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A Unified Perspective on Traffic Flow Theory Part I: The Field Theory

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Abstract

Over more than half a century, traffic flow theorists have been pursuing two goals: (1) simple and efficient models to abstract vehicular traffic flow and (2) a unified framework in which existing traffic flow models fit and relate to each other. Continuing these efforts, we report our humble understanding in a trio of papers. This paper (Part I) introduces a Field Theory with an emphasis on traffic flow modeling at the microscopic level. In this theory, highways and vehicles are perceived as a field by a driver whose driving strategy is to navigate through the field along its valley. A special case of the theory, Longitudinal Control Model, was formulated with specific function form to aid practical applications. Also discussed are properties of the model and its connection to existing knowledge base.

Keywords: Mathematical modeling, field theory, traffic flow theory

1 Introduction

Three systems are of particular interest: a physical system, a transportation system, and a social system, illustrated in Figure 1. The physical system typically consists of non-living objects whose motion and interaction are subject to physical laws such as Newton's laws of motion. In contrast, the social system involves living entities such as humans whose behavior vary widely among the population but generally follow some loosely defined rules (e.g. seeking gains and avoiding losses). As such, physical science is recognized as 'hard" since

it is more objective, rigorous, and accurate, while social science is perceived as "soft" because of its subjectivity, looseness, and inexactness. Straddling the above two goes the transportation system which involves both living entities (human drivers) and non-living objects (roadways and vehicles). Hence, transportation science can be perceived as "firm" (for the lack of a proper word between "hard" and "soft") since it deals with both physical laws and social rules. In addition, it is close to the "soft" end when strategic planning is concerned, while it migrates toward the "hard" end if tactical decision and particularly operational control are of interest.

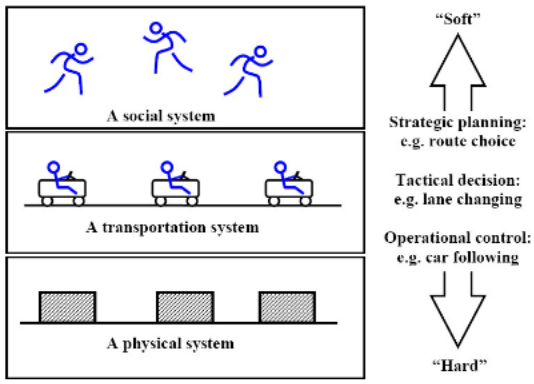

Figure 1: Three systems

In this paper, our attention shall be devoted to the modeling of driver operational control in a transportation system, i.e. the motion and interaction of driver-vehicle units on a long homogeneous highway. By combining physical laws and social rules, a phenomenology is postulated which represents the driving environment perceived by a driver as a field. In this field, objects (e.g. roadways and vehicles) are each represented as a component field and their union represents the overall hazard that the driver tries to avoid. Hence, the modeling of vehicle motion is simply to seek the least hazardous route by navigating through the field along its valley.

The paper is arranged as follows. In Section 2, a list of physical phenomena in traffic flow are analyzed in connection with social rules, i.e. the motivation behind driving decision. The purpose is to demonstrate the similarity between the physical system and the transportation system. As such, a basis is provided that motivates the formulation of the Field Theory in Section 3. Particular attention is drawn to a special case of the Field Theory in Section 4 to facilitate the application of the Field Theory. Properties of the special case is discussed in Section 5 including its model parameters, ability to describe multiple driving regimes, steady-state behavior under equilibrium, and its connection to existing knowledge base.

2 Physical Basis of Traffic Flow

Many traffic flow phenomena are analogous to those in the physical system, yet the transportation system has something special to distinguish itself.

2.1 Mechanics Phenomena

In Physics, forces are the cause of change of motion. In addition, they are measurable and their effects reproducible. For example, Newton's second law of motion stipulates that the velocity of an object changes if it is subject to a non-zero external force; Newton's third law says that, for every action, there is an equal and opposite reaction. Similarly, "forces" exist in traffic flow, but such forces and subsequently fields are all perceived by the subject driver and hence are subjective matters. As recognized by Lewin (1) and Gibson (2), the distinction between a perceived force and a physical force is that the former impinges from the inside of the driver through motivation as opposed to through external compulsion. For example, a fast driver feels a 'force" (a stress in the driver's mind) when approaching a slow vehicle. Conversely, the slow driver may or may not be subject to the "reaction force" depending on whether the driver pays attention and respond to it, i.e., Newton's third law may or may not take effect in this case. More examples of mechanics phenomena are provided below:

M1: Directional flow Traffic always flows in a pre-determined direction much like free objects always fall to the ground. The reason why free objects fall is because they are constantly subject to earth gravity. Similarly, it is reasonable to imagine that vehicles in the traffic are subject to a 'gravity" along the roadway. Such a roadway gravity is, again, a subjective matter since it exists mentally in drivers and is not measurable, but it is recognized that the gravity is related to factors such as driver personalities (e.g. aggressiveness), vehicle properties (e.g. engine power), and road conditions (e.g. freeways vs. streets).

M2: Free flow An object in free fall will eventually settle on an equilibrium speed due to air resistance, so does a vehicle in free flow. In this case, the 'resistance" comes from the driver's willingness to comply with traffic rules (e.g. speed limits) as oppose to rolling, grade, and air resistances. Unlike the free fall speed which is deterministic and replicable given the same condition, the free speed of a vehicle is, once again, a subjective matter because it is largely the driver's choice. Given the same condition, the choice may vary among different drivers or at different times. In addition, different roadways support different free speeds. To avoid confusion, the free speed chosen by a driver is called 'desired speed", whereas the free speed aggregated over a group of vehicles is called 'free-flow speed". Generally, the desired speed is

related to driver personalities and road conditions, while the free flow-speed is affected by road conditions and driver population.

