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Irregular repeat-accumulate codes for volume holographic memory systems

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We investigate the application of irregular repeat-accumulate (IRA) codes in volume holographic memory (VHM) systems. We introduce methodologies to design efficient IRA codes. We show that a judiciously designed IRA code for a typical VHM can be as good as the optimized irregular low-density-parity-check codes while having the additional advantage of lower encoding complexity. Moreover, we present a method to reduce the error-floor effect of the IRA codes in the VHM systems. This method explores the structure of the noise pattern in holographic memories. Finally, we explain why IRA codes are good candidates for the VHM systems. © 2004 Optical Society of America

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1. Introduction

Several issues, such as coding, modulation, and equalization, are important for volume holographic systems (VHM) systems. In this paper, we are concerned with choosing efficient error-correcting codes for the VHM systems. The storage capacity of a VHM system is defined as the number of information bits stored under the condition that the bit error rate (BER) is lower than a required value. The information theoretical capacity is the highest theoretically achievable capacity. Since the diffraction efficiency of the recorded holograms decreases with an increasing of the number of pages, the BER increases when we increase the number of stored pages. To increase the storage capacity, we can store more pages and use error-control coding to decrease the BER to the desired value. If we increase the number of stored pages by a factor f , the capacity of the system is increased by the factor $f \times R$, where R is the code rate (the ratio of the number of information bits to the total number of bits). Thus, for a constant number of pages, to have the highest storage capacity we need to find a code with the highest rate that provides us with the required output BER. The op-

timization of the number of pages was studied in Ref. 1.

Error-control coding for VHM systems has been studied extensively.¹⁻⁶ The application of capacity-achieving codes for these systems was introduced in Ref. 7. Capacity-achieving codes provide the highest storage capacity for VHM systems. In Ref. 7, it was shown that by use of proper low-density parity-check (LDPC) codes a considerable improvement in the storage capacity is achievable in comparison with Reed-Solomon (RS) codes together with an interleaver. Codes that are used in holographic memories have high rates and can have large block lengths. There are several requirements for these codes. The first and most important requirement is that the codes must perform near the Shannon limit. This is because we want to get the highest capacity. Second, these codes must not suffer from the error-floor effect. The error floor can be explained in the following way. Generally when the signal-to-noise ratio (SNR) increases, the output BER of the decoder decreases. However, in many capacity-approaching codes, the BER seems to stop decreasing at a certain value, e.g., 10^{-6} . This is very undesirable in VHM systems. In fact, a BER of 10^{-12} is required in these systems. Third, they must have efficient encoding algorithms. Finally, they must have efficient decoding algorithms.

LDPC codes, if constructed appropriately, seem to satisfy all requirements for the VHM systems, except for encoding complexity. Practically speaking, it is very important that the codes we use have low encoding and decoding complexity. The decoding complexity of LDPC codes is linear with respect to the

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code length. However, their encoding complexity is quadratic in the code length. Although in Ref. 8 the authors proposed a method to reduce the encoding complexity, a simpler encoding process is desirable for the VHM systems. This is because we would like to use codes with very large (10^5 bits) block lengths to get the best performance versus complexity.⁷

Our aim in this paper is to show that carefully designed irregular repeat-accumulate (IRA) codes satisfy all of the required properties. As oppose to LDPC codes, IRA codes have linear encoding complexity. Meanwhile, although ordinary IRA codes may suffer from the error-floor problem, we take advantage of the specific properties of the noise pattern in the VHM systems to solve this issue. It is worth noting that for simplicity we ignored the intersymbol interference. Intersymbol interference can be considered in both designing and decoding of IRA codes.⁹

2. Background on Irregular Repeat-Accumulate Codes

In this section we provide a brief review of IRA codes, following the terminology in Ref. 10, where IRA codes were first introduced. These codes are in fact a special case of LDPC codes.^{11–15} An error-correcting code with k input bits with $n = k + r$, is said to be a (k, n) code. An ensemble of $(k, n = k + r)$ IRA codes is determined by the parameters $(f_1, f_2, \dots, f_J; a)$, where $f_i \geq 0$, $\sum_i f_i = 1$, and a is a positive integer. To encode the information bits $(u_1, u_2, \dots, u_k)$, we repeat each bit several times. In particular, for any i , kf_i of bits are repeated i times. Thus we obtain a vector of length $ra = k \sum_i if_i$. Call this vector $W = (w_1, w_2, \dots, w_{ra})$. We then permute the vector W to obtain the vector $V = (v_1, v_2, \dots, v_{ra})$. The permutation is chosen beforehand, randomly from all permutations of ra elements and is made fixed. That is, to choose a code from the ensemble $(f_1, f_2, \dots, f_J; a)$, we first choose a permutation and fix it for the code. Thus any code from the ensemble $(f_1, f_2, \dots, f_J; a)$ is determined by a random permutation. Now we can generate the parity bits $x_1, x_2, \dots, x_r$ using the following recursion:

$$x_j = x_{j-1} + \sum_{i=1}^a v_{(j-1)a+i}, \quad (1)$$

for $j = 1, 2, \dots, r$. Note that the addition operation in Eq. (1) is the XOR operation. The codeword is then $(u_1, u_2, \dots, u_k, x_1, x_2, \dots, x_r)$, i.e., the code is systematic. Therefore the code rate is $R = k/(k + r)$. Practically speaking, the above encoding is very efficient.¹⁰

