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We report dynamic regimes supported by a sharp quasi-one-dimensional (1D) (“razor”), pyramid-shaped (“dagger”), and conical (“needle”) potentials in the 2D complex Ginzburg–Landau (CGL) equation with cubic-quintic nonlinearity. This is a model of an active optical medium with respective expanding antiwaveguiding structures. If the potentials are strong enough, they give rise to continuous generation of expanding soliton patterns by a 2D soliton initially placed at the center. In the case of the pyramidal potential with M edges, the generated patterns are sets of M jets for $M \leq 5$, or expanding polygonal chains of solitons for $M \geq 6$. In the conical geometry, these are concentric waves expanding in the radial direction. © 2010 Optical Society of America

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Complex Ginzburg–Landau (CGL) equations are well known as basic models of the pattern formation in various nonlinear dissipative media [1–3]. The CGL equation with cubic-quintic (CQ) nonlinearity was first proposed in two-dimensional (2D) form by Petviashvili and Sergeev [4]. Later, they found numerous applications [5–8], a notable one being modeling the dynamics of laser cavities. The studies in this field have been focused on complex stable patterns [9–17] and interactions of localized pulses [18–21]. Recently, adding external potentials in CGL models has been a theme of extensive studies, expanding the already wide spectrum of relevant applications [22–24]. Desirable patterns of the refractive-index modulation in materials described by CGL equation, which may induce the effective potentials, can be achieved by means of various techniques, such as optics induction [25] and writing patterns by streams of ultrashort laser pulses [26].

In this Letter, we introduce 2D CGL with external wedge-shaped potentials in the form of a sharp quasi-1D “razor,” pyramidal “dagger” with a polygonal cross section, and an axisymmetric conical potential (“needle”). We consider the action of these potentials on a 2D soliton initially placed at the central position (apex of the respective potential). In the cases of the razor and dagger, the result is continuous emission of fundamental 2D solitons along symmetry directions of the potential, while the needle gives rise to the generation of an array of concentric annular solitons.

The CQ CGL equation is written in “optical” notation [15], as the evolution equation for the amplitude of the electromagnetic wave in an active bulk optical medium:

$$iu_z + i\delta \cdot u + (1/2 - i\beta)(u_{xx} + u_{yy}) + (1 - i\varepsilon)|u|^2u - (\nu - i\mu)|u|^4u = F(x, y)u, \quad (1)$$

where z and (x, y) are the propagation distance and transverse coordinates. The coefficients of the diffraction and cubic self-focusing nonlinearity are scaled, respectively, to be 1/2 and 1, while $\nu < 0$ accounts for the quintic self-defocusing, δ is the coefficient corresponding to linear loss ($\delta > 0$) or gain ($\delta < 0$), $\mu > 0$ is the quintic-loss parameter, $\varepsilon > 0$ is the cubic-gain coefficient, and $\beta > 0$ accounts for an effective diffusion (viscosity). The last term on the right-hand side of Eq. (1) introduces the potential in the transverse plane, $F(x, y)$. In fact, the three above-mentioned types of splitting potentials—razors, daggers, and needles—represent antiwaveguiding structures in the optical medium. Generic results may be adequately represented by setting $\delta = 0.5$, $\mu = 1$, $\varepsilon = 2.7$, $\beta = 0.5$, and $\nu = 0.01$ in Eq. (1).

First, we consider a wedge-shaped potential that depends on one coordinate (razor):

$$F(x, y) = -a|x|^n, \quad (2)$$

where a and n indicate the strength and steepness of the potential. The initial state is a stable 2D soliton solution of the CQ CGL equation without the external potential, with the center placed at the apex of the razor potential. Then simulations of Eq. (1) reveal five typical dynamic regimes, obtained by varying a and n , as shown in Fig. 1(b) (regions c , d , e , f , and g). In region c , continuous generation of pulse streams by the initial soliton is observed; see Figs. 1(c1) and 1(c2). The gain term in the CQ CGL equation is the source of the energy necessary for this. The emitted pulses self-trap into fundamental solitons, which slide along slopes of the potential. The comparison of Figs. 1(c1) and 1(c2) demonstrates that the stronger potential provides for a higher emission rate.

In region d , a very sharp tip of the potential splits the initial soliton into two secondary solitons, which roll down along the slopes, as shown in Fig. 1(d). Further, in region e [see a typical example in Fig. 1(e)], the generated pulses rapidly dissipate, failing to self-trap into

secondary solitons, because the potential's slope exceeds the critical value, admitting steady motion of the dissipative solitons [24], while the central soliton survives, despite the fact that it sits on the potential maximum. In region *f*, the combination of the sharp tip and steep slopes completely destroys the central soliton, without generating new pulses, as shown in Fig. 1(f). Finally, in region *g*, a less steep razor potential stretches the central soliton without splitting it [see Fig. 1(g)], the stretching force increasing with *a*.

The viscosity term in Eq. (1), $\sim\beta$, plays an important role in maintaining the position of the central soliton. With little or zero viscosity, this soliton is always subject to the drift instability, starting to roll down from the tip of the potential. On the other hand, with $\beta = 0.5$, which is the value adopted in the simulations reported here, a random shift of the initial soliton's position by $|\Delta x| < 1.2$ does not affect the established dynamic regime.

