Is First-Order Vector Autoregressive Model Optimal for fMRI Data?

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Abstract

We consider the problem of selecting the optimal orders of vector autoregressive (VAR) models for fMRI data. Many previous studies used model-order of one and ignored that it may vary considerably across datasets depending on different data dimensions, subjects, tasks and experimental designs. Besides, the classical information criteria (IC) used, e.g. the Akaike IC (AIC) are biased and inappropriate for the high-dimensional fMRI data typically with small sample size. We examine the mixed results on the optimal VAR orders for fMRI, especially the validity of the order-one hypothesis by a comprehensive evaluation using different model selection criteria, over three typical data types: a resting-state, an event-related-design and a block-design dataset, with varying time-series dimensions obtained from distinct functional brain networks. We use a more balanced criterion, the Kullback’s IC (KIC) based on the Kullback’s symmetric divergence combining two directed divergences. We also consider the bias-corrected versions (AICc and KICc) to improve the VAR model-selection in small samples. Simulation results show better small-sample selection performance of the proposed criteria over the classical ones. Both bias-corrected ICs provide more accurate and consistent model order choices than their biased counterparts which suffer from over-fitting, with KICc performing the best. Results on real data show that orders greater than one were selected by all criteria across all datasets, for the small to moderate dimensions, particular from small specific networks such as the resting-state default mode network and the task-related motor networks. Whereas, low orders close to one but not necessarily one were chosen for the large dimensions of full-brain networks.

Keywords – fMRI, vector autoregressive models, model selection, AIC, Kullback information criterion
1. Introduction

Vector autoregressive (VAR) modeling of fMRI time series data has been widely used to infer directed connectivity networks of distinct brain regions (Harrison et al., 2003; Valdés-Sosa, 2004; Deshpande et al., 2009; Valdés-Sosa et al., 2005; Gorrostieta et al., 2012; Gorrostieta et al., 2013; Seghouane & Amari, 2012; Ting et al., 2014; Ting et al., 2015). Determining the optimal model order $p$ for fMRI data modeling is an important practical as well as a theoretical issue when using VAR models. Our focus in this paper is the identification of the optimal VAR model orders when modeling fMRI time series, and approaches to estimate these orders. This is potentially important in the analysis of directed functional connectivity using constructs like Granger causality (Seghouane & Amari, 2012). Granger causality (usually) rests upon autoregressive models and inferences about Granger causal dependencies depend upon optimizing the model order. Although we do not expand upon this application, the implications of our work reach into the domain of directed connectivity through estimates of autoregression coefficients - that underlie granger causality, directed coherence and estimates of transfer entropy (Seghouane & Amari, 2012).

The estimation of VAR orders is often carried out as model selection based on information criteria (IC) (e.g. Akaike IC (AIC) and Bayesian IC (BIC)) to find an optimal model balancing between the goodness-of-fit and the model complexity. Using these classical criteria, order of $p=1$ was suggested by some studies to be the optimal, e.g. for a high-dimensional voxel-wise data of a large-scale brain network during the resting state (Valdés-Sosa, 2004). However, it was also claimed as valid for low-dimensional datasets obtained from specific small networks of only a few functionally linked regions-of-interest (ROIs), e.g. an event-related-design data of a 6-ROI motor network (Deshpande et al., 2009). The suggested optimality of VAR of order one has been used, automatically without any further tests, in later studies as a true hypothesis for analyzing fMRI data, regardless of different datasets, subjects,
dimensions of the networks under study and the experimental conditions. These studies include, for example, a whole-brain network of 116 ROIs for a block-design visual task (Valdés-Sosa et al., 2005), a 9-ROI motor-visual network with an event-related task (Satoa et al., 2010), among many others (Lund et al., 2005).

Our primary concern is whether the optimality of VAR(1) model for fMRI, as claimed by the above-mentioned studies, is too strong. We raise questions on its validity that might be limited to the particular datasets being studied. A contradictory result in Gorrostieta et al. (2012) showed that the optimal order selected by BIC for a 7-ROI event-related motor-visual dataset was $p = 2$. Another study of 5-ROI motor-decision network of stroke patients (Gorrostieta et al., 2013) argued that VAR(1) was not sufficient to capture the temporal dynamics in the fMRI data, based on diagnostic tests on the fitted residuals which suggested a VAR(2) instead. Order $p = 4$ was chosen by Bayesian evidence for a block-design data from a 6-ROI visual-attentional network (Harrison et al., 2003). Therefore, there has been a lack of consensus in the literature on the optimal model orders for fMRI data. The selected VAR orders may vary considerably with different ICs, even so across datasets of varied dimensions, subjects, tasks and experimental designs.