Figure 1 shows the Tanner graph representation of the codes from the ensemble defined by $(f_1, f_2, \dots, f_J; a)$. The circles at the left are information nodes (i.e., $u_1, u_2, \dots, u_k$), whereas the right circles are parity nodes ($x_1, x_2, \dots, x_r$). We call the union of the information nodes and the parity nodes, the variable nodes. The quadrangles show the check nodes. Any check node corresponds to one parity-check equation. From Eq. (1) we see that the check node

Fig. 1. Tanner graph representation of the codes from the ensemble defined by $(f_1, f_2, \dots, f_J; a)$.

that is connected to the parity bits x_j and x_{j-1} is associated with the equation

$$x_j + x_{j-1} + \sum_{i=1}^a v_{(j-1)a+i} = 0. \quad (2)$$

Since the class of IRA codes is a subclass of LDPC codes, IRA codes can be decoded by message-passing algorithms. Message-passing algorithms are very efficient and run in linear time with respect to the code length. It is shown in Ref. 10 that these codes, when decoded by use of a message-passing algorithm called belief propagation, are capacity achieving for the binary erasure channel. It has also been verified experimentally that for other channels these codes can approach the Shannon limit.¹⁰

3. Design of Irregular Repeat-Accumulate Codes for Volume Holographic Memory Systems

The design procedure that we propose for IRA codes in VHM systems is similar to the design we proposed for LDPC codes in Refs. 7 and 16. However, extra care is needed for handling the error-floor problem. In this section, we first discuss the design and the analysis of the IRA codes over nonuniform channels. The design of codes for the VHM systems is specifically considered as a special case of the code design for the nonuniform channels. We then discuss the error-floor effect and introduce a method to alleviate this problem.

A. Irregular Repeat-Accumulate Codes for Nonuniform Channels

In holographic data storage, the information is recorded and retrieved in the form of two-dimensional data pages (i.e., two-dimensional patterns of bits). These bits are subject to different sources of noise. More specifically, SNR decreases as we move from the center to the corner of the page. For example, the raw BER might vary between 10^{-3} and 10^{-6}

over a page.¹ Similar to the work in Ref. 7 we divide a page into several regions such that the BER is almost constant in each region. Let these regions be $R_1, R_2, \dots, R_{k_r}$. In Ref. 7 we proposed to use a specific ensemble of LDPC codes that is suitable for nonuniform error correction. Instead of assuming a single degree distribution¹⁵ for all nodes, we considered the ensemble of graphs in which the bits from different regions in the page may have different degree distributions. It was shown that this ensemble has several advantages over the ordinary ensembles.¹⁷ For IRA codes we propose to use a similar ensemble.

Let n be the block length and $(x_1, x_2, \dots, x_n)$ be a codeword. Let also $W^{(j)}$ be the set of the bits in the j th region in the codeword (i.e., $W^{(j)} = \{x_i: x_i \in R_j\}$) and $|W^{(j)}| = n^{(j)}$ where $|\cdot|$ denotes the cardinality of a set. The asymptotic performance of an ensemble of IRA codes is determined by its parameters $(f_1, f_2, \dots, f_j; a)$. To optimize the code performance we find the density evolution formulas^{15,18} for the IRA codes over nonuniform channels.

Density evolution is an algorithm developed by Richardson *et al.*^{14,15} to find the densities of the messages exchanged between variable nodes and check nodes in the message-passing algorithms. In this method, the distributions of messages from variable nodes to check nodes at two consecutive iterations of belief propagation are connected by a recursive formula. This method is used to determine the performance of LDPC and IRA codes and to find optimum degree distributions for them. For simplicity, we give the density evolution formulas for the binary erasure channel (BEC). The formulas can be simply generalized for other memoryless channels. In the rest of this section, we provide the density evolution analysis for the completeness of the paper. We assume that the interested reader is familiar with the fundamentals of density evolution presented in Refs. 10, 15, and 18.

Previously, we provided the density evolution formulas when the channel is a nonuniform BEC. Formally, a BEC can be defined in the following way: a BEC is determined by its erasure probability ϵ . When a bit is transferred over the BEC, the bit is erased with probability ϵ . The bit is received correctly at the receiver with probability $1 - \epsilon$. Thus, at the receiver, with probability $1 - \epsilon$ we are certain about the value of the transmitted bit and with probability ϵ we do not get any information about the transmitted bit. In a nonuniform BEC, different bits may have different erasure probabilities. In the above setting we assume that bits in the set $W^{