A Spatial Model of Optimal Water Conveyance

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Most water projects suffer from losses in conveyance. Because conveyance has public good characteristics, investment in reducing conveyance losses must be provided by a central authority. This paper develops a spatial model to determine optimal conveyance investment, water allocation, and investment in firm-specific conservation technology. The efficiency and distributional characteristics of the optimal solution are compared to projects with (i) well-developed water markets and (ii) spatially uniform water prices, both with sub-optimal conveyance. A numerical illustration is provided. © 1995 Academic Press, Inc.

1. INTRODUCTION

It is now commonly recognized that unlike in the past, water management problems can no longer be solved simply by investing in new project capacity. The rising cost of generating new supplies and scarcity of untapped water sources has led to an increased emphasis on better management of existing projects rather than the construction of new facilities [10, 13]. In keeping with this view, economists have argued that the main thrust of water policy reform should be the introduction of well-defined, tradeable property rights to water [2]. Once water could be freely bought and sold, markets will allocate the resource to its most valuable use and water prices will reflect opportunity costs, thereby generating private incentives to apply and save water efficiently.

An important factor frequently ignored by economists is the role of conveyance losses. If conveyance losses are high, as in most water projects, even perfectly functioning water markets may result in a suboptimal use of resources. This is because there are increasing returns to scale in conveyance, and left to themselves, individual firms that receive water from the project will not invest optimally in distribution canals.

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This paper develops a von Thunen-like spatial model of a water project in which a regulated utility supplies water to individual firms and invests in the distribution system (canals). Optimal water prices, investment in conveyance, and each firm's investment in irrigation technology are determined at each location. This solution is then compared with stylized situations in which the utility provides water but fails to make conveyance investments. Two scenarios are examined: one with legal impediments to water trading such that the utility charges spatially uniform prices for water, and another which allows for water trading between firms.

The comparisons show that conveyance improvements, together with optimal water pricing, have strong efficiency and "equity" effects. Under plausible assumptions, optimal conveyance investments lead to an increase in aggregate water use, which is somewhat surprising because intuitively one would expect conveyance improvements to "save" water and therefore use less of it. It is shown that compared to the uniform pricing model, conveyance improvements lead to lower water use and increased water conservation by individual firms, and aggregate service area and aggregate output increase. Firms also accrue lower land rents under the optimal solution. Similar (but weaker) results hold under water markets, in which case the distribution of water, output, and land rents is skewed in favor of firms located closer to the water source. Finally, an illustration with stylized data from western U.S. agriculture suggests that net benefits from a project with optimal conveyance are likely to be substantially higher than those with no centralized investment in water distribution.

In terms of policy, these findings imply that if conveyance losses are high, the introduction of water markets without adequate provision of conveyance skews the distribution of project benefits in favor of firms located closer to the water source and against firms located near the project boundary. Therefore, policies that aim to rehabilitate existing water projects by providing conveyance investments and creating water markets may have to provide lump-sum compensation to firms that are adversely affected by the improvement.

This research substantially generalizes earlier works on the economics of conveyance [5, 14] which take conveyance losses as given. Partly because of this restriction, these studies do not determine optimal investment in conveyance and its impact on allocation and distribution within the project or compare it with alternative institutional arrangements, the focus of this paper. Section 2 develops the notation and characterizes the optimal solution. Section 3 compares the optimal outcome with models without conveyance.\(^3\) Section 4 provides the illustration. Section 5 concludes the paper.

2. OPTIMAL RESOURCE ALLOCATION BY A WATER UTILITY

This section examines the case of a central planner or a water utility (e.g., a water district or mutual ditch company) that invests optimally in the water distribution system and charges individual firms the associated shadow prices. The resulting optimal solution is characterized to determine conveyance and firm-

\(^3\)The complete mathematical development of the arguments made in Sections 2 and 3 are presented in a previous, unabridged version of the paper [6] which can be obtained by writing to the authors.
specific conservation investment at each location and the spatial distribution of water use, output, and land and water rents.

The Model

Consider a simple one-period (i.e., one cropping season) model of a water project that abstracts from uncertainty. Water is supplied by the utility from an in situ point source (e.g., a dam or a diversion structure) into a canal. Firms are located over a continuum on either side of the canal on land of uniform quality. Firms draw water at various locations \( x \) along the canal, where \( x \) is the distance measured from the source. In order to maintain the simplest possible setting, we assume these firms to be identical in all respects except location. Define \( \alpha \) to be the constant width of the project area at any \( x \).

Let the amount of water available at the source be \( z_0 \), which is endogenously chosen by the utility. The cost of generating water at source \( g(z_0) \) represents the per period capital expenditures as well as the costs of operation (e.g., pumping) and maintenance of the head works. It may vary with specific water generation or diversion technologies, projects, or geographical regions. However, we assume that it is an increasing, twice-differentiable, convex function, \( g'(z_0) > 0, g''(z_0) > 0 \). The quantity of water delivered (per unit land area) to a firm at location \( x \) is denoted \( q(x) \), with \( q(x) \geq 0 \). The fraction of water lost in conveyance per unit length of canal is given by \( a(x) \), with \( a(x) \geq 0 \). Let \( z(x) \) be the residual quantity of water flowing in the canal at location \( x \), \( z(x) \geq 0 \). Then

\[
z'(x) = -q(x)\alpha - a(x)z(x),
\]