Besides, the above studies used classical criteria which are un-reliable for VAR order selection for fMRI data, typically of large dimensions $N$ with small sample size $T$, especially in the full-brain network analysis. The classical AIC is an asymptotically unbiased estimator of Kullback’s directed divergence, a dissimilarity measure between the true and the fitted model. However, it becomes strongly biased for small samples or when the number of fitted parameters $k$ is a moderate to large fraction of $T$. This bias might lead to the selection of over-fitted models, as already shown for the univariate AR model selection with small samples (Hurvich & Tsai, 1989). It becomes more critical for the VAR case (Hurvich & Tsai, 1993) when $N$ is large, which involves fitting much more parameters i.e. $p \times N^2$.

This paper aims to examine the validity of the optimality of first-order
autoregression for fMRI data in particular, and to address the mixed results in the
literature in general. We present a comprehensive study on selecting the optimal VAR
order for modeling connectivity in fMRI data, with particular focus on two issues not
fully explored in the literature: (1) sensitivity of the selected orders to datasets of
different tasks, experimental designs and data dimensions, and (2) performance of
different ICs for small-sample selection particularly in the high-dimensional settings.
We performed a careful cross-dataset evaluation using a resting-state dataset and two
motor task-related datasets with event-related and block-designs. We compared the
selected orders for the large-dimensional data of the whole brain with the small data
from specific functional networks i.e. the default mode and the motor networks.

We use a bias-corrected version of classical AIC, AIC\(_c\) (Hurvich & Tsai, 1989;
Hurvich & Tsai, 1993) to improve the small-sample VAR order selection. Simulation
results showed a significant bias-reduction in AIC\(_c\) compared to the AIC, and hence a
better performance in order selection in small samples. We further consider a new
class of criteria called Kullback information criterion (KIC) which is based on the
Kullback’s symmetric divergence summing up two single directed divergences with
alternated roles of the true and the fitted candidate models in the measure
(Cavanaugh, 1999). The AIC with unidirected divergence is biased to selecting over-
fitted models. While the KIC with combined information from both directions, is
arguably more balanced and able to detect both over-fitted and under-fitted models.
The bias-corrected version KIC\(_c\) has been proposed for linear regression model
selection in finite samples (Cavanaugh, 2004) and extended for univariate and
multivariate AR models in our earlier studies (Seghouane & Bekara, 2004;
Seghouane, 2006). Simulation results showed that KICs provide more accurate VAR
order choices than their AIC analogues, and KIC\(_c\) performs the best in the small-
sample situations (Seghouane, 2006). In this study, the performance of these different
criteria is compared via simulations and across the three real fMRI datasets.
2. VAR Order Selection

2.1 VAR model

Let \( \mathbf{y}_t = (y_{t1}, \ldots, y_{tN})^T \) be a \( N \times 1 \) vector of fMRI time series observations measured from \( N \) voxels or ROIs (defining the nodes of a brain network) at time \( t \), for \( t = 1, \ldots, T \). The dimension of the time series \( N \) is usually large, or even larger than sample size \( T \). Suppose the observations \( \mathbf{y}_1, \ldots, \mathbf{y}_T \) is generated by an \( N \)-dimensional VAR(\( p \)) process

\[
\mathbf{y}_t = \sum_{\tau=1}^{p} \mathbf{A}_\tau \mathbf{y}_{t-\tau} + \mathbf{\varepsilon}_t, \quad t = 1, \ldots, T
\]

