Auctioning Many Similar Items

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Examples of auctioning similar items
- Treasury bills
- Stock repurchases and IPOs
- Telecommunications spectrum
- Electric power
- Emissions permits

Ways to auction many similar items
- Sealed-bid: bidders submit demand schedules
  - Pay-as-bid auction (traditional Treasury practice)
  - Uniform-price auction (Milton Friedman 1959)
  - Vickrey auction (William Vickrey 1961)

Pay-as-bid Auction:
All bids above $P_0$ win and pay bid

Uniform-Price Auction:
All bids above $P_0$ win and pay $P_0$

Vickrey Auction:
All bids above $P_0$ win and pay opportunity cost
Payment rule affects behavior

More ways to auction many similar items

• Ascending-bid: Clock indicates price; bidders submit quantity demanded at each price until no excess demand
  – Standard ascending-bid
  – Ausubel ascending-bid (Ausubel 1997)

Standard Ascending-Bid Auction:
All bids at $P_0$ win and pay $P_0$

Ausubel Ascending-Bid:
All bids at $P_0$ win and pay price at which clinched

More ways to auction many similar items

• Ascending-bid
  – Simultaneous ascending auction (FCC spectrum)

• Sequential
  – Sequence of English auctions (auction house)
  – Sequence of Dutch auctions (fish, flowers)

• Optimal auction
  – Maskin & Riley 1989

Research Program
How do standard auctions compare?

• Efficiency
  – FCC: those with highest values win

• Revenue maximization
  – Treasury: sell debt at least cost
Efficiency
(not pure common value; capacities differ)

• Uniform-price and standard ascending-bid
  – Inefficient due to demand reduction
• Pay-as-bid
  – Inefficient due to different shading
• Vickrey
  – Efficient in private value setting
  – Strategically simple: dominant strategy to bid true demand
  – Inefficient with affiliated information
• Ausubel ascending-bid
  – Same as Vickrey with private values
  – Efficient with affiliated information

Inefficiency Theorem
In any equilibrium of uniform-price auction,
with positive probability objects are won by
bidders other than those with highest values.

• Winning bidder influences price with positive probability
• Creates incentive to shade bid
• Incentive to shade increases with additional units
• Differential shading implies inefficiency

Inefficiency from differential shading

Vickrey inefficient with affiliation

• Winner’s Curse in single-item auctions
  – Winning is bad news about value
• Winner’s Curse in multi-unit auctions
  – Winning more is worse news about value
  – Must bid less for larger quantity
  – Differential shading creates inefficiency in Vickrey

What about seller revenues?

Uniform price may perform poorly

• Independent private values uniform on [0,1]
• 2 bidders, 2 units; L wants 2; S wants 1
• Uniform-price: unique equilibrium
  – S bids value
  – L bids value for first and 0 for second
  – Zero revenue; poor efficiency
• Vickrey
  – price = v(2) on one unit, zero on other
Standard ascending-bid may be worse

- 2 bidders, 2 units; L wants 2; S wants 2
- Uniform-price: two equilibria
  - Poor equilibrium: both L and S bid value for 1
    - Zero revenue; poor efficiency
  - Good equilibrium: both L and S bid value for 2
    - Get \( v_1 \) for each (max revenue) and efficient
- Standard ascending-bid: unique equilibrium
  - Both L and S bid value for 1
    - S’s demand reduction forces L to reduce demand
    - Zero revenue; poor efficiency

Efficient auctions tend to yield high revenues

**Theorem.** With flat demands drawn independently from the same regular distribution, seller’s revenue is maximized by awarding good to those with highest values.

Generalizes to non-private-value model with independent signals:

\[ v_i = u(s_i, s_{-i}) \]

Award good to those with highest signals if downward sloping MR and symmetry.

Downward-sloping demand:

\[ p_i(q_i) = v_i - g_i(q_i) \]

**Theorem.** If intercept drawn independently from the same distribution, seller’s revenue is maximized by

- awarding good to those with highest values if constant hazard rate
- shifting quantity toward high demanders if increasing hazard rate

Note: uniform-price shifts quantity toward low demanders

But uniform price has advantages

- Participation
  - Encourages participation by small bidders (since quantity is shifted toward them)
  - May stimulate competition
- Post-bid competition
  - More diverse set of winners may stimulate competition in post-auction market

Auctioning Securities

* A pure common-value model with affiliation

- \( n \) risk-neutral symmetric bidders
- Each bidder has pure common value \( V \) for security and can purchase any quantity (flat demand curve w/o capacity)

Models

- Common uncertainty
  - Bidders have no private information
- Affiliated private signals
  - Bidder \( i \) gets signal \( S_i \)
  - Random variables \( V, S_1, \ldots, S_n \) are affiliated
Results: Common Uncertainty

**Proposition.** (Wilson ‘79; Maxwell ‘83; Back & Zender ‘93)
- Wide range of prices can be supported as equilibrium in uniform-price auction, even if supply is stochastic; highest yields EV

**Proposition.** (Wang & Zender ‘96)
- Many equilibria in pay-as-bid auction, even if supply is stochastic; highest yields EV
- Indeterminacy avoided if set reserve price (even 0)

Results: Affiliated Private Signals

- With affiliated signals, each auction format has a “simple equilibrium” where bidders submit flat demand curves
- Conjecture: These simple equilibria provide upper bounds on revenues from each format
- Std. ascending-bid > Uniform > Pay-as-bid

Results: Affiliated Private Signals

Vickrey and Ausubel ascending-bid eliminate bottom end of revenue indeterminacy:

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Conclusion

- Efficient auctions should be favored
- Treasury should try Ausubel ascending-bid
- IPOs should be auctioned