where the first and second terms on the right-hand side indicate, respectively, total water delivered and water lost in conveyance at each location \( x \). The loss function \( a(x) \) depends on \( k(x) \), defined as the conveyance investment per unit surface area of the canal, which is the sum of annualized investment cost and the cost of operation and maintenance. Note that \( k \) is a function of \( x \) and can therefore vary with location. If no investment is made in improving canal quality, i.e., \( k(x) = 0 \) (e.g., earthen canals), then the fraction of water lost at \( x \), \( a(x) \) equals the base loss rate \( a_0 \). On the other hand, a positive value of \( k(x) \) (e.g., water transmission through concrete or metal pipes) causes the loss function \( a(x) \) to be smaller than \( a_0 \). Annualized investment in conveyance is then given by \( v \cdot k(x) \) where \( v > 0 \) is a constant that represents the ratio of the canal perimeter to its cross-sectional area \( z \).

Let the reduction in the conveyance loss rate obtained by investing \( k(x) \) be given by \( m(k(x)) \). We then obtain

\[
a(x) = a_0 - m(k(x)).
\]

Assume \( m(\cdot) \) to be an increasing, twice differentiable function with decreasing returns to scale in \( k \), \( m(\cdot) \in [0, a_0], m'(\cdot) > 0, m''(\cdot) < 0, \lim_{k \to \infty} m(k) = a_0, \lim_{k \to 0} m'(0) = \infty \). The last two limits indicate that conveyance losses can only be driven to zero in the limit and that the marginal returns to conveyance investment

\footnote{Ideally, the canal perimeter to be lined should increase at a decreasing rate with the amount of water \( z \) flowing in the canal, i.e., \( v'(z) > 0, v''(z) < 0 \). For an analysis of this problem, see [6].}
approach infinity when conveyance investment goes to zero. From (2), \( a(x) \in (0, a_0] \).

Condition (1) implies that \( z'(x) \leq 0 \), or the residual flow of water in the canal decreases away from the source. The total stock of water must equal the sum of aggregate water delivered and lost in conveyance. If \( X \) is the length of the project area (to be chosen endogenously), then \( z(X) = 0 \), i.e., the flow of water in the canal reduces to zero at the project boundary. By integrating (1) we obtain

\[
z_0 = \int_0^X [q(x) \alpha + a(x) z(x)] \, dx.
\]

Firms can make private investment in technology that conserves water on their land and thereby increases the efficiency of the water delivered, \( q(x) \). These techniques may include land smoothing, reuse of tail water that runs off the downstream end of fields, or matching the rate of water inflow to the soil's infiltration capacity through drip or sprinkler irrigation [11]. Other yield-increasing inputs such as high-yielding seeds, fertilizer, or farm machinery, can also be modeled in the same fashion, although not attempted here. Let \( \hat{l}(x) \) be the amount of firm-specific investment in water conservation. Then the conservation efficiency function \( h(\hat{l}(x)) \) gives the proportion of water delivered that actually reaches the plant, and is assumed to be increasing, twice differentiable, and concave, i.e., \( h(\cdot) \in (0, 1); h'(\cdot) > 0; h''(\cdot) < 0 \). The price of \( l \) is taken to be unity.

Assuming monocropping, we can then specify a two-input production function of crop yield per unit land area for each firm that is a multiplicative function \( f(\hat{q}h(l)) \) of water delivered, \( \hat{q} \), and conservation efficiency, \( h(l) \). Although it is beyond the scope of this paper, multimicrocropping can be accommodated by indexing on the production function for each crop type. The production technology is assumed to exhibit constant returns to scale with respect to land and all other production inputs. Let \( f(\cdot) \) be twice differentiable with the following properties: \( f(\cdot) > 0; \partial f/\partial q > 0; \partial f/\partial l > 0; a f^2/\partial q^2 < 0; \) and \( a \partial f^2/\partial l^2 < 0 \). For notational convenience, define \( e = \hat{q}h(l) \) where \( e \) is the "evapotranspiration" requirement of the crop (see [17]). Thus, each firm withdraws \( q \) units of water from the canal but only \( e \) units are actually used by the crop. Similar distinctions between "delivered" and "actual" input use have been made elsewhere, such as in the case of energy-conserving appliances [9]. The above assumptions imply \( f'(e) > 0, f'(e) < 0 \). In order to ensure strict concavity of the production function, the elasticity of marginal product of applied water \( \eta_e = f''(\hat{q}h/f') \) is assumed to be in the range \(-\infty < \eta_e < -1\).

**The Optimization Problem**

Let \( p \) be the constant output price of the crop. The assumption of price-taking behavior might be reasonable for relatively large regional markets or export-oriented agriculture. Given a value of \( z_0 \), the water utility or central planner

\footnote{Since \( \eta_e \) must be negative, restricting it from taking values in \([-1, 0) \) implies that the region outside the production phase where marginal product is near its maximum value and increasing with small input levels, is ignored. Similar restrictions have been imposed elsewhere in the water literature (e.g., [4]).}
chooses functions $q(x), k(x), I(x)$, and $X$ so as to maximize net benefits from the project as

$$
\text{maximize } \frac{NB(z_0)}{q, k, I, X} = \int_0^X \left[ \left( p(I) qh(I) - l \right) \alpha - kv \right] dx
$$

