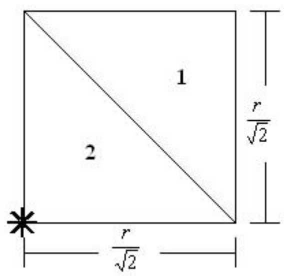

Fig. 3. Regions 1 and 2 for Algorithm 1.

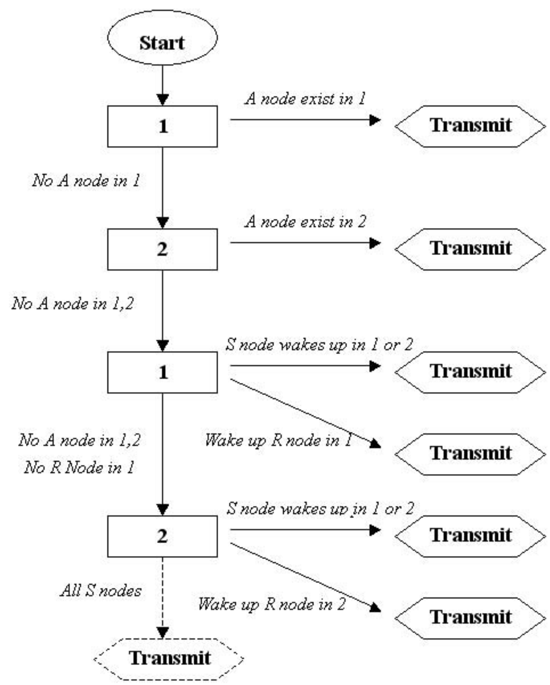

Fig. 4. Flowchart for Algorithm 1.

We now derive probabilities for each case of transmission based on the above algorithm. We denote the event that region i has at least one X node using X_i, where $X =$*A*ctive, *R*esting or *S*leeping. SA_i denotes the event that at least one sleeping node awakens to become active in region i over a period of $2t$. The complement of event X is denoted by $\overline{X}$. The quantities $(d,\ e,\ t)$ following each case number 1-7 refer to the average effective distance the data packet has traveled along the diagonal, the energy expended and the time taken to complete the transmission respectively. It is assumed that the average distance traveled at each hop, an r.v. denoted by d, concentrates around an average that coincides with the distance between the sending node and the centroid of the region. The number of nodes in each triangular region 1 or 2 is denoted by M. The use of $\delta(c)$ allows a general case, which accounts for both connected and disconnected networks, to be formulated.

Case 1 $[\frac{2}{3}r, E_t, t]$: $\Pr(A_1) = 1 - e^{-\frac{np_A r^2}{4}}$

Case 2 $[\frac{1}{3}r, E_t, t]$: $\Pr(\overline{A}_1, A_2) = e^{-\frac{np_A r^2}{4}}(1 - e^{-\frac{np_A r^2}{4}})$

Case 3 $[\frac{2}{3}r, E_{tw}, \frac{(T_S - Mt)e^{-\frac{Mt}{T_s}} + T_S}{(1+M)(1-e^{-\frac{Mt}{T_S}})}]$:$\Pr(\overline{A}_1, \overline{A}_2, R_1, SA_1) = e^{-\frac{1}{2}np_A r^2}(1 - p_S^{\frac{1}{4}nr^2} e^{\frac{1}{4}np_A r^2})(1 - e^{\frac{tnr^2 p_S}{4T_S(p_S+p_R)}})$

Case 4 $[\frac{2}{3}r, E_{tw}, 2t]$: $\Pr(\overline{A}_1, \overline{A}_2, R_1, \overline{SA}_1) = e^{-\frac{1}{2}np_A r^2}(1 - p_S^{\frac{1}{4}nr^2} e^{\frac{1}{4}np_A r^2}) e^{\frac{tnr^2 p_S}{4T_S(p_S+p_R)}}$

Case 5 $[\frac{1}{2}r, E_{tw}, \frac{(T_S - 2Mt)e^{-\frac{2Mt}{T_s}} + T_S}{(1+2M)(1-e^{-\frac{2Mt}{T_S}})}]$: $\Pr(\overline{A}_1, \overline{A}_2, \overline{R}_1, R_2, SA_{1,2}) = e^{-\frac{1}{2}np_A r^2} p_S^{\frac{1}{4}nr^2} e^{\frac{1}{4}np_A r^2}(1 - \frac{\delta(c) p_S^{\frac{1}{2}nr^2} e^{\frac{1}{2}np_A r^2}}{1 - e^{\frac{1}{4}np_A r^2} p_S^{\frac{1}{4}nr^2}})(2 - e^{-\frac{tnr^2}{4T_S}} - e^{-\frac{tnr^2 p_S}{4T_S(p_S+p_R)}})$

Case 6 $[\frac{1}{3}r, E_{tw}, 2t]$:$\Pr(\overline{A}_1, \overline{A}_2, \overline{R}_1, R_2, \overline{SA}_{1,2}) = e^{-\frac{1}{2}np_A r^2} p_S^{\frac{1}{4}nr^2} e^{\frac{1}{4}np_A r^2}(1 - \frac{\delta(c) p_S^{\frac{1}{2}nr^2} e^{\frac{1}{2}np_A r^2}}{1 - e^{\frac{1}{4}np_A r^2} p_S^{\frac{1}{4}nr^2}})(e^{-\frac{tnr^2}{4T_S}} + e^{-\frac{tnr^2 p_S}{4T_S(p_S+p_R)}} - 1)$

Case 7 $[\frac{1}{2}r, E_t, \frac{T_S}{M+1}]$: $\Pr(all\ S\ nodes\ ) = \delta(c) p_S^{\frac{nr^2}{2}}$

An r.v. of interest is m, the number of hops the data packet will take to reach the sink. Using a martingale argument, m is taken to concentrate around an average [23] and is given by $m = \frac{\sqrt{2}}{\mathbf{E}(d)}$.

