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Notes, Comments, and Letters to the Editor

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Starrett has made a significant contribution to our understanding of the mathematics of externalities,¹ but the nonconvexity which he identifies has a remedy in law. The mechanism by which the law circumvents this nonconvexity has gone unnoticed in the literature, as evidenced by a recent exchange in this Journal.² This note relates nonconvexity to the law of liability.

THE PROBLEM

We shall identify the nonconvexity by recalling Arrow's formulation of a market for externalities and then describing the remedy.³ Let y_{ik} be the i th firm's net supply of the k th good, where $i = 1, 2, \dots, m$ and $k = 1, 2, \dots, n$. Let $y_{i(-)}$ indicate the net supply of all firms except the i th: $y_{i(-)} \equiv (y_{11}, \dots, y_{(i-1)n}, y_{(i+1)1}, \dots, y_{mn})$. Externality is indicated by a production function whose arguments include the net supply of other firms: $0 \geq f^i(y_{i(-)}, y_{in}; y_{i(-)})$. Arrow defines a variable y_{jtk} which may be described as j 's

¹ David Starrett, Fundamental non-convexities in the theory of externalities, *J. Econ. Theory* 4 (1972), 180-199.

² In an exchange on the Coase theorem, the Editors of this Journal remark that Starrett has shown that there are "non-convexities inherent in the problem of external diseconomies, non-convexities which destroy the validity of the Coase Theorem." These comments are a response to Shapiro's example where setup costs invalidate the Coase theorem if exchange opportunities are restricted to marginal bargains. See David Shapiro, A note on rent and the Coase theorem, *J. Econ. Theory* 7 (1974), 125-128; and the Editors, Addendum to "A note on rent and the Coase theorem" by David Shapiro, *J. Econ. Theory* 14 (1977), 221-222.

³ K. J. Arrow, The organization of economic activity: Issues pertinent to the choice of market versus non-market allocation, in U. S. Congress, Joint Economic Committee, "The Analysis and Evaluation of Public Expenditures: The PPB System," Vol. 1, pp. 47-64, Government Printing Office, Washington, D.C.

observation of i 's net supply of good k ; by an obvious extension of notation we let $y_{i(i)}$ indicate i 's observation of $y_{i(i)}$. We imagine that institutions can be established so that there is an equivalence relation between net supplies and their observation: $y_{jik} = y_{ik}$ for all j, i, k . Consequently we may eliminate the externality by substituting observations for commodities in the production function: $0 \geq f^i(y_{i1}, \dots, y_{in}; y_{i(i)})$.

The equivalence relation is interpreted by Arrow as the market clearing condition for observations. Obviously there is bilateral monopoly in these markets, so competitive behavior is not to be anticipated; Starrett observed that there is also a nonconvexity. The difficulty is that externalities can be observed beyond the point where own production is nil, as indicated by the example of the laundry and smoke in Fig. 1. Equilibrium may fail to exist in the market for observations of smoke because of this nonconvexity.

FIG. 1. Starrett's nonconvexity.

THE LIABILITY SOLUTION

The institutional problem is to eliminate the externality by a mechanism which can distinguish between global and local optima. The courts can solve this problem by assigning liability rights according to a calculation of benefits and costs, as we shall show for the laundry and smokey electric company. Benefits and costs are measured by the profit functions for the two firms. Let y_{js} indicate j 's smoke with price $p_s = 0$, where $(p_1, p_2, \dots, p_n)$.

$$\text{Laundry: } \pi^i \equiv \max_{y_{ik}} \sum_k p_k y_{ik} \quad \text{s.t.} \quad f^i(y_{i1}, \dots, y_{in}; y_{js}) \leq 0,$$

$$\text{Electric Company: } \pi^j \equiv \max_{y_{jk}} \sum_k p_k y_{jk} \quad \text{s.t.} \quad f^j(y_{j1}, \dots, y_{jn}) \leq 0.$$

The cost of pollution is the minimum loss in profits to the laundry caused by the smoke, or the loss when the laundry maximizes profits, written

$\pi^i(p, 0) - \pi^i(p, y_{js})$.⁴ If the electric company is liable for the full cost of smoke, then its profit net of compensation is⁵

$$\bar{\pi}^i \equiv \max_{y_{js}} \sum_k p_k y_{jk} - (\pi^i(p, 0) - \pi^i(p, y_{js})) \quad \text{s.t.} \quad f^j(y_{j1}, \dots, y_{jn}) \leq 0.$$

The term $\pi^i(p, 0)$ is constant, so the electric company is maximizing joint profits as it would if the firms merged. Thus the externality is internalized. The joint maximization problem exhibits Starrett's nonconvexity, as illustrated in Fig. 2, but it is likely that the electric company will be able to identify the global maximum if the laundry divulges information about profits.⁶