As the first example of proper 2D potentials, we take the one with linear slopes [24], shaped as a pyramid (dagger) whose cross section is an equilateral polygon with *M* edges; see the right panels in Fig. 2. The analytical form of the polygonal potential is

Fig. 1. (Color online) (a) 2D profile of one of the solitons (marked by the arrow) forming the generated pattern in panel (c1). (b) Regions of different dynamic regimes in the plane of *n* and *a*. Other panels display examples of the evolution: (c1), (c2) in region *c*, for $n = 1$, $a = 0.2$ and $n = 1$, $a = 0.4$; (d) in region *d*, for $n = 0.5$, $a = 2$; (e) in region *e*, for $n = 1$, $a = 0.85$; (f) in region *f*, for $n = 1$, $a = 2$; (g) in region *g* for $n = 1$, $a = 0.1$.

$$F(x, y) = -a \left| \sin \left[\left(\frac{1}{2} - \frac{1}{M} + \frac{2(n-1)}{M} \right) \pi x \right] + \cos \left[\left(\frac{1}{2} - \frac{1}{M} + \frac{2(n-1)}{M} \right) \pi y \right] \right|, \\ -1 + \frac{2(n-1)}{M} < \frac{\theta}{\pi} \leq -1 + \frac{2\pi n}{M}, \quad n = 1, 2, \dots, M, \quad (3)$$

where θ is the angular coordinate and *a* is the strength of potential. In this case, the simulations reveal similar dynamic regimes for $M \leq 5$. In one case, the central soliton continuously generates *M* streams of secondary solitons, as shown in Figs. 2(a) and 2(b) for $M = 3$, $a = 0.18$, and $M = 4$, $a = 0.2$. It is worthy to note a small spontaneous shift of the position of the central soliton at $z = 150$ in Fig. 2(a), due to the fact that the polygonal potential with smaller *M* holds the central soliton weaker than its counterpart with larger *M*. Naturally, the generation rate increases with the *a*. However, at $a > 0.8$ [for $n = 1$ Eq. (2)], because the potential's slope exceeds the critical value, the generated pulses rapidly dissipate, failing to self-trap into secondary solitons.

The increase of the number of edges in the pyramidal potential leads to a smaller separation between the emerging in-phase secondary solitons; hence, the attraction between them strengthens. Under the action of the attraction, *M* jets of the secondary solitons at first fuse into polygonal ring-shaped patterns, at $M \geq 6$. The rings subsequently expand and eventually break. A typical example is shown in Fig. 2(d) for $M = 6$, $a = 0.4$.

In the same polygonal geometry but with a weaker dagger-shaped potential, $a < 0.12$, jets of secondary solitons are not generated. Instead, the simulations demonstrate a gradual stretch of the central soliton, as shown in Fig. 2(c), with $a = 0.1$.

Fig. 2. (Color online) Dynamic regimes in the model including the pyramid-shaped potentials whose cross sections are displayed in panels at the right edge. (a), (b) Continuous generation of three and four soliton jets, respectively. The regime of the four-jet generation is observed at values of the potential's strength $0.13 < a < 0.8$. (c) Gradual stretching of the central soliton, without the generation of secondary solitons, in the potential with the square-shaped cross section, at smaller values of the potential strength than in (b), $0 < a < 0.13$. (d) Generation of stretching and eventually breaking ring-shaped hexagonal patterns $z = 0, 20, 40, 60, 80$, and 100 .

Fig. 3. (Color online) Evolution initiated by the central soliton in the case of the conical (needle-shaped) potential; see Eq. (3): (a) continuous generation of expanding annular solitons, at $a = 0.25$ in Eq. (3), (b) stretch of the central soliton, for $a = 0.08$, (c) transformation of the initial central soliton into a stretching ring, at $a = 0.13$.

Next, we consider a conical potential with the circular cross section (the needle):

$$F(x, y) = -ar, \quad r \equiv \sqrt{x^2 + y^2}. \quad (4)$$

For $0.1 \leq a \leq 1.24$, the simulations demonstrate that the central soliton continuously generates ring-shaped patterns, which expand in the radial direction and do not break; see an example in Fig. 3(a) for $a = 0.25$. The generation rate increases with a . The weaker potential (4), with $0 < a < 0.1$, causes stretching of the central soliton, without emitting concentric waves, as shown in Fig. 3(b), for $a = 0.08$. On the other hand, a stronger potential, with $1.24 < a \leq 1.53$, transforms the central soliton into a single ring, expanding in the radial direction, as shown in Fig. 3(c), for $a = 1.3$. Finally, at $a > 1.53$, the central soliton suffers a decay.

In conclusion, we have introduced a version of the 2D CGL equation with the CQ nonlinearity, which includes sharp potentials of three types: quasi-1D (razor), pyramid-shaped (dagger), and conical (needle). Typically, these settings give rise to the generation of arrays of secondary solitons, if the slope of the potential is steep enough. The generation rate increases with the strength of the potential. If the potential is weak, the stretch of the central soliton is observed instead. In the case of the pyramidal potential, whose cross section is the equilateral polygon with $M \leq 5$ edges, it generates M jets of secondary solitons. At $M \geq 6$, stretching polygonal chains are, instead, generated, which break in the course of the expansion. The conical (axisymmetric) potential readily generates arrays of concentric waves expanding in the radial direction.

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