(4)

subject to condition (1), where $NB(\cdot)$ represents net benefit from the project. The problem defined in (4) is cast in a standard optimal control format where $z(x)$ is the state variable, and $q(x), k(x),$ and $I(x)$ are control functions. Associating auxiliary function $\lambda(x)$ to the differential equation (1), the Hamiltonian can be written as$^6$

$$
H(q, k, I, z, \lambda) = \left[ p(I) qh(I) - l \right] \alpha - kv - \lambda [q \alpha + az].
$$

(5)

For the purposes of this paper, we assume that the necessary convexity requirements are always satisfied and, therefore, an interior optimal solution exists. Let $q^*, k^*, I^*, z^*, X^*$ denote the corresponding optimal values. Then the following conditions hold at equilibrium:$^7$

$$
\frac{\partial H}{\partial q} = \left[ pf'(I) \lambda h(I) - \lambda \right] \alpha = 0 \quad (6)
$$

$$
\frac{\partial H}{\partial k} = -v + \lambda z \lambda'/(k) = 0 \quad (7)
$$

$$
\frac{\partial H}{\partial I} = \left[ pf' qh'(I) - 1 \right] \alpha = 0 \quad (8)
$$

$$
\lambda'(x) = \lambda a \quad (9)
$$

and

$$
H(X) = 0. \quad (10)
$$

Define the optimal net benefit function given any value of $z_0$ as $NB^*(z_0)$. Then the optimal stock of water at source $z_0^*$ is obtained by solving the problem

$$
\text{Maximize } \frac{NB^*(z_0) - g(z_0)}{z_0}\quad (11)
$$

which suggests that $z_0^*$ must satisfy $NB^*(z_0) = g'(z_0)$. Obviously, $\lambda(x)$ is the shadow price of each unit of delivered water at location $x$. Defining $\lambda_0$, the shadow price of the stock of water at source, as $\lambda_0$, we obtain the “salvage value” condition

$$
\lambda_0 = g'(z_0^*). \quad (12)
$$

Equations (6)–(8) give us the usual marginal conditions. Condition (6) equates the shadow price of water at each location $x$ to its marginal benefit in crop

$^6$To avoid notational clutter, writing the independent variable $x$ is avoided whenever convenient.

$^7$The rest of this section deals only with the optimal values of the choice variables. Labeling the variables with an asterisk is avoided whenever possible in order to keep the notation simple.

$^8$Two-step optimization is not necessary and is only done to facilitate comparison with the sub-optimal models in Section 3.
production which is the marginal value product of a unit of water $p_f(\cdot)$ times the proportion of water $h(f)$ that reaches the crop. In (7) the benefits from water saved at location $x$ by an additional unit of $k$, given by $\lambda m'(k)$ ($m'(k)$ is the amount of water saved by the marginal dollar invested in conveyance) is equated to its cost $v$ at each location. Finally, in (8) the marginal cost of firm-specific investment in conservation, which is unity, equals its marginal benefit at location $x$, the latter being equal to the value of marginal product of each unit of delivered water ($p_f(\cdot)$) times the marginal efficiency of each additional unit of investment in conservation, $h'(f)$.

At the project boundary $X^*$, (10) implies that net benefits (i.e., total revenue less investment in conservation and conveyance) from extending the project into one more unit of land must equal the shadow value of water at the boundary. If a non-zero opportunity rent for land were incorporated into the model, that would imply a smaller $X^*$.

A central planner can implement the optimal solution by charging each firm at location $x$, $\lambda(x)$ per unit of delivered water and investing $vk(x)$ in conveyance at each location. Condition (9) implies that the rate of change of shadow prices is equal to the optimal loss rate of water in conveyance. Solving the differential equation (9) and using condition (12), it is easy to see that the price of water is just the marginal cost of water generation $g'(z_0)$ at the source, and it increases exponentially away from the source. In other words, the opportunity cost of delivering water to any location increases with distance because of conveyance losses.

Characterization of the Optimal Solution

In what follows, we provide a heuristic discussion of the optimal solution defined above. First, an increase in the shadow price $\lambda(x)$ of water from head (upstream) to tail (downstream) causes a decrease in the amount of water ($q(x)$) used by each firm. However, an increase in the value of water also makes increased investment in conservation ($H(x)$) profitable. So firms situated downstream of the project receive less water, but spend more on conservation relative to those located upstream (see Proposition 1 (i, ii) in the Appendix). This result is consistent with the empirical observation that higher water prices lead to an increased use of conservation technology [1].

The spatial distribution of conveyance investment $k(x)$ is jointly determined by two effects: the increasing scarcity value of water away from the source measured by the shadow price $\lambda(x)$ and the decreasing residual volume of water $z(x)$. The former "scarcity" effect tends to increase conveyance investment while the latter "volume" effect causes it to decrease. However, as is clear from Proposition 1 (iii), the volume of water flowing in the canal falls faster than the rate of increase of shadow price, because an amount $q(x)$ is being withdrawn at each location. In other words, the total value of the residual water in the canal given by $\lambda(x)z(x)$ is decreasing away from the source. Therefore conveyance investment also decreases away from the source.

There may be a jump discontinuity in $\lambda(x)$ at $x = X^*$ which is ignored for the purposes of this paper.
Recall that the net amount of water per unit area "delivered" to the firm at $x$ is $q(x)$. Define $Q(x)$ as the corresponding gross amount of water per unit area "sent" from source to a firm at location $x$. At the source, $q(0) = Q(0)$ because there are no conveyance losses. Furthermore, since optimal water prices are given by $\lambda(x)$, define total water charges $WC(x)$ paid by a firm at location $x$ as $WC(x) = \lambda(x)q(x)\alpha$. Then as shown in Proposition 2 (see Appendix), the water sent to each location $Q(x)$ increases with distance as the net outcome of two opposing effects: water lost in conveyance at the rate $a(x)$ and water delivered to each location which decreases at a rate less than $a(x)$ because firms substitute into conservation but do not reduce water use as much. From the definition of $WC(x)$, this suggests that water charges increase with distance. The utility collects water charges from each firm and pays for water generation at the source and conveyance at each location.

