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ABSTRACT

If temperature and salinity are mixed at different rates, the mixing efficiencies in flows with the same stratification and forcing can vary if the contributions of temperature and salinity to the density differ. Two models are used to examine the effect of differential transport of salt and heat on the mixing efficiency. The first model assumes constant eddy diffusivities for heat and salt and examines the effect of the density ratio $R_\rho = \alpha\Delta T/\beta\Delta S$ and the diffusivity ratio $d = K_S/K_T$ on the mixing efficiency. The model predicts that the effect of differential transport can be as large as that due to stratification and the type of process generating the turbulence. The second model incorporates the effect of stratification on the mixing by using results from laboratory experiments on entrainment across a sharp density interface. The model predicts that the mixing efficiency depends on the density ratio and a Richardson number Ri_0 based on the density jump and velocity and length scales of the turbulence near the interface. Because the laboratory measurements show that salt and heat are entrained at equal rates for Richardson numbers less than a transition value Ri_c, the mixing efficiency initially increases with increasing Ri_0 for all density ratios. However, for $\mathrm{Ri}_0 > \mathrm{Ri}_c$, the efficiency decreases (past a peak at Ri_c) for low density ratio and increases monotonically for high density ratio. These results suggest that the generation of fine structure in diffusively stable regions of the ocean can depend on the density ratio.

1. Introduction

The mixing efficiency, or the ratio of the rates of potential energy change and work input, is an important quantity for assessing the ability of a turbulent flow to mix a stratified fluid. Oceanographers and limnologists use a mixing efficiency based on rates of turbulent potential energy change and work input to estimate vertical mixing in lakes and the ocean (e.g., Osborn 1980; Ivey and Imberger 1991). Engineers use an efficiency based on the bulk change of potential energy and work done to design destratification schemes for lakes and reservoirs. Many oceanographic studies of vertical mixing assume a constant efficiency in applying the Osborn (1980) model (e.g., Gregg 1987), even though the efficiency varies with the process generating the turbulence and the relative strengths of stratification and forcing (Linden 1979; Ivey and Imberger 1991; Yamazaki and Osborn 1993). Linden found that the process generating the turbulence can cause the mixing efficiency to vary by as much as a factor of 2 while the effect of stratification, measured by a Richardson or Froude number, can cause the efficiency to vary between 1% and 25%. In this paper, we examine another possible cause of variations in the mixing efficiency—differential transport of salt and heat.

The main idea can be described simply: If temperature and salinity are mixed at different rates, the mixing efficiencies in flows with the same stratification and forcing can vary if the contributions of temperature and salinity to the density differ. In this case, two key parameters are the density ratio $R_\rho = \alpha\Delta T/\beta\Delta S$, where ΔT and ΔS are the magnitudes of the temperature and salinity differences and α and β are the coefficients of thermal expansion and saline contraction, and the diffusivity ratio $d = K_S/K_T$, where K_S and K_T are the eddy diffusivities for salt and heat. For example, if the eddy diffusivity of temperature is greater than that of salt and the stratification is mostly determined by temperature (large density ratio), the fluid's potential energy will change more quickly than in cases with smaller density ratio.

Differential transport of heat and salt has been observed in the laboratory, in numerical computations, and

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in the field. Turner (1968) and Altman and Gargett (1987) found that the entrainment of heat across a density interface exceeded the entrainment of salt when the stratification was strong. Because the eddy diffusivity is proportional to the entrainment rate across the density interface (Linden 1979), Turner's measurements can be used to show that $0.3 < d < 1$ (e.g., Gargett and Holloway 1992; Gargett and Ferron 1996). Recent measurements in a system with stable, linear profiles of temperature and salinity yield a similar range of d (Jackson 2001). In a similar fashion, the numerical simulations of Merryfield et al. (1998) show greater transport of heat than salt for weak turbulence in a strongly stratified fluid. Recent field measurements by Nash and Moum (2002) suggest differential transport can also occur in moderately turbulent flows. The diffusivity ratio varied between 0.6 and 1.1, but, because of measurement uncertainty, Nash and Moum (2002) could not rule out $d \approx 1$.

