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Neutron Scattering and the Dispersion Relation in He II at Small Momentum Transfer*

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Weighted-least-squares fits to the small momentum transfer ($k \leq 1 \text{ \AA}^{-1}$) results of Woods and Cowley have been carried out. We find that the neutron data presently available (i) are not of sufficient accuracy to determine whether or not quadratic terms are present in the dispersion relation and (ii) *weakly* favor positive dispersion in the absence of the quadratic term.

The detailed form of the dispersion relation in He II has recently been the subject of intense interest.¹⁻⁷ The present discussion⁸ was motivated by the work of Molinari and Regge² in which they suggested the dispersion curve in He II might possess a term quadratic in the momentum. Their work was based on theoretical efforts as well as on computer fits to the data of Cowley and Woods¹ as shown in Fig. 1. In the work we shall describe here we have also carried out fits to the data of Cowley and Woods. It is our feeling that questions concerning the lower-order coefficients of the dispersion relation in He II can be best addressed by an examination of the small-momentum-transfer portion of the neutron scattering data. Hence, in our case we have restricted attention to the small-angle data ($k \leq 1.0 \text{ \AA}^{-1}$).

As a basis for the present comments, let us introduce the following expansion for the dispersion relation $E(k)$:

$$E(k) = \hbar^2 k^2 \left[1 + \sum_{j=1}^P \alpha_j \left(\frac{\hbar k}{mc} \right)^{n_j} \right]. \quad (1)$$

The present discussion is concerned with whether or not the existing neutron-scattering experiments support the conclusions that (i) the dispersion curve has a small upward concavity³ and (ii) the above expansions contain odd⁹ powers of k (i.e., can $n=1$?). The very recent work of Roach *et al.*⁵ seems to establish beyond doubt that for $n=1$ the coefficient α_1 in (1) above is either absent or else is very small. If the disper-

sion curve can in fact be represented by analytic expression such as Eq. (1), then the results obtained by Roach *et al.* can be expected to remain valid for larger values of the momentum. To maintain perspective, however, it should be remembered that the measurements of Roach *et al.* at the highest frequency (90 MHz) correspond to phonon energies of $4 \times 10^{-3} \text{ K}$ and k values of $2 \times 10^{-4} \text{ \AA}^{-1}$. These are to be compared with the neutron results of interest here which span energies up to 12 K and momentum transfer values as large as $1 \text{ \AA}^{-1}$ (the smallest being $0.1 \text{ \AA}^{-1}$).

Based on our fits to the neutron-scattering data at small momentum transfer ($k \leq 1 \text{ \AA}^{-1}$) we find that (a) these data are not of sufficient accuracy to determine whether or not one should take $n=1$ or $n=2$ in Eq. (1), (b) if one *demand*s an expansion of the form of Eq. (1) with $n=2$, then the neutron data favors $\alpha_1 > 0$ and finally (c) if one *demand*s an expansion of the form of Eq. (1) with $n=1$ but with $\alpha_1 = 0$, then $\alpha_2 > 0$ is favored and again the dispersion is positive.

These conclusions can be seen rather clearly by reference to the tables. Table I displays the relative errors of the various fits for several representative sets of the data. The table can be most easily understood by an example. Focus attention on the three-parameter fits for $E(k)$. Take the case of $M=25$. The entries indicate that an even-power expansion (2, 4, 6) is preferred, an expansion of mixed powers starting with k^2

FIG. 1. Data of Woods and Cowley. The work of Molinari and Regge (Ref. 2) was confined to the region $k \geq 1.1 \text{ \AA}^{-1}$. We have focused attention on the region $k \leq 1.0 \text{ \AA}^{-1}$.

(2, 3, 4) is not as good and the (1, 2, 3) expansion is the worst of the three. The entries are the sum square deviations of the data from the given fit relative to the best fit for the given number of parameters and M value under consideration. The larger the value of M in the table, the more data points were fit (see Fig. 2.) and consequently the larger the values of the momentum transfer which were included. It is our view that the table supports the conclusion that the present neutron data (at small momentum transfer) makes no unambiguous statement about the presence or absence of the quadratic term. Table II is a very schematic portrayal of the preferred sign of the term $(\hbar k/mc)^2$ in the various fits. If we focus attention on those fits which are of the form $(1.0 + \alpha_2(\hbar k/mc)^2 + \dots)$ the conclusion is that the present neutron data tends to weakly support the results of Maris. It should be emphasized here that the correct statement is that the presently available scattering results *tend* to support the conclusions of Maris when attention is confined to small momentum transfer.