Thus downstream firms not only pay higher prices and receive less water, but their total water charges are also higher relative to those located upstream. An increase in the price of water away from the source causes firms to substitute into conservation. Since the two inputs are imperfect substitutes, the overall effect of a rise in the input price of water is a decrease in "effective" input use $e(x)$ and per unit output $y(x)$ in downstream locations (see Proposition 3).

Define quasi-rents to land $R_1(x)$ accruing to a firm at location $x$ by $R_1(x) = [p(\lambda(1)) - 1] \alpha - \lambda q(x) \alpha$, and quasi-rents to water $R_w(x)$ accruing to the utility at each location $x$ as $R_w(x) = \lambda(x)q(x)\alpha - \nu k(x)$. Then the increase in shadow prices causes a decrease in land rents and an increase in water rents over distance (Proposition 3). However, water rents at any given location need not accrue to firms located there, except under specific property rights arrangements, such as when the ownership of land and water rights are parceled together.

Monotone decreasing output and land rent functions indicate that the von Thünen results for land use are preserved. It also suggests a skewed distribution of resource rents over space. Land rents are higher at upstream locations while rents per unit water increase downstream of the source. If the planning agency charges shadow prices, the resulting income distribution will favor upstream firms. This "inequity" may be resolved by distributing surplus water rents as lump-sum compensation. By differentiating the rent functions, one can derive comparative statics results (see Proposition 4) which suggest that an increase in conveyance investment saves water and increases water rents, while simultaneously decreasing land rents. Similarly, higher conservation by firms has the opposite effect of transferring resource rents from water to land owners.

The above discussion indicates that investment in conveyance is likely to shift economic rents from owners of land to holders of water rights, and from upstream to downstream firms within a project. The effect of conveyance in reducing aggregate land rents leads us to conjecture that to the extent political power lies with the landowning firms, they will oppose rehabilitation of water projects through pricing reform and "public" conveyance investment. These pressures will be stronger if, as is often the case, the larger and hence more powerful firms are located at the head reaches [18]. To some extent, these factors might explain the low level of political interest in allocating funds for maintenance. On the other hand, better conveyance may be politically favored if the more influential firms are located at the tail. In either case, successful implementation of policies aimed at rehabilitating existing water projects may have to be accompanied by appropriate
mechanisms that distribute the surplus accruing to the utility from the introduction of optimal water pricing. For instance, firms that are adversely affected by conveyance improvements could be made shareholders of the water utility so that they are able to appropriate a portion of the water rents. The distribution of shares, for example, could be inversely related to distance from the water source.

3. COMPARISON WITH PROJECTS WITHOUT PROVISION OF CONVEYANCE

In this section the above solution is compared with a stylized model of a water project characterized by underprovision of conveyance. In reality, water allocation and pricing schemes vary considerably across legal and institutional regimes, and across (often within) countries. Thus, the comparison is mainly illustrative, and is meant to capture only the essential features of resource allocation and pricing behavior commonly found in water projects.

Consider the case of a utility that supplies water to firms but fails to invest optimally in the distribution canals. Such projects could include those in which canals and related control structures may have been working efficiently during commissioning of the project but have deteriorated over time because of poor or no maintenance. This problem often arises because public maintenance budgets are generally spread thinly over too many projects, while collection of water charges (rarely enough to cover annual operation and maintenance) is generally done by the revenue department and goes directly into the state coffers.\(^\text{10}\) Various surveys of the performance of water projects attest to the above scenario (see [13], and studies cited therein). Thus, two different pricing structures are examined, as follows.

The Uniform Pricing Solution

The utility is assumed to charge a spatially uniform price per unit of water delivered, the price being equal to the marginal cost of water generation at source. The qualitative results derived for this case are likely to hold for other uniform pricing schemes such as when each individual firm pays a fixed water charge (unrelated to water use) based on output produced or when the water charge is proportional to the amount of land utilized by each firm.

Without centralized canal investments, an individual firm is assumed to be sufficiently “small” such that it obtains negligible benefit from investing in conveyance at its own location, although it may benefit firms located downstream from it. The presence of increasing returns to scale in conveyance imply that no individual firm will invest in conveyance from the source to its own location. Thus under uniform pricing, firms do not invest in conveyance, i.e., \( k = 0 \), implying that the distribution canals will be unlined with a water loss rate of \( \alpha_0 \). Denoting firm profits under uniform pricing \( \Pi \), each firm located at \( x \) solves:

\[
\max_{q, I, k} \Pi = [pf(q_0(I)) - \lambda q - I_0] \alpha. \tag{13}
\]

\(^{10}\)In order to alleviate this problem, Thailand has enacted legislation that commits water fees to project operation and maintenance. This option is being considered by other countries as well [12].
Observe that (13) is independent of \( x \), and the necessary conditions are obtained as

\[
\frac{\partial \Pi}{\partial q} = pf'(\cdot)h(I) = \lambda_0, \tag{14}
\]

and

\[
\frac{\partial \Pi}{\partial I} = pf'(\cdot)qh'(I) = 1. \tag{15}
\]

