Water allocation under distribution losses: comparing alternative institutions

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Water allocation under distribution losses: Comparing alternative institutions

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Abstract

The distribution of water resources is characterized by increasing returns to scale. Distribution links water generation to its end-use. Standard economic analysis overlooks the interaction among these micro-markets – generation, distribution and end-use. We compare water allocation when there is market power in each micro-market. These outcomes are compared with benchmark cases – social planning and a competitive business-as-usual regime. Simulations suggest that institutions with market power in generation and end-use generate significantly higher welfare than the distribution monopoly and the competitive regime. However, if the policy goal is to maximize the size of the grid, a distribution monopoly is preferred.

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1. Introduction

Water management has been called the most significant challenge of the 21st century. Most water is publicly provided by vertically integrated state utilities because the distribution of water is characterized by increasing returns to scale. Various studies have shown that public ownership and management of water have led to serious inefficiencies including “weak incentives to reduce costs, implement marginal cost pricing or maintain water systems” (Cowen and Cowen, 1998).¹

Water reform has often meant the creation of markets at the downstream end where water users may buy and sell water as well as the management of generation and distribution facilities. However, market behavior downstream is closely linked to the upstream generation and distribution of water. Market power may occur at the supply, distribution or end-use stage. The organizational structure in any given micro-market affects the performance of the system as a whole. The focus of this paper is the integrated analysis of the microstructure of such markets in which the overall effect of market power in any one segment of the market can be examined.

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¹ They suggest that the record of governmental provision in the water sector is “extremely poor.” In developing countries, tariffs are routinely set below cost recovery levels, often less than half the water supplied is paid for by beneficiaries, and large segments of the population are not connected to the grid.

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There is a significant body of literature that suggests the existence of market power in the generation, distribution, and end-use of water. Agricultural water districts in California buy water from the government and the Bureau of Reclamation and often wield monopsony power in the water market.\(^2\) Global water companies such as Vivendi of France operate in over 100 countries and act as distribution monopolies that supply water to users. Giant agribusiness firms as well as producer cooperatives in the US, which by law are exempt from anti-trust regulation, organize production and have market power in the output market.\(^3\) In this paper, we develop a model that examines how market power in water generation, distribution, and end-use affects social welfare, equilibrium output and price, volume of service and grid size. We compare these stylized institutions to two polar cases: social planning and a competitive regime characterized by market failure in distribution. The main policy implication is that given increasing returns to scale in distribution, the introduction of water markets need not automatically lead to efficiency. The choice of a second-best institutional regime may depend upon the precise characteristics of the microstructure of the water market.

In our spatial framework, water is generated at a given location and sold to a utility that maintains the distribution network. This utility supplies water to firms located along the distribution system, which in turn use it to produce output. The analysis focuses on market power in generation, distribution, and in the final product market. We show that under competition in distribution (CID) and an end-use monopoly, less water is likely to be provided in the aggregate. Both regimes result in a higher end-use price, lower output, and a smaller-than-optimal service area. To produce the same aggregate output, the monopoly uses less water than the competitive regime.

Simulations with stylized data suggest that institutions with market power in generation and end-use generate significantly higher welfare than the distribution monopoly and business-as-usual regime. However, the distribution monopoly maximizes service area coverage. Using the same amount of water, it serves twice the geographical area than an output cartel. If the policy objective is maximizing the service area and bringing more consumers into the grid, the distribution monopoly may be preferred even though it produces lower economic welfare. It may be the regime of choice in water scarce regions. The analysis suggests that proposals for intervention in water markets must be informed by the choice of an appropriate institutional delivery system. Which institutional performs better depends upon the conditions in each of the micro-markets and their interlinkages through technology.\(^4\)

Section 2 describes a vertically integrated water model with distribution. In Section 3, we examine the specific institutional alternatives and compare their characteristics. Section 4 provides an illustration. Section 5 concludes the paper.

2. The model

For ease of exposition, we develop the model for water distribution in farming, which is by far the largest user of water.\(^5\) The insights may hold with some modifications, for other commodities and services distributed over space. The basic structure of the model is similar to that of Chakravorty et al. (1995), here referred to as CHZ. We extend their framework by modeling market structure at the generation, distribution and end-use markets.\(^6\)

Consider a simple one-period model in which water is generated at a point source, e.g., a dam or a diversion in a river or a groundwater source. Let \(z(0)\) denote the amount of water generated at this location at cost \(g(z(0))\), assumed to be an increasing, differentiable, convex function, \(g’(z(0)) > 0, g''(z(0)) > 0\). If there are multiple sources, then the cheapest one is selected for each marginal unit generated. This water is sold to the canal authority which manages the distribution system.\(^7\) Generation and distribution may be operated independently or by one vertically integrated firm, which we discuss below.

The canal company supplies water to identical users located at distance \(x\) over a continuum on either side of the canal, where \(x\) is measured from the source. Let \(r\) be the opportunity rent per unit area of land. Without loss of generality, let the constant width of land be unity. Each firm occupies a unit of land, so that the number of producers is proportional to the length of the canal. Let the price of water charged by the canal company at any location \(x\) be \(p_w(x)\) and the quantity delivered be \(q(x)\). The fraction of water lost in distribution per unit length of canal is \(a(x)\). Let \(z(x)\) be the residual water in the canal at location \(x\). Then

\[ z’(x) = -q(x) - a(x)z(x) \tag{1} \]
where the right-hand side sums the water delivered and lost in distribution at x. The residual flow of water in the canal must decrease with x, \( z'(x) \leq 0 \). Let X be the canal length to be determined. Then \( z(0) = \int_0^X [q(x) + a(x)z(x)] dx \) so that \( z(X) = 0 \).

The loss function \( a(x) \) depends on \( k(x) \), defined as the annualized capital and operation and maintenance expenditure in distribution, which varies with location. Let the reduction in the distribution loss rate be given by \( m(k(x)) \). Then

\[
a(x) = a(0) - m(k(x))
\]

where \( m(\cdot) \) is assumed to be increasing with decreasing returns to scale in \( k \), i.e., \( m'(k) > 0 \), \( m''(k) < 0 \). Firms located along the canal use water \( q(x) \) to produce output \( y = f(q) \) where \( f(\cdot) \) is concave, i.e., \( f(q) > 0 \), \( f'(q) < 0 \), and \( \lim_{q \to 0} f'(q) = \infty \). The production technology is assumed to exhibit constant returns to scale with respect to all other inputs. Let \( Y \) denote aggregate output given by \( Y = \int_0^X f(q(x)) dx \).

