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Rotational response of superconductors: magneto-rotational isomorphism and rotation-induced vortex lattice

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The analysis of nonclassical rotational response of superfluids and superconductors was performed by Onsager (in 1949) [1] and London (in 1950) [2] and crucially advanced by Feynman (in 1955) [3]. It was established that, in thermodynamic limit, neutral superfluids rotate by forming—without any threshold—a vortex lattice. In contrast, the rotation of superconductors at angular frequency Ω —supported by uniform magnetic field $\mathbf{B}_L \propto \Omega$ due to surface currents—is of the rigid-body type (London Law). Here we show that, neglecting the centrifugal effects, the behavior of a rotating superconductor is identical to that of a superconductor placed in a uniform fictitious external magnetic field $\tilde{\mathbf{H}} = -\mathbf{B}_L$. In particular, the isomorphism immediately implies the existence of two critical rotational frequencies in type-2 superconductors.

Superconductors and superfluids are distinguished by their Non-Classical Rotational Response (NCR), strikingly different from the rotational response of ordinary states of matter. The NCRs have a high degree of universality rooted in the fact that superconductors and superfluid break $U(1)$ symmetry and, at long-length scales, are described by a complex scalar field theory. Rotating superfluids form vortex lattices [1, 3], which is routinely used to demonstrate superfluid properties.

By contrast, a rotational response of a superconductor—known as the London Law [2]—is different. It is derived from the minimal, constant-density ($n = \text{const}$) model of superconductivity

$$F[\phi, \mathbf{A}] = \frac{n}{4m} \left(\hbar \nabla \phi - \frac{2e}{c} \mathbf{A} \right)^2 + \frac{1}{8\pi} (\text{curl} \mathbf{A})^2 = \frac{\gamma n}{2} (\nabla \phi + q \mathbf{A})^2 + \frac{1}{2} (\text{curl} \mathbf{A})^2, \quad (1)$$

where F is the free energy density, ϕ is the superconducting phase, $\mathbf{A}$ is the vector potential, m and e are the fundamental constants: the mass and charge of the electron. To simplify notation we employ the units $\hbar = c = 1$, in which q and γ are the absolute value of bare electric charge and bare inverse mass of two electrons. According to London [2], the superconductor, rotating with the angular velocity Ω , generates the magnetic field

$$\mathbf{B}_L = -\frac{2mc}{e} \Omega = \frac{2}{\gamma q} \Omega. \quad (2)$$

Current experimental observations are consistent with the universal character of London’s Law (see, e.g., [4, 5]). London effect also contributes to the magnetic field of pulsars, which are rotating protonic superconductors.

Below we show that the state of a rapidly rotating type-2 superconductor becomes different from the London one. The general solution to the problem is readily obtained by observing that there is an isomorphism between rotating superconductor and a non-rotating superconductor in a uniform external magnetic field.

Assuming electroneutrality as a natural physical condition and confining ourselves, for simplicity, with the constant-density regime, we put superconducting matter field onto a uniformly rotating uniformly charged background. (In superconducting metals, the background charge is associated with the crystal lattice of positively charged ions. In the case of protonic superconductivity in a neutron star, the background charge comes from normal electrons.) In a neutral superfluid, rotation is equivalent to introduction of a fictitious vector potential (and also the centrifugal potential, which we ignore here). Thanks to electroneutrality, the equivalence directly applies to our case as well. The key circumstance enforced by electroneutrality is the invariance of the net electric current $\mathbf{J}_{\text{net}} = \mathbf{J} + \mathbf{J}_n$, where $\mathbf{J}$ is the supercurrent and $\mathbf{J}_n$ is electric current of the normal background. When going to the rotating frame, $\mathbf{J}_{\text{net}}$ remains the same, thus implying the same vector potential field $\mathbf{A}$.

In the rotating frame, the free energy density thus reads:

$$F[\phi, \mathbf{A}] = \frac{\gamma n}{2} (\nabla \phi + q \mathbf{A} - \gamma^{-1} \mathbf{W})^2 + \frac{1}{2} (\text{curl} \mathbf{A})^2, \quad (3)$$

with $\mathbf{W} = \Omega \times \mathbf{r}$ the rotating-frame fictitious vector potential. Note that while the uniformly rotating uniformly charged background does not explicitly enter the free-energy functional, the expression is senseless in the absence of the background.

For a type-1 superconductor, there will be a critical rotation frequency when the London’s magnetic field energy density $B_L^2/2$, coming from (2), becomes equal to superconducting condensation energy. At this rotation frequency, the type-1 superconductor experiences a first-order phase transition from London regime to a normal state. Within Ginzburg-Landau model for a type-1 superconductor, the value of this magnetic field coincides with the thermodynamical critical magnetic field given by $H_c = \Phi_0/(4\pi\xi\lambda)$, where ξ and λ are coherence and magnetic field penetration lengths, and Φ_0 is the mag-

netic flux quantum. Thus

$$\Omega_c = \frac{\gamma q \Phi_0}{8\pi \xi \lambda}. \quad (4)$$

For a type-2 superconductor, Eq. (3) allows superconducting state to persist at higher rotation frequencies by forming a vortex lattice. The picture becomes immediately clear by the following mapping. Introduce *shifted* vector potential

$$\tilde{\mathbf{A}} = \mathbf{A} - (q\gamma)^{-1} \mathbf{W} \quad (5)$$

and observe that, in terms of $\tilde{\mathbf{A}}$, Eq. (3) becomes isomorphic to free energy of a superconductor in external field $\tilde{\mathbf{H}} = -(q\gamma)^{-1} \text{curl} \mathbf{W} = -\mathbf{B}_L$:

$$F[\phi, \tilde{\mathbf{A}}] = \frac{\gamma n}{2} (\nabla \phi + q \tilde{\mathbf{A}})^2 + \frac{1}{2} \left[\text{curl} \left(\tilde{\mathbf{A}} + \frac{\mathbf{W}}{q\gamma} \right) \right]^2. \quad (6)$$