The utility is assumed to choose aggregate water use \( z_0 \) by solving problem (11) with the additional constraint \( k = 0 \).\(^\text{11}\) Let the subscript \( u \) denote parameter values for this model. Then for any given level of \( z_0 \), aggregate net benefits in the constrained model must be lower than in the optimal model by the Le Chatelier Principle. Therefore, the optimal marginal net benefit function \( MNB^*(z_0) \) must be higher than the constrained function \( MNB^u(z_0) \) at least for small values of \( z_0 \) (see Fig. 1). Thus, in regions with water scarcity, for which it is plausible to assume that optimal values of \( z_0 \) are not too large, the marginal cost curve \( g'(z_0) \) cuts the constrained marginal benefit function below the optimal. This results in a smaller equilibrium value of \( z_0 \) in the uniform pricing model and a lower shadow price of water at source, as shown in the figure. Firms respond to the lower price of water by using too much of it and reducing spending on conservation which is now relatively more expensive. Cheap water creates super-normal land rents at each location. The combined effect of a lower water stock, excess withdrawals and higher conveyance losses in the uniform pricing model results in a smaller project area (i.e., a shorter canal).

It is possible that the marginal net benefit functions (Fig. 1) never intersect in which case our results hold. Alternatively, in the less likely case of \( z_0^u < z_0^s \), the price of water at the source will be higher than optimal, hence water use will be lower and on-farm investment higher. Because of uniform pricing, however, aggregate water losses in conveyance will be higher. Aggregate output is likely to be

\(^{11}\) Under uniform pricing, the utility's budget may not balance and its operations may have to be subsidized, a phenomenon commonly observed in practice [13].
lower than optimal. The effect on the length of canal is indeterminate and will depend on the conveyance loss rate.

The Water Market Solution

It is now assumed that there is a well-functioning market for the transfer of water among firms but no conveyance is provided by the central authority. We abstract from specifying the actual process of transition to a market and assume that the necessary legal and institutional restrictions to trading in water rights have been removed such that the price of traded water at each location equals its shadow price. Clearly, in terms of resource allocation, the water market solution is equivalent to the case in which the utility charges the shadow prices derived from a model with zero conveyance investment. The maximization problem for each firm is exactly similar to (13) except that the price of water is now $\lambda(x)$. Conditions (14, 15) also hold with this added qualification.

The underprovision of conveyance in the water market case leads to a lower than optimal shadow price of water at source just like in the uniform pricing case. Cheaper water in upstream regions gives rise to a resource allocation pattern similar to that under uniform pricing, i.e., firms increase water use, reduce conservation, and produce higher output relative to the optimal case. However, in contrast to the uniform pricing model, the introduction of a water market causes the shadow price of water to increase further away from the source, at a higher than optimal rate (equal to the conveyance loss rate, $a_0$; check condition (9)). Therefore, as shown in Fig. 2a, the water market shadow price path cuts the optimal from below. Downstream firms under a water market pay higher prices for water, invest more in conservation, produce less output (Fig. 2b), and accrue lower land rents (Fig. 2c) relative to the optimal. A higher rate of increase in the shadow price ensures that the length of the canal is smaller than optimal. These results suggest that a water market with inadequate provision of conveyance leads to a skewed distribution of project benefits that favors firms located upstream, while those located downstream are adversely affected. As is clear from Fig. 2, the reduction in the project area also implies that some downstream firms which are serviced under the optimal solution will be excluded in the water market setting.

4. AN ILLUSTRATION

This section presents a simple illustration of the optimal model and the two cases described in Section 3 by using typical cost and demand parameters for western U.S. agriculture. A quadratic production function for California cotton is derived in terms of effective water $e$ such that a maximum yield of 1500 lbs. can be obtained when $e = 3.0$ acre–ft while $e = 2.0$ acre–ft produces an yield of 1200 lbs. [8]. Using cotton prices of US$0.75 per lb. gives the revenue function

$$ pf(e) = -0.2224 + 1.0944 \cdot e - 0.5984 \cdot e^2, \quad (16) $$

where revenue is in US$, and $e$ is in $m^3/m^2$ of water. Differentiating (16) with
respect to \( e \) gives the value of marginal product function

\[
pf'(e) = 1.0944 - 1.1968 \cdot e. \tag{17}
\]

The on-farm conservation function is approximated from cost estimates of investing in irrigation technologies in California as shown in Table I. When furrow irrigation is applied, it is assumed that there is no investment cost to the farmer so that \( h(0) = 0.6 \), i.e., 60% of the water delivered at the farm-gate reaches the plant. The function \( h(I) \) increases at a decreasing rate as more sophisticated technologies such as sprinkler and drip are employed, and is approximated as a continuous function of \( I \) as

\[
h(I) = 0.6 + 21.67 \cdot I - 333.3 \cdot I^2, \tag{18}
\]

where \( I \) is in \$/m². Fixed costs for irrigated farming are taken to be $433 per acre
or $0.107/m²$ [16]. A quadratic function for conveyance investment was constructed from average lining and piping costs in 17 states in the western United States [15, Table 15, p. 87]. An investment of $200/m length of canal in piped systems results in zero conveyance losses in the system. Concrete lining with an investment of $100/m attains a loss factor of $10^{-5}/m$ or a conveyance efficiency of 0.8 over a 20-km length of canal. For simplicity, we assume a relatively "flat" canal cross section, so that $v$ is approximated to be unity. When $k = 0$, the loss factor is $4 \times 10^{-5}/m$, giving an overall conveyance efficiency of 0.2. Thus we get

$$a = 4 \cdot 10^{-5} - (4 \cdot 10^{-7}k - 10^{-9}k^2)$$

so that from condition (2), $a_0 = 4 \times 10^{-5}$, and

$$m(k) = 4 \times 10^{-7}k - 10^{-9}k^2, \quad 0 \leq k \leq 200.$$  

The conveyance loss figures are consistent with findings from engineering studies [3]. The exact loss coefficient, however, would depend on soil characteristics, ambient temperatures, and other environmental factors. The results were found to be generally insensitive to variations in the value of $a_0$.