2.1. Resource allocation by an integrated water utility

Let the social planner’s total cost of producing a given output level \( Y \) be \( C(Y) \), which can be expressed as the sum of the cost of water generation, distribution and land rent:

\[
C(Y) = g(z(0)) + \int_0^X [k(x) + r] dx
\]

The planner minimizes the cost of producing a given output level \( Y \) by choosing \( q(x) \), \( k(x) \) and values for \( X \) and \( z(0) \) as follows:

\[
\text{Minimize } g(z(0)) + \int_0^X [k(x) + r] dx
\]

subject to \( z'(x) = -q(x) - a(x)z(x) \)

\[
Y'(x) = f(q),
\]

And

\[
z(0)\text{free}, \ z(X) = 0, \ X\text{free}
\]

The corresponding Lagrangian is

\[
L = k + r + \lambda_w(q + az) - \lambda_f f(q)
\]

where \( \lambda_w(x) \) and \( \lambda_f(x) \) are shadow prices of water and output associated with \( 4(b) \) and \( 4(c) \). The necessary conditions for an interior solution are

\[
\lambda_f f'(q) - \lambda_w = 0
\]

\[
\lambda_w zm'(k) - 1 = 0
\]

\[
\lambda_w'(x) = \lambda_w(x)a(x)
\]

\[
\lambda_f' = 0
\]

\[
\lambda_w(0) = g'(z(0))
\]

and

\[
L(X) = 0
\]

Let \( C'(Y) \) denote the solution to the above problem. Define the consumers’ inverse demand function for aggregate output \( Y \) as \( D^{-1}(Y) \) with \( D^{-1}'(Y) < 0 \). The equilibrium aggregate output \( Y^* \) and price \( p^* \) (see Fig. 1) solve \( \max_Y \int_0^Y D^{-1}(\theta) d\theta - C'(Y) \) which yields

\[
D^{-1}(Y^*) - C''(Y^*) = 0
\]

---

\( ^9 \) Let \( m(0) = 0 \) and \( \lim_{k \to 0} m(k) = \infty \), which suggests that marginal returns to distribution investments approach infinity as \( k \) goes to zero, and \( a(x) = a(0) \) when \( k = 0 \). Then \( 0 < a(x) < a(0) \). Let \( m(0) = 0 \) and \( \lim_{k \to 0} m(k) = \infty \), which suggests that marginal returns to distribution investments approach infinity as \( k \) goes to zero, and \( a(x) = a(0) \) when \( k = 0 \). Then \( 0 < a(x) < a(0) \).

\( ^{10} \) At the boundary, \( \lambda_w(X) - \lambda_f(X) = \beta \), where \( \beta \) is a constant.

---

The assumptions on \( f'(0) \) and \( m'(0) \) suggest that \( q(x) > 0 \) and \( k(x) > 0 \) and thus (6) and (7) hold with equality. We avoid unnecessary complications by not attaching a multiplier to the state constraint \( z(x) > 0 \). Since \( z(x) \) is decreasing, the state constraint is never tight except possibly at \( x = X \). Since \( g(z(0)) \) is strictly convex, \(-a(z)\) is convex in \( k \) for given \( z \), and \( \lambda_w(x) \) and \( \lambda_f(x) \) are nonnegative, so the control problem satisfies the Mangasarian sufficiency theorem. Strict convexity implies uniqueness.
i.e., price equal to marginal cost.$^{11}$ From the maximum principle, $\dot{\lambda}_{w}(x)$ is continuous on $[0, X)$, and $q(x)$ and $k(x)$ are continuous except at $x = X$. We interpret $\dot{\lambda}_{w}(x)$ as the shadow price of delivered water at location $x$ and (8) says that it increases away from the source because of the cost of distribution. As established by CHZ, water use, output, and investment in distribution decrease away from the water source. An increase in the shadow price of water from head to tail causes a decrease in water use by firms. The shadow price increases, but the volume of water carried by the distribution system decreases at a higher rate because of water withdrawals by firms and distribution losses. The net effect is a decrease in the "value" of the residual water flowing in the system, causing a decrease in distribution investment. At the boundary of the grid $X$, (12) gives

$$L(X) = k(X) + r + \dot{\lambda}_{w}(X)[q(X) + az(X)] - \lambda_{yf}(q(X)) - \dot{\lambda}_{w}(X)z(X) = 0.$$ Substituting $z(X) = 0$ and $k(X) = 0$ and rearranging terms yields

$$\lambda_{yf}(q(X)) - \dot{\lambda}_{w}q(X) = r$$

which implies that net benefits from expanding service by one unit must equal the opportunity rent of land, $r$. A lower $r$ would imply a greater service area measured by $X$.

### 3. Alternative regimes for water management

We now discuss the systemwide effects of alternative institutional mechanisms for the generation, distribution and end-use of water. If government intervention is prohibitively costly or infeasible (e.g., when the government is weak), then this socially optimal allocation may not be achieved. The alternative institutions we consider may emerge through reform or because of some evolutionary process that we take as given. A polar extreme may entail total decentralization in which a utility may supply water but the distribution of water is delegated to individual firms at each location. Because of increasing returns to scale each firm will compete and attempt to free-ride by underinvesting in distribution. We compare the social planner and the competitive solution above with three regimes with market power in either water generation, distribution, or end-use. These are: a water-users association, which has market power in water generation and maintains the distribution system while charging each firm the true marginal cost of supplying water (Dosi and Easter, 2003); a canal operator that owns the generation facility (or buys water in a competitive market) but is a monopoly seller of water to individual firms; and finally, a vertically integrated utility that buys water competitively or owns the generation facility, supplies water at competitive prices to firms, and buys the output for sale in the product market, where it has market power.$^{12}$ These three cases highlight the effect of market power in the three micro-markets-water generation, distribution, and end-use-on welfare and resource allocation in the entire system.

Table 1 provides a taxonomy of the different institutional arrangements. For example, the model with CID may involve average or marginal cost pricing of water at each location. We consider different combinations of market microstructures (competition and monopoly) in the generation, distribution, and end-use markets. Below we discuss only a small subset of those shown in the table.