M3: Stopping at a red light Much like a moving object being stopped by a wall, a vehicle decelerates to a stop in front of a red light. The "repelling force" in the latter case resides in the driver and, if he or she ignores the red light, the consequence is costly (e.g. an accident or a ticket). Unlike the moving object which always stops in the same fashion in repeated experiments, drivers are entitled to decelerate at his or her comfortable rate to a stop and, in some extreme cases, drivers may forget to stop.

M4: Road barriers Vehicles moving in the same direction on a roadway are separated by lane lines. To avoid colliding with vehicles in adjacent lanes, a driver must keep his or her lane as if he or she were guided by barriers at both edges of the lane. If, however, the driver departs from the current lane, he or she would perceive some stress which motivates him or her to steer back in lane as if a correction "force" from the barrier acts on the vehicle and pushes it toward the center of the lane. If the driver is blocked by a slow vehicle, the desire for mobility would motivate the driver to change lane as if he or she were energized or elevated above the barrier so he or she can cross it and land on the adjacent lane. Running off the road is discouraged, so barriers at road edges are typically higher than lane barriers. Encroaching into the opposite direction of travel is so dangerous that the barrier at the center line is very high, see an illustration of the barriers in Figure 2. It should be noted that these barriers are not real objects, but only imaginary in drivers' minds.

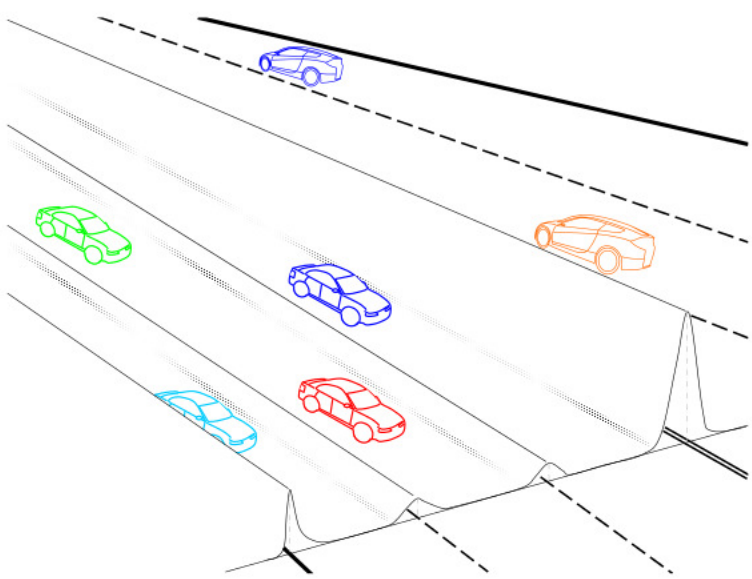

Figure 2: Road barriers

2.2 Electromagnetic Phenomena

An object can exert a force on another object in either of the following two ways: collision and action at a distance. For example, hitting a ball with a bat is the former and finding a needle using a magnet is the latter. Though collisions are not uncommon on highways, action at a distance is what vehicles normally interact with each other and examples of this kind include some of the above mechanics phenomena as well as the following:

E1: Car following Back to the example in the beginning of Section 2 where a fast vehicle catches up with a slow vehicle, the fast driver perceives an imminent collision if he or she keeps driving at that speed. The cost and fear of the collision motivates the fast driver to take actions in advance. If lane change is not an option and the slow driver does not speed up, the fast driver has to decelerate and eventually adopts the slow vehicle's speed separated by a safe following distance. This is analogous to moving a charge A toward a like charge B. According to Coulomb's law, the electric force between them is directly proportional to the product of their charges and inversely proportional to their distance squared. Similarly, in car following the "force" (stress) acting on the fast driver is larger if he or she approaches the slow vehicle faster and the distance between them is shorter. However, the same opposite force may or may not act on the slow driver as he or she may or may not notice the vehicle approaching from behind.

E2: Tailgating Continuing the above example and assuming that the fast vehicle tailgates (i.e. following at a dangerously short distance), it is likely that the opposite force is perceived by the slow driver who may respond by speeding up or giving his or her way to the fast follower. Back to the analogy, charge B is now driven (or driven away) by charge A and Newton's third law holds in this case. In general, a "force" must be perceived by a driver before the force takes effect on the person. In addition, a driver's ability to perceive depends on where he or she scans and how frequent this happens.

E3: Shying away If two vehicles happen to run in parallel, one or both drivers may feel intimidated. The fear of a side collision motivates them to spread out in space (longitudinally or laterally). Such a shying-away effect becomes more evident when one of the vehicles is a heavy truck.

2.3 Wave Phenomena

W1: Harmonic wave A platoon of vehicles on a roadway is like a harmonic wave. The platoon is characterized by flow (in vehicles per hour), traffic speed (in km per hour), and density (in vehicles per km), while the wave is determined by frequency (in Hz or cycles per second), wave speed (in meters per second), and wave length (in meters). One immediately recognizes that flow is equivalent to frequency, traffic speed is equivalent to wave speed, and the

