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FIG. 2. Magnetic surfaces $\tilde{\psi} = \text{Ai}(-\rho) \cos ky = \text{const}$. The parameters are such that $\beta = (2\epsilon)^{-1}$.

we see from Fig. 1 that as $U(\rho_2) \rightarrow 0$, $\rho_1 \rightarrow -\infty$. Also $(dU/d\rho)_{\text{max}} \rightarrow 0.7$, and $\rho_0 \rightarrow U_{\text{max}}^{-2} \approx 4$. From (22) and (23) we have

$$\beta = -\rho_1/8\epsilon$$

in the limit $\rho_1 \rightarrow -\infty$.

We conclude that an equilibrium exists for all β . However, as $\beta \rightarrow \infty$, $\lambda \rightarrow -1$, and since j_ϕ is proportional to $x + \lambda$ the current must reverse direction across the plasma. The condition that

the current not reverse is $\lambda > 1$. We find that the condition $\lambda = 1$ corresponds to $b = 1.3$, $\rho_2 = 2.0$, $dU(\rho_2)/d\rho = 0.5$, and $p_0 = 4.0$, and obtain

$$\beta = 0.6\epsilon^{-1} \quad (25)$$

as the critical β above which the current reverses.

When β is of order ϵ^{-1} , we have seen that λ is of order unity. This means that the equilibrium is primarily maintained by the poloidal current. From (17) we have

$$\tilde{I}(\tilde{\psi}) \approx \frac{1}{2}\beta\theta\tilde{p}(\tilde{\psi}).$$

Differentiating and using (3), we find

$$j_\phi B_\phi \approx c \nabla p.$$

A typical equilibrium is shown in Fig. 2.

¹V. D. Shafranov, *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 2.

²*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965).

Small-Momentum-Transfer Structure Factor of Quantum Liquids*

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The liquid structure factor of ^3He has been determined in the momentum-transfer range 0.2 to $1.0 \text{ \AA}^{-1}$. At temperatures near 0.4 K there is evidence for a gentle change of slope of the structure factor in the region near $0.35 \text{ \AA}^{-1}$. Comparison with the work on helium-4 suggests that the observed behavior is a general feature of the two fluids.

We have measured¹ the intensity of copper $K\alpha$ x rays scattered from liquid ^3He ² as a function of scattering angle at temperatures of 0.36 and 0.41 K . By normalizing against neon at 77 K a determination of the liquid structure factor of ^3He was made. The results suggest that the structure-factor curve for ^3He possesses a gentle shoulder near the momentum-transfer value of $0.35 \text{ \AA}^{-1}$. Comparison of the present results with earlier work on ^4He by Hallock³ indicates that the change of slope reported for ^4He is not peculiar to ^4He , but rather is a general feature of the fluids.

The intensity of x-rays scattered from a fluid at a temperature T and momentum transfer k is given by⁴ the expression

$$I(k, T) = A\tau N[\sigma_e S(k, T) + \sigma_i], \quad (1)$$

where A is the Thompson cross section for the scattering of an x ray by an electron, τ is the transmission factor, N is the number of atoms visible to both the source of x rays and the detector, and σ_e and σ_i are the coherent and incoherent scattering factors. S is the liquid structure factor and provides a measure of the departure of the fluid from ideal gas behavior. Since for an ideal gas $S \equiv 1$ for all angles of scattering, one can obtain S for a fluid by comparing the scattering from the fluid in question to that produced by an ideal gas. If one denotes the intensity scattered from the ideal gas by

$$I = A\tau N(\xi_e + \xi_i), \quad (2)$$

then one finds that

$$S(k, T) = \frac{I_L T_G \rho_G}{I_G T_L \rho_L} \frac{\xi_e + \xi_i}{\sigma_e} - \frac{\sigma_i}{\sigma_e}, \quad (3)$$

where the ratio N_G/N_L has been replaced by the ratio of number densities since for a given scattering angle the scattering volume will be a constant. Our normalization gas for the ^3He runs was neon at 77 K and 446 Torr (^3He at $T=0.36$) and 77 K and 680 Torr (^3He at $T=0.41$). The density of the liquid ^3He was taken⁵ as 0.0819 g/cm^3 , while the density of the neon was computed from the Berlin⁶ virial coefficients. The scattering factors for helium were taken from the work of Kim and Inokuti,⁷ while those for neon were taken from the work of Tovard, Nicolas, and Roualt.⁸