---

$^{11}$ For sufficiency we must have $D^{-1}(Y^{*}) - C^{*}(Y^{*}) < 0$. Since $f'(0)$ is large for small $q$, $D^{-1}(0) > g(0)$ ensures that a positive aggregate output will be produced.

$^{12}$ In reality, these various forms of organization may have some degree of market power in both factor and output markets. An alternative formulation may involve an oligopolistic structure in the input and output markets, left for future work.
Table 1

<table>
<thead>
<tr>
<th>Institution</th>
<th>Generation</th>
<th>Distribution</th>
<th>End-use</th>
<th>Water pricing to user</th>
</tr>
</thead>
<tbody>
<tr>
<td>CID</td>
<td>Competitive</td>
<td>Minimal</td>
<td>Competitive</td>
<td>Average cost</td>
</tr>
<tr>
<td>CHZ</td>
<td>CID water market</td>
<td>Competitive</td>
<td>Minimal</td>
<td>Marginal cost</td>
</tr>
<tr>
<td>Privatization</td>
<td>Social planner</td>
<td>Competitive</td>
<td>Competitive</td>
<td>Marginal cost</td>
</tr>
<tr>
<td>X</td>
<td>Water Users Assoc (WUA)</td>
<td>Monopsony</td>
<td>Competitive</td>
<td>Marginal cost</td>
</tr>
<tr>
<td>X</td>
<td>Water seller</td>
<td>Monopoly</td>
<td>Competitive</td>
<td>Marginal cost</td>
</tr>
<tr>
<td>X</td>
<td>Canal operator</td>
<td>Competitive</td>
<td>Monopoly</td>
<td>Monopoly pricing</td>
</tr>
<tr>
<td>X</td>
<td>Integrated water company</td>
<td>Monopoly</td>
<td>Competitive</td>
<td>Monopoly pricing</td>
</tr>
<tr>
<td>X</td>
<td>Producer cartel</td>
<td>Competitive</td>
<td>Monopoly</td>
<td>Marginal cost</td>
</tr>
<tr>
<td>X</td>
<td>WUA+cartel</td>
<td>Monopsony</td>
<td>Competitive</td>
<td>Marginal cost</td>
</tr>
</tbody>
</table>

Note: The institutions marked ‘X’ are analyzed in the paper. Markets with market power are shown in italics.

3.1. Competition in Distribution (CID)

This is the benchmark model that aims to capture the situation when there is no centralized distribution of water. Firms withdraw water from a rudimentary distribution system and make private investments in distribution taking investment by other firms as given. As we show below, this causes the usual free-rider problems since firms do not accrue all the benefits from their investment. Water losses in distribution will thus be higher than under social planning. For convenience, we assume that individual firms can engage in trading in water rights and thus pay spot shadow prices for all the benefits from their investment. This represents a setting in which the number of buyers of water in the generation market is small. We compare investment in distribution at any location given by \( \text{Max } p(f(q(x))) = \lambda_w(x) \). Investment at each location \( I \) is given by \( m'((k(l)+k_{-}(l))z(l)) = 1 \). Firm \( i \) chooses \( k_i \) to satisfy the above condition. Adding over all \( i \), the condition for investment in distribution at any location \( x \in [0, X] \) is \( m((\sum k(l))z(0)) = 1 \). Compare this condition to socially optimal investment in distribution given by \( (7) \), rewritten as \( \lambda_w'z^2m(k) = 1 \), where \( \lambda_w' \) represents the values of variables in the optimal model. CHZ show that \( \lambda_w' > \lambda_w \), and \( z(0) > z(0) \), i.e., the shadow price of water at source as well as aggregate water stock is higher in the optimal model. Then at locations close to the source, we have \( m((\sum k(l)) > m(k) \) so that \( (k(l) < k(x) \) since \( m(k) < 0 \). Since \( z(x) \) declines faster than \( z'(x) \), \( \lambda_w \) increases faster than \( \lambda_w' \) by \( (8) \), and there is sub-optimal investment in water distribution in the CID model.

Let the cost function for aggregate output be \( C^d(Y) \). Equilibrium aggregate output \( Y^d \) and price \( P^d \) are then obtained as \( \text{Max} \int_0^Y D^{-1}(y)d\theta - C^d(Y) \) which as in \( (13) \) yields \( D^{-1}(Y^d) - C^d(Y^d) = 0 \) and \( D^{-1}(Y^d) - C^d(Y^d) < 0 \). For the CID case, output price equals the constrained marginal cost of producing output with sub-optimal distribution.

3.2. The Water Users Association (WUA) – an input monopsony

The WUA is an entity whose market power lies in the market for water generation. It is a price-taker in the output market. This represents a setting in which the number of buyers of water in the generation market is small. We compare the WUA with the output cartel and show how market power in the input and output markets may differentially affect resource allocation. Examples of such “input cooperatives” are California water districts which acquire and store water and

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13 Wade (1987) provides evidence on the gradual breakdown of a canal system in South India because of corruption and rent-seeking. Poor maintenance of canals induces losses from leakage as well as theft. Often the upstream beneficiaries steal water from the canals, depriving downstream users of their entitlements. Ray and Williams (2002) discuss the incentives for water theft and cooperation along a canal.

14 An alternative model with uniform pricing over space, e.g., a water charge in the form of a land or output tax, has been examined by CHZ. Our assumption of marginal cost pricing is conservative, since uniform pricing is less efficient and will lead to greater inefficiencies.

15 Each firm buys water at cost at the source and chooses investment in distribution.
allocate it to users. “These quasi-governmental entities can appropriate water, construct reservoirs and distribution systems, and enter into contracts with federal or state local suppliers” (Congressional Budget Office, 1997). The biggest of them serves more than half a million acres. They contract to buy surface water supplies from either the State or the Bureau of Reclamation. “Water is allocated in large blocks controlled by strong local districts. They influence the terms under which virtually all of the water in California’s Central Valley is used” (National Research Council, 1992). In other states, the most common type of water supply organization is a mutual water (or ditch) company. “Mutuals” are non-profit cooperative organizations that sell stocks or shares to members. These organizations determine retail water prices. They recover their costs by charging prices that may or may not be independent of the quantity used. More importantly, retail prices may be much higher than what the districts pay to suppliers such as the Bureau of Reclamation (Congressional Budget Office, 1997).