A rising long-run marginal cost function for water supply was constructed from average water supply cost data from 18 irrigation projects in the western United States [19] as

$$g'(z_0) = 0.003785 + (3.785 \times 10^{-11}z_0),$$

where marginal cost is in $ and $z_0$ is in m³. It gives a marginal cost of 0.003785$/m³ ($4.67 per acre-ft) when $z_0 = 0$, and marginal cost values in the range 0.068 to 0.16 $/m³ (93.34 to 195.9 $/acre-ft) for the various models analyzed (see Table II). A linear functional form was assumed to keep the formulation simple. For computational purposes, the width of the rectangular cropped area $a$ is taken to be $10^5$ m. The width, of course, does not affect the relative orders of magnitude across models.

A computer algorithm was written that starts by assuming an initial value of $z_0$, and computes $\lambda_0$ from (12). At $x = 0$, (7) gives $m'(k)$. By iterating on $k$, we compute $k(x)$ that satisfies (20), and (19) gives $a(x)$. Knowing $\lambda_0$, (6) and (8) solved simultaneously yield $I(x)$, $q(x)$, and thus $e(x)$, $y(x)$, and $R(x)$, respectively. Next, when $x = 1$, using $a(0)$ and $\lambda_0$ in the solution to (9) gives $\lambda(1)$, and $z(1)$ is obtained from (1) by subtracting the water already used up previously. Again, $\lambda(1)$


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimal</td>
</tr>
<tr>
<td>Aggr. Net Benefit (10^8 $)</td>
<td>3.34</td>
</tr>
<tr>
<td>Area (10^3 ha)</td>
<td>490</td>
</tr>
<tr>
<td>Water Stock (10^5 m^3)</td>
<td>41</td>
</tr>
<tr>
<td>Aggr. Output (10^5 $)</td>
<td>13.28</td>
</tr>
<tr>
<td>Aggr. Land Rent (10^4 $)</td>
<td>0.137</td>
</tr>
<tr>
<td>k_{head} ($/m)</td>
<td>199.39</td>
</tr>
<tr>
<td>k_{slip} ($/m)</td>
<td>165.07</td>
</tr>
<tr>
<td>q_{head} (m/m^2)</td>
<td>0.835</td>
</tr>
<tr>
<td>q_{slip} (m/m^2)</td>
<td>0.835</td>
</tr>
<tr>
<td>I_{head} ($/m^3)</td>
<td>0.023</td>
</tr>
<tr>
<td>($/acre)</td>
<td>(93.1)</td>
</tr>
<tr>
<td>I_{slip} ($/m^3)</td>
<td>0.023</td>
</tr>
<tr>
<td>($/acre)</td>
<td>(93.1)</td>
</tr>
<tr>
<td>λ_{head} ($/m^3)</td>
<td>0.1589</td>
</tr>
<tr>
<td>($/acre-ft)</td>
<td>(195.9)</td>
</tr>
<tr>
<td>λ_{slip} ($/m^3)</td>
<td>0.1593</td>
</tr>
<tr>
<td>($/acre-ft)</td>
<td>(196.5)</td>
</tr>
<tr>
<td>R_{head}(10^8 $)</td>
<td>0.28</td>
</tr>
<tr>
<td>R_{slip}(10^8 $)</td>
<td>0.25</td>
</tr>
</tbody>
</table>

*a Since the algorithm is discrete, "tail" values are given for the nearest integer km. Therefore, these values are not at the boundary, but in its neighborhood.

*b The length of the canal is different in each model, so the location of the tail is unique in each case.

and z(1) give k(1) from (7) and the cycle is repeated to give q(1), I(1), etc. The process is continued with increasing values of x until exhaustion of z_0 terminates the cycle, and a new value of z_0 is assumed. Aggregate land rents are calculated for each z_0 by summing over R_i(x) and aggregate rents to water are computed similarly. The algorithm selects the value of z_0 that maximizes total net benefits (given by (11)), which must also equal the sum of aggregate land and water rents. The algorithm was modified suitably for the water market and uniform pricing models.

**Simulation Results**

Simulations were run for the three models discussed earlier. The results are shown in Table II. They can be summarized as follows:

1. Net benefits (output) from the project with optimal conveyance are more than two (three) times those from the models with no conveyance. Project area and, therefore, the length of canal (since width is assumed constant) are also more than three times in the optimal case.

2. Aggregate land rents are much lower in the model with conveyance and are highest in the uniform pricing model where firms pay a fixed rate for water that does not vary with location. Similarly, land rents accruing to upstream firms are
considerably lower when conveyance is provided. This suggests that upstream firms
would tend to lose from an improved water distribution system. However, land rent
changes will be positive for those downstream firms which are brought under the
project because of project improvements.