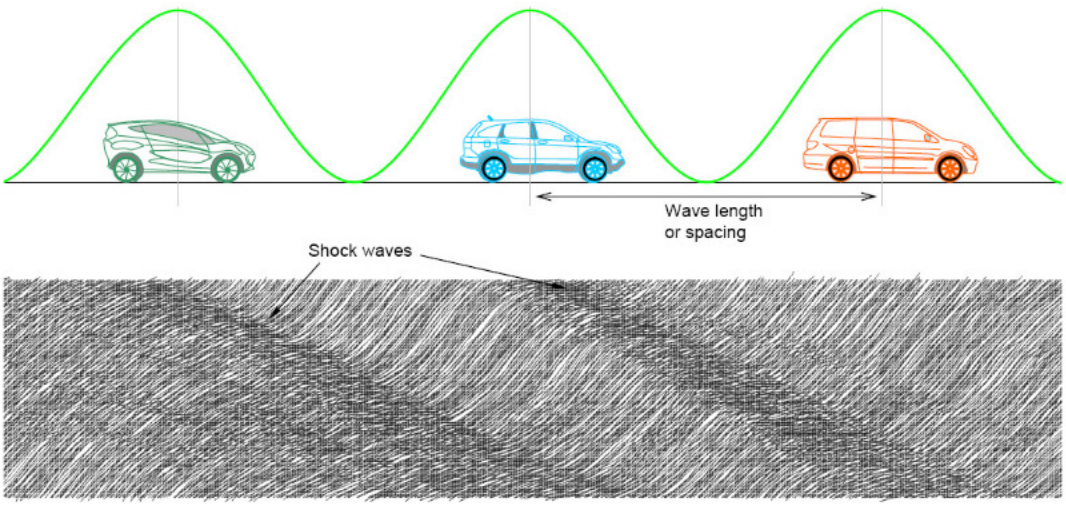

Figure 3: Traffic and waves

spacing (the reciprocal of density) is equivalent to wave length. The upper part of Figure 3 shows a platoon of vehicles as a harmonic wave.

W2: Signal propagation The signal here does not mean a traffic signal, rather it refers to any quantity that clearly defines the location and speed of a perturbation in a medium. When the leading vehicle of a compact platoon brakes briefly, a kinematic wave forms and propagates against the platoon, where the signal here is the brief speed reduction. When a platoon of fast vehicles catch up with a platoon of slow vehicles, a shock wave is generated and propagates against the traffic, where the signal here is the interface between fast and slow vehicles. The bottom part of Figure 3 illustrates a few shock waves observed in vehicle trajectories.

W3: Wave-particle duality All matter, particularly small-scale objects, exhibits both wave-like and particle-like properties. The latter is prominent when individual particles are concerned (e.g. the photoelectric effect), while the former becomes significant when the behavior of many particles is viewed collectively (e.g. diffraction of waves). In traffic flow, individual vehicles act like particles (e.g. car following and lane changing), while a platoon or platoons of vehicles exhibit wave property (e.g. kinematic waves and shock waves).

2.4 Statistical Mechanics Phenomena

Traffic flow has been modeled by many as a one-dimensional compressible fluid, such as a gas. In gases, the speeds of gas molecules follow Maxwell-Boltzmann distribution, see an illustration in Part 1 of Figure 4. Remarkable in the distribution is the increase in average speed and speed variance as temperature becomes higher. In contrast, traffic flow exhibits a different trend. Empirical

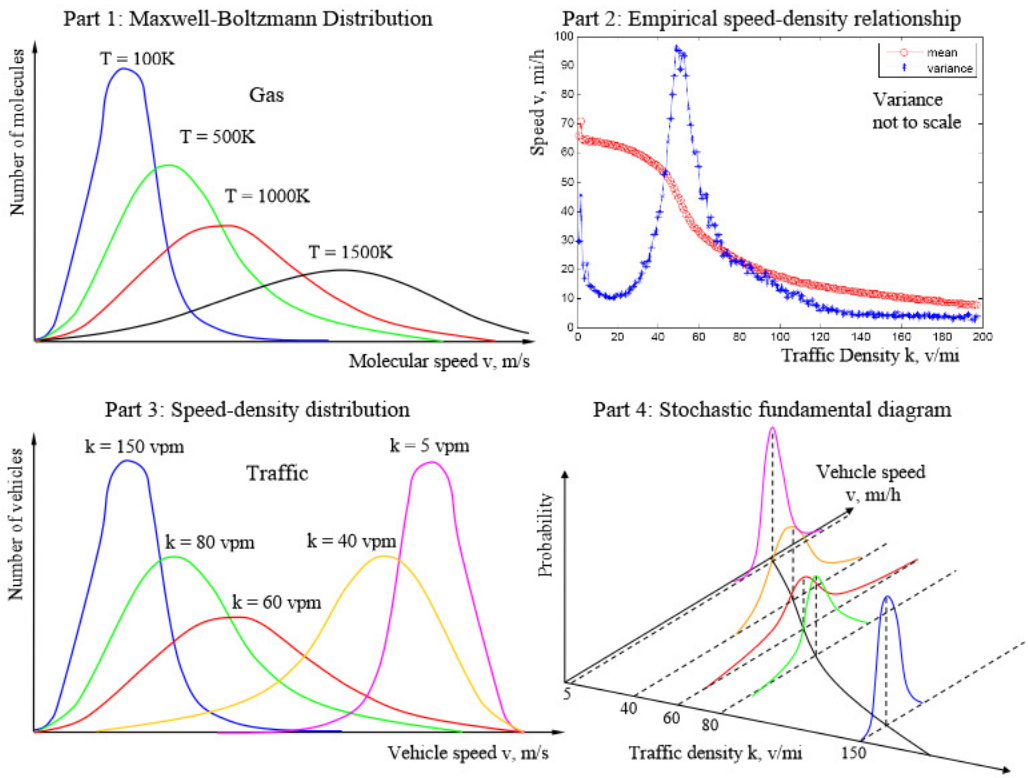

Figure 4: Maxwell-Boltzmann distribution in traffic flow

observations in Part 2 of the figure show that the variance of traffic speed peaks around optimal density (where capacity flow occurs) and drops at both ends. Such a distribution is illustrated in Part 3 of the figure in contrast to Maxwell-Boltzmann distribution and further elaborated in a three-dimensional model in Part 4 which forms the basis of a stochastic fundamental diagram.