(1)

where \( p \) is the model order, \( \mathbf{A}_\tau \) is a \( N \times N \) matrix of AR coefficients at time lag \( \tau \) and \( \{\mathbf{\varepsilon}_t\} \) are \( N \times 1 \) i.i.d Gaussian noise with mean zero and \( N \times N \) variance-covariance matrix \( \mathbf{\Sigma}_e \). The AR coefficient matrices \( \mathbf{A}_\tau = \{a_{ij}\}_{i,j=1}^N \) can characterize the network of temporal interdependencies between different brain regions for different time lags \( \tau = 1, \ldots, p \). There exists a causal influence that node \( j \) exerts on node \( i \) after a time interval of \( \tau \) if \( a_{ij}^\tau > 0 \). Model (1) can be re-written in the form of a multivariate linear model

\[
\mathbf{Y} = \mathbf{X}\mathbf{\beta} + \mathbf{E}
\]

(2)

where

\[
\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1' \\ \vdots \\ \mathbf{y}_T' \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} \mathbf{y}_0' & \cdots & \mathbf{y}_{(p-1)}' \\ \vdots & \ddots & \vdots \\ \mathbf{y}_{(T-1)}' & \cdots & \mathbf{y}_{(T-p)}' \end{bmatrix}, \quad \mathbf{\beta} = \begin{bmatrix} \mathbf{A}_1' \\ \vdots \\ \mathbf{A}_p' \end{bmatrix} \quad \text{and} \quad \mathbf{E} = \begin{bmatrix} \mathbf{\varepsilon}_1' \\ \vdots \\ \mathbf{\varepsilon}_T' \end{bmatrix}.
\]

The total number of unknown parameters comprises the AR coefficients in \( \mathbf{\beta} \) and noise variance-covariances in \( \mathbf{\Sigma}_e \), \( k = pN^2 + N(N+1)/2 \). Note that a small increment in \( p \) will cause a rapid increase in the number of parameters, by order of \( N^2 \) especially when \( N \) is large. The maximum likelihood estimation of VAR model (1) is equivalent to the ordinary least squares (OLS) regression of \( \mathbf{Y} \) on \( \mathbf{X} \). The OLS estimators of \( \mathbf{\beta} \) and \( \mathbf{\Sigma}_e \) are defined by \( \hat{\mathbf{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \) and \( \hat{\mathbf{\Sigma}_e} = (1/T)(\mathbf{Y} - \mathbf{X}\hat{\mathbf{\beta}})'(\mathbf{Y} - \mathbf{X}\hat{\mathbf{\beta}}) \) respectively.
2.2 Model-order selection criteria

Let \( p(y|\theta_p) \) denotes the VAR model of order \( p \) that generates the observations \( y \triangleq y_1, \ldots, y_T \), where \( \theta_{kp} \) is a vector of the corresponding \( k \) model parameters. We use \( \theta_k \triangleq \theta_{kp} \) for simplicity of notation. Given \( y \), our objective is to determine the true model order \( p_0 \). This can be cast as a model selection problem of searching, among a set of \( P \) fitted candidate VAR models from orders 1 to \( P \{ p(y|\hat{\theta}_p), 1 \leq p \leq P \} \), an optimal model of order \( \hat{p} \) that best approximates the true model \( p(y|\theta_0) \) of order \( p_0 \), according to some discrepancy measures between \( p(y|\hat{\theta}_p) \) and \( p(y|\theta_0) \). Various model selection criteria were developed as estimators of these dissimilarity measures. The candidate model corresponding to the minimum value of a criterion is considered optimal.

One such measure is the Kullback’s directed divergence (Shibata, 1989)

\[
2I(\theta_0, \hat{\theta}_p) = d(\theta_0, \hat{\theta}_p) - d(\theta_0, \theta_0)
\]

(3)

where \( d(\theta_i, \theta_j) = E_{\theta_j} \{-2\ln p(y|\theta_i)\} \) and \( E_{\theta_j} \{ . \} \) denotes the expectation with respect to \( p(y|\theta_j) \). The constant \( d(\theta_0, \theta_0) \) can be dropped without affecting the comparison between candidate models. AIC is an asymptotically unbiased estimator of \( d(\theta_0, \hat{\theta}_p) \) and, for VAR model, is given as

\[
AIC = -2\ln p(y|\hat{\theta}_p) + 2k \\
= T \ln |\Sigma_\epsilon| + 2 \left\{ pN^2 + \frac{N(N + 1)}{2} \right\}.
\]

(4)

The classical criteria such as AIC and BIC can also be treated as approximations to the Bayesian model evidence, as shown by Penny et al. (2004) and Penny (2012). The (Bayes) optimal model order is the order that maximizes model evidence. The model evidence is the probability of the data given a particular model order. In other words, it is the integrated or marginal likelihood of the data, having integrated out uncertainty about the (autoregressive) model parameters. It is trivial to show that
model evidence comprises accuracy minus complexity (Penny, 2012). Invariably, the criteria used to optimize model order approximate this mixture of accuracy and complexity under various simplifying assumptions. In this paper, we will only consider the ad hoc criteria i.e. the AIC and BIC as baseline, noting that these criteria can always be expressed in terms of accuracy (first term) and a (computationally cheap) approximation to complexity (second term).