Differential transport can affect predictions of the ocean circulation. Gargett and Holloway (1992) explored the effects of a variation in the diffusivity ratio from unity using the Geophysical Fluid Dynamics Laboratory (GFDL) circulation model. They noted that the model, which has assumed $d = 1$, produces unrealistically homogeneous deep-water properties. Gargett and Holloway (1992) found that slight differences in the eddy diffusivities can produce large differences in thermohaline circulation predicted by the GFDL circulation model. The magnitude and sense of the thermohaline circulation, water mass characteristics, and intermediate and deep-water stability distributions were extremely sensitive to the assumption of equal mixing rates for heat and salt. Gargett and Ferron (1996) studied the effects of differential diffusion with a diffusive box model and concluded that further study of differential diffusion is required to improve the predictions of ocean models.

We examine the kinematic effects of differential transport on the mixing efficiency with two simple models and discuss the implications for vertical mixing in the ocean. Section 2 describes an eddy diffusion model that simply takes K_T and K_S to be constant and analyzes the effect of R_ρ and d on the mixing efficiency; representative values of d are taken from previous laboratory and field measurements. The model described in section 3 accounts for the effect of stratification on the transport of heat and salt by using results from the grid-mixing entrainment experiments. Section 4 contains a summary and conclusions.

2. Uniform eddy diffusion

To estimate the kinematic effects of differential diffusion on mixing efficiency, we solve eddy diffusion equations for temperature and salinity and compare the rate of change of mean potential energy $d\text{PE}/dt$ to a constant rate of work dW/dt. The system is stratified

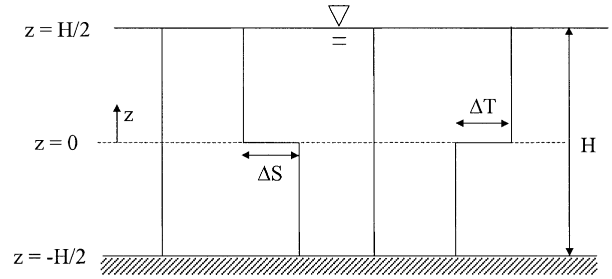

FIG. 1. Initial salinity and temperature profiles used in the models. The mean temperature and salinity are T_0 and S_0, respectively, and the temperature and salinity differences, ΔT and ΔS, are taken to be positive. The length, width, and depth of the fluid are L, B, and H, respectively.

with stable profiles of both salinity and temperature. We consider a fluid initially made up of two layers (Fig. 1), although the main result applies to all cases in which the ratio of the turbulent fluxes of αT and βS does not vary in space, or—for mixing described by eddy diffusion—cases in which $(T_0 - T)/\Delta T$ and $(S - S_0)/\Delta S$ are initially equal.

For simplicity, the diffusivity ratio is specified and the eddy diffusivities are assumed to be constant in time and space. Although this model allows the effects of density ratio and nonunity diffusivity ratio to be estimated, it ignores the dampening effect of stratification on the turbulence. As seen in previous experiments and simulations described in the introduction, this effect allows molecular diffusion to influence the mixing when the turbulence is weak. A model that accounts for the effect of stratification is described in the next section.

By solving the diffusion equations and computing the potential energy of the fluid, the mixing efficiency can be calculated as

$$R_f = \frac{d\text{PE}/dt|_{t=0}}{dW/dt} = \frac{\rho_0 LBHK_T N^2}{dW/dt}\left(\frac{d + R_\rho}{1 + R_\rho}\right), \quad (1)$$

where L, B, and H are defined in Fig. 1, the buoyancy frequency is $N = [g(\alpha\Delta T + \beta\Delta S)/H]^{1/2}$, with g being the acceleration of gravity, and the rate of potential energy change is computed at time $t = 0$ by following the method of Rottman and Britter (1986). We base the mixing efficiency at $d = 1$, which we call $R_{f,\text{eq}}$, on the eddy diffusivity for temperature because K_T is usually measured in oceanographic microstructure studies; this definition essentially holds K_T constant and allows K_S to vary. Thus, the normalized mixing efficiency is

$$\frac{R_f}{R_{f,\text{eq}}} = \frac{d + R_\rho}{1 + R_\rho}. \quad (2)$$

Equation (2) allows the consequences of ignoring differential diffusion to be estimated. The normalized mixing efficiency is shown in Fig. 2 as a function of the density ratio (and Turner angle) for several values of the diffusivity ratio. Density ratios between zero and infinity are considered, and values of d between 0.4 and