3 The Field Theory

Lewin (1) defined the 'field at a given time" in psychology and Gibson (2) further identified the 'field" in automobile driving. However, both offered their discoveries qualitatively without mathematical rigor. Hence, the objective of this section is to formulate the Field Theory of traffic flow. Since the transportation system involves both living entities (e.g. human drivers) and non-lining objects (e.g. roadways and vehicles), it is subject to both physical laws and social rules. As such, the Field Theory is formulated progressively based on a set of postulates, two of which (1 and 3) are physical and another two (2 and 4) social.

Postulate 1: A road is a physical field

Postulate 1 is motivated by phenomena M1, M2, and M4 in Section 2. In the longitudinal (x) direction, a driver-vehicle unit i is subject to a gravity along the road:

$$G_i = m_i g_i \tag{1}$$

where G_i is the roadway gravity acting on the unit, m_i is the mass of the unit, and g_i is the acceleration of roadway gravity perceived by driver i. As discussed in M1, g_i is a function of driver personalities, vehicle properties, and road conditions. Meanwhile, the unit is also subject to a resistance R_i perceived by the driver due to his or her willingness to observe traffic rules (e.g. speed limits). As discussed in M2, R_i is related to the driver's perceived difference between his or her actual speed $\dot{x}_i$, and desired speed v_i which in term is related to the free-flow speed of the road v_f (subscript f here means free flow, not a vehicle ID), i.e. $R_i = R_i(\dot{x}_i, v_i(v_f))$. Therefore the net force acting on unit i in the longitudinal direction can be expressed as:

$$m_i \ddot{x}_i = G_i - R_i \tag{2}$$

where $\ddot{x}_i$ is the acceleration of unit i. Since the right-hand side represents the amount of net force that can be used to accelerate the unit, it can be interpreted as the driver's unsatisfied desire for mobility. As the unit speeds up, the right-hand side decreases because R_i increases. Eventually, the right-hand side varnishes, at which time the unit reaches its desired speed v_i. If somehow a random disturbance brings the unit's speed above v_i, the right-hand side becomes negative. In this case, the unit decelerates and finally settles back to v_i.

In the lateral direction of the road, there are lane lines, road edges, and center lines to guide and separate traffic. As discussed in M4 of Section 2, these cross-section elements of the road can be mapped into a roadway potential field U_i^R perceived by the driver. When the unit deviates from its lane, the unit is subject to a correction force N_i which can be interpreted as the stress on the driver to keep his or her lane, see illustrations in Figures 2 and 6. The effect of such a force is to push the unit back to the center of the current lane. Based on physical principles, the force can be determined as the derivative of the roadway field, U_i^R, with respect to the unit's lateral displacement y_i:

$$N_i = -\frac{\partial U_i^R}{\partial y_i} \tag{3}$$

Postulate 2: A driver responds to his surrounding anisotropically

The interaction between two driver-vehicle units differs from the collision of two objects in two ways: one pertains to Newton's third law of motion which is discussed below and the other concerns non-contact forces which is the subject of the next postulate.

In classical physics, Newton's third law of motion holds when two objects collide with each other. However, the law generally does not hold in the interaction between two driver-vehicle units. For example, when a fast vehicle catches up with a slow vehicle, the fast driver perceives a "repelling force" (i.e. stress) as the gap closes up. The smaller the gap, the greater the force. Conversely, the reaction force may or may not be perceived by the slow driver depending on whether he or she notices the approaching fast vehicle and his or her willingness to respond. Since drivers all sit facing front, it is the driver behind who is responsible for watching safety distances and held liable for a rear-end collision should it happen. Therefore, it is not uncommon that a leading driver does not respond to situations happening behind such as an approaching fast vehicle.

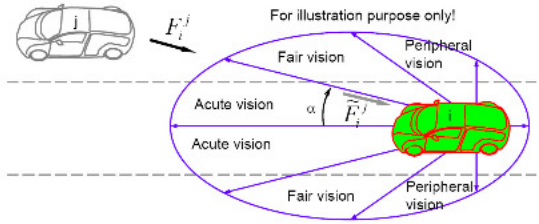

Figure 5: Distribution of driver's attention

In general, a driver's responsiveness to his or her surrounding vary with his or her viewing angle and scanning frequency, see Figure 5. For example, the area right in front of the driver, especially in the same lane, falls into the driver's acute vision zone. The driver is responsible for watching this area constantly and responding to a situation promptly. Roughly in the driver's fair vision zones, the frontal areas in side lanes receive considerable amount of the driver's attention since vehicles in the side lanes may change to the subject lane and the driver needs to watch these area when making a lane change. In comparison, the driver scans less frequently at both sides of his or her vehicle (roughly the driver's peripheral vision zones) unless the driver needs to make a lane change or avoid parallel running. The last and least attended area is the rear of the vehicle not only because it is difficult to access (indirectly by means of side or rear mirrors) but also because the responsibility rests on drivers behind. Therefore, it is reasonable to assume that the driver's directional response to his or her surrounding, ζ_i, is a function of his or her

viewing angle α_i, i.e.$\zeta_i = \zeta(\alpha_i)$. Consequently, the force that actually acts on the unit, $\tilde{F}_i$, is the product of the force that might have been perceived by the driver if he or she had paid full attention, F_i, and his or her directional response ζ_i, i.e.,

$$\tilde{F}_i = F_i \times \zeta(\alpha_i) \tag{4}$$

where $\alpha_i \in [-\pi, \pi]$ is the viewing angle. For example, if one chooses $\zeta(0) = 1$ and $\zeta(\pi) = 0$, the driver responds to F_i in full when it comes from the vehicle in front (i.e., $\alpha_i = 0$) and ignores F_i when it comes from the vehicle behind (i.e., $\alpha_i = \pi$ or $-\pi$), respectively.