An AIC bias-corrected version has been developed for small-sample size settings (Hurvich & Tsai, 1993)

\[
AIC_c = T \ln|\hat{\Sigma}_e| + 2b \left( pN^2 + \frac{N(N+1)}{2}\right)
\]

(5) 

where \( b = T / (T - (pN + N + 1)) \). AIC\(_c\) differs from AIC only by a scale factor \( b \) which is non-negligible when \( T \) is small relative to \( N \). Simulation results showed that AIC\(_c\) improves VAR order selection in small samples compared to AIC.

An alternative measure is Kullback’s symmetric divergence combining two directed divergences with alternated role of true and the approximating models in (3)

\[
2J(\theta_0, \hat{\theta}_p) = 2I(\theta_0, \hat{\theta}_p) + 2I(\hat{\theta}_p, \theta_0)
\]

\[
= d(\theta_0, \hat{\theta}_p) - d(\theta_0, \theta_0) + d(\hat{\theta}_p, \theta_0) - d(\hat{\theta}_p, \hat{\theta}_p)
\]

(6) 

Cavanaugh (1999) proposed the KIC as an asymptotically unbiased estimator of the quantity \( K(\theta_0, \hat{\theta}_p) = d(\theta_0, \hat{\theta}_p) + d(\hat{\theta}_p, \theta_0) - d(\hat{\theta}_p, \hat{\theta}_p) \), defined for the VAR model as (Seghouane, 2006)

\[
KIC = -2 \ln p(y \mid \hat{\theta}_p) + 3k
\]

\[
= T \ln|\hat{\Sigma}_e| + 3 \left( pN^2 + \frac{N(N+1)}{2}\right)
\]

(7) 

which imposes stronger penalty term on the number of parameters than the AIC to prevent over-fitting. The bias-corrected version has been derived for VAR model (Seghouane, 2006)

\[
KIC_c = T \ln|\hat{\Sigma}_e| + \frac{TN(2pN + N + 1)}{T - pN - N - 1} + \frac{TN}{T - Np - (N-1)/2} + \frac{2N^2p + N^2 - N}{2}
\]

(8)
where the penalty term is a function of $T$ to account for small-sample conditions. KIC$_c$ was shown to outperform KIC and AICs in simulation study of small-sample order selection studies (Seghouane & Bekara, 2004; Seghouane, 2006). Note that the constant terms $T\ln(2\pi T)+T\ln N$ in the original formulae which play no practical role in model selection, have been already dropped in (4)-(5) and (7)-(8).

3. Experimental Results

In this section, we first compare the small-sample performance of various selection criteria in estimating the orders of VAR models, based on simulated data. We aim to demonstrate the effectiveness of our proposed bias-corrected and symmetric Kullback’s divergence criteria in providing more accurate estimates of the model orders under the small-sample size settings. Having established this, we then generalize the simulation findings by applications to real data to verify the appropriateness of order one in VAR modeling of fMRI signals.

3.1 Simulations

We consider two VAR model structures for simulated data generation: (1) low-dimensional models with high orders and (2) higher-dimensional models with low orders. The first is bivariate VAR models with a range of known orders $p_0 = 2, 4, 6, 8$. We fixed the coefficient matrices for the first two lags for $p_0 = 2$

$$A_1 = \begin{bmatrix} 0.50 & -0.30 \\ 0.20 & 0.65 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.50 & 0.30 \\ 0.0 & -0.40 \end{bmatrix}$$

