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Chaos in oil prices? Evidence from futures markets

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Abstract

We test for the presence of low-dimensional chaotic structure in crude oil, heating oil, and unleaded gasoline futures prices from the early 1980s. Evidence on chaos will have important implications for regulators and short-term trading strategies. While we find strong evidence of non-linear dependencies, the evidence is not consistent with chaos. Our test results indicate that ARCH-type processes, with controls for seasonal variation in prices, generally explain the non-linearities in the data. We also demonstrate that employing seasonally adjusted price series contributes to obtaining robust results via the existing tests for chaotic structure. Maximum likelihood methodologies, that are robust to the non-linear dynamics, lend support for Samuelson's hypothesis on contract-maturity effects in futures price-changes. However, the tests for chaos are not found to be sensitive to the maturity effects in the futures contracts. The results are robust to controls for the oil shocks of 1986 and 1991. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

It has been well documented that non-linear relationships that are deterministic can yield highly complex time paths that will pass most standard tests of randomness (see Brock, 1986 for a survey). Such random-looking but deterministic series

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have been termed ‘chaotic’ in the literature (e.g. Devaney, 1986; Guckenheimer and Holmes, 1986). Chaotic dynamics are necessarily non-linear and may be able to explain a richer array of time series behavior. For example, linear models may not properly capture sudden movements and wide fluctuation in stock prices, exchange rates, or in other financial and economic series, while chaotic models may be quite suitable in explaining these behaviors. Furthermore, modeling non-linear processes may be considered less restrictive than linear structural systems as the former are not dependent on the knowledge of the specific underlying structures.

Direct applications of chaos to economic theory has been initiated only in the last 20 years (e.g. Stutzer, 1980; Benhabib and Day, 1981, 1982), with researchers such as Brock and Sayers (1988) employing relatively new techniques to test the null of chaos in a number of macroeconomic series (such as the US unemployment rate).¹ The evidence of chaos in economic time series such as GNP and unemployment has thus far been weak (e.g. Brock and Sayers, 1988).

On the other hand, the few studies on the structure of commodity prices, employing a range of statistical tests, have generally found evidence consistent with low dimension chaos.

Lichtenberg and Ujihara (1988) apply a non-linear cobweb model to US crude oil prices; Frank and Stengos (1989) estimate the correlation dimension and Kolmogorov entropy for gold and silver spot prices; Blank (1991) estimate the Lyapunov exponent for soybean futures; DeCoster et al. (1992) apply correlation dimension to daily sugar, silver, copper and coffee futures prices; Yang and Brorsen (1993) employ correlation dimension and the Brock, Dechert, and Scheinkman (BDS) test on several futures markets.²

Why is the evidence of chaos stronger for commodity prices than for economic time series? Baumol and Benhabib (1989) suggest that disaggregated variables (such as commodity prices or production levels), that are inherently subject to resource constraints will make far better candidates for chaotic structure. There may be other reasons for this disparity. Prior studies on the structure of commodity prices suffer from a mixture of short data spans and fairly coarse tests for chaos. Several studies have also failed to control for seasonal variations in commodity prices. To what extent have these factors contributed to the evidence of chaos in commodity prices?

In this paper, we provide evidence on the structure of commodity prices while addressing such questions. We examine the non-linear dynamics and their explanations for three important energy futures contracts: crude oil, heating oil, and unleaded gasoline, from the early 1980s. There is a surprising lack of evidence on

¹See Baumol and Benhabib (1989) for a more complete review on the application of chaos to economic theory.

²Other papers document evidence of non-linearity for various financial and economic time series. For instance, see Hsieh (1989), and Aczel and Josephy (1991) for evidence on exchange rates, Scheinkman and LeBaron (1989), Hsieh (1991) for stock returns, Mayfield and Mizrach (1992) for S & P index prices, and Chwee (1998) for natural gas futures. However, the findings in these papers are not always consistent with chaos. Hsieh (1993) shows that conditional variance effects that are satisfactorily captured by ARCH-type models explain non-linearities in several currency futures contracts.

possible non-linear dynamics in energy prices.³ These commodities play an obviously critical role in the world economy, and are necessarily subject to pricing pressures arising from world demand and supply conditions.⁴ Futures trading volume and prices in these commodities are known to be fairly seasonal, with prices generally rising in spring and early fall, in anticipation of increased demand. However, given that many disparate domestic and international factors affect the prices of these contracts, and that quantifiable information on such factors is generally incomplete, the accurate structural modelling of such commodities could be considered impossible.⁵ The evidence of chaos will offer some scope for modelling price behavior by simply employing the time series of prices.

Testing for chaotic structure in commodity prices should be considered a meaningful exercise for other reasons. It has been speculated that technical analysis may succeed in forecasting short-term price behavior of various financial series because these series may be non-linear and/or chaotic (see for example, Bohan, 1981; Brush, 1986; Pruitt and White, 1988; Pruitt and White, 1989; Clyde and Osler, 1997). Several researchers show that technical analysis produces superior outcome relative to linear models in predicting price behavior of many financial instruments and economic time series (see LaBaron, 1991; Brock et al., 1992; Taylor, 1994; Blume et al., 1994; Chang and Osler, 1995; Kumar, 1992, among others). Clyde and Osler (1997) conclude that it is worth investigating chaotic behavior because, unlike random processes, chaotic series are more conducive to technical analysis. Therefore, it would be informative to analyze the behavior of various financial data in order to determine the source of non-linearities, if they exist. If the non-linearity stems from chaos, then perhaps technical analysis could be applicable in the short-run for prediction purposes. However, chaos would also imply that while prices are deterministic, long-range prediction based on ‘technical’ or statistical forecasting techniques becomes treacherous, as the slightest errors in function formulation will multiply exponentially. More is said on the matter in the next section.