Hence, type-2 superconductor will have first and second critical rotation frequencies:

$$\Omega_{c1} = \frac{\gamma q}{8\pi} \left(\frac{\Phi_0}{\lambda^2} \right) \ln \left(\frac{\lambda}{\xi} \right), \quad \Omega_{c2} = \frac{\gamma q \Phi_0}{8\pi \xi^2}. \quad (7)$$

Restoring dimensional units we get

$$\Omega_{c1} = \frac{e}{8\pi m c} \left(\frac{\Phi_0}{\lambda^2} \right) \ln \left(\frac{\lambda}{\xi} \right), \quad \Omega_{c2} = \frac{e \Phi_0}{8\pi c \pi \xi^2}. \quad (8)$$

At Ω_{c1} the rotating vortex lattice will appear, and at Ω_{c2} the system will become normal. The London state—characterized by an ideal diamagnetic response to the fictitious magnetic field $\tilde{\mathbf{H}}$ —emerges as a counterpart of the Meissner state. Under typical conditions, Ω_{c1} is extremely high. However $\Omega_{c1}(T \rightarrow T_c) \rightarrow 0$ because $\lambda(T \rightarrow T_c) \rightarrow \infty$. Also, Ω_{c1} can be low enough in thin superconducting films, where the effective penetration length is $\lambda_f = 2\lambda_b^2/d$, with λ_b the bulk penetration length and d the film thickness. Given that the length λ_f can be as large as $\sim 1\text{cm}$, it is clear that attaining the critical value Ω_{c1} even of order of 10 Hz in thin films is experimentally feasible. A young neutron star is a rotating gradually cooling system, which undergoes a superconducting phase transition for protons. If protonic system forms type-2 superconductor, then rotation-induced protonic vortex lattices should be generically present in a neutron star at a certain stage of its evolution.

Some physical aspects are worth a discussion. First, observe that, in the limit $\Omega \rightarrow \Omega_{c2}$, the vortices of the lattice carry no magnetic flux. In this limit, the fictitious vector potential $(q\gamma)^{-1} \mathbf{W}$ is compensated by the vortex phase windings in $\nabla \phi$, rather than by $\mathbf{A}$. The only relevant vector potential here is $\mathbf{W}$ and standard Feynman relationship between the flux of corresponding fictitious field and the vortex density holds. Physically, the fluxless vortex lattice in this state mimics the solid-body rotation of the superfluid matter field, co-rotating

with oppositely charged normal component. Apart from possible short-length-scale effects, there is no net transfer of electric charge, $\mathbf{J}_{\text{net}} = 0$, and thus $\mathbf{A} = 0$.

In a general case, the rotational response will be a combination of London response and vortex lattice, the number of vortices (antivortices, if negative) inside the system satisfying the relation

$$N = (\Phi - \tilde{\Phi})/\Phi_0, \quad (9)$$

where $\Phi_0 = -2\pi/q$ is the magnetic flux quantum, Φ is the magnetic flux through the system, and $\tilde{\Phi} = (q\gamma)^{-1} \oint_{\text{syst.}} \mathbf{W} \cdot d\mathbf{l}$ is the flux of the London field $\mathbf{B}_L$ through the system. The total magnetic flux per vortex is not quantized due to the existence of London background field. Nevertheless, an addition of a vortex (antivortex) *at fixed rotational frequency* amounts to addition (subtraction) of exactly one flux quantum Φ_0 to (from) the total magnetic flux.

The rotational response of a superconductor is summarized as follows. Slow rotation results in creation of uniform magnetic field, in accordance with the London picture. The uniformity of the field sets in at the lengths scale λ from the boundary of the system. At the first critical rotation frequency of type-2 superconductor, vortex lattice appears. Each vortex *reduces* the total magnetic flux through the system by one flux quantum—in contrast to the case of Abrikosov lattice [6], where a vortex does the opposite. Close to the second critical rotation frequency, the vortex lattice is essentially free of magnetic flux.

The overall electrical neutrality allows one to use similar approach—fictitious gauge field in the rotating frame—in a more general case of multicomponent systems. For example, in multiband superconductors, the normal background neutralizes several superconducting components originating in different bands. If, in the rotating frame, all these components have similar coupling to the fictitious gauge field, the rotational response of such system is isomorphic to its magnetic response, allowing, in particular, vortex clusters in the so-called type-1.5 superconductors [7]. On the other hand, in the case of a mixture of components with different masses and $U(1) \times U(1)$ or higher broken symmetry, a rotational response of the system may be different from the magnetic response because of different couplings to the fictitious vector potential.

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- [1] L. Onsager, *Nuovo Cimento, Suppl.* **6**, 279 (1949).
[2] F. London, *Superfluids*, Wiley, New York, 1950; page 78.
[3] R. P. Feynman, *Progress in Low Temperature Physics* **1**, 17, (1955).
[4] A. F. Hildebrandt Phys. Rev. Lett. **12**, 190 (1964)
[5] A. A. Verheijen, J. M. van Ruitenbeek, A. de Bruyn Ouboter, and L. J. de Jongh, *Nature* **345**, 418 (1990).
[6] A. A. Abrikosov, Sov. Phys. JETP **5**, 1174 (1957).
[7] E. Babaev and M. Speight, Phys. Rev. B **72**, 180502 (2005).