Starrett was concerned that the pollutee (laundry) might supply an infinite number of pollution rights to a competitive market, but this problem does not arise under liability. The laundry may sell its legal claim for compensation by signing a contract which waives its right to recover damages. We may describe this transaction as selling coupons granting their owner the right to pollute. The laundry moves down the production frontier as it sells coupons to the electric company, until further damages are impossible. The right to recover further damages is worthless in the nonconvex region, because further damages are nil. In effect, the electric company must pay a positive price for coupons up to the point where the laundry can sustain no further damages, and then the price becomes nil. Exchange of pollution rights is not competitive under the liability solution, because the price varies with the quantity.

⁴ It is essential for economic efficiency to compensate for the minimum profit loss, written $\pi^i(p, 0) - \pi^i(p, y_{js})$, rather than the *actual* profit loss, written $\pi^i(p, 0) - \sum_k p_k y_{ik}$. If actual losses are the basis of compensation, then the Laundry's profit net of compensation is constant, $\sum_k p_k y_{ik} + (\pi^i(p, 0) - \sum_k p_k y_{ik}) = \pi^i(p, 0)$, and it has no incentive for efficient production. It is difficult in practice for courts to distinguish between actual and minimum losses.

⁵ This presentation is simplified by not discussing court costs or settlement costs; we avoid the theory of litigation.

⁶ Here is a simple proof on nonconvexity. Let the objective function in the optimization problem defining $\bar{\pi}^i$ be denoted $\Psi_p(y_j) = \sum_k p_k y_{jk} - (\pi^i(p, 0) - \pi^i(p, y_{js}))$. We shall prove that $\Psi_p(y_j) \geq \alpha$ is a nonconvex set.

(i) Starrett's nonconvexity, as drawn in Fig. 1, implies that laundry output approaches zero continuously as smoke approaches some finite level. (The proof is slightly different when this level is infinite.) Hence, there exist $(\hat{p}, \hat{y})$ such that $\pi^i(\hat{p}, y_{js}) > 0$ for $y_{js} < \hat{y}_{js}$ ($= 0$ otherwise). Choose $\bar{y}_j$ and $\bar{\bar{y}}_j$ so that $\bar{y}_{js} < \hat{y}_{js} < \bar{\bar{y}}_{js}$, and $\sum_k \hat{p}_k (\bar{y}_{jk} - \bar{\bar{y}}_{jk}) + \pi^i(\hat{p}, \bar{y}_{js}) = 0$. Hence for $p = \hat{p}$ we have $\Psi_{\hat{p}}(\bar{y}_j) = \Psi_{\hat{p}}(\bar{\bar{y}}_j) = \alpha$.

(ii) Choose t so that $0 < t < 1$ and $t \bar{y}_{js} + (1 - t) \bar{\bar{y}}_{js} = y_{js}$.

(iii) We wish to show that $\Psi_{\hat{p}}(t \bar{y}_j + (1 - t) \bar{\bar{y}}_j) < \alpha$. Assume the contrary; then $\Psi_{\hat{p}}(t \bar{y}_j + (1 - t) \bar{\bar{y}}_j) \geq \alpha = t \Psi_{\hat{p}}(\bar{y}_j) + (1 - t) \Psi_{\hat{p}}(\bar{\bar{y}}_j) \Rightarrow \pi^i(\hat{p}, t \bar{y}_{js} + (1 - t) \bar{\bar{y}}_{js}) \geq t \pi^i(\hat{p}, \bar{y}_{js}) + (1 - t) \pi^i(\hat{p}, \bar{\bar{y}}_{js}) \Rightarrow 0 \geq t \pi^i(\hat{p}, \bar{y}_{js}) > 0$, which is a contradiction.

FIG. 2. Joint profits.

One source of confusion concerns whether the electric company will have enough profits to compensate the laundry. This is a question of whether quasi-rents are adequate in the short run, or rents are adequate in the long run. The solution is essentially the same in both cases; we shall explain the long-run case. In the long run the electric company's profits net of compensation are nonnegative because it can produce zero output and pay no compensation: $\bar{\pi}^i \geq 0$. In other words the electric company's rent is always sufficient to compensate for the laundry's lost rent at the optimum. The rule of pollutor liability creates sufficient rent for efficient compensation.