Our paper is distinguishable from previous studies on chaos in commodity futures markets in that: (i) relatively long price histories are examined;⁶ (ii) unlike

³Exceptions are Lichtenberg and Ujihara (1988) and Chwee (1998).

⁴The US is the second largest producer of crude after Saudi Arabia, with approximately 6.6 million barrels a day. All but a handful of states are oil producers.

⁵For instance, the supply and marketing of gasoline and distillate fuels can be as predictable as a major integrated refiner shipping its refinery output to a distribution terminal. Or, the chain can be long and varied, involving a combination of integrated or independent refiners, importers, resellers, and retailers. There may be similar unknowns at the production stage of these commodities. These may range from the variations in OPECs production quotas, to the agreements between producers and refiners on ‘posted prices’ set by the refiners.

⁶Lichtenberg and Ujihara (1988) employ yearly data for crude from the mid-1960s to the mid-1980s; Chwee (1998) employs daily data for natural gas futures from 1990 to 1996. Yang and Brorsen (1993) examine the non-linear dynamics in daily futures prices for various commodity futures over the 1979–1988 interval. Blank (1991) examines only 2 years of data for soybean futures (the November 1986 contract). DeCoster et al. (1992) cover an interval more comparable to ours, from October 1972 to March 1989, for the silver, copper, sugar, and coffee futures.

prior papers, the data are subject to adjustments for seasonalities and maturity effects that may otherwise have led to an erroneous conclusion of deterministic structure; (iii) a wider range of ARCH-type models are considered as explanations to the non-linearities; and (iv) alternate statistical techniques are employed to test the null of chaotic structure. Like most prior studies we present strong evidence that commodity futures prices exhibit non-linear dependencies. Unlike these earlier studies, however, we find evidence that is clearly inconsistent with chaotic structure. The explanations for these inconsistencies may be attributed to differences in data size and methodology. We also make a case that employing seasonally adjusted price series may be important in obtaining robust results via the existing tests for chaotic structure. While we find notable maturity effects in the futures contracts, these effects were not found to be important to the chaos tests. We identify some commonly known ARCH-type processes that satisfactorily explain the non-linearities in the data. The GARCH model of Bollerslev (1986) and Exponential GARCH model of Nelson (1991) are found to generally perform the best in accounting for the non-linear dynamics in the commodities analyzed.

The next section motivates the tests for chaos and further discusses the implications of chaotic structure in commodity prices. Simulated chaotic data are employed to highlight some important properties of chaos. Section 3 describes the procedures that this paper employs to test the null of chaos. Section 4 presents the test results for the three energy futures. Section 5 closes with a summary of the results.

2. Chaos: concepts and implications for commodity markets

As the concepts of chaos are well developed in the literature, our descriptions are brief relative to some papers that we reference here. There are several definitions of chaos in use. A definition similar to the following is commonly found in the literature (e.g. Devaney, 1986; Brock, 1986; Deneckere and Pelikan, 1986; Brock and Dechert, 1988; Brock and Sayers, 1988; Brock et al., 1993): the series a_t has a chaotic explanation if there exists a system (h, F, x_0) where $a_t = h(x_t)$, $x_{t+1} = F(x_t)$, x_0 is the initial condition at $t = 0$, and where h maps the n -dimensional phase space, R^n , to R^1 , and F maps R^n to R^n . It is also required that all trajectories, x_t , lie on an attractor, A , and nearby trajectories diverge so that the system never reaches an equilibrium or even exactly repeats its path.

For chaotic time series, if one knows (h, F) and could measure x_t without error, one could forecast x_{t+i} and, thus, a_{t+i} perfectly. In that respect, chaos (or strangeness) is the opposite of the process that is instantaneously unpredictable. With respect to the divergence property and attractor, A : in order that F generates random-looking behavior (which is deterministic), nearby trajectories must diverge exponentially; moreover, in order that F generates deterministic behavior, locally diverging trajectories must eventually fold back on themselves.

The attractor A may be thought of as a subset of the phase space towards which sufficiently close trajectories are asymptotically attracted (e.g. Brock and Sayers, 1988).

Chaotic time paths will have the following properties that should be of special interest to commodity market observers:⁷ (i) the universality of certain routes (such as the period folding over of trajectories) that are independent of the details of the map; ii) time paths that are extremely sensitive to microscopic changes in the initial conditions; this property is often termed sensitive dependence upon initial condition or SDIC;⁸ and (iii) time series that appear stochastic even though they are generated by deterministic systems; i.e. the empirical spectrum and empirical autocovariance functions of chaotic series are the same as those generated by random variables, implying that chaotic series will not be identified as such by most standard techniques (such as spectral analysis or autocovariance functions).

The presence of chaos will hamper the success of technical traders and long-range forecasting models. Both basic forecasting devices — extrapolation and estimation of structural forecasting models — become highly questionable in chaotic systems (also see Baumol and Benhabib, 1989). Similarly, if a price series is chaotic, it is fair to say that regulators must have some knowledge of F, h to effect meaningful and more-than-transitory changes in the price patterns. Then too, it is not obvious that regulators will succeed in promoting their agenda. Without highly accurate information of F and h , and the current state x_0 , chaos would imply that regulators cannot extrapolate past behavior to assess future movements. In effect, they would only be guessing as to the need for regulation. In other words, the sensible technical analyst and policy maker ought to be indifferent to whether or not the non-linear structure is chaotic, unless of course, she had detailed knowledge of the underlying chaotic structure.

It should be noted, however, that chaotic systems may provide some advantage to forecasting/technical analysis in the very short-run (say a few days when dealing with chaotic daily data). As indicated earlier, a deterministic chaotic system is, in some respects, polar to an instantaneously unpredictable system. For instance, Clyde and Osler (1997) simulate a chaotic series and demonstrate that the heads-over-shoulder trading rule will be more consistent at generating profits (relative to random trading) when applied to a known chaotic system. However, the results in Clyde and Osler also indicate that this consistency declines dramatically, so that the frequency of ‘hits’ employing the trading rule is not distinguishable from that of a random strategy after just a few trading periods (days).⁹

⁷See Brock et al. (1993) for a more complete description of the properties.