It is generally known⁹ that for finite temperatures in the limit of vanishing momentum transfer the structure factor can be written as

$$\lim_{k \rightarrow 0} S(k, T) = \rho k_B T X_T, \quad (4)$$

where ρ is the number density of the sample, k_B the Boltzmann constant, T the temperature, and X_T the isothermal compressibility. It is clear from this expression that so long as T remains nonzero, the structure factor will be nonzero. This expression allows us to find the intercept $S(0, T)$ for any fluid. In particular, for an ideal gas $X_T = (\rho k_B T)^{-1}$ and hence $S(0, T) = 1$ for such a case. In general the value of $S(0, T)$ is a very sensitive test of whether or not a gas is ideal. Nonideal character causes a departure of $S(k)$ from 1, and the most severe departure occurs for $k \rightarrow 0$. For neon at 680 mm and 77 K, the value of $S(0, T)$ obtained from the virial coefficients is 1.004. Hence, neon at the temperature and pressures of this experiment can be considered, for our purposes, to be a strictly ideal gas.

The spectrometer used in this work differs only slightly from the one used for our earlier work on ^4He which has been described in detail elsewhere.¹⁰ The target cell for the data taken at 0.36 K was a Mylar cylinder similar to those¹¹ we have used in the past. For the data taken at 0.41 K we have used a cylindrical beryllium cell, $\frac{1}{2}$ in. in diameter and 2 in. in length.¹² The wall thickness over $1\frac{5}{8}$ in. of the length is 0.010 in. We chose beryllium for this work since we wished to study the structure of gaseous ^4He and ^3He at elevated pressures during the same run.¹³ The use of a beryllium cylinder also allowed us to obtain data from two targets of different construction. The runs at the two temperatures thus represent two completely different experimental determinations of the structure factor of liquid ^3He . As in my previous work, a General Electric CA-7H x-ray tube is used as the source of x rays. The main beam is derived from the line

source and collimated by a series of soller slits (to limit the vertical divergence) and knife edges (to define the beam horizontally). The beam spot on the target measures 0.350 in. high by 0.060 in. wide. The target cell is mounted at the base of a closed-cycle ^3He refrigerator and is centered so that its vertical axis is coincident with the rotation axis of the scattered-beam detector. The temperature was measured by two calibrated carbon-composition resistors¹⁴ by means of a standard bridge circuit. Power dissipated in the resistors during temperature measurements was about 10^{-10} W . The detector¹⁵ was moved to provide angular increments of about 0.5° for the data at $T=0.41 \text{ K}$. The angular resolution for all of this work is about 0.6° . The method of data acquisition is identical to that reported earlier¹⁶ and need not be discussed again here.

Some of our results for ^3He are shown in Fig. 1 along with our low-temperature results³ for ^4He for comparison. The dots represent data taken at $T=0.41 \text{ K}$ while the crosses¹⁷ represent data taken at 0.36 K. Equation (4) demands that in the limit of $T \rightarrow 0$ we must have $S(0, T) \rightarrow 0$. Further, in the temperature range below about 1 K the structure factor is temperature independent¹⁸ for values of k greater than about $0.3 \text{ \AA}^{-1}$. Thus, our

FIG. 1. Small- k liquid structure factor of ^3He as compared to ^4He . The closed circles and the crosses represent data taken on ^3He at $T=0.41 \text{ K}$ and $T=0.36 \text{ K}$, respectively. The open circles represent the ^4He data (Ref. 3) taken at $T=0.38 \text{ K}$. In each case the intercepts are obtained from Eq. (4). Note that even though the temperature is nearly the same for ^3He as for ^4He , the difference in the compressibility makes $S(0, T)$ different. The solid line represents the Bijl-Feynman results $S(k) = \hbar k / 2mc$ for ^4He . To avoid confusion, only representative error bars (due to counting statistics) have been shown. No smoothing has been applied to any of the data.