Postulate 3: A driver interacts with others by action at a distance

As described in E1, E2, and E3 in Section 2, a driver is able to sense the presence of other vehicles and obstacles in its vicinity and take preventive actions to avoid a collision. It is postulated that such an action at a distance is mediated by a field which is perceived by the driver as the hazard of collision. One may imagine the field as a hill; the higher and steeper the hill is, the more difficult it is to climb. The base of the hill/field delineates a region, outside of which the driver is not influenced by the field. For example, the dash-dotted oval (labeled as 'Base j") in Figure 6 represents the base of the field perceived by driver i due to unit j. One may also interpret the field as the personal space of unit j, into which intrusion is discouraged. The deeper unit i intrudes, the stronger repelling force it receives. The longitudinal section of the field is illustrated as the curve above the x axis.

Similarly, unit k represents another field (whose base is labeled as 'Base k") which also exerts an influence on unit i. Since unit k is at the side lane of unit i, the influence is not in longitudinal x direction but in lateral y direction, i.e., the field results in a repelling force, F_i^k, on unit i which motivates it to shy away from unit k.

The above fields and, consequently, forces are related to driver personalities and vehicle dynamics. For example, since an aggressive driver accepts shorter car-following distances, the field perceived by such a driver covers a smaller base. On the other hand, the faster a unit moves, the more hazard it imposes on neighboring vehicles, and thus the larger and steeper the field it creates.

Postulate 4: A driver tries to achieve gains and avoid losses

A driver's strategy of moving on roadways is to achieve mobility and safety (gains) while avoiding collisions and violation of traffic rules (losses). Such a strategy can be represented using an overall potential field U_i which consists

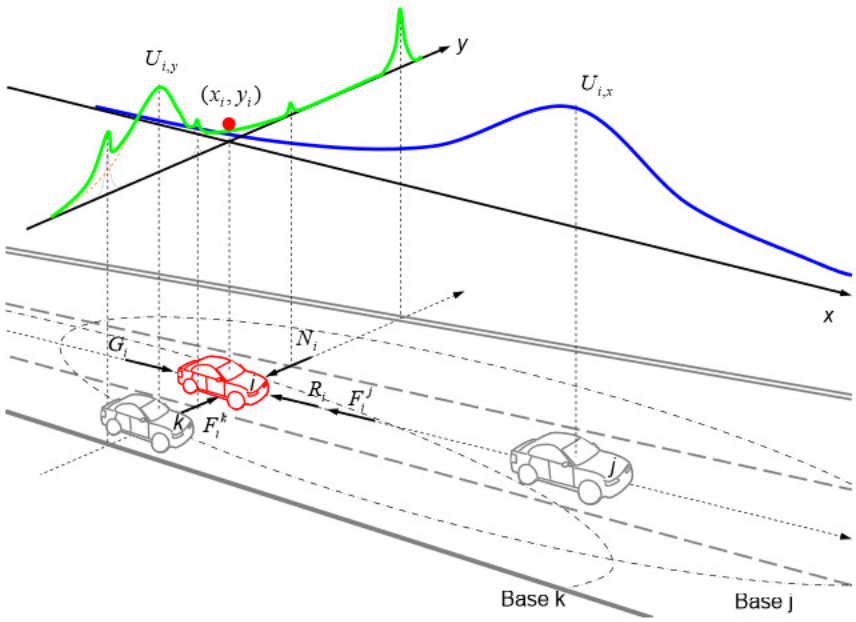

Figure 6: The illustration of a field

of component fields such as those due to moving units U_i^B, roadways U_i^R, and traffic control devices U_i^C, i.e.,

$$U_i = U(U_i^B, U_i^R, U_i^C) \tag{5}$$

If U_i is viewed as a mountain range whose elevation denotes the risk of losses, the driver's strategy is to navigate through the mountain range along its valley, i.e. the least stressful route. For example, Figure 6 illustrates two sections of such a field. Perceived by driver i, the longitudinal x section of the field, $U_{i,x}$, is dominated by unit j since it is the only neighboring vehicle in the center lane. Unit i is represented as a ball which rides on the tail of curve $U_{i,x}$ since the vehicle is within unit j's field. Therefore, unit i is subject to a repelling force F_i^j which is derived from $U_{i,x}$ as:

$$F_i^j = -\frac{\partial U_{i,x}}{\partial x} \tag{6}$$

The effect of F_i^j is to push unit i back to keep safe distance. By incorporating the driver's unsatisfied desire for mobility $(G_i - R_i)$, the net force in the x direction can be determined as:

$$m_i \ddot{x}_i = \quad F_{i,x} = G_i - R_i - F_i^j = (m_i g_i - R_i) + \frac{\partial U_{i,x}}{\partial x} \tag{7}$$

The section of U_i in the lateral y direction, $U_{i,y}$ (the bold curve), is the union of two components: the cross section of the field due to unit k (the

dashed curve) and that due to the roadway field (the dotted curve). The former results in a repelling force F_i^k which makes unit i to shy away from unit k and the latter generates a correction force N_i should unit i depart its lane center. Therefore, the net effect can be expressed as:

$$m_i \ddot{y}_i = \quad F_{i,y} = F_i^k - N_i = -\frac{\partial U_{i,y}}{\partial y} \tag{8}$$

By incorporating time t, driver i's perception-reaction time τ_i, and driver i's directional response ζ_i, Equations 7 and 8 can be expressed as:

$$m_i \ddot{x}_i(t+\tau_i) = \quad \tilde{F}_{i,x}(t) = \zeta_i^0[G_i(t) - R_i(t)] + \zeta(\alpha_i^j)\frac{\partial U_{i,x}}{\partial x} \tag{9a}$$

$$m_i \ddot{y}_i(t+\tau_i) = \quad \tilde{F}_{i,y}(t) = -\zeta(\alpha_i^k)\frac{\partial U_{i,y}}{\partial y} \tag{9b}$$

where $\zeta_i^0 \in [0,1]$ represents the unit's attention to its unsatisfied desire for mobility (typically $\zeta_i^0 = 1$), α_i^j and α_i^k are viewing angles which are also functions of time. The above system of equations summarizes the Field Theory in generic terms and constitutes the basic law that governs a unit's motion on a planar surface.