⁸This property follows from the requirement that local trajectories must diverge; if they were to converge, the system would be stable to disturbance, and non-chaotic. Often, researchers illustrate the property of sensitivity of initial conditions by demonstrating that, in a chaotic process, microscopic changes in the value of the parameter(s) of the system can result in very deviant time paths. This idea is later presented at greater length (Baumol and Benhabib, 1989).

⁹It is also noteworthy that short-term forecasting techniques, such as locally weighted regressions, are known to perform better for chaotic data than for random data (e.g. Hsieh, 1991).

3. Testing for chaos

The known tests for chaos attempt to determine from observed time series data whether it is deterministic. There are three tests that we employ here: the correlation dimension of Grassberger and Procaccia (1983) and Takens (1984), and the BDS statistic of Brock et al. (1987), and a measure of entropy termed Kolmogorov–Sinai invariant, also known as Kolmogorov entropy. Among this group, Kolmogorov entropy probably represents the most direct test for chaos, measuring whether nearby trajectories separate as required by chaotic structure. However, this and other tests of SDIC (e.g. Lyapunov exponent) are known to provide relatively fragile results (e.g. Brock and Sayers, 1988). Thus, the need for the alternate tests for chaos. We briefly outline the construction of the tests, but we do not address their properties at length, as they have been well established (for instance, Brock et al., 1987 and Brock et al., 1993).

3.1. Correlation dimension

Consider the stationary time series x_t , $t = 1 \dots T$.¹⁰ One imbeds x_t in an m -dimensional space by forming M -histories starting at each date t : $x_t^M = \{x_t, x_{t+1}, \dots, x_{t+M-1}\}$. One employs the stack of these scalars to carry out the analysis. If the true system is n -dimensional, provided $M \geq 2n + 1$, the M -histories can help recreate the dynamics of the underlying system, if they exist (Takens, 1984). One can measure the spatial correlations among the M -histories by calculating the correlation integral. For a given embedding dimension M and a distance ε , the correlation integral is given by

$$C^M(\varepsilon) = \lim_{T \rightarrow \infty} \left\{ \frac{\text{the number of } (i, j) \text{ for which } |x_i^M - x_j^M| \leq \varepsilon}{T^2} \right\} \quad (1)$$

where $|\cdot|$ is the euclidean norm.¹¹ For small values of ε , one has $C^M(\varepsilon) \sim \varepsilon^D$ where D is the dimension of the system (see Grassberger and Procaccia, 1983). The correlation dimension in embedding dimension M is given by

$$D^M = \lim_{\varepsilon \rightarrow 0} \lim_{T \rightarrow 0} \{ \ln C^M(\varepsilon) / \ln \varepsilon \} \quad (2)$$

and the correlation dimension is itself given by

$$D = \lim_{M \rightarrow 0} \ln D^M \quad (3)$$

¹⁰ It is known that non-stationary processes can generate low dimensions even when not chaotic (e.g. Brock and Sayers, 1988). To rule out non-stationarity as a 'cause' for low dimension, one may difference the original series if it contains a unit root.

¹¹ In practice T is limited by the length of the data which in turn places limitations on the range of the values of ε and M to be considered.

If the value of D^M stabilizes at some value (D) as M increases, then D is the correlation dimension. If D^M continues to rise as M is raised, then the system is to be regarded as stochastic, since for practical purposes, there is no difference between high-dimensional system and randomness. Furthermore, one's computations can only be of finite resolution and data sets are of limited length, limiting the embedding level. On the other hand, if a stable low value of D^M is obtained (substantially lower than 10), there is evidence that the system is deterministic.¹²

A problem associated with the implementation of Eqs. (3) and (4) is that, with the limited length of the data, it will almost always be possible to select sufficiently small ε so that any two points will not lie within ε of each other (e.g. Ramsey and Yuan, 1987). A popular approach to overcome this difficulty is to instead estimate the statistic

$$SC^M = \frac{\{\ln C^M(\varepsilon_t) - \ln C^M(\varepsilon_{t-1})\}}{\{\ln(\varepsilon_t) - \ln(\varepsilon_{t-1})\}} \quad (4)$$

for various values of M (e.g. Brock and Sayers, 1988). The SC^M statistic is a local estimate of the slope of the C^M vs. ε function. Following Frank and Stengos (1989), we take the average of the three highest values of SC^M for each embedding dimension.

There are at least two ways to consider the SC^M estimates. First, the original data may be subjected to shuffling, thus destroying any chaotic structure if it exists. If chaotic, the original series should provide markedly smaller SC^M estimates than their shuffled counterparts (e.g. Scheinkman and LeBaron, 1989).¹³ Second, along with the requirement (for chaos) that SC^M stabilizes at some low level as we increase M , we also require that linear transformations of the data leave the dimensionality unchanged (e.g. Brock, 1986). For instance we would have evidence against chaos if AR errors provide SC^M levels that are dissimilar to that from the original series.

3.2. BDS statistic

BDS (1987) employ the correlation integral to obtain a statistical test that has been shown to have strong power in detecting various types of non-linearity as well as deterministic chaos. BDS show that if x_t is IID with a non-degenerate distribution,

$$C^M(\varepsilon) \rightarrow C^1(\varepsilon)^M, \quad \text{as } T \rightarrow \infty \quad (5)$$

¹²Grassberger and Procaccia (1983) determine the correlation dimension of the Logistic map at 1.00 ± 0.02 , the Henon map at 1.22 ± 0.01 , and the Mackey Glass equation at 7.5 ± 0.15 . For further discussion, see Brock et al. (1993).