To reach these conclusions we have carried out a series of weighted-least-squares polynomial fits to the data of Cowley and Woods.¹ We used their 33 data points as presented in their plot of velocity versus k^2 (see Fig. 9 of Ref. 1) for the momentum-transfer range $k \leq 1.0 \text{ \AA}^{-1}$, rather than

FIG. 2. Small-momentum-transfer dispersion curve for He II. The data shown here is that we have used in our various fits. It was obtained from Ref. 1 in the manner described in the text. M refers to the number of points used in a given fit. In all of our fits we have set E equal to zero at $k=0$.

the 9 points in this range as presented in their Table 3. Figure 2 displays these points translated into the more familiar format of the dispersion curve. Our reason for adopting this procedure was that the values presented in Table 3 (of Ref. 1) are "best values" and not actual data points.¹⁰ We have also carried out several fits using the "best-values" data. In the cases where $k_{\max}$ coincided with that used by Molinari and Regge, our fits produced essentially the same coefficients

TABLE I. Relative errors of a representative sample of the closed form fits. The fits to the elementary excitation spectrum show no definite preference for a given leading term in the fit. The columns headed by parameters indicate the number of coefficients in the fit. M refers to the number of data points used in the fit. For $M=30$ nearly all of the data $k \leq 1 \text{ \AA}^{-1}$ is used. The numbers in the first row refer to the powers of $(\hbar k/mc)$ used in the fit in the column below.

M	Two parameters			Three parameters			Four parameters		
	1, 2	2, 3	2, 4	1, 2, 3	2, 3, 4	2, 4, 6	1, 2, 3, 4	2, 3, 4, 5	2, 4, 6, 8
10	1	1	1	1	1	1	1	1	1
15	1	1.02	1.04	1.05	1.02	1	1	1	1.01
20	1.01	1	1.03	1	1.01	1.02	1	1	1
25	1.08	1	1.12	1.02	1.01	1	1.02	1.01	1
30	1	1.18	1.40	1.10	1.03	1	1.01	1	1

TABLE II. Sign of the $(\hbar k/mc)^2$ term in the various fits. This table is highly schematic and represents the sign of the $(\hbar k/mc)^2$ term favored by the fits with M above 25 in the various cases. As can be seen, the neutron data in this range is consistent with positive dispersion. As indicated in the text, however, this table must be viewed with caution.

Two parameters	Three parameters	Four parameters
1, 2 (-)	1, 2, 3 (-)	1, 2, 3, 4 (+)
2, 3 (+)	2, 3, 4 (+)	2, 3, 4, 5 (+)
2, 4 (+)	2, 4, 6 (+)	2, 4, 6, 8 (+)

as theirs.

Several comments on the details of the fitting are in order. Our fits were to closed form polynomial expressions of the sort given by Eq. (1). We have carried out fits with maximum values of P [see Eq. (1)] ranging from 2 to 5. For each value of P we have carried out fits over increasing ranges of the data starting with the lowest 6 ($M=6$) data points and working up to the full 33 ($M=33$) points which completely cover the momentum-transfer range $k \leq 1 \text{ \AA}^{-1}$. In most cases the resulting coefficients were highly correlated. These coefficients represent a set of numbers too large to be published here.

Complementary to the fitting of the neutron data of Cowley and Woods we have carried out the same

sort of fitting on the x-ray $S(k)$ measurements of Hallock.¹¹ In the case of the liquid-structure-factor data, we have used from 6 to 33 points, covering in the same manner as before the momentum-transfer span $k \leq 1.125 \text{ \AA}^{-1}$. The results of this study suggest that a linear term and odd powers for $S(k)$ is slightly preferred.

Finally we have also carried out a series of small-momentum-transfer fits to determine the suitability of terms⁴ like $k^4 \ln(1/k)$ in the expressions for both $E(k)$ and $S(k)$. We find that for $k \leq 1 \text{ \AA}^{-1}$ neither the energy data nor the structure-factor data is accurate enough to allow a determination as to whether or not the $k^4 \ln(1/k)$ terms are present.

The conclusion we wish to draw here is that while the neutron-scattering results of Woods and Cowley are capable of answering many questions concerning the nature of superfluid helium those results are not accurate enough to allow unambiguous answers to the questions we have posed here. It seems clear that to answer these questions from neutron-scattering data alone, one requires data of higher accuracy than presently available.

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¹R. A. Cowley and A. D. B. Woods, Can. J. Phys. **49**, 177 (1971).

²A. Molinari and T. Regge, Phys. Rev. Lett. **26**, 1531 (1971).

³H. J. Maris, Phys. Rev. Lett. **28**, 277 (1972).

⁴H. Gould and V. K. Wong, Phys. Rev. Lett. **27**, 301 (1971).

⁵P. R. Roach, B. M. Abraham, J. B. Ketterson, and M. Kuchnir, Phys. Rev. Lett. **29**, 32 (1972).

⁶N. E. Phillips, C. G. Waterfield, and J. K. Hoffer, Phys. Rev. Lett. **25**, 1260 (1970).

⁷C. Zasada and R. K. Pathria, Phys. Rev. Lett. **29**, 988 (1972).

⁸The results of this work appear in *Proceedings of the Thirteenth International Conference on Low Temperature Physics, Boulder, Colo., 1972*, edited by R. H. Kropschot and K. D. Timmerhaus (University of Colorado Press, Boulder, Colo., 1973). See also, Bull. Am. Phys. Soc. **17**, 610 (1972).

⁹E. Feenberg, Phys. Rev. Lett. **26**, 301 (1971).

¹⁰A. D. B. Woods (private communication).

¹¹R. B. Hallock, Phys. Rev. A **5**, 320 (1972).