which were used by (Hurvich & Tsai, 1993) to evaluate the bias-corrected AIC, and based on these, set recursively $A_\tau = 0.8A_{\tau-2}$ for $\tau = 3, \ldots, 8$ to emulate the decaying temporal dependencies at the longer time-lags for the higher-order models $p_0 > 2$. The second is 20-dimensional VAR models with $p_0 = 1$ and $p_0 = 2$, with block-diagonal coefficient matrices each formed by two $10 \times 10$ sub-blocks along the main diagonal. The non-zero entries of the sub-blocks were randomly generated from a scaled
uniform distribution i.e., for \( i \) and \( j \) in the same block, \( \lambda^{\alpha} a_{ij} \sim U[-0.5, 0.5] \) with a scaling constant \( \lambda \), set here as 1.35. All other off-diagonal entries are zeros. Similar structure with a lower dimension has been used in a simulation study of order selection for fMRI using the Bayesian evidence (Harrison et al., 2003). The above AR parameters were chosen to ensure stability of the simulated processes. Noise covariance matrix \( \Sigma_e = I \) was used for all models.

A thousand simulated realizations of sample size \( T \) were generated from each model with the known order \( p_0 \). The values of \( T \) were chosen to represent small-sample scenarios, compared to the large number of parameters \( k \). We also investigated the impact of increased ratios \( k/T \) on the selection performance, using the values of ratio 0.16 for all the bivariate models (Table 1), 2 and 4 for the 20-dimensional VAR(1) and VAR(2) models (Table 2), respectively. For each realization, candidate VAR models of orders \( p = 1, \ldots, P \) were fitted by OLS method, and various criteria were used to select among the candidates an estimator \( \hat{p} \) to recover the true model order \( p_0 \). The results are presented as a confusion matrix of frequencies of the selected orders for the 1000 realizations, producing an approximate posterior model distribution \( p(p|y) \) over \( p \). Table 1 and 2 show the estimation results by various criteria with maximum model order considered \( P = 12 \) and \( P = 8 \), for the VAR model structures (1) and (2) respectively.

From both Tables, the Kullback’s symmetric divergence-based criteria, KICs generally show the best performance in selecting the correct model orders, followed by AICc and BIC, for the true models considered under the small-sample settings. The KICc consistently achieves the highest accuracy among all criteria with percentage of correct selections above 90%, except for the bivariate VAR(8) which reveals mild over-fitting tendency of the criterion. These are consistent with the results in Seghouane (2006), in terms of the prediction error (ME) of the fitted models. Table 1 shows that BIC initially performs better than the AICc and KIC for small orders.
Table 1: Frequency of order selected by various criteria over 1000 realizations for bivariate VAR models with known orders $p_0 = 2, 4, 6, 8$ and different sample sizes $T$. Grey shadings indicate the highest counts.

<table>
<thead>
<tr>
<th>Model</th>
<th>$T$</th>
<th>Criterion</th>
<th>Selected Model Order</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1  2  3  4  5  6  7  8  9  10 11 12</td>
</tr>
<tr>
<td>VAR(2)</td>
<td>50</td>
<td>BIC</td>
<td>30 934 24 1 1 1 0 0 0 2 3 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC</td>
<td>0 211 40 14 11 20 9 11 15 35 78 556</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC_c</td>
<td>11 858 89 23 8 6 2 1 0 2 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC</td>
<td>18 776 56 12 9 6 4 5 5 10 18 81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC_c</td>
<td>23 945 30 1 1 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>VAR(4)</td>
<td>100</td>
<td>BIC</td>
<td>34 7 31 919 9 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC</td>
<td>0 0 3 639 128 60 40 23 25 16 24 42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC_c</td>
<td>0 0 3 870 89 24 9 4 1 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC</td>
<td>0 0 6 917 56 16 5 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC_c</td>
<td>0 2 9 949 34 4 2 0 0 0 0 0</td>
</tr>
<tr>
<td>VAR(6)</td>
<td>150</td>
<td>BIC</td>
<td>92 67 121 9 23 686 2 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC</td>
<td>0 0 0 1 4 708 143 53 31 25 15 20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC_c</td>
<td>0 0 2 1 7 861 98 17 8 5 1 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC</td>
<td>0 2 7 3 11 907 60 5 3 2 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC_c</td>
<td>0 3 13 7 14 932 29 1 1 0 0 0</td>
</tr>
<tr>
<td>VAR(8)</td>
<td>200</td>
<td>BIC</td>
<td>78 59 518 68 60 9 5 203 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC</td>
<td>0 0 0 1 4 0 5 722 141 57 36 34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC_c</td>
<td>0 0 0 3 8 2 7 843 99 21 7 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC</td>
<td>0 0 14 14 26 8 14 860 49 9 3 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC_c</td>
<td>0 1 29 24 49 10 17 843 25 2 0 0</td>
</tr>
</tbody>
</table>

Table 2: Frequency of order selected by various criteria over 1000 realizations for 20-dimensional VAR models with known orders $p_0 = 1$ and $p_0 = 2$. Grey shadings indicate the highest counts.