¹³As discussed earlier, chaos is associated with lower dimensions than found in randomness.

for fixed M and ε . Employing this property, BDS show that the statistic

$$W^M(\varepsilon) = \sqrt{T[C^M(\varepsilon) - C^1(\varepsilon)^M] / \sigma^M(\varepsilon)} \quad (6)$$

where σ^M , the standard deviation of $[.]$, has a limiting standard normal distribution under the null hypothesis of IID. W^M is termed the BDS statistic. Non-linearity will be established if W^M is significant for a stationary series void of linear dependence. The absence of chaos will be implied if it is demonstrated that the non-linear structure arises from a known non-deterministic system. For instance, if one obtains significant BDS statistics for a stationary data series, but fails to obtain significant BDS statistics for the standardized residuals from an Auto Regressive Conditional Heteroskedasticity (ARCH) model, it can be said that the ARCH process explains the non-linearity in the data, precluding low dimension chaos.

Brock et al. (1993) employ this premise in their technique to test for chaos. The authors employ a two-step process. First, they extract the standardized residuals from an ‘appropriate’ ARCH-type model (generally GARCH, Exponential GARCH, or GARCH in Mean). Then the authors test for non-linear dependence on the standardized residuals. An absence of non-linear dependence implies an absence of chaos, since the ARCH-type model has ‘captured’ the non-linearities in the data (with no remaining non-linearities to be explained by other processes — such as chaos). Others who have employed the same methodology of precluding chaos when an ARCH-type process explains the non-linear behavior of the series include Hsieh (1989); Hsieh (1991).

Brock et al. (1993) examine the finite sample distribution of the BDS statistic and find the asymptotic distribution will well approximate the distribution of the statistic when: the sample has 500 or more observations; the embedding dimension is selected to be 5 or lower; and ε is selected to be between 0.5 and 2 S.D. of the data. The authors also find that the asymptotic distribution does not approximate the BDS statistic very well when applied to the standardized residuals of ARCH-type models (also see Brock et al., 1987). This is noteworthy as financial and commodity price movements are often found to have ARCH processes. The authors suggest bootstrapping the null distribution to obtain the critical values for the statistic when applying it to standardized residuals from these models.

3.3. Kolmogorov entropy

Kolmogorov entropy quantifies the concept of sensitive dependence on initial conditions. Imagine two trajectories representing time paths are extremely close so as to be indistinguishable to a casual observer. As time passes, however, the trajectories diverge so that they become distinguishable. Kolmogorov entropy (K) measures the speed with which this takes place.¹⁴

¹⁴Correlation entropy estimates are based on the natural logarithms of the C^M .

Grassberger and Procaccia (1983) devised a measure for K , which is more implementable than earlier measures of entropy. The measure is given by

$$K_2 = \lim_{\varepsilon \rightarrow \infty} \lim_{m \rightarrow \infty} \lim_{N \rightarrow \infty} \ln \left(\frac{C^M(\varepsilon)}{C^{M+1}(\varepsilon)} \right). \quad (7)$$

If a time series is non-complex and completely predictable, $K_2 \rightarrow 0$. If the time series is completely random, $K_2 \rightarrow \infty$. That is, the lower the value of K_2 , the more predictable the system. For chaotic systems, one would expect $0 < K_2 < \infty$, at least in principle.

4. Evidence from energy futures markets

We employ daily prices of the nearby (expiring) futures contracts written at New York Mercantile Exchange (NYMEX) on crude oil, heating oil, and unleaded gasoline.¹⁵ The commodities were selected for the study given their relatively long futures price histories (see Table 1 for the intervals studied). To obtain a spliced, continuous price series for each commodity, we follow common practice in tracking a particular contract until the last day of the pre-expiration month, at which point the series switch to the next nearby contract (e.g. Adrangi and Chatrath, 1999). We focus our tests on daily returns, which are obtained by taking the relative log of prices as in, $R_t = \ln(P_t/P_{t-1}) \times 100$, where P_t represents the closing price (at 15.10 h US central time) on day t .¹⁶

Table 1 presents the R_t diagnostics for the four series. The series are found to be stationary employing the Augmented Dickey Fuller (ADF) statistics. The series are found to exhibit linear and non-linear dependencies as indicated by the $Q(12)$ and $Q^2(12)$ statistics, and Autoregressive Conditional Heteroskedasticity (ARCH) effects are strongly suggested by Engle's ARCH chi-square statistic (Engle, 1982). Thus, as expected, there are clear indications that non-linear dynamics are generating the price series. Whether these dynamics are chaotic in origin is the question that we turn to next.

To eliminate the possibility that the linear structure or seasonalities may be responsible for the rejection of chaos by the tests employed, we first estimate autoregressive models for each of the three commodities with controls for possible seasonal effects, as in

$$R_t = \sum_{i=1}^p \beta_i R_{t-i} + \sum_{j=1}^n \gamma_j M_{jt} + \varepsilon_t, \quad (8)$$

¹⁵The data are obtained from the Futures Industry Institute, Washington, DC.

¹⁶We do not employ smoothing models to detrend the data, as we feel that the imposed trend reversions may erroneously be interpreted as structure (see Nelson and Plosser, 1982).