4 A Special Case

Though the generic form of the Field Theory is able to explain some traffic phenomena qualitatively, rigorous modeling of traffic flow requires a specific form which is the focus of this section. In the generic theory, the functional forms of the field U_i, roadway gravity G_i, and resistance R_i are undetermined. It appears that the generic theory can take many specific forms and it is impractical to enumerate all of them. In choosing a specific form, Occam's razor appears to be a good rule to follow which basically says that 'entities should not be multiplied unnecessarily." Hence, the razor gives rise to the following considerations: (1) the chosen specific form should make physical sense, to which empirical observations are a good test (this is responded in Part III of the paper), (2) it should take a simple functional form (responded in Equation 14) that involves physically meaningful parameters but not calibration coefficients (responded in Section 5), and (3) it should provide a sound microscopic basis for aggregated behavior, i.e. macroscopic equilibrium models (responded in Section 5 and Part III). With the above considerations, a specific form of the theory is formulated by focusing on longitudinal forces which receive full attention while lateral forces is only considered during a lane change.

4.1 Motion in the Longitudinal Direction

Major forces to be considered are unsatisfied desire for mobility due to roadway gravity and resistance and action at a distance due to the leading vehicle.

4.1.1 Unsatisfied desire for mobility

The term $(G_i - R_i)$ explains a driver-vehicle unit's unsatisfied desire for mobility. Intuitively, when a unit starts from stand-still, i.e., $\dot{x}_i = 0$, its unsatisfied desire for mobility is the greatest. As the unit speeds up, $(G_i - R_i)$ decreases accordingly but is still positive, i.e., it still accelerates the unit to higher speeds. When the unit achieves its desired speed, i.e., $\dot{x}_i = v_i$, its desire for mobility has been fully satisfied and, hence, $G_i - R_i = 0$ which means that the unit settles at v_i if no other forces act on it. If a random perturbation brings $\dot{x}_i$ over v_i, the unit's desire for mobility is over-satisfied and $G_i - R_i$ becomes negative which decelerates the unit back to v_i. With the above understanding, a specific form of the unsatisfied desire for mobility can be formulated as:

$$G_i - R_i = m_i g_i [1 - (\frac{\dot{x}_i}{v_i})^{\delta}] \tag{10}$$

where δ is a calibration parameter.

4.1.2 Action at a distance

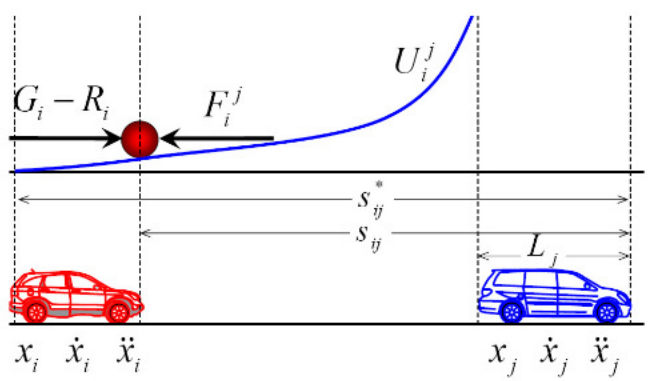

Figure 7: Action at a distance

When a fast unit i (with displacement x_i, speed $\dot{x}_i$, and acceleration $\ddot{x}_i$) catches up a slow unit j (with x_j, $\dot{x}_j$, and $\ddot{x}_j$), the former is subject to a non-contact force, F_i^j, from the latter. Such a non-contact force varies with the spacing between the two units, $s_{ij} = x_j - x_i$. For example, the force virtually has no effect on unit i when it is distant; it takes effect when unit i becomes close (e.g. within the range of its desired spacing s_{ij}^*); it increases as

the spacing becomes even shorter ($s_{ij} \downarrow$); it reaches maximum when $s_{ij} \rightarrow l_j$ where l_j represents the minimum 'safety room" required by unit j. In addition, the effect of the force is also related to the speeds and relative speed of units i and j. Such an effect can be incorporated into the formulation of driver i's desired spacing s^*_{ij}.

Therefore, a simple way to represent the force is to use an exponential function. The general idea of this model is to set the desired spacing s^*_{ij} as a base line, beyond which the intrusion by unit i is translated exponentially to the repelling force acting on the unit. As such, a more specific but still quite generic form of the force can be:

$$F^j_i = f\,(e^{s^*_{ij} - s_{ij}}) \tag{11}$$

where $s^*_{ij} - s_{ij}$ represents how far unit i intrudes into s^*_{ij}.