We model the WUA as a monopsonist that buys \( z(0) \) units of water to maximize total net benefits given by

\[
R(z(0)) \quad D_w(t) \quad d_t / C_0 - C_0(z(0)) z(0)
\]

where \( D_w(z(w(0))) \) is the derived demand for aggregate water for the association. The necessary condition is:

\[
D_w(z(w(0))) = MFC(z(w(0))) z(w(0)) + C_0(z(w(0))) / C_17
\]

where \( MFC(z(0)) \) is the marginal factor cost of \( z(0) \) and \( z(w(0)) \) is the solution to the above problem (see Fig. 2). The WUA allocates this aggregate level of service \( z(w(0)) \) efficiently in production. Given the imperfect input market, the cost of producing aggregate output is likely to be higher than for the social planner. The WUA is competitive in the output market so that price equals marginal cost of output.

3.3. The canal operator – a distribution monopoly (DM)

The canal operator owns the canal, buys water from the generator and sells it as a monopolist to individual firms along the canal.\(^\text{16}\) The operator also manages the distribution system. Firm output is sold competitively. There is substantial evidence to suggest that past privatization efforts in water have meant the sale of distribution assets from state-owned utilities to private monopolies which then own and manage the canal system. For example, British Waterways, a state-owned water utility is seeking private partners for its 2000-mile national water network. Its privatization will lead to sales of water to industrial users and other water businesses and restore “stretches of the canal system” to move water to areas including Wales and Scotland (Beautiful Britain, 2004). Suez Lyonnaise, and Vivendi, two French Fortune 100 companies are major private distributors of water and have acquired several US water distribution companies including United Water Resources and US Filter Corporation. At one time, Enron acquired several water distribution companies around the globe in a failed attempt at water privatization.\(^\text{17}\)

Let \( \lambda_w(x) \) be the marginal cost of water at location ‘x’ and \( p^c \) be the equilibrium output price. The operator is a water monopoly and charges a water price \( p_w(x) \) at each location. It chooses aggregate water use \( z^*(0) \), distribution

---

\(^\text{16}\) This model is similar to the Independent System Operator (ISO) with market power in electricity transmission (see Joskow and Tirole, 2000). Bardhan and Mukherjee (2006) examine an infrastructure delivery model in which a local government procures the service (e.g., water or electricity) from the utility and allocates it to heterogeneous groups (rich and poor) of users. Their focus is on financial decentralization and the effect of bureaucratic capture on service delivery.

\(^\text{17}\) Enron’s efforts to set up markets for trading water are described in Smith and Lucchetti (2000).
investment $k^c(x)$ and water supplied at each location $q^c(x)$ to solve

$$\text{Maximize} \quad \int_0^x [p_w(q)q(x) - k(x)]dx - g(z(0))$$

subject to conditions (1) and (2), where $p_w(q)$ is the derived demand for water at each location given by $D^{-1}(Y)f(q,q_i)$. The necessary condition for $q^c(x)$ is $p^c f''(q^c(x))q^c(x) + p^c f'(q^c(x)) = \lambda_0^w(x)$, which implies that the marginal revenue at each location equals marginal cost (which varies by location). Investment in distribution is given by (7), with $\lambda_0^w(x)$ as the relevant marginal cost. Aggregate water use is determined by $\lambda_0^w(0) = g(x(0))$. Aggregate output $Y^c$ is given by $D^{-1}(Y) - C^c(Y) = 0$ where $C^c(Y)$ is the marginal cost of output for the operator.

3.4. The producer cartel (PC) – an output monopoly

The PC organizes water distribution and production. It may either own the generation facility, or equivalently, buy aggregate water at marginal cost. It supplies water at each location through a trading or rationing scheme and has market power in the end-use market where it operates as a monopoly. Examples of such market power include large agribusiness firms such as Archer Daniels Midland, Dole Pineapple, Tyson Chicken, ConAgra, Cargill and the European dairy giant Parmalat, as well as various producer cooperatives. The Capper-Volstead Act of 1922 grants anti-trust exemption to producer cooperatives in the US so that they could overcome “destructive competition” among independent farmers.

For each output level $Y$, both the social planner and the PC solve problem 4(a)–4(d). Their total cost of producing $Y$ is identical since both allocate internal resources efficiently, including in distribution. Hence the cartel cost function is also given by $C^c(Y)$. Cartel output $Y^c$ is obtained by maximizing profits $I^c = D^{-1}(Y)Y - C^c(Y)$ so that $Y^c$ solves $MR(Y^c) - C^c(Y^c) = 0$ and $MR(Y^c) - C^{\text{opt}}(Y^c) < 0$. Let $p^c$ be the output price for the cartel. Then $p^c = D^{-1}(Y^c)$.

3.5. Comparison of alternative institutions

We first compare the cost of producing output for each regime. This can be stated as

**Proposition 1.** For any aggregate output level $Y$: (a) the PC has the same cost as the social planner, i.e., $C^c(Y) = C^s(Y)$; (b) the total and marginal costs of the competitive regime, the canal operator, and the WUA are higher than for the social planner. The slope of their marginal cost functions is also greater; i.e., $C^c(Y) > C^s(Y)$, $C^c(Y) > C^{\text{opt}}(Y)$, and $C^c(Y) > C^i(Y)$, where superscript $i$ denotes the three regimes in (b); and (c) the marginal cost functions $C^0(Y)$, $C^c(Y)$, $C^{\text{opt}}(Y)$ and $C^i(Y)$ are positively sloped.

**Proof.** See Appendix A.

Marginal costs for the social planner and the competitive (CID) case are shown in Fig. 1. Marginal costs for both the social planner and the PC are given by $C^c(Y)$. The marginal cost of output for CID is everywhere higher than optimal. The equilibrium price and output for the three regimes is also shown. The cartel produces a higher output at a lower price than the CID regime. The converse could also happen. The cartel could charge a higher price than the CID model if demand were relatively inelastic or if water losses contributed significantly to raising the marginal cost of output, or returns to investments in distribution were low. We now compare the CID model with social planning:

**Proposition 2.** Competitive distribution leads to lower than optimal output at a higher price, less aggregate water use and it serves a smaller number of firms.