Table 1
Return diagnostics^a

	Crude	Heating	Unleaded
Interval	10/1/83	1/02/85	1/02/85
	03/31/95	03/31/95	03/31/95
Obs.	2887	2575	2575
Mean	−0.015	−0.015	−0.004
S.D.	2.12	2.12	2.35
ADF	−26.13 ^b	−25.18 ^b	−23.38 ^b
ADF(T)	−26.13 ^b	−25.18 ^b	−23.38 ^b
$Q(12)$	68.82 ^b	88.19 ^b	53.26 ^b
$Q^2(12)$	177.32 ^b	185.16 ^b	203.87 ^b
ARCH(6)	89.18 ^b	66.29 ^b	97.14 ^b

^aThe table presents the return diagnostics for three commodity futures contracts (daily data). Returns are given by $R_t = \ln(P_t/P_{t-1}) \cdot 100$, where P_t represents closing price on day t . ADF, ADF(T) represent the augmented Dickey–Fuller tests (Dickey and Fuller, 1981) for unit roots, with and without trends, respectively. The $Q(12)$ and $Q^2(12)$ statistics represent the Ljung–Box (Q) statistics for autocorrelation of the R_t and R_t^2 series respectively. The ARCH(6) statistic is the Engle (1982) test for ARCH (of order 6) and is χ^2 distributed with 6 degrees of freedom

^bRepresents a significance level of 0.01.

where M_{jt} represent the 12 month-of-the-year dummies. The lag length for each series is selected based on the Akaike (1974) criterion. The residual term (ε_t) represents the price movements that are purged of linear relationships and seasonal influences.

Table 2 reports the results from the OLS regressions. There is evidence of seasonal effects in each of the three returns. Returns are found to be significantly positive in August, and negative in the winter months of November through January. There is also significant linear structure in the returns, up to 10 lags for crude oil, heating oil, and unleaded gasoline contracts. The Q -statistics indicate that the residuals are free of linear structure.

4.1. Correlation dimension estimates

Table 3 reports the correlation dimension (SC^M) estimates for various components of the returns' series alongside that for the Logistic series developed earlier. We report dimension results for embeddings up to 20 in order to check for saturation.¹⁷ An absence of saturation provides evidence against chaotic structure. For instance, the SC^M estimates for the Logistic map stay close to 1.00, even as we increase the embedding dimensions. Moreover, the estimates for the Logistic series

¹⁷Yang and Brorsen (1993), who also calculate correlation dimensions for various commodity futures, compute SC^M only up to $M = 8$.

Table 2
Linear structure and seasonality^a

	Crude	Heating oil	Unleaded gas
R_{t-1}	0.014 (0.78)	0.037 ^d (1.89)	0.033 ^d (1.68)
R_{t-2}	−0.019 (−1.03)	−0.005 (−0.03)	0.009 (0.49)
R_{t-3}	−0.089 ^b (480)	−1.060 ^b (−5.40)	−0.091 ^b (−4.62)
R_{t-4}	−0.039 ^c (2.08)	−0.006 (−0.33)	−0.004 (−0.02)
R_{t-5}	−0.079 ^b (4.26)	−0.075 ^b (−3.79)	−0.018 (−0.91)
R_{t-6}	−0.001 (−0.07)	−0.045 ^b (−2.57)	−0.048 ^c (−2.44)
R_{t-7}	0.023 (1.25)	0.016 (0.79)	−0.004 (−0.19)
R_{t-8}	−0.084 ^b (4.50)	−0.091 (−4.61)	0.071 ^b (−3.57)
R	—	0.009 (0.45)	—
R_{t-10}	—	0.048 ^c (2.44)	—
Jan	−0.164 (−1.24)	0.444 ^b (−3.13)	0.045 (0.29)
Feb	−0.328 ^c (−2.35)	−0.399 ^b (−2.72)	−0.241 (−1.49)
Mar	0.090 (0.69)	−0.008 (−0.06)	0.381 ^c (2.52)
Apr	0.226 ^d (1.62)	0.108 (0.74)	0.377 ^c (2.30)
May	0.073 (0.53)	0.005 (0.04)	−0.006 (−0.04)
Jun	−0.157 (−1.14)	−0.091 (−0.63)	−0.285 ^d (−1.78)
Jul	0.031 (0.22)	0.180 (1.24)	−0.164 (−1.01)
Aug	0.240 ^d (1.77)	0.373 ^b (2.61)	0.259 ^d (1.64)
Sep	0.154 (1.09)	0.245 ^d (1.67)	−0.074 (−0.45)
Oct	−0.065 (−0.49)	−0.023 (−0.17)	0.102 (0.65)
Nov	−0.239 ^d (1.74)	−0.079 (−0.53)	−0.409 ^c (−2.45)
Dec	−0.082 (−0.61)	−0.084 (−0.58)	−0.171 (−1.05)
R^2	0.025	0.037	0.024
$Q(12)$	14.23	12.29	12.23
LM(1)	0.73	0.81	0.06

^aThe coefficients and residual diagnostics are from the OLS regressions of returns on prior returns and 12 month-of-year dummies. The lag-length was selected based on the criterion of Akaike (1974). The $Q(12)$ statistic represents the Ljung–Box (Q) statistics for autocorrelation in the residuals. The Lagrange multiplier test (LM) is for the null of one degree of autocorrelation less than the χ^2 statistic with one degree of freedom. Statistics in () are t -values.

^bRepresents a significance level of 0.01.

^cRepresents a significance level of 0.05.

^dRepresents a significance level of 0.10.

do not change meaningfully after AR transformation. Thus, as should be expected, the SC^M estimates are not inconsistent with chaos for the Logistic series.

For the three commodity series, on the other hand, the SC^M estimates provide evidence against chaotic structure. For instance, if one examines the estimates for the crude oil returns alone, one could (erroneously) make a case for low dimension chaos: the SC^M statistics seem to ‘settle’ at much lower than 10. However, the estimates are substantially higher for the seasonally adjusted residual series. Thus, the correlation dimension estimates suggest that there is no chaos in crude oil prices. Patterns for the other two contracts are even more pronounced in the sense