3. Conveyance improvements reduce water losses which in turn even out
shadow price differentials across locations. Thus, the variation in head–tail shadow
prices are small and differences in water and technology investments from head to
tail are negligible (they equal zero when rounded off to two decimal places). The
significant difference between head and tail allocations in the water market model
suggests that water trading under large conveyance losses will contribute to spatial
"inequity" between upstream and downstream firms.

5. CONCLUDING REMARKS

The above results have several policy implications. They suggest that upgrading
existing projects through lining of canals and optimal pricing would release water
that could be used either to expand acreage and output, or for meeting competing
urban and industrial needs. These policies can also help mitigate adverse environ-
mental impacts such as excessive seepage and waterlogging caused by dysfunctional
water projects. The magnitude of gains from rehabilitation imply that future water
policy should aim toward improvement of existing systems rather than building
expensive new water projects [10]. However, for any such policy to succeed, the
negative distributional effects on upstream beneficiaries will have to be recognized
and taken into account during implementation.

Although the present analysis is static, the problem highlighted is essentially
dynamic in nature. It is conceivable that soon after the commissioning of a project,
project structures are still in place so that conveyance losses are low and the system
behaves somewhat like our optimal model. But with time, if maintenance is poor,
the control structures deteriorate, and conveyance losses increase. Effective project
coverage is reduced, and firms located toward the tail of the project get left out as
the project "shrinks." Land rents at the head rise, but overall output and net
benefits decrease. The performance of the system diverges from optimality and
moves toward the stylized cases examined in Section 3.

It is straightforward to observe that in a dynamic setting, an increase in the
interest rate will increase conveyance costs, thereby decreasing conveyance invest-
ments and causing increased water losses, shorter canals, and more intensive water
use nearer the source. An increase in maintenance costs over time will have an
analogous effect. In that case, the utility may need to carry out periodic overhaul of
the distribution system. Before each such overhaul, there may be periods when
increased maintenance costs increase water prices and cause "shrinking" of the
system.

One important factor ignored in the analysis is that water lost in conveyance
often ends up as groundwater recharge [7]. Then the cost of seepage through
canals is not the foregone marginal product of water but the cost of groundwater
pumping which then has to be compared to conveyance costs. In water districts
such as in California's Central Valley which practice conjunctive use, frequently
the seepage of the wet years replenishes the groundwater aquifer which is then
used in periods of drought.\footnote{We would like to thank an anonymous referee for drawing our attention to this issue.} However, seepage may also contribute to waterlogging in which case conveyance losses may impose additional cost. These added benefits and costs from water losses need to be incorporated in an extended model of conveyance.

Similarly, a more sophisticated model should consider the costs of administering the conveyance system. These costs could be higher under optimal conveyance because of (i) the need to undertake regular operation and maintenance, (ii) computing and implementing a price system that varies over location (although spatially uniform water prices that equal the shadow price at source may serve as a good approximation, as shown by Table II), and (iii) set-up costs in communication and negotiation that may be incurred in upgrading project facilities. Other administrative costs such as in monitoring and enforcement and in the measurement of water deliveries may also be higher under the optimal regime. It is possible that in regions where conveyance losses are low (say, because of topography) and the cost of upgrading the management system is high, transition to optimal conveyance may reduce aggregate welfare. However, it is less likely that these two conditions may both be satisfied by the same project. For instance, in developed countries, the additional costs of public management may be relatively low. Similarly, in developing countries, although improving management may be relatively costly, the additional benefits from conveyance may also be higher. In addition, administrative costs could be falling over time because of technological change. These issues are important, and need to be examined formally in future work.

The proposed model can also be extended by considering the possibility of supplementing water supply through rainfall or groundwater sources. If private groundwater extraction were allowed, optimal extraction might increase with distance from the source, similar to the spatial distribution of conservation technology. The stochastic nature of water supply and storage as well as land quality and topographical variations within the project are other dimensions that need to be explicitly considered in the model.

\section*{APPENDIX}

\textbf{PROPOSITION 1.} (i) $q'(x) < 0$; (ii) $I'(x) > 0$; and (iii) $k'(x) < 0$.

\textit{Proof.} (i) Differentiating (8) which is an identity, we get

\begin{equation}
I'/I = -q'/q\left[(\eta_I + 1)/(\eta_I\eta_h + \eta_h^2)\right],
\end{equation}

where $\eta_h = h'(1)/h(1) > 0$ and $\eta_I = h''(1)/h'(1) < 0$ are the elasticities of the firms' investment and marginal returns to investment in conservation, respectively. Similarly, condition (6) yields $\eta_I q'/q + [(1 + \eta_I)\eta_h]I'/I = a$. Simplifying, and substituting for $I'/I$ from (A1), we obtain

\begin{equation}
q'/q[\eta_I\eta_h - \eta_h - 2\eta_I\eta_h] = [\eta_I\eta_h + \eta_h]a < 0
\end{equation}

since $\eta_I < -1$. Noting that $|\eta_h| < |2\eta_I\eta_h|$ yields $q'(x) < 0$.\footnote{We would like to thank an anonymous referee for drawing our attention to this issue.}
(ii) Substitute \( q'(x) < 0 \) in (A1) and use proof of (i).

(iii) Differentiating (7) gives \( \eta_\mu'(k)\alpha = -k'/\lambda - z'/z = q(x)\alpha/z \) using (1) and (9), where \( \eta_\mu(k) = mm'(k)/m'(k) < 0 \). Therefore \( k'(x) < 0 \).