**Proof.** See Appendix A.

Poor distribution not only implies a lower level of service (less aggregate water use) but a higher price in the end-use market since the cost of producing a given level of service is higher. The geographical coverage is smaller than optimal. The following compares the cartel to the socially optimal and CID regimes:

**Proposition 3.** (a) The cartel produces less output at a higher price relative to the social planner. It uses less aggregate water and services fewer firms. (b) To produce the same level of output, the cartel uses less aggregate water and services fewer firms than the CID regime.

**Proof.** See Appendix A.

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18 Sufficiency implies $p^c f''(q^c(x))q^c(x) + 2p^c f'(q^c(x)) < 0$.

19 Total net business volume of US producer cooperatives in 2000 was over $100$ billion (USDA, 2002).

20 A recent anti-trust analysis of dairy cooperatives states “we now see a small number of huge cooperatives, that have extended their power beyond the assemblage of farmers, stretching vertically by ownership and alliances through the chain of production and distribution, all the way to the retail level... we are finding higher and higher levels of concentration... that increasingly present a model quite different from market competition” (Miyakawa, 2004).
Given its market power in the end-use market, the cartel will price its product higher. However, the level of aggregate service (quantity of water) provided by the cartel is also lower, and service is restricted to a smaller area than for the social planner. This is true even though the cartel is a monopoly either in the market for water generation or distribution. The cartel provides more water at each location than in the model with poor distribution, because it enjoys higher water efficiencies.

The relationship between the WUA and the social planner is shown in Fig. 2. The aggregate marginal net benefit when water is distributed optimally is given by $D^w(z(0))$. The WUA chooses the aggregate stock using (16) and pays the supplier price $p_w$, which is lower than the optimal price $\lambda^*_w(0)$ and reflects its monopsony power. Aggregate water use is smaller than optimal. Aggregate output and grid size will also be lower than optimal. It is not clear if its output is lower or higher than the cartel or the CID regime. This depends on the specific characteristics of the input and end-use markets and the cost of distribution. If the cost of water generation is relatively high (flat $g(z(0))$), then the WUA may have a smaller degree of market power in the input market. Its behavior will be closer to that of the social planner. If the generation cost is inelastic, water use will be significantly below optimal, and grid size will be smaller.

Which regime may be a preferred second-best outcome? If the factor market is relatively elastic, as may happen in a region with abundant water resources, the buying price of water will be low even though the aggregate water use is lower. By (8), the price of water paid by a firm at any location will also be relatively low. This will mean a higher output at each location. If the returns from distribution investments are high, then the marginal benefit from using a larger stock of aggregate water is likely to be high (see Fig. 2) leading to a relatively flat marginal benefit curve. In that case monopsony power in the input market will lead to a significant shrinking of the service area. If end-use markets are elastic, as in the case of an export market, the cartel may perform better than other second-best alternatives since deadweight losses in the retail end are likely to be lower. On the other hand, in regions where distribution losses are relatively low (e.g., less porous soils) or the returns from distribution investments are small (high construction or maintenance costs relative to the cost of water), a CID regime might be an acceptable substitute for costly deadweight losses in an institution with market power. Even with market failure in distribution, a low maintenance competitive system may outperform a privatized system with market distortions. This may also happen if both input and output markets are relatively inelastic.

4. An illustration

This section presents a simple illustration of the various institutional alternatives using typical cost and demand parameters for the Western United States. Although the parameters used may differ across regions or countries, the purpose is to show that the relative differences in welfare and resource allocation across the various models may actually be quite significant. Firm demand for water is derived from a quadratic production function for California cotton\(^22\) in terms of water $q$ such that a maximum yield of 1500 lb is obtained with 3.3 acre-feet of water, and a yield of 1200 lb with 2.2 acre-feet (Hanemann et al., 1987).\(^{23}\) The production function (in lb) is given by $f(q) = -0.2965 + 1.3134q - 0.6463q^2$ where $q$ is in m/m\(^2\) of water. Differentiating gives the marginal product $f'(q) = 1.3134 - 1.2926q$. Firm fixed costs denoted by $F$ are $433$ per acre or $0.107/m^2$ (University of California, 1988). A quadratic function for distribution investment was constructed from average lining and piping costs in 17 states in Western United States (US Department of Interior, 1979, Table 15, p. 87).\(^{24}\) An investment of $200 per meter length of canal in piped systems is assumed to lead to zero distribution losses in the system. Concrete lining with an investment of $100/m attains a loss factor of 0.2. We get

\[
a = 4 \times 10^{-5} - (4 \times 10^{-7}k - 10^{-9}k^2)
\]

\(^{21}\) In this case, the Nash Equilibrium investment at each location may be close to the optimal.

\(^{22}\) Cotton is one of the most important cash crops in the world, grown in over 70 countries. It is grown in 17 states in the United States, which along with China ranks as the world’s two largest producers.

\(^{23}\) We assume a field efficiency of 0.9, i.e., 10% of the water allotted to the firm is lost through leakage, etc.