Table 3
Correlation dimension estimates^a

	$M =$			
	5	10	15	20
Logistic	1.02	1.00	1.03	1.06
Logistic AR	0.96	1.06	1.09	1.07
Crude returns	2.82	4.12	4.38	7.02
Crude AR(8)	2.88	3.95	5.06	7.91
Crude AR(8),S	2.88	4.75	4.82	9.53
Crude shuffled	3.84	6.11	13.52	17.28
Heating oil returns	4.16	7.78	7.20	12.37
Heating oil AR(10)	4.13	8.13	6.80	14.10
Heating oil AR(10),S	4.13	7.89	7.62	14.42
Heating oil shuffled	3.97	7.79	7.12	13.71
Unleaded gas returns	4.06	7.18	6.34	13.95
Unleaded gas AR(8)	4.09	7.25	10.04	9.29
Unleaded gas AR(8),S	4.02	7.45	10.55	14.82
Unleaded gas shuffled	3.77	7.54	6.74	15.56

^aThe table reports SC^M statistics for the logistic series ($w = 3.750$, $n = 2000$), daily crude oil, heating oil, and unleaded gasoline returns, and their various components over four embedding dimensions: 5, 10, 15, 20. $AR(p)$ represents autoregressive (order p) residuals, $AR(p)S$ represents residuals from autoregressive models that correct for month-of-year effects in the data.

that the correlation dimension estimates do not settle for any of the estimated models.

It is notable, however, that while the SC^M estimates for the $AR(p)$ series are similar to the estimates for seasonally corrected $AR(p),S$ series for heating oil, the estimates are quite dissimilar for crude oil and unleaded gasoline. Note that the estimates are generally much higher for the $AR(p),S$ series in the crude and unleaded gasoline series, especially for the higher dimensions (notably, $M = 20$). Thus, the correlation dimension estimates are found to be fairly sensitive to controls for seasonal effects. This has important implications for future tests for chaos employing correlation dimension.

4.2. BDS test results

Following the technique in Brock et al. (1993) and Hsieh (1991), we also test for chaos by examining whether the non-linear dependence noted in the data can be explained away by alternate non-chaotic processes. As in Brock et al. (1993) and Hsieh (1991), we first fit the data to alternate ARCH-type systems known to satisfactorily explain the non-linear dependence in several financial and commodity price series. Then, the standardized residuals are tested for non-linearities employ-

Fig. 1. Kolmogorov Entropy Estimates.

ing the BDS test statistic. As in Brock et al. (1993), if ARCH-type models ‘explain’ the non-linearities in the data, we conclude that the non-linear process is non-chaotic.

Table 4 reports the BDS statistics for [AR(p),S] series, and standardized residuals ($\varepsilon/\sqrt{h}$) from three sets of ARCH-type models with their respective variance equations, GARCH (1,1):

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \beta_2 TTM_t, \quad (9)$$

Exponential GARCH(1,1):

$$\log(h_t) = \alpha_0 + \alpha_1 \left| \frac{\varepsilon_{t-1}}{h_{t-1}} \right| + \alpha_2 \left| \frac{\varepsilon_{t-1}}{h_{t-1}} \right| + \beta_1 \log(h_{t-1}) + \beta_2 TTM_t, \quad (10)$$

Asymmetric component GARCH (1,1):

$$h_t = q_t + \alpha(\varepsilon_{t-1}^2 - q_{t-1}) + \beta_1(h_{t-1} - q_{t-1}) + \beta_2(\varepsilon_{t-1}^2 - q_{t-1})d_{t-1} + \beta_3 TTM_t$$

$$q_t = \omega + \rho(q_{t-1} - \omega) + \phi(\varepsilon_{t-1}^2 - h_{t-1}), \quad (11)$$

where $d_{t-1} = 1$ if $\varepsilon_t < 0$; 0 otherwise, the return equation which provides ε_t is the same as in Eq. (8), and TTM represents time-to-maturity (in days) of the futures contract.¹⁸ The time to maturity variable is intended to control for any maturity effects in the series (Samuelson, 1965).¹⁹ The BDS statistics are evaluated against critical values obtained by bootstrapping the null distribution for each of the GARCH models (see Appendix A). The estimates from the above variance equations are discussed later in the paper.

The BDS statistics strongly reject the null of no non-linearity in the AR(p),S errors for crude and heating oil futures, however, the evidence on unleaded futures is weaker. This evidence, that the energy futures have non-linear dependencies, is consistent with the findings in Table 1, and in Lichtenberg and Ujihara (1988) and Chwee (1998), among others. The BDS statistics for the standardized residuals from the ARCH-type models, however, indicate that the source of the non-linearity in the three commodities is *not* chaos. For instance, for the three contracts, the BDS statistics are dramatically lower (relative to those for the AR(p),S errors) for all the standardized residuals, and consistently insignificant for the GARCH (1,1) model. On the whole, the BDS test results further support the notion that the non-linear dependence in energy futures is explained by dynamics other than chaos. The evidence is compelling that the non-linear dependencies in commodity futures arise from ARCH-type effects, rather than from complex, chaotic structures.

Finally, it is also noteworthy that the asymmetric GARCH models also performed well for each contract, and the exponential GARCH model seemed to satisfactorily fit the crude oil and unleaded gasoline series.

4.3. Entropy estimates

Fig. 1 plots the Kolmogorov entropy estimates (embedding dimension 15–32) for the Logistic map ($w = 3.75$, $x_0 = 0.10$), [AR(p),S] for crude oil, heating, and unleaded gasoline series.²⁰

The entropy estimates for a shuffled crude oil return series are also presented

¹⁸ The return equation from the ARCH-type systems provided coefficients similar to those in Table 2. We also estimated another familiar model, GARCH in Mean (GARCHM). The BDS statistics from the GARCHM and GARCH (1,1) models were found to be very similar. In the interest of brevity, we do not provide the results from the GARCHM model. The GARCH model is due to Bollerslev (1986), the exponential model (EGARCH) is from Nelson (1991), and the asymmetric component ARCH model is a variation of the Threshold GARCH model of Rabemananjara and Zakoian (1993).

¹⁹ It is noteworthy that, in each of the above models, the *TTM* variable is found to be significant and in support of the Samuelson hypothesis: volatility (conditional variance) rises as one approaches contract maturity.