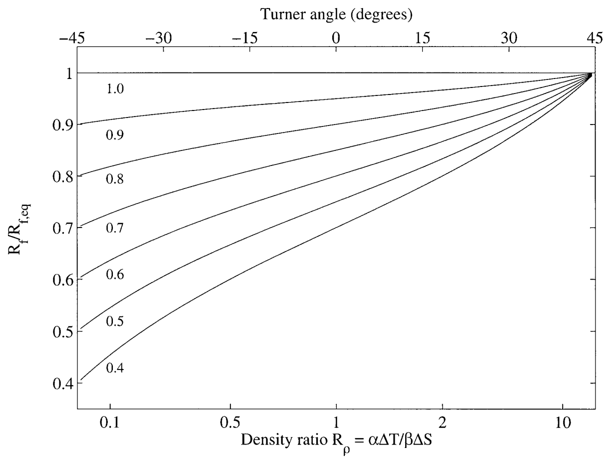

FIG. 2. Normalized mixing efficiency predicted by the uniform eddy diffusion model as a function of the initial density ratio for various values of the diffusivity ratio $d = K_S/K_T$. Values of d are shown below each curve. The baseline mixing efficiency $R_{f,\mathrm{eq}}$ is the mixing efficiency when heat and salt mix equally, or $d = 1$. The diffusivity ratio d was varied by holding K_T constant and allowing K_S to vary. The values of d and R_ρ reflect ranges from previous observations in diffusively stable flows. The curves are plotted as function of Turner angle Tu $=$ arctan$[(R_\rho - 1)/(R_\rho + 1)]$ to cover the range $0 < R_\rho < \infty$; the scale of the lower axis is therefore nonuniform.

1 are used to represent the ranges discussed in section 1. The effects of the density ratio and the diffusivity ratio predicted by the model agree with the intuition and results of previous work discussed in the introduction. When heat mixes faster than salt ($K_T > K_S$ or $d < 1$), the mixing efficiency increases with increasing R_ρ or Turner angle Tu because more of the stratification is due to temperature. Also, a larger deviation of the diffusivity ratio from unity leads to larger changes in the normalized mixing efficiency. Because of the choice of $R_{f,\mathrm{eq}}$, the normalized efficiency equals the diffusivity ratio when $R_\rho = 0$ (or Tu $= -45°$) and approaches unity at high density ratios. When the stratification is entirely due to salinity, the eddy diffusivity for temperature is irrelevant, and only the eddy diffusivity for salinity affects the efficiency, as Eq. (2) shows. When the stratification is mostly due to temperature, the effect of salt flux, or d, on the overall mixing becomes relatively small. In a fluid with no salinity stratification ($R_\rho \to \infty$ or Tu $= 45°$) the value of the eddy diffusivity of salt becomes irrelevant.

Despite the simplicity of the model, it shows that differential diffusion can change the mixing efficiency substantially. Microstructure methods used in ocean and lake mixing studies typically assume the efficiency to be a constant—either the maximum in the range Osborn (1980) suggested or some other value. Efficiencies based on turbulent fluxes (Ivey and Imberger 1991) and total changes (Linden 1979) vary between 0 and about 25%, depending on the Richardson or Froude number and the mechanism generating the turbulence. The eddy diffusion model predicts that, for the range of d chosen to represent experimental results, the mixing efficiency can vary by as much as a factor of 2.5. The largest effects occur at low density ratios: when the stratification is mostly due to temperature ($R_\rho > 2$), assuming that salt and heat mix at equal rates leads to a mixing efficiency within 25% of the true value. Thus, in some diffusively stable regions of the ocean, differential transport can cause as large a change in the mixing efficiency as other processes can.