<table>
<thead>
<tr>
<th>Model</th>
<th>$T$</th>
<th>Criterion</th>
<th>Selected Model Order</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1  2  3  4  5  6  7  8</td>
</tr>
<tr>
<td>VAR(1)</td>
<td>200</td>
<td>BIC</td>
<td>1000 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC</td>
<td>0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC_c</td>
<td>1000 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC</td>
<td>713 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC_c</td>
<td>1000 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>VAR(2)</td>
<td>200</td>
<td>BIC</td>
<td>807 193 0 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC</td>
<td>0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AIC_c</td>
<td>0 1000 0 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC</td>
<td>0 20 0 0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KIC_c</td>
<td>1 999 0 0 0 0 0 0</td>
</tr>
</tbody>
</table>
\( p_0 = 2 \) and \( p_0 = 4 \), however, it is surpassed by the AICc and KIC for high orders. The BIC is more prone to under-fitting compared to other criteria.

The bias-corrected KICc and AICc clearly outperform their original versions KIC and AIC which tend to choose over-fitted models, with gross over-parameterization by the AIC, especially evident for the bivariate VAR(2) model. When the parameter counts \( k \) increase rapidly compared to \( T \) with the larger dimensions \( N \), the performance of the biased criteria further deteriorates with more substantial amount of over-fitting, as shown in Table 2. This is due to the increased bias of the criteria as estimators of the Kullback divergences. Whereas, the results re-confirm that the bias reduction in AICc and KICc, which implies stronger penalty functions to prevent over-fitting under finite samples, can provide more accurate model-orders than their original versions, as demonstrated in (Hurvich & Tsai, 1989; Hurvich & Tsai, 1993; Seghouane & Bekara, 2004; Seghouane, 2006). Moreover, we can see that the bias-corrected criteria particularly the KICc perform better in terms of consistency, i.e. convergence of the selected orders in probability to true values, compared to the larger variance of the distribution of the biased AIC and KIC estimates.

3.2 Evaluation on fMRI data

Here, we check the validity of previous findings that order one is sufficient for VAR modeling of fMRI data. We performed a comprehensive evaluation on real fMRI data using different selection criteria over (1) three typical data types: a resting-state, an event-related-design and a block-design motor task dataset; and (2) varying dimensions of time-series obtained from distinct functional brain networks of different sizes.

3.2.1 Resting-state data

We used resting-state fMRI data of the first subject from the first session obtained from Shehzad et al. (2009). The data were acquired with a Siemens Allegra 3-Tesla
scanner. During scans, subjects were asked to relax with eyes open, with word ‘Relax’ projected in white on a black background. Functional MR images were obtained using a Blood oxygenation level-dependent BOLD-weighted echo planar imaging (EPI) sequence (TR/TE = 2000/25 ms, flip angle (FA) = 90°, voxel size = 3x3x3 mm³, matrix size = 64x64, 39 contiguous slices). $T = 197$ EPI volumes were collected for each scan.

3.2.2 Event-related finger-tapping data

We used an event-related finger-tapping-task dataset of a single subject, provided by Lee et al. (2011). The subject was instructed to perform right finger flexion. The session began with a 30 s preparation and a 30 s resting period, followed by 40 repeated trials of a task-period and a resting period with duration of 28 s each trial, always starting with the task-period, and ended with an additional 30 s resting period. Within the repeated trials, the resting period consisted of inter-stimulus interval between 4 and 20s with an average of 12s. Functional images were acquired on a 3.0-T ISOL system, Korea (TR/TE = 2000/35 ms, FA = 80°, voxel size 3.44x3.44x4 mm³, matrix = 64x64, 24 slices). A total of $T = 326$ EPI volumes were collected.