²⁰ Correlation entropy may not be singularly reliable for differentiating chaotic from stochastic processes. The correlation entropy will converge to infinity (for stochastic processes) only if ε tends to zero. In practice, K_2 is estimated for finite ε . For this reason, most researchers seek a preponderance of evidence for chaos rather than relying on the entropy measure alone.

Table 4
BDS statistics^a

Panel A: Crude ε/σ	M			
	2	3	4	5
AR(8), S, E.				
0.50	14.76 ^b	20.13 ^b	26.26 ^b	34.90 ^b
1.00	16.03 ^b	20.79 ^b	24.89 ^b	28.89 ^b
1.50	16.78 ^b	21.00 ^b	23.95 ^b	26.07 ^b
2.00	16.21 ^b	19.75 ^b	22.23 ^b	23.62 ^b
GARCH(1,1) S.E.				
0.50	-0.93	-0.74	-0.34	-1.39
1.00	-1.27 ^c	-1.07	-0.23	-0.63
1.50	-1.67	-1.44	-0.82	-0.17
2.00	-2.09 ^b	-1.83 ^c	-1.27	-0.86
EGARCH(1,1) S.E.				
0.50	-1.17	-1.02	-0.09	0.74
1.00	-1.24	-1.10	-0.28	0.44
1.50	-1.36	-1.06	-0.39	0.20
2.00	-1.41	-0.94	-0.24	0.23
AGARCH(1,1) S.E.				
0.50	0.64	0.57	1.82	2.98
1.00	0.38	0.22	1.17	2.05
1.50	-0.04	-0.16	0.52	1.10
2.00	-0.53	-0.60	0.04	0.36
Panel B: Heating oil ε/σ				
AR(10), S, E.				
0.50	9.12 ^b	11.81 ^b	13.10 ^b	17.08 ^b
1.00	11.22 ^b	14.11 ^b	16.50 ^b	18.67 ^b
1.50	13.17 ^b	16.08 ^b	18.15 ^b	19.63 ^b
2.00	13.51 ^b	16.42 ^b	18.25 ^b	19.28 ^b
GARCH(1,1) S.E.				
0.50	0.09	0.29	0.83	1.15
1.00	0.11	0.14	0.53	0.78
1.50	0.08	0.11	0.46	0.70
2.00	0.03	0.03	0.04	0.62
EGARCH(1,1) S.E.				
0.50	1.65	2.18	2.52 ^b	2.69
1.00	2.00	2.42	2.92 ^b	3.11
1.50	2.75 ^b	3.43 ^b	3.90 ^b	4.13 ^b
2.00	3.06 ^b	3.95 ^b	4.66 ^b	4.92 ^b
AGARCH(1,1) S.E.				
0.50	0.31	0.75	1.55	2.02 ^c
1.00	0.50	0.71	1.17	1.48 ^c
1.50	0.52	0.65	1.11	1.41 ^c
2.00	0.56	1.87 ^b	1.19	1.46 ^c

Table 4 (Continued)

Panel C: Unleaded gas ε/σ	M			
	2	3	4	5
AR(8), S, E.				
0.50	0.75	0.91	0.77	0.56
1.00	1.23	2.42 ^c	3.24 ^b	3.62 ^b
1.50	0.90	2.05 ^c	3.17	4.12 ^b
2.00	0.47	1.15	1.87 ^d	2.61
GARCH(1,1) S.E.				
0.50	−0.40	−0.17	−0.66	−0.57
1.00	−0.44	0.24	0.93	1.03
1.50	−0.44	0.01	0.42	0.32
2.00	−0.41	0.01	0.24	0.04
EGARCH(1,1) S.E.				
0.50	−1.12	−1.01	−0.08	−0.54
1.00	−1.16	−0.71	−0.22	−0.32
1.50	−0.86	−0.41	−0.06	−0.26
2.00	−0.58	−0.16	−0.03	−0.18
AGARCH(1,1) S.E.				
0.50	−1.84	−1.30	−0.62	−0.17
1.00	−1.79	−0.90	−0.32	−0.09
1.50	−1.97	−1.18	−0.88	−0.89
2.00	−1.82	−1.12	−0.97	−1.06

^aThe figures are BDS statistics for AR(p),S residuals, and standardized residuals $\varepsilon/\sqrt{h}$ from three ARCH-type models. The BDS statistics are evaluated against critical values obtained from Monte Carlo simulation (Appendix A).

^bRepresents a significance level of 0.01.

^cRepresents a significance level of 0.05.

^dRepresents a significance level of 0.10.

for comparison. The estimates for the Logistic map and the shuffled series provide the benchmarks for a known chaotic, and a generally random series. The entropy estimates for the [AR(p),S] crude oil, heating oil and unleaded gasoline series show little signs of ‘settling down’ as do those for the Logistic map. They behave much more like the entropy estimates for the shuffled series: a general rise in the K2 statistic as one increases the embedding dimension. The plots in Fig. 1 reaffirm the correlation dimension and BDS test results: there is no evidence of low dimension chaos in energy futures prices.

4.4. ARCH and maturity effects in futures markets

It is apparent from the BDS statistics presented in Table 4, that the GARCH (1,1) model effectively explained the non-linearities in the three contracts. Furthermore, we can reexamine the Samuelson hypothesis on the relationship between

Table 5
ARCH dynamics in crude oil, heating oil, and unleaded gasoline futures^a

	Crude [h_t]	Heating oil [h_t]	Unleaded gas [h_t]
Constant	0.029 ^b (1.73)	0.24 ^b (7.05)	0.51 ^b (10.19)
ε_{t-1}	0.11 ^b (13.75)	0.086 ^b (12.90)	0.12 ^b (14.56)
h_{t-1}	0.88 ^b (107.04)	0.89 ^b (97.21)	0.86 ^b (88.92)
TTM	−0.007 (−0.52)	−0.014 ^b (−5.96)	−0.03 (−11.82)
LL	−5339.77	−5060.54	−5310.60
Chi-square	1752.96 ^b	924.22 ^b	276.32 ^b

^aThe maximum likelihood estimates are from GARCH (1,1) models fitted to the three futures returns. The variance parameters estimated are from Eqs. (9) and (11). Statistics in () are *t*-values. TTM represents time to maturity in days. The Chi-square test statistic is given by $2[LL(\text{GARCH})-LL(\text{OLS})]$, where LL represents the Log-likelihood function.