LOCATION

Another source of confusion concerns the optimal location of plants. Locational choice does not change the essential nature of the problem, but it increases the computational difficulties. Let z_r be a variable indicating the site chosen for plant r and let $w(z_r)$ be the corresponding rent. Location affects production by changing the transportation inputs needed for a particular output. Furthermore, the amount of smoke observed by the laundry y_{ijs} depends upon the electric company's emissions y_{js} , and the location of the plants: $y_{ijs} = y_{ijs}(z_i, z_j, y_{js})$. Thus the laundry solves

$$\pi^i \equiv \max_{y_i, z_i} \sum_k p_k y_{ik} - w(z_i) \quad \text{s.t.} \quad f^i(y_i, z_i; y_{ijs}(z_i, z_j, y_{js})) \leq 0.$$

Under full liability, the electric company solves

$$\bar{\pi}^j \equiv \max_{y_j, z_j} \sum_k p_k y_{jk} - w(z_j) - (\pi^i(p, 0, z_j) - \pi^i(p, y_{js}, z_j))$$

$$\text{s.t.} \quad f^j(y_j, z_j) \leq 0.$$

regardless of location, so $\pi^i(p, 0, z_i)$ is constant with respect to z_j .

The electric company is choosing net supplies and location to maximize joint profits, as required for social efficiency, but the computations are difficult when location is a choice variable. Efficiency requires that compensation be based upon the losses which the laundry would suffer when its location maximizes its profits. The electric company and the courts may have difficulty distinguishing between losses at the laundry's actual location and hypothetical losses at its ideal location.

There is an added difficulty in identifying the global maximum, due to a locational nonconvexity which is different from Starrett's nonconvexity⁷. Suppose that the laundry and electric company are located far apart, and the production set is convex over the region where laundry is produced in positive quantities, as shown in Fig. 3. As the laundry changes its location and moves closer to the electric company, the production frontier dips toward the origin for all interior points, as shown, which indicates that smoke is interfering more with laundry production. However, the corners of the frontier are unaltered. If this process continues as the laundry is moved ever closer, the production frontier's interior eventually becomes nonconvex, as shown.

FIG. 3. Baumol's nonconvexity from locating plants closer together.

LIMITED INFORMATION AND BARGAINING

The liability solution requires courts to base compensation upon the pollutee's minimum profit loss, as given by the profit function, rather than the actual profit loss. (See footnote 4.) The courts may lack the information needed for such a computation. It is possible to construct models in which

⁷ See W. J. Baumol, On taxation and control of externalities, *Amer. Econ. Rev.* 62 (1972), 307-322.

the courts overcome limited information by a sequence of court decisions which converge to the optimum, but convergence assumes full convexity, which is absent in our case.⁸ Consequently, it would be desirable if efficiency could be achieved by private bargaining, irrespective of the law's structure, as postulated by the Coase theorem.⁹

Even in the two-person case, the nature of bargaining is such that efficient or group rational outcomes cannot be anticipated with confidence. For example, the parties may fail to agree because each holds out for more than half of the surplus from cooperation. In such a strategic setting, the electric company may be unable to extract the information needed to distinguish between global and local maxima. The likelihood of efficiency is still lower when there are several polluters and pollutees, all of whom must become parties to the bargain.

How does the existence of several polluters or pollutees influence the liability solution? The essential nature of the problem is unchanged when there are several laundries and a single electric company, which corresponds to class action against a single defendant. Adding additional terms to the joint profit function requires the electric company to have more information, but the nature of its maximization problem is unchanged. However, adding several electric companies creates serious difficulties for the full liability solution. The courts would need to compute the global optimum in order to divide damages among the polluters in a way which would achieve efficiency. It would be difficult for the courts to make such a computation.¹⁰

ACKNOWLEDGMENT

I would like to thank an anonymous reviewer for correcting errors in an earlier draft.

⁸ See R. Cooter, L. Kornhauser, and D. Lane, Liability rules, limited information, and the role of precedent, *Bell J. Econ.* (Spring 1979).

⁹ The symmetry of compensation rules is asserted by Coase in his classic paper, The problem of social cost, *J. Law and Econ.* 3 (1960). His failure to consider the implications of the strategic problems in bargaining is discussed in R. Cooter, The cost of believing Coase's theorem, *UCB Working Paper No. 76*, revised 1978.

¹⁰ Let $y_{1s}^*, \dots, y_{ns}^*$ be the efficient pollution. Firm j will choose the efficient value for y_{js} if compensation is based upon the damages j 's smoke imposes when other polluters are at the efficient point. Specifically, y_{js}^* is the value of y_{js} which solves $\bar{\pi}^j \equiv \max_{y_{js}} \sum_k p_k y_{jk} - \sum_i \pi^i(p, 0) - \pi^j(p, y_{j1}^*, \dots, y_{jn}^*)$ s.t. $f^j(y_{j1}, \dots, y_{jn}) \leq 0$. With nonconvexities, it is difficult to think of an interactive process by which the courts could discover the globally efficient pollution levels; see footnote 8.