\(^{24}\) These numbers for construction costs, although somewhat dated, do not change appreciably over time. Even if they did, it is not the level of costs but the variation across alternative regimes which is important for our comparisons. We also perform sensitivity analysis with higher distribution costs, see below.
where marginal cost is in US dollars and \( \bar{p}_c \) are not able to recover their fixed costs at that price. So, the monopolist is assumed to engage in perfect price marginal factor cost. The canal monopolist charges a water price the price of water equals the marginal cost at a given solution to (15). For the output monopoly, supply equals marginal revenue in the output market. For the input monopsony, \( z(0) \) absolute value of demand elasticity. To solve for the social planner, the algorithm assumes an initial value of output price \( 1.76 1.55 0.00 0.01 0.03 \) such that demand when \( x(1) \) that satisfies the derivative of (18), and (17) gives \( a(x) \). Using \( \lambda(0) \), (6) and (7) used simultaneously yield \( q(x) \) and \( y(x) \). When \( x = 1 \), using \( a(0) \) and \( \lambda(0) \) in the solution to (8) gives \( \lambda(1) \), and \( z(1) \) is obtained from (1) by subtracting the water used up previously. Next we get \( k(1) \) from (7) and the cycle is repeated to obtain \( q(1) \), etc. The process is continued with increasing values of \( z(0) \) terminates the cycle, and a new \( \lambda(0) \) is assumed. The algorithm selects the \( \lambda(0) \) from (11). At \( x = 0 \), (8) gives \( m(k) \). By iterating on \( k \), we compute \( k(x) \) that satisfies the derivative of (18), and (17) gives \( a(x) \). Using \( \lambda(0) \), (6) and (7) used simultaneously yield \( q(x) \) and \( y(x) \). When \( x = 1 \), using \( a(0) \) and \( \lambda(0) \) in the solution to (8) gives \( \lambda(1) \), and \( z(1) \) is obtained from (1) by subtracting the water used up previously. Next we get \( k(1) \) from (7) and the cycle is repeated to obtain \( q(1) \), etc. The process is continued with increasing values of \( z(0) \) terminates the cycle, and a new \( \lambda(0) \) is assumed. The algorithm selects the \( z(0) \) that minimizes total cost (given by (4a)). For each vector \( (p,z(0)) \), output \( Y \) is computed to generate the supply function \( Y(p) \). Finally, the equilibrium price and quantity is computed from (13). The algorithm was modified suitably for the other models. For the CID case, given the relatively high cost of distribution, the Nash Equilibrium investment \( k(x) \) is zero, i.e., \( k(l) = 0 \) is a corner solution to (15). For the output monopoly, supply equals marginal revenue in the output market. For the input monopsony, the price of water equals the marginal cost at a given \( z(0) \), but the shadow price of water at source equals the corresponding marginal factor cost. The canal monopolist charges a water price \( p^C_w \) at each location. It turns out empirically that firms are not able to recover their fixed costs at that price. So, the monopolist is assumed to engage in perfect price discrimination, charging price \( p^C_w \) at which each firm exactly covers its total cost and makes zero profit. This pricing mechanism will not change any of the institutional comparisons developed earlier, since the quantity of water allocated at each location by the monopolist is still the same (see below).

25 The exact loss coefficient, however, would depend on environmental factors such as soil characteristics, ambient temperatures, etc. The results were found to be generally insensitive to variations in the value of \( a(0) \).
4.1. Simulation results

The institutional differences are evaluated when demand for the end-use becomes more elastic, which may be representative of water use in another region or an end-use with different production characteristics, see Table 2 and Figs. 3 and 4. Welfare effects are obviously highest under a social planner, but are closely followed by the PC and the WUA.\(^{26}\) The distribution monopoly yields the lowest total welfare in all four cases. Looking at the components of social welfare, surplus from water generation is maximized under the WUA, while distribution surplus peaks when there is a distribution monopoly. The generation surplus is also high under social planning, mainly because of the large volume of aggregate water used, which creates a cost surplus over the intra-marginal units. The distribution monopoly uses nearly half the water under social planning, but covers nearly the same area when elasticity is low \(\varepsilon = -2\). This is because the former allocates low volumes of water at each location. If land availability is an issue, then this regime may not be the appropriate institutional choice. Except for the distribution monopoly, water use at each location is quite homogenous across all regimes.

As output demand becomes more elastic, the PC increases output and reduces commodity prices, while both price and output for the CID decline. Aggregate welfare is always higher under a PC than for CID. With increasing demand elasticity, the cartel covers a larger area and uses more water in the aggregate. The cartel always generates higher social welfare than the WUA, and performs almost like the social planner when elasticity is high \(\varepsilon = -4\). In this case, a cartel may produce greater output than the WUA. Therefore, for high elasticity in the end-use market (e.g., production for export), the PC may be a preferred alternative not only from a welfare consideration but also in terms of aggregate output.

The distribution monopoly and the CID model are consistently the weak performers, even in the high elasticity case \(\varepsilon = -4\). However, the CID regime always performs better of the two. This suggests that poor distribution may be preferred to monopoly power in distribution. Water in the CID regime is relatively expensive since transmission losses are high. Thus, the equilibrium price of the end-use commodity is high, and at higher demand elasticities, consumer surplus declines rapidly. As demand becomes more elastic, the underperformance of both these regimes is more pronounced.\(^{27}\)

It is likely that higher water prices under a distribution monopoly will lead to increased conservation through use of more efficient technology, not modeled in this paper. This may lead to more efficient use of water. Water prices are also

\[^{26}\text{Aggregate welfare for the social planner is decomposed into consumer surplus given by } \int_0^1 D^{-1}(\theta)d\theta - D^{-1}(Y)Y, \text{ and aggregate producer surplus into three individual components: } D^{-1}(Y)Y - C(Y) = \left[ \int_0^1 \left[ \int_0^1 (D^{-1}(Y)f(q) - i(x) - f(\phi)d\phi - g(z(0))z(0) + \int_0^1 g'(\phi)d\phi \right] dx \right] + \int_0^1 \left[ i(x) - f(\phi) - g(z(0))z(0) + \int_0^1 g'(\phi)d\phi \right] dx \right] + \left[ \int_0^1 \left[ i(x) - f(\phi) - g(z(0))z(0) + \int_0^1 g'(\phi)d\phi \right] dx \right] + \left[ \int_0^1 \left[ i(x) - f(\phi) - g(z(0))z(0) + \int_0^1 g'(\phi)d\phi \right] dx \right].\]

\[^{27}\text{If water prices were based on average costs in the CID case, it may underperform relative to the distribution monopoly. Trading of water rights even with competitive distribution improves CID resource allocation.}\]
End-use commodity prices are always highest in the CID regime. Any institutional reform is likely to reduce commodity prices, because of the resulting investment in distribution. Privatization for example, mostly results in an increase in water prices, because ex ante, water prices are lower than the true marginal cost because of policy distortions. Moving from the CID to any of the regimes with distribution investments significantly increases (by as much as 70–80%) the efficiency of water use as measured by output per unit water generated. In general, consumer surplus also increases significantly from reform, even though the move to a distribution monopoly generates the smallest gain.28

4.2. Design of appropriate regulatory policies

It may be useful to discuss the type of regulatory policies that can be adopted to achieve second best outcomes. For example, it is unreasonable to expect that the canal operator will not have full monopoly power and may be subject to some form of price cap or rate of return regulation. The government may specify a fixed water price plus a conveyance fee that the DM can charge to customers. Investment in distribution by the DM will then depend upon how this fee is specified. Given that each consumer may be charged a different price because of location, the regulation may be related to an average price. Alternatively, rate of return regulation that guarantees the monopoly a designated rate of return on investment may lead to overinvestment in conveyance, which given the rapid decline in marginal benefits from distribution, will lead to a misallocation of resources. Because investment in distribution is observable at low cost, rate of return regulation may be feasible, especially if the contract period is sufficiently large so that the distribution monopoly can recoup costs. It is easy to make prices contingent on investment in assets rather than unverifiable expenditures or investment in less useful assets (such as higher salaries and perks). In the case of water distribution, yardstick regulation could apply, in which case price is set equal to the average cost of comparable utilities. Since water generation is often local or regional and distribution is costly, comparable facilities may be easily identified, especially in other commodities with significant distribution costs such as electricity.