data indicate that the small- k structure factor of ^3He will have a gentle shoulder in the region near $k = 0.35 \text{ \AA}^{-1}$. The open circles represent the data³ for ^4He at $T = 0.38 \text{ K}$. The solid line represents the Bijl-Feynman relation $S(k) = \hbar k / 2mc$ for ^4He . The intercept obtained from Eq. (4) is indicated by the symbol on the ordinate in each case. Note that while the temperatures are nearly the same, the intercept for ^3He is higher than that for ^4He because X_T is larger for ^3He than it is for ^4He . It should be emphasized that no smoothing or fitting of the data has been done to reduce the scatter in any of the figures.

The appearance of a gentle change of slope in the case of ^3He indicates that the shoulder observed in ^4He is not peculiar to that system, but rather is a general feature of the quantum fluids and not a consequence of a *particular* type of quantum statistics.

It is interesting to investigate the detailed shape of the very small-momentum-transfer structure factor of ^3He as the temperature approaches zero. In particular, for a noninteracting (ideal) Fermi system the structure factor should have the form¹⁹

$$S(k) = \frac{3}{2} (k/2k_F) \text{ as } k \rightarrow 0. \quad (5)$$

Recently Widom and Sigel²⁰ have shown that the Landau theory of density oscillations in a Fermi liquid predicts that

$$S(k) = \frac{3}{2} \alpha (k/2k_F) \text{ as } k \rightarrow 0, \quad (6)$$

where $\alpha = 0.575$. The behavior of these two limiting forms for $S(k)$ is shown in Fig. 2 along with

FIG. 2. Expected behavior of $S(k)$ for a noninteracting Fermi gas (dashed line) and the Landau theory (solid line). The open triangles represent the data of Achter and Meyer on ^3He at $T = 0.56 \text{ K}$. Again, the dots and crosses are our data on ^3He at 0.41 and 0.36 K . The ordinates are calculated from Eq. (4).

our current results and the results of Achter and Meyer.²¹ Because of the large compressibility of ^3He , the finite-temperature effects remain significant even at temperatures as low as those reported here. It is clear that as $T \rightarrow 0$ there can be no question about the presence of a bend in $S(k)$. Our work is in disagreement with the work of Achter and Meyer on the question of the strength of that bend.

The detailed form of the structure factor at very small k is another matter. Widom and Sigel claim that their result, Eq. (6), along with the results of Achter and Meyer, constitutes strong evidence for the validity of the Landau theory of density oscillations in Fermi liquids. We view our results as very suggestive in this regard.²² However, in view of Fig. 2, it is clear that in order to speak to this question conclusively one needs data at lower temperatures and perhaps smaller scattering angles. To this and other ends we are constructing a dilution refrigerator and will attempt to explore the very low-temperature behavior of the ^3He structure factor at both small and large momentum transfer.

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¹A very preliminary report on some of our results for liquid ^3He will appear: R. B. Hallock, in *Proceedings of the Twelfth International Conference on Low Temperature Physics*, Kyoto, 1970 (to be published).

²The ^3He was purified in a separate apparatus (built by H. D. Cohen at Stanford University) and had about 100 ppm of ^4He impurity.

³R. B. Hallock, *Phys. Rev. A* (to be published).

⁴See, for example, N. S. Gingrich, *Rev. Mod. Phys.* **15**, 90 (1943).

⁵See, for example, J. Wilks, *The Properties of Liquid and Solid Helium* (Oxford U. Press, Oxford, England, 1969).

⁶See, for example, *Argon, Helium and the Rare Gases*, edited by G. A. Cook (Interscience, New York, 1971), p. 262.

⁷Y. K. Kim and M. Inokuti, *Phys. Rev.* **165**, 39 (1967).

⁸C. Tovard, D. Nides, and M. Roualt, *J. Chim. Phys. Physicochim. Biol.* **64**, 540 (1967).

⁹See, for example, L. Goldstein, *Phys. Rev.* **84**, 466 (1951).