3. Mixing with Richardson number dependence

The calculations discussed in this section will address some of the limitations of the eddy diffusion model in section 2. To represent the mixing more realistically, the model in this section incorporates the effect of stratification on the turbulence by using measurements from laboratory experiments. Using the laboratory data also improves upon another limitation of the model in section 2. Whereas the eddy diffusion model provides mixing efficiencies relative to the efficiency in the case with no differential transport, the analysis of this section allows the magnitude of the efficiency to be estimated.

Again we consider a two-layer fluid stably stratified with temperature and salinity (Fig. 1). The eddy diffusion model had uniform turbulence levels throughout the water column, whereas this model assumes the turbulence is generated both above and below the interface, and it diffuses toward the interface, as in the laboratory experiments of Altman and Gargett (1987) and Turner (1968) on entrainment across a density interface. As Turner (1968) discusses, such an experiment is used to study the deepening of a surface mixed layer bounded on the bottom by a thermocline. The advantage of using Turner's (1968) measurements, instead of those from another experiment, is that measurements for both temperature and salinity are available. As in the eddy diffusion model, we compute the mixing efficiency from Eq. (1) by estimating the rates of potential energy change and work input.

In an entrainment experiment with double-sided stirring, the temperature and salinity interfaces remain sharp and stationary while the temperature difference ΔT and salinity difference ΔS across the interface decrease in time. Applying conservation of heat content and salt mass to one of the layers produces relations between the scalar entrainment velocities u_e^T and u_e^S and the rate of change of the scalar differences across the interface (e.g., Turner 1968). For the case of equal layer depths,

$$\frac{d\Delta T}{dt} = -\frac{4}{H}\Delta T u_e^T \quad \text{and} \quad \frac{d\Delta S}{dt} = -\frac{4}{H}\Delta S u_e^S. \qquad (3)$$

With this description of the entrainment experiment, the rate of change of potential energy can be expressed as

$$\frac{d\mathrm{PE}}{dt} = \frac{1}{2}\rho_0 g L B H(\alpha \Delta T u_e^T + \beta \Delta S u_e^S). \qquad (4)$$

Table 1. Parameters for the analysis based on entrainment experiments. Physical parameters of the experiments were taken from Hopfinger and Toly (1976).

f (Hz)	2.0
C	0.25
S (cm)	4.0
M (cm)	5.0
g (cm s^{-2})	981.0
a	0.93
b	1.5
n	−0.95
m	−1.37
Ri_c	3.18
γ	0.24
C_D	1.0
H (cm)	50

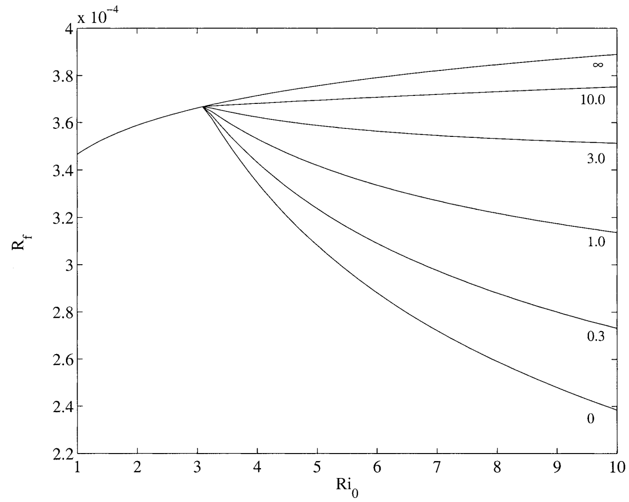

FIG. 3. Mixing efficiency computed from entrainment experiments as a function of the Richardson number for various values of the density ratio, shown below each curve. For $\mathrm{Ri} < \mathrm{Ri}_c$, heat and salt are entrained at equal rates.