Q.E.D.

**Proposition 2.** (i) \( Q'(x) > 0 \); and (ii) \( WC'(x) > 0 \).

**Proof.** (i) Consider a small strip of canal of length \( dx \). If \( Q \) units of water enter the strip from one end, the conveyance loss \( dQ \) in flowing through this strip is \( Qa(x) \cdot dx \). Hence \( dQ = -Qa(x) \cdot dx \). Rearranging and integrating both sides gives

\[
Q(x) = q(x)\exp\left(\int_0^x a(\tau) \, d\tau\right).
\]

(A3)

Integrating (A2) gives

\[
q(x) = q(0)\exp\left(\int_0^x A(a(\tau)) \, d\tau\right),
\]

(A4)

where \( A = [\eta_\mu'\eta_\epsilon + \eta_\mu]/[\eta_\mu'\eta_\epsilon - \eta_\mu - 2\eta_\mu'\eta_\epsilon] < 0 \) by the assumption \( \eta_\mu' < -1 \).

Combining (A3) and (A4) yields

\[
Q(x) = q(0)\exp\left(\int_0^x a(1 + A) \, d\tau\right).
\]

(A5)

Differentiating (A5) and using the fact that \( |A| < |\eta_\mu'\eta_\epsilon + \eta_\mu'|/|\eta_\mu'\eta_\epsilon - \eta_\mu - 2\eta_\mu'\eta_\epsilon| < 1 \) gives \( Q'(x) > 0 \).

(ii) By the definition of \( WC(x) \), using (9) and (A2) yields \( WC'(x) = [A(x)q(x) + \lambda(x)q'(x)]\alpha = q\alpha(1 + A)\lambda a > 0 \), because \( |A| < 1 \).

**Proposition 3.** (i) \( e'(x) < 0 \); (ii) \( y'(x) < 0 \); (iii) \( R_1'(x) < 0 \); and (iv) \( R_2'(x) > 0 \).

**Proof.** (i) Differentiating the definition of \( e (= qh(1)) \) gives \( e'/e = q'/q + \eta_\mu I'/I - [(\eta_\mu - \eta_\epsilon)/(\eta_\mu'\eta_\epsilon + \eta_\mu)]q'/q < 0 \) using (A1), canceling terms and substituting \( q'(x) < 0 \) and \( \eta_\mu' < -1 \).

(ii) Again, by definition, \( y'(x) = f(e)e'(x)\alpha < 0 \) by part (i).

(iii) By differentiation, \( R_1'(x) = (pf'h'I'q + pf'h'q' - I')\alpha - \lambda q(x)\alpha - \lambda q'(x)\alpha = -\lambda'(x)q(x)\alpha < 0 \) using (6), (8), and (9) and canceling terms.

(iv) Differentiating \( R_2(x) \) and using (9) gives \( R_2'(x) = aq\lambda a(1 + q'/qa) \sim vk' \).

Since \( vk' < 0 \) by Proposition 1(iii), noting that \( 1 + q'/qa = 1 + A > 0 \) by proof of Proposition 2(i) yields \( R_2'(x) > 0 \).

Q.E.D.

In what follows, we make the plausible assumption that the ratio of the perimeter to the sectional area of the canal \( v \) is small in relation to the width of the project area \( \alpha \), i.e., \( v/\alpha \approx 0 \).

**Proposition 4.** (i) \( \partial R_2/\partial k > 0 \); (ii) \( \partial R_2/\partial k < 0 \); (iii) \( \partial R_2/\partial I < 0 \); and (iv) \( \partial R_1/\partial I > 0 \).

**Proof.** (i) By the definition of \( R_2 \), \( \partial R_2/\partial k = \partial/\partial k[\lambda q\alpha] - v = \alpha[\partial/\partial k(\lambda q) - v/\alpha] = \alpha\partial/\partial k(\lambda q) \) by assumption. So \( \partial R_2/\partial k = \alpha\partial/\partial k[\lambda q + \alpha(q + q) + q] \).

From (7), \( \partial\lambda/\partial k = \lambda m''/z(m'i)^2 > 0 \), and \( \lambda \partial q/\partial k + q = q(\lambda q' + q) > 0 \) by (6) and \( \eta_\mu' < -1 \). Therefore \( \partial R_2/\partial k > 0 \).
(ii) Similarly, \( \partial R_N / \partial k = \alpha [ \rho f^2 h (I) \partial q / \partial k + \rho f^2 h (I) \partial I / \partial k - \partial I / \partial k - \partial (\lambda q) / \partial k] \). Using (6) and (8) and canceling terms, \( \partial R_N / \partial k = \alpha \left[ \lambda \partial q / \partial k - \partial (\lambda q) / \partial k \right] = - \alpha \lambda \partial q / \partial k < 0 \) since \( \partial q / \partial k > 0 \) from proof of part (i).

(iii) Using the chain rule, \( \partial R_w / \partial I = \partial R_w / \partial k \cdot \partial k / \partial I < 0 \) using part (i), \( \partial q / \partial k > 0 \) from proof of part (i) and \( \partial q / \partial I \) (obtained from (6)).

(iv) Again, \( \partial R_N / \partial I = \partial R_N / \partial k \cdot \partial k / \partial I > 0 \) by part (ii) and proof of part (iii).

Q.E.D.

REFERENCES