Using results from [22] and [24], the connectivity condition is found to have the form $r \geq \sqrt{\frac{k ln(n)}{n}}$ for some k.

B. Asymptotic Analysis of Delay and Energy

Theorem 4: If $r = \sqrt{\frac{k\ln(n)}{n}}$, then

$$\lim_{n\to\infty} \mathbf{E}(D) = \theta(\sqrt{\frac{n}{\ln(n)}}).$$

Proof: We derive the lower bound based on the propagation of the data packet along the diagonal of length $\sqrt{2}$ and adopt an approach similar to the proof of Theorem 1. We observe that $\mathbf{E}(D) \geq \mathbf{E}(\sqrt{2}D_P(r))$. By Lemma 1, $\lim_{n\to\infty} \mathbf{E}(D) \geq \sqrt{\frac{2n}{k\ln(n)}}t$.

We derive the upper bound using Algorithm 1. Since Algorithm 1 is sub-optimal, $\mathbf{E}(D) \leq m \sum_{i=1}^{7} \tau_i$ Pr(Case *i*) , where τ_i represents the average time taken for the complete transmission of a data packet for Case i. Since $\lim_{n\to\infty}\mathbf{E}(D) \leq 3\sqrt{\frac{n}{2k\ln(n)}}t$, we can conclude that $\lim_{n\to\infty}\mathbf{E}(D) = \theta(\sqrt{\frac{n}{\ln(n)}})$. Space limitations necessitate the omission of details. ■

Theorem 5: If $r = \sqrt{\frac{k\ln(n)}{n}}$ and $E_t = c_2 \in \Re$, then

$$\lim_{n\to\infty}\mathbf{E}(E_{dynamic}) = \theta(\lambda(n)\sqrt{\frac{n}{\ln(n)}}),$$

where $\lambda(n)$ is the arrival rate of the data packets.

Proof: Adapting the proofs to Theorems 2 and 4, observe that $\mathbf{E}(E_{dynamic}) \geq \mathbf{E}(\sqrt{2}E_P(r))\lambda(n)T_0$. By Lemma 2, $\lim_{n\to\infty}\mathbf{E}(E_{dynamic}) \geq T_0 E_t \lambda(n)\sqrt{\frac{n}{k\ln(n)}}$.

For the upper bound, we observe $\mathbf{E}(E_{dynamic}) \leq m\sum_{i=1}^{7}\varepsilon_i$ Pr(Case *i*), where ε_i represents the energy expended for the complete transmission of a data packet for Case i. Hence, $\lim_{n\to\infty}\mathbf{E}(E_{dynamic}) \leq 3T_0 E_t \lambda(n)\sqrt{\frac{n}{2k\ln(n)}}$ This completes the proof. ■

Theorem 6: If $r = \sqrt{\frac{k\ln(n)}{n}}$ and $E_t = \beta r^\alpha$, then

$$E_{dynamic}(n) = \theta(\lambda(n)[\frac{1}{D(n)}]^{\alpha-1})$$

where $\lambda(n)$ is the arrival rate of the data packets.

Proof: Proof follows from the proof to Theorem 5 and substituting $E_t = \beta r^\alpha$, which gives $\lim_{n\to\infty}\mathbf{E}(E_{dynamic}) = \theta(\lambda(n)\sqrt{\frac{\ln(n)}{n}}^{\alpha-1})$. ■

V. Discussion and Further Work

Recall that total energy $E = E_{static} + E_{dynamic}$ and $E_{static} = \theta(n)$. The dependence of $E_{dynamic}$ on $\lambda(n)$ suggests the presence of a threshold $\lambda_0 = \theta(f(n))$ beyond which $E_{dynamic}$ dominates for the case where E_t is constant. With this insight, based on the nature of the arrival rate $\lambda(n)$, the network designer can choose to optimize the parameters so as to minimize the dominant energy component and in so doing, maximizes the lifetime of the network.

For line networks, the threshold is given by $\lambda_0 = \theta(\ln(n))$ when E_t is a constant. In the case where $E_t = \beta r^\alpha$, the threshold is given by $\lambda_0 = \theta(\frac{n^\alpha}{[ln(n)]^{\alpha-1}})$. Theorem 3 describes the tradeoff between D and E when the threshold is exceeded and $E_{dynamic}$ dominates. It should be noted that the effect of $\lambda(n)$ cannot be neglected. In some cases, $\alpha = 2$ as according to the distance-squared law, we obtain $E(n) = \theta([\frac{\lambda(n)}{D(n)}])$.

For 2D networks and in the case where E_t is a constant, $E_{dynamic}$ dominates when the arrival rate exceeds the threshold $\lambda_0 = \theta(\sqrt{n\ln(n)})$. For the case where $E_t = \beta r^\alpha$, the threshold is given by $\lambda_0 = \theta(\frac{n^{\frac{1}{2}(\alpha+1)}}{[ln(n)]^{\frac{1}{2}(\alpha-1)}})$. Once the arrival rate exceeds this threshold, the energy-delay tradeoff is described by Theorem 6.

The scientific community will benefit from further studies on multi-state WSNs that accounts for queuing delay in addition to transmission time. The effects of channel contention for cases where multiple source-to-sink data packet transmissions occur concurrently can also be investigated in future studies.

More sophisticated state transition models, possibly with more states, can also be explored to investigate the gains and advantages, if any, for doing so.

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