^bRepresents a significance level of 0.01.

contract maturity and variance employing the appropriately modelled variance structure. The Samuelson hypothesis implies that the volatility in futures price-changes increases as a contract's delivery date approaches. If the Samuelson hypothesis were to be valid, proper valuation of futures and futures options would require that the term-structure of the volatility be estimated (also see Bessembinder et al., 1996).

Table 5 reports the maximum likelihood estimation results for the three energy contracts. In the interest of brevity, we do not present the results from the mean equations. The results indicate strong ARCH effects in all contracts. The Samuelson hypothesis is clearly supported for both the heating oil and unleaded gasoline contracts: the time to maturity (TTM) variable is negative and significant in the two variance equations. The same negative relationship is shown for the crude oil contract, but is not statistically significant. Thus, there is strong evidence that as we get closer to maturity (as TTM falls), the conditional variance (h_t) increases. However, it is notable that while TTM is found to be significant in the variance equation, this variable did not play a large role as a 'control variable' in the tests for chaos: the BDS statistics remained almost unchanged when we employed standardized residuals from models without TTM. In other words, the correlation–integration based tests for chaos are not as sensitive to controls for TTM.

It should be noted that the evidence on strong ARCH-type effects (and a lack of chaos) is robust to controls for structural changes in the data. For instance, given that 1986 and 1991 witnessed tremendous volatility in oil prices, we also considered extensions to the above ARCH type models that included controls for the abnormal price shocks over the 2 years. We formed a dummy variable which takes on the value of 1 for those years (1986 and 1991) and zero otherwise. The ARCH type models for the three contracts were reestimated, and their standardized residuals extracted. The BDS statistics for the standardized residuals were virtually identical to those reported in Table 2. These results are available from the authors.

5. Conclusion

Financial research in the past two decades has focused on the chaotic behavior of various price series for a number of reasons. Several research papers document evidence of non-linearity and chaos for various financial and economic variables. Chaotic dynamics are necessarily non-linear, and may be quite suitable in explaining sudden movements and wide fluctuations in stock prices, exchange rates, and myriad of other financial and economic series may not be properly captured by linear models. Furthermore, there is growing belief that technical analysis may succeed in forecasting short-term price behavior of chaotic time series. For instance, there is some evidence that simple and common technical trading rules (e.g. the heads-over-shoulders trading rule) will provide a short-term advantage to the trader when the price series is chaotic.

Employing daily data for over a decade, we conduct a battery of tests for the presence of low-dimensional chaotic structure in the crude oil, heating oil, and unleaded gasoline futures prices.

Daily returns data from the nearby contracts are diagnosed employing correlation dimension tests, BDS tests, and tests for entropy. While we find strong evidence of non-linear dependence in the data, the evidence is not consistent with chaos. Our test results indicate that ARCH-type processes may explain the non-linearities in the data. We also make a case that employing seasonally adjusted price series is important to obtaining robust results via the existing tests for chaotic structure. The evidence that the three oil commodities are driven by ARCH-type processes (and not chaos) is robust to controls for the oil shocks of 1986 and 1991.

For the three contracts several ARCH-type models adequately explain non-linear dependencies, and the maturity-effect in futures prices. The GARCH (1,1) results for the three futures contracts provide evidence in favor of the Samuelson hypothesis: volatility in futures returns increases as contracts approach maturity. However, the tests for chaos were found not to be as sensitive to controls for futures contract-maturity as they were to controls for seasonality.

Appendix A. Simulated critical values for the BDS test statistic

The figures represent the simulated values of the BDS statistic from Monte Carlo simulations of 2000 observations each. The simulations generated the 250 replications of the GARCH model ($\alpha_1 = 0.10$, $\beta_1 = 0.80$), the exponential GARCH model ($\alpha_1 = 0.05$, $\alpha_2 = 0.05$, $\beta_1 = 0.80$), and the asymmetric component model ($\alpha = 0.05$, $\beta = 0.10$, $\rho = 0.80$, $\phi = 0.05$). BDS statistics for four embedding dimensions and $\varepsilon = 0.5, 1, 1.5$ and 2 S.D. of the data were then computed for the 2500 simulated series. The critical values represent the 97.5th and 2.5th percentile of the distribution of the simulated statistics.

<i>M</i>	ε/σ			
	0.5	1.0	1.5	2.0
GARCH (1,1) (97.5% critical values)				
2	1.62	1.53	1.42	1.25
3	1.76	1.63	1.45	1.44
4	2.35	2.21	2.16	1.97
5	2.42	2.28	2.25	2.10
Exponential GARCH (97.5% critical values)				
2	2.75	2.54	2.10	1.83
3	3.30	3.07	2.42	2.38
4	3.48	3.31	2.66	2.56
5	3.66	3.47	2.97	2.61
Asymmetric Component GARCH (2.5% critical values)				
2	−2.86	−2.29	−1.78	−1.74
3	−3.51	−2.89	−2.49	−2.26
4	−3.64	−3.01	−2.81	−2.55
5	−3.67	−3.12	−3.08	−2.64
Asymmetric Component GARCH (97.5% critical values)				
2	1.40	1.13	1.02	0.80
3	1.47	1.27	1.17	0.93
4	1.62	1.28	1.22	1.00
5	1.82	1.40	1.31	1.07

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