3.2.3 Block-design hand-movement data

A single-subject block-design hand pressing dataset acquired from website http://www.psychology.gatech.edu was used. The subject was asked to press left hand at some time points and right hand at other times. The same hand was pressed continuously for a time-block of 12s with a resting block of 36s. The experimental paradigm started with the left hand press, with a 12s resting period between the left and the right hand pressing. Only the right-hand data was considered here. Functional images were acquired on a Siemens TrioTim system (TR/TE = 2000/32 ms, FA = 90°, voxel size 3x3x3 mm³, matrix size = 64x64, 36 slices). A total of $T = 188$ EPI volumes were collected.
3.2.4 Data pre-processing and analysis

The datasets were pre-processed using SPM8 and Matlab with the following steps (Khalid & Seghouane, 2014): realignment to the first image for motion correction, normalization to a standard Talarach template, and spatial smoothing with an 8-mm full-width half maximum (FWHM) Gaussian kernel. The low-frequency drifts in the data were removed using a discrete cosine transform (DCT) filter (with a cut-off frequency of 1/128 Hz). The resting-state data was temporally band-pass filtered (0.01 – 0.08 Hz) to retain its low-frequency fluctuations which reflect spontaneous neuronal activity. For the task-related data, the high-frequency components were temporally smoothed by a 1.5s FWHM Gaussian filter.

We investigated the sensitivity of the selected orders to the increased time-series dimensions, from small-scale specific functional networks to the whole-brain networks. For the resting-state data, 79 time-series belonging to the default mode network (DMN) were selected based on the strongest correlations with a seed ROI based on the posterior cingulate cortex (PCC). For both event-related and block-design datasets respectively, 29 time-series from the activated motor regions were selected. Figure 1 shows the spatial maps for the areas of functional networks under study. Figure 1(a) shows the major regions of the resting-state DMN which are strongly correlated with the PCC as a hub, comprising the medial prefrontal cortex (MFC), left and right inferior parietal lobe (IPL). Both the right finger-tapping and the right hand-pressing tasks induced the activation of the left primary motor cortex areas, as shown in Figure 1(b) and (c). For each dataset, we also evaluated two whole-brain networks of 116 and 200 ROIs parcellated based on the automated anatomical labeling (AAL) (Tzourio-Mazoyer et al., 2002) and the Craddock’s spectral clustering (Craddock et al., 2012) atlases, respectively. The AAL and Craddock based ROIs were prepared using DPARSF toolbox (Yan & Zang, 2010). 100 random selections of half of the ROI time-series were carried out for each data type and each network. The sampling interval and a small sample size are fixed TR = 2s and T = 180 across all datasets for a consistent comparison. The VAR model order selection analysis was
Figure 1: Spatial maps of functional networks being modeled. (a) PCC-seeded correlation map of the DMN during rest (converted to z-statistics and threshold at $p<0.05$. (b)-(c) F-statistics activation maps of the motor regions, threshold respectively at $p<0.0001$ and $p<0.00001$ for the event-related right finger-tapping task (b) and the block-design right-hand pressing task (c).

performed on the pre-processed time series, for the block-design data, it was performed on their residuals having regressed out the task-related mean levels of activation, modeled by a general linear model (GLM).

3.2.5 Results

Table 3 shows the frequencies of VAR orders selected by each criterion over the 100 replications for the different data types and the increasing time-series dimensions from distinct functional networks. The selection is performed among candidate models of orders $p=1,...,10$. It is clear that, as the dimension $N$ increases, all criteria tend to select lower model orders for all three datasets, due to the penalty imposed on the model complexity to prevent over-fitting. For the relatively low dimensions $N=15$ and $N=30$ of the small functional networks, orders of much higher than one are suggested consistently by all criteria for all datasets, with the highest $p=7$ for the resting-state data from the DMN, followed by $p=5$ for both the event-related and block-design data from the motor networks. This explains why higher orders were found to be more optimal by the studies (Harrison et al., 2003; Gorrostieta et al., 2012; Gorrostieta et al., 2013) which fitted only a few ROI time-series. For the
Table 3: Frequency of VAR model-order $\hat{p}$ selected by various information criteria for resting-state, event-related-design and block-design hand-movement fMRI data from functional brain networks of distinct sizes. (over 100 independent runs of random selection of $N$ time-series for each network). The sampling rate and sample size are fixed TR = 2s, T = 180. Grey shadings indicate the highest counts.