The dependence of the entrainment velocities on the Richardson number is obtained from laboratory measurements. Turner (1979, p. 291) presented normalized entrainment velocities of heat and salt as a function of the Richardson number $\mathrm{Ri}_0 = g'\ell_1/u_1^2$, where g' is reduced gravity and ℓ_1 and u_1 are length and velocity scales of the turbulence near the interface. As discussed below, Ri_0 can be related to parameters used more commonly in oceanography, such as $\varepsilon/\nu N^2$, where ε is the rate of dissipation of turbulent kinetic energy and ν is the kinematic viscosity. For the flow in Fig. 1, $g' = g(\alpha\Delta T + \beta\Delta S)$. For Richardson numbers less than some transition value Ri_c, heat and salt are entrained at equal rates. For $\mathrm{Ri}_0 > \mathrm{Ri}_c$, the entrainment rate for heat is larger. The dependence of the entrainment rate on the Richardson number is usually expressed as a power law; the general forms of these entrainment relationships for Turner's experiment are

$$u_e^T/u_1 \cong a\mathrm{Ri}_0^n \quad \text{and} \tag{5}$$

$$u_e^S/u_1 \cong \begin{cases} a\mathrm{Ri}_0^n & \text{for } \mathrm{Ri}_0 < \mathrm{Ri}_c \\ b\mathrm{Ri}_0^m & \text{for } \mathrm{Ri}_0 \geq \mathrm{Ri}_c, \end{cases} \tag{6}$$

where a and b are coefficients and m and n are parameters. Parameters of power laws fit to the data are listed in Table 1. Although Turner's experiments treated heat and salt separately, experiments with dual-component stratification show that the two components mix as if they were separate (Altman and Gargett 1987).

The turbulent length scale ℓ_1 and velocity scale u_1 can be taken from the experiments of Hopfinger and Toly (1976). Like Turner (1968), they stirred a two-layer fluid with a horizontal grid that oscillated vertically. The length scale of the turbulence was proportional to the distance z' from a ''virtual origin'':

$$\ell_1 = \gamma z', \tag{7}$$

where γ is a coefficient. Hopfinger and Toly (1976) also found that the velocity scale of the turbulence could be related to z' and to grid parameters such as stroke S, frequency f, and mesh size M by

$$\frac{u_1}{fS} = C\frac{S^{1/2}M^{1/2}}{z'}, \tag{8}$$

where C is a coefficient. For simplicity, we assume that the virtual origins of the turbulence are at the water surface for the upper grid and the tank bottom for the lower grid—that is, $z' = H/2$.

The rate of work done by the grids on the fluid can be estimated using a drag coefficient and the experimental grid parameters used by Turner (1968) and Hopfinger and Toly (1976):

$$\frac{dW}{dt} = \rho_0 C_D LB(fS)^3, \tag{9}$$

where C_D is a drag coefficient based on the tank cross-sectional area. Then the mixing efficiency can be expressed as

$$R_f = \begin{cases} A\mathrm{Ri}_0^{n+1} & \text{for } \mathrm{Ri}_0 < \mathrm{Ri}_c \quad \text{(10a)} \\ A\mathrm{Ri}_0^{n+1}\left(\dfrac{\dfrac{b}{a}\mathrm{Ri}_0^{m-n} + R_\rho}{1 + R_\rho}\right) & \text{for } \mathrm{Ri}_0 > \mathrm{Ri}_c, \quad \text{(10b)} \end{cases}$$

where

$$A = \frac{8aC^3}{\gamma C_D}\left(\frac{S^{1/2}M^{1/2}}{H}\right)^3. \tag{11}$$

Table 1 shows the values of the grid parameters used to compute the mixing efficiency.

Figure 3 shows the mixing efficiency as a function of the Richardson number for various values of the density ratio. For Richardson numbers that are less than Ri_c, the mixing efficiency increases with increasing Ri_0; the curves collapse to a single curve because salt and heat are entrained at equal rates. For a fixed Richardson number that is greater than Ri_c, the mixing efficiency is larger for larger density ratios. As discussed in section 2, when heat is mixed at a greater rate than salt, the efficiency will be larger when the contribution of heat to the density increases. Also, when $\mathrm{Ri}_0 > \mathrm{Ri}_c$, the density ratio affects the mixing efficiency's dependence on Ri_0. For low R_ρ, the efficiency reaches a peak at Ri_c and decreases with increasing Ri_0 for $\mathrm{Ri}_c < \mathrm{Ri}_0 < 10$. For high R_ρ, the mixing efficiency monotonically increases with increasing Ri_0. The values of the density ratio separating the different types of behavior depend on the equations fit to the laboratory data, but the qualitative behavior does not change.