28 We have performed sensitivity analysis with an outward shift in demand which may represent changing demand conditions over time, and with an increase in the cost of water generation and distribution. The order of performance is generally preserved. The effect of increases in generation and distribution costs is to shift up the supply curve which in turn means a higher output price. The WUA is most impacted because with an increase in the cost of water generation, there is a 30% decline in aggregate water use. Increased distribution costs affect the optimal model the most by reducing aggregate water use and service area. The complete sensitivity analysis results are available separately from the authors.
Rate of return regulation applied to a monopoly generator of water is likely to increase water generation, and thus lead to a bigger service area and lower output prices, whatever the market structure in the downstream end. In comparison to monopoly power in distribution, this latter policy option may generate higher benefits. Regulation of a generation monopsony may involve legislating long term contracts so that frequent hold-up problems do not occur. Another option both in generation and distribution may be to auction off franchises that transfer ownership of the assets for a limited period of time, but sufficiently long to induce firm investment.

Since investments in water distribution have a relatively long life, it is important to discuss how the alternative regimes may compare in a dynamic context. Without explicitly developing a dynamic model, it may be useful to make some informed speculation based on the static results above (see also Chakravorty and Umetsu, 2003). One important reason for preferring the CID regime over the DM is in settings where the lost water can be reused at relatively low cost. The consideration of these third party effects may make CID the preferred arrangement, even though DM is the dominant regime for the project in question. If the demand for water were to increase in the future, then the value of this externality will also increase, and CID may dominate even more. A dynamic model may also be able to capture the opportunity rent of land, which may change exogenously. For example, an increasing land rent function (say in alternative uses) will imply that over time, the system may shrink, possibly after the DM has upgraded the system. In this case, the monopolist may not be able to recoup their sunk costs. These dynamic issues may need to be considered during ex ante contracting.

5. Concluding remarks

The production of water has often been viewed as one monolithic entity. The standard recommendation made is that the introduction of water markets leads to efficiency. However, increasing returns to scale in water distribution suggests that a water market will not lead to optimal resource allocation. Improving water management may mean a variety of alternative institutional choices. This paper shows that the choice of an institutional regime, in turn, may depend upon the characteristics of the microstructure of the water market. The analysis is relevant for the delivery of any infrastructure service (water, sanitation, electricity, natural gas, etc.) that entails significant distribution costs.

The paper yields insights into which institutional alternatives may be appropriate in a given situation. For example, when generation of the service is relatively elastic, a users association with market power in generation may perform closer to the optimal solution. On the other hand, elastic demand for the final product may indicate that a cartel or cooperative with market power in the output market may be preferred to the status quo. Similarly in locations where the scale of the service is important, a distribution monopoly may ensure delivery over a relatively large service area, even though it performs poorly according to conventional welfare measures. Monopoly power in distribution may also induce private conservation. Often a competitive distribution mechanism characterized by market failure may be preferred to a monopoly in distribution.

The numerical illustration shows that institutions may have differential impacts upon the geography of the region. Moving from competition to a distribution monopoly may mean that the service area may expand significantly, yet output may increase only marginally and welfare may fall. Maximizing the number of beneficiaries and maximizing welfare may be divergent goals. The distribution monopoly always maximizes the extensive margin yet performs poorly in the aggregate welfare and production measures. Relative to competition, a privatized regime with market power in the input or output market is always Pareto-improving. This suggests that laws that support WUAs or provide anti-trust exemption to output cartels may generate ancillary benefits in terms of mitigating problems associated with natural monopolies.

A major assumption of the model is that the water lost cannot be retrieved elsewhere. The value of this externality may depend on specific factors such as pumping costs. These considerations should improve the performance of the CID regime. The dynamics of water use and its possible treatment as a renewable or a nonrenewable resource (e.g., groundwater) may also complicate the analysis. Multiple uses of water as in power generation and hydroelectricity under imperfect competition could be considered in future work (Crampes and Moreaux, 2001).

Appendix A. Proofs of propositions

**Proposition 1.** (a) To produce a given output \( Y \), both regimes allocate resources efficiently and therefore have the same cost. (b) The cost function \( C'(Y) \) is the total cost of producing output \( Y \) when investment in distribution \( k(x) \) is sub-optimal. \( C'(Y) \) is the minimum cost of producing \( Y \). Therefore, \( C'(Y) \) must be no greater than \( C^d(Y) \). Similarly, the canal operator is a monopoly in distribution, and the WUA is a monopoly in generation. Hence, \( C'(Y) \) must be no greater than \( C^d(Y) \) and \( C^m(Y) \). To establish the second inequality, for the same level of aggregate output \( Y \), the CID model uses more aggregate input, \( z^d(0) > z^v(0) \) because investments in water distribution are sub-optimal. Since \( g'(z^v(0)) > 0 \), we have \( g'(z^d(0)) > g'(z^v(0)) \), i.e., the CID marginal cost of water generation is greater than optimal. We can write the identity

\[
C^d'(Y) = \frac{dC^d}{dz^d(0)} \frac{dz^d(0)}{dY} = g'(z^d(0)) \frac{dz^d(0)}{dY}
\]

Similarly, \( C^v'(Y) = g'(z^v(0)) \frac{dz^v(0)}{dY} \). But \( g'(z^d(0)) > g'(z^v(0)) \) and sub-optimal distribution investment implies more water is required to produce incremental output in the CID case, i.e., \( (dz^d(0)/dY) > (dz^v(0)/dY) \). This yields \( C^d(Y) > C^v(Y) \).
Finally, the slope of the unconstrained marginal cost function must be lower than the slope of the constrained marginal cost function since the former is an envelope of the latter (Silberberg, 1990, p. 234). The proof of the other two cases is similar. (c) The relation \( C(Y) = g'(z(0))dz(0)/dY \) and the discussion in (b) above imply that \( C'(Y) \) is positive. The remaining cases are similar. □