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whole-brain networks of moderate dimensions $N = 50$, orders of $p = 3$ to $p = 4$ are selected for all datasets. For the high-dimensional case of $N=100$, $p = 1$ is suggested as optimal for the resting-state data, in consistency with the findings in Valdés-Sosa et al. (2005) which studied similar dimensions of 116 time-series. However, order close to one but not one, i.e. $p = 2$ is chosen for both the event-related and block-design data. Among the ICs, AIC and KIC tend to select much higher orders than their bias-corrected counterparts, e.g. for the DMN. As illuminated by the simulation results, these are possibly over-fitted models which fail to be detected due to the bias of the classical criteria in finite-sample cases. Both bias-corrected ICs perform comparably in detecting the over-fitted models, with slightly better performance by the KIC$_c$.

4. Discussion

We have presented a careful interrogation of the optimal temporal orders of VAR for modeling fMRI data using various information criteria, across different tasks, experimental designs and dimensions of the brain networks being modeled. Based on our results, we suggest, albeit with due caution, a lower bound of the optimal VAR orders for fMRI data (with small sample size $T \sim 200$) as $p \geq 5$ for low dimensions $N \leq 15$, across all type of datasets with higher orders for the non-task-related resting-state data. Orders of $p = 3$ to $p = 4$ are suggested for the moderate dimensions $30 \leq N \leq 50$, while orders close to one but not necessarily one for the high dimensions comparable to $T$, $N > 100$ depending on the data types. Bias-corrected criteria AIC$_c$ and KIC$_c$ should be used in favor of AIC and KIC to improve the small-sample order selection. Our results imply that Valdes-Sosa's assertion “For this data, the GCV criterion indicated that the most appropriate model order for the spatio-temporal multivariate autoregressive model (ST-MAR) was $p = 1$” (Valdés-Sosa, 2004), which was adopted indiscriminately by subsequent studies (Valdés-Sosa et al., 2005; Satoa et al., 2010; Lund et al., 2005) among others as a conclusively-true hypothesis for fMRI data, is no longer valid across different conditions. Therefore,
we conclude that when modeling fMRI time-series with VAR, the optimal orders which have been shown to vary considerably with different datasets, experiments and the time-series dimensions, should always be carefully analyzed using appropriate model selection criteria (Hurvich & Tsai, 1993; Seghouane, 2006), and not taken as $p=1$ automatically.

Acknowledgement
This work was supported by the Universiti Teknologi Malaysia (UTM) and the Ministry of Education (MOE) Malaysia under Fundamental Research Grant Scheme (FRGS) –R.J130000.7809.4F668.

References
or activity: their role in intra- and inter-subject variation in fMRI. *Neuroimage*,
26(3), 960-964.

Hurvich C. M. & Tsai C. L. (1989). Regression and time series model selection in

Hurvich C. M. & Tsai C. L. (1993). A corrected Akaike information criterion for

Cavanaugh J. E. (1999). A large-sample model selection criterion based on


based on the Kullback symmetric divergence. *IEEE Trans. Signal Process.*, 52(12),
3314–3323.

samples using Kullback’s symmetric divergence. *IEEE Trans. Circuits and
Systems-I Regular Papers*, 53(10), 2327-2335.

Shehzad Z., Kelly A. M., Reiss P. T., Gee D. G., Gotimer K., Uddin L. Q., Lee S. H.,
Margulies D. S., Roy A. K., Biswal B. B., Petkova E., Castellanos F. X. & Milham
2209-2229.

sparse dictionary learning with MDL criterion, *IEEE Trans. Medical Imaging*,
30(5), 1076-1089.

algorithm for fMRI data analysis. *IEEE Statistical Signal Process. Workshop (65-
68). Piscataway, NJ: IEEE.*

Tzourio-Mazoyer N., Landeau B., Papathanassiou D., Crivello F., Etard O., Delcroix

A whole brain fmri atlas generated via spatially constrained spectral clustering.*

Yan C. & Zang Y. (2010). Dparsf: a matlab toolbox for ”pipeline” data analysis of