4.1.3 The Longitudinal Control Model

Combining the above, the effort that is required by driver i to control his or her vehicle in the longitudinal direction can be expressed as:

$$m_i \ddot{x}_i = m_i g_i [1 - (\frac{\dot{x}_i}{v_i})^\delta - f\,(e^{s^*_{ij} - s_{ij}})] \tag{12}$$

if the coefficient of term F^j_i is chosen properly. Though Equation (12) can be further detailed in many possible ways, the following special case is of particular interest. Putting time t and response delay τ back on and eliminating vehicle mass m at both sides, Equation (12) can take the following special form:

$$\ddot{x}_i(t + \tau_i) = g_i [1 - (\frac{\dot{x}_i(t)}{v_i})^\delta - e^{\frac{s_{ij}(t)^* - s_{ij}(t)}{Z}}] \tag{13}$$

If one chooses $\delta = 1$ and $Z = s_{ij}(t)^*$, the above equation reduces to a more specific form:

$$\ddot{x}_i(t + \tau_i) = g_i [1 - (\frac{\dot{x}_i(t)}{v_i}) - e^{\frac{s_{ij}(t)^* - s_{ij}(t)}{s_{ij}(t)^*}}] \tag{14}$$

No further motivation for this special case is provided other than the following claims: (1) it takes a simple functional form that involves physically meaningful parameters but not arbitrary coefficients (see Section 5), (2) it makes physical and empirical sense (see Part III), and (3) it provides a sound microscopic basis to macroscopic behavior (see Section 5 and Parts II and III).

The determination of desired spacing $s_{ij}(t)^*$ admits safety rules. An example is that one should leave a time gap of at least one perception-reaction time in front to ensure safety:

$$s^*_{ij}(t) = \dot{x}_i(t)\tau_i + l_j \tag{15}$$

Another example can be the following: the desired spacing should allow vehicle i to stop behind its leading vehicle j after a perception-reaction time τ_i and a deceleration process (at rate $b_i > 0$) should vehicle j applies an emergency brake (at rate $B_j > 0$). After some math, the desired spacing can be determined as follows:

$$s^*_{ij}(t) = \frac{\dot{x}_i^2(t)}{2b_i} + \dot{x}_i\tau_i - \frac{\dot{x}_j^2(t)}{2B_j} + l_j \tag{16}$$

4.2 Motion in the Lateral Direction

A driver-vehicle unit's motion in the lateral y direction involves decision at two two levels: lane change and gap acceptance. A lane change decision concerns the driver's desire to change to an adjacent lane to better achieve his or her goals such as mobility and safety. A gap acceptance decision addresses the execution of the lane change decision by physically moving the vehicle into the target lane when an opportunity comes up (e.g. a safe gap is available in the target lane). Figure 8 illustrates a scenario where units i and j are in the right lane and a trailing neighbor p in the left lane. The top part of the figure positions unit i (the ball) in its perceived potential fields: U_i^j due to unit j in the same lane, U_i^p due to unit p in the left lane, and U_i^R due to road barrier (lane line). Driven by its desire for mobility, unit i climbs up onto U_i^j, during which unit i has to adapt to unit j's speed while achieving a balance between $(G_i - R_i)$ and F_i^j. Under this circumstance, unit i reaches its decision on a lane change in order to satisfy its desire for mobility. With this decision, driver i begins to seek opportunities in adjacent lanes. In this example, the right side is obviously not an option since it is prohibitive to move off-road. Hopefully, an opportunity exists in the left lane because the elevation of unit i (where the ball rides) is higher than both lane barrier U_i^R and the front of field U_i^p. Therefore, unit i initiates a smooth transition by laterally rolling off the tail of U_i^j, crossing over U_i^R, and landing onto the front of U_i^p, the effect of which is shown in the middle part of Figure 8.

The above analysis can be simplified by reducing a smooth field to a binary "personal space". Therefore, a discrete version of lane change is resulted, see the bottom part of Figure 8.

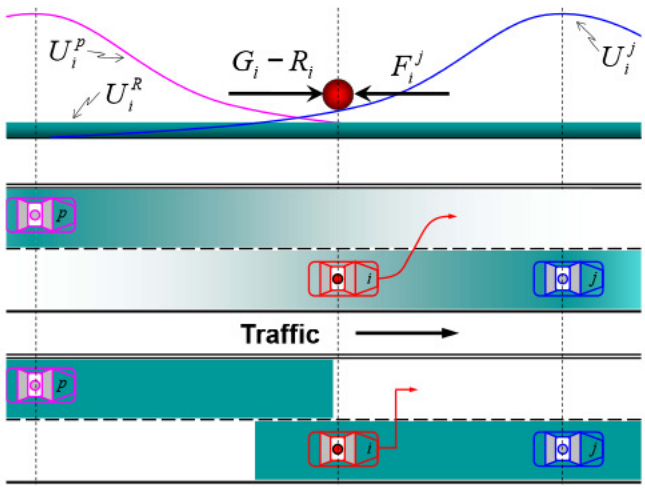

Figure 8: Lane change

5 Properties of the Theory

5.1 Steady-State Behavior under Equilibrium

If a system is in steady state, any property of the system is unchanging in time. More specifically, a traffic system in steady state would consists of homogeneous vehicles which exhibit unform behavior over time and space. Therefore, under steady-state condition, vehicles lose their identities (e.g. $\tau_i \rightarrow \tau$, $b_i \rightarrow b$, $B_j \rightarrow B$), vehicles travel at uniform speed (i.e. $\dot{x}_i = \dot{x}_j = v$ and $\ddot{x}_i = 0$), and drivers' desired speeds converge to the free-flow speed of the road (i.e. $v_i \rightarrow v_f$). As such, the special case in Equation (14) can be aggregated to the following equilibrium equation:

$$v = v_f\,[1 - e^{1-\frac{k^*}{k}}] \tag{17}$$

where $s_{ij} \rightarrow s = \frac{1}{k}$, $s^*_{ij} \rightarrow s^* = \frac{1}{k^*}$, $s^* = \frac{v^2}{2b} + v\tau - \frac{v^2}{2B} + l$ if Eq.16 is adopted, $l = \frac{1}{k_j}$ average minimum spacing, and k_j jam density. If one further ignores the difference in braking (i.e. $b = B$) and considers the effect that drivers are more vigilant at higher speeds (e.g., $\tau = \tau e^{-v/v_f}$), a more specific form of Equation (17) is resulted:

$$v = v_f\,[1 - e^{1-\frac{1}{k(v\tau + l)}}] \tag{18}$$

While an explicit speed-density function is unavailable, an explicit density-speed relation does exist. Solving Equation (18) for k yields:

$$k = \frac{1}{(v\tau + l)[1 - \ln(1 - \frac{v}{v_f})]} \tag{19}$$

Figure 9 illustrates the fundamental diagram resulted from the above equilibrium model. Four plots are generated: speed-density (v-k), speed-flow (v-q), flow-density (q-k), and speed-spacing (v-s). The slope of the v-s curve is found by evaluating $\frac{dv}{ds}$ at point $(s = l, v = 0)$ which yelds $\frac{dv}{ds} = \frac{1}{\tau + \frac{L}{v_f}}$. Capacity flow (i.e. $q = q_m$ achieved when $k = k_m$ and $v = v_m$) can be determined by setting derivative of $q = kv$ with respect to either k or v to zero. Unfortunately, the solution is difficult to find analytically, but hopefully solvable numerically. The plots in Figure 9 resemble closely to those curves typically observed in empirical data (see empirical results in Part III). In particular, both q-k and v-q curves are concave, which are desirable.

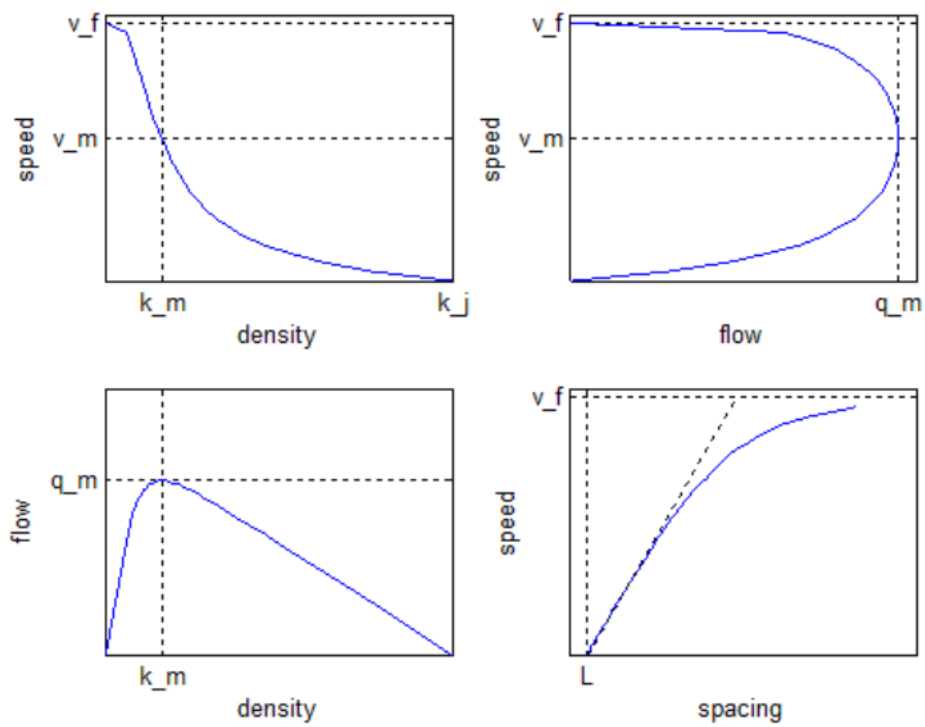

Figure 9: The fundamental diagram

5.2 Connection to Existing Knowledge Base

The Longitudinal Control Model suggests a potential function very similar to Lennard-Jones potential which plays an important role in granular flow and molecular dynamics:

$$U(r) = 4\ [(\frac{\sigma}{r})^{12} - (\frac{\sigma}{r})^{6}] \tag{20}$$

where U is Lennard-Jones potential due to particle interaction, r is the distance between two particles, is the depth of the potential well, and σ is the distance at which the inter-particle potential is zero. The equation is actually a union of two terms: a long-range attraction term $(\frac{\sigma}{r})^6$ (similar to driver's desire for mobility) and a short-range repulsion term $(\frac{\sigma}{r})^{12}$ (similar to inter-vehicle reaction).

Meanwhile, Equation (9) is a special form of Newton's second law of motion if one ignores driver's perception-reaction time τ and directional response ζ. In addition, action at a distance as a means of interaction between drivers becomes a hard collision if driver's need for safety disappears (i.e. the potential field as a function of spacing $U(s_{ij})$ becomes a spike). Moreover, Newton's third law of motion holds if drivers respond to their surroundings isotropically. Furthermore, isotropic response, together with hard collision, gives rise to laws of momentum and energy conservation. Therefore, the Field Theory represents a special form of classical physics involving human drivers. With its interpretation of mean free path (i.e. desired car-following distance s^*_{ij}) and molecular collision (i.e. action at a distance between vehicles), the Field Theory allows the application of other physical principles (such as kinetic theory) to further understand transportation systems.

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References

[1] Kurt Lewin. Defining the 'Field at a Given Time". Psychological Review, 50(3):292–310, 1943.

[2] James J. Gibson and Laurence E. Crooks. A Theoretical Field-Analysis of Automobile-Driving. The American Journal of Psychology, 51(3):453–471, 1938.

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