Like the uniform eddy diffusion model, the analysis based on results from entrainment experiments predicts a significant variation in mixing efficiency with density ratio. At small Richardson numbers, variations due to differential diffusion are small; at larger Richardson numbers, mixing efficiencies can vary by up to 50%. Because Ri_0 is based on scales of the large eddies of the turbulence, it is related to the generalized overturn Froude number of Ivey and Imberger (1991) or the Froude number used by Yamazaki (1990). Nash and Moum (2002) related Ri_0 from Turner's (1968) experiments to $\varepsilon/\nu N^2$; from their estimates, $\varepsilon/\nu N^2 = C_1/\mathrm{Ri}_0$, where C_1 is between 90 and 270. Thus, the range $1 < \mathrm{Ri}_0 < 10$ in Fig. 3 corresponds to $O(10) < \varepsilon/\nu N^2 < O(100)$, and $\mathrm{Ri}_0 = \mathrm{Ri}_c$ corresponds to $\varepsilon/\nu N^2 \approx 50$.

The results in Fig. 3 suggest that differential diffusion can affect the generation of fine structure in diffusively stable parts of the ocean. Phillips (1972) and Posmentier (1977) argued that if the buoyancy flux increases with the buoyancy gradient (or the mixing efficiency increases with Ri_0), perturbations to the density profile will be diffused away. However, if the flux decreases with increasing stratification strength, perturbations will be amplified and a series of layers and strongly stratified interfaces will appear. Our analysis, based on the laboratory measurements of Turner (1968), produces efficiency–Ri_0 relationships assumed in previous layering theories, and it shows that the layering can depend on R_ρ. In particular, if the Phillips–Posmentier mechanism causes the layering (see Holford and Linden 1999), then fine structure can appear at low values of the density ratio. When temperature controls the density, the layer generation is suppressed.

4. Summary

We used two simple models to examine the effect of differential transport of salt and heat on the mixing efficiency of turbulence in a stratified fluid. If temperature and salinity are mixed at different rates, the mixing efficiencies in flows with the same stratification and forcing can vary if the contributions of temperature and salinity to the density, which are measured by the density ratio $R_\rho = \alpha\Delta T/\beta\Delta S$, differ. The first model assumes constant eddy diffusivities for heat and salt and examines the effect of the density ratio and the diffusivity ratio $d = K_S/K_T$ on the mixing efficiency. The model predicts that for the range of d chosen to represent experimental results the mixing efficiency can vary by as much as a factor of 2.5; thus, the eddy diffusion model shows that the effect of differential transport on the mixing efficiency can be as large as that due to the stratification and the type of process generating the turbulence.

To address the main limitation of the eddy diffusion model, the second model incorporates the effect of stratification on the mixing by using results from laboratory experiments on entrainment across a sharp density interface. The mixing efficiency predicted by the second model depends on both the Richardson number $\mathrm{Ri}_0 = g'\ell/u^2$ and the density ratio. For Richardson numbers less than a critical value (corresponding to $\varepsilon/\nu N^2 \approx 50$), the mixing efficiency increases with increasing Ri_0 for all density ratios because salt and heat are entrained at equal rates. For a fixed Richardson number above the critical value, the mixing efficiency is larger for larger density ratios, as in the results of the eddy diffusion model for $d < 1$. Also, when $\mathrm{Ri}_0 > \mathrm{Ri}_c$, the density ratio affects the mixing efficiency's dependence on Ri_0. When cast in terms of models of layering in strongly stratified fluids, these results suggest that steps in temperature and salinity profiles in diffusively stable regions of the ocean can occur for low density ratios.

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