¹⁰R. B. Hallock, thesis, Stanford University, Stanford, Calif., 1969 (University Microfilms, Ann Arbor, Mich.).

¹¹R. B. Hallock, *Rev. Sci. Instrum.* **41**, 1107 (1970).

¹²Beryllium cylinders have been produced for us in various sizes by Speedring Corp., 7111 East Eleven Mile Road, Warren, Mich.

¹³R. B. Hallock, to be published.

¹⁴Speer grade 1002, 5%, $\frac{1}{2}$ Ω .

¹⁵Reuter-Stokes model RG-61 (1 atm xenon-methane fill).

¹⁶R. B. Hallock, *Phys. Rev. Lett.* **23**, 830 (1969), and Ref. 3.

¹⁷Experimental difficulties limited the time available for data collection to that necessary for a single sweep of angles at low temperatures in the run at $T=0.36$ K and hence the error bars on those data are larger than those on the data taken at $T=0.41$ K. One of these difficulties was a slow temperature oscillation of the order of ± 50 mK. The temperature of 0.36 K is thus an average temperature (several oscillations generally oc-

curred in the time required to accumulate data at one angle of scattering). In view of the reasons stated in the text, such small excursions in temperature have no observable effect on $S(k)$ for k values greater than about $0.2 \text{ \AA}^{-1}$. The temperature remained stable at $T=0.41$ K throughout the collection of the data represented by the dots.

¹⁸This fact is clearly illustrated in Ref. 3 for the case of ^4He . There the measured structure factor for the case of $T=1.45$ K becomes indistinguishable from that for $T=0.38$ K above $k=0.3 \text{ \AA}^{-1}$. The data of Ref. 3 extend over the range $0.133 \leq k \leq 1.125 \text{ \AA}^{-1}$.

¹⁹See, for example, E. Feenberg, *Quantum Theory of Fluids* (Academic, New York, 1969); D. Pines and P. Nozières, *Theory of Quantum Liquids* (Benjamin, New York, 1966).

²⁰A. Widom and J. L. Sigel, *Phys. Rev. Lett.* **24**, 1400 (1970).

²¹E. K. Achter and L. Meyer, *Phys. Rev.* **188**, 291 (1969).

²²A reduction of the temperature to the order of 50 mK should result in a downward trend for the data at lowest angles. The data above about $0.25 \text{ \AA}^{-1}$ should remain virtually unaffected by the temperature reduction. The intercept $S(0, T)$ will drop by a factor of 8. The existence of such data down to $k \approx 0.1 \text{ \AA}^{-1}$ would, in our opinion, constitute confirmation (in the spirit of Fig. 1 for ^4He) of the Landau theory.

Anomalous Diffusion in Axisymmetric Toroidal Systems

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We consider the anomalous diffusion in an axisymmetric toroidal plasma due to low-frequency instabilities. The radial, cross-field diffusion is mainly due to the nonconservation of the azimuthal canonical angular momentum. In particular, the electric field component along the magnetic line of force could contribute significantly to the cross-field diffusion by driving a change of the canonical angular momentum. For trapped particles, this diffusion could be $(B_T/B_p)^2$ larger than the $\vec{E} \times \vec{B}$ diffusion for comparable $E_{\parallel}$ and $E_{\perp}$, where B_T and B_p are the toroidal and poloidal magnetic fields, respectively.

The importance of geometric effects on the transport processes in toroidal plasmas has been generally recognized. The major effects, such as the enhancement of the classical diffusion¹ and the inward convection due to the inductive electric field,² are associated with the trapped particles in the magnetic well. The trapped particles acting as a "dynamic impurity" are also responsible for a class of low-frequency instabilities—the trapped-particle instabilities.³ The turbulence resulting from these low-frequency instabilities could lead to anomalous diffusion.⁴ Recently, Yoshikawa⁵ pointed out that the anomalous diffusion coefficients in toroidal systems depend critically on the geometric factors. Such dependence was further suggested as a possible explanation for the observed difference in confinement time in various toroidal devices.⁵

In this Letter, we consider the anomalous diffusion in an axisymmetric system due to low-frequency,