**Proposition 2.** The first part follows from Proposition 1 and is clear from Fig. 1. Since the marginal cost function in the CID case \( C(Y) \) is everywhere higher than the optimal \( C'(Y) \), the equilibrium output price under CID management is higher and aggregate output is lower than optimal. On the other hand, CHZ show that a model with no conveyance is likely to have lower aggregate marginal benefits which cuts the marginal cost curve below the optimal, leading to a lower value of \( x^*(0) \), i.e., \( x^*(0) < x^l(0) \). By (13), this implies a lower marginal cost of water at the source, i.e., \( \lambda w^l(0) \leq \lambda w^l(0) \). Applying (14) to the respective boundaries of the two regimes we have \( [p^*f(q^l) - z^l q^l]_{x0X} - [p^*f(q^l) - z^l q^l]_{XdX} \). Assume \( \lambda w(X^l) < \lambda w(X^d) \). Since \( p^* > p^q \), and \( q(x) \) is a monotone decreasing function, \( p^*(q^l) > p^q(q^l) \). Since the product \( \lambda q \) increases with \( \lambda \), \( \lambda q^l > \lambda q^d \). Substituting these inequalities suggests that the boundary condition will not be satisfied which is a contradiction. Thus, \( \lambda w(X^l) < \lambda w(X^d) \). Now let \( X^l > X^* \). Since \( \lambda w^l(x) \) is lower than \( \lambda w^l(x) \) at the beginning and at the boundary, then \( Y^d = \int_0^{X^d} y^d(x) \text{d}x > \int_0^{X^*} y^*(x) \text{d}x = Y^* \), which is a contradiction. Hence \( X^l \leq X^* \). □

**Proposition 3.** (a) The first part is obvious since the cartel is a monopolist in the output market. For the second part, both the cartel and social planner allocate resources efficiently. Since the former produces less aggregate output it uses less aggregate water. Thus, \( z^l(0) < z^l(0) \). Finally we show that the cartel uses less land area. Suppose \( X^p > X^* \). Then \( p^l(0) < p^l(0) \) by (13). Since \( \lambda w^l(x) \) is continuous on \( [0,X] \), \( \lambda w^l(x) < \lambda w^l(x) \) at locations close to the source. A higher price of water implies less water use, hence \( q^p(x) > q^l(x) \). But this implies that output at every location close to the source is higher in the cartel case. Since its aggregate output is lower than socially optimal, and \( X^p > X^* \), there must exist an interval \( L \subseteq [0,X^p] \cap [0,X^*] \) where socially optimal output is higher for all \( x \in L \). By continuity of \( \lambda w^l(x) \), \( \lambda w^l(x) \) and \( \lambda w^l(x) \) must cross. Thus, socially optimal output is lower than cartel output upstream but higher downstream of the water source. Since \( Y^d < Y^* \), we have \( \int_0^{X^d} y^d(x) \text{d}x > \int_0^{X^*} y^*(x) \text{d}x > \int_0^{X^d} y^d(x) \text{d}x > \int_0^{X^d} y^*(x) \text{d}x \) so that \( \int_0^{X^d} y^d(x) \text{d}x < \int_0^{X^*} y^*(x) \text{d}x \) since the first term in the last integral are non-negative. That is, cartel production beyond \( X^d \) in the interval \( M = [X^d,X^p] \) is lower than the deficit in production in the cartel (relative to optimal) in the interval \( N = [0,X^d] \). Thus, the cartel can mimic the optimal allocation of resources by transferring production from area \( N \) to area \( M \) and save on distribution losses since interval \( N \) is closer to the source. The last inequality implies that this rearrangement is feasible. Therefore the cartel is not efficient to begin with, which is a contradiction. So \( X^p \leq X^* \). (b) The first part of the proof is straightforward. To produce the same level of output relative to CID, the monopolist invests efficiently in water distribution. Aggregate water losses are therefore lower, so that to produce the same output, the monopolist must use a lower aggregate amount of water. Hence, \( z^p(0) < z^p(0) \). Next, we need to show that \( X^p \leq X^d \). Assume \( X^p > X^d \). Then (13) and the last inequality yields \( X^p(0) = g(x^p(0)) < g(x^d(0)) = X^d(0) \). As in the proof of part (a), \( \lambda w^l(x) < \lambda w^l(x) \) at locations close to the source, which implies \( q^p(x) > q^l(x) \). Since the aggregate outputs are equal, and \( X^p > X^d \), there must exist an interval \( L \subseteq [0,X^p] \cap [0,X^d] \) where CID output is higher, i.e., \( q^p(x) > q^d(x) \) and \( y^p(x) < y^d(x) \) for all \( x \in L \). Let \( X^1 \) be the location at which \( \lambda w^p(x) \) and \( \lambda w^d(x) \) cross. Since \( Y^d = Y^d \), we have \( \int_0^{X^1} y^p(x) \text{d}x + \int_{X^1}^{X^d} y^p(x) \text{d}x > \int_0^{X^1} y^p(x) \text{d}x + \int_{X^1}^{X^d} y^d(x) \text{d}x \) so that \( \frac{X^1}{X^1} y^p(x) \text{d}x = \int_0^{X^1} (y^p(x) - y^d(x)) \text{d}x + \int_{X^1}^{X^d} y^d(x) \text{d}x < \int_{X^1}^{X^d} (y^d(x) - y^d(x)) \text{d}x = y^d(x) \text{d}x \) since the first term on the right of the equality sign is strictly positive. The cartel produces in the region \( U = [X^1,X^p] \) while the CID does not, and this output is lower than the deficit in production in the cartel relative to the CID in the interval \( V = [X^1,X^p] \). Thus, the cartel can mimic the CID allocation of resources by transferring production from area \( U \) to area \( V \), and save on distribution losses since \( V \) is closer to the source. The last inequality implies that this rearrangement is feasible. Thus the cartel is not efficient to begin with, which is a contradiction. So, \( X^p \leq X^d \). □

**References**


29 That \( q^l(x) \) is monotone decreasing and \( \lambda q^l(x) \) is monotone increasing follows immediately from the proofs of Propositions 1 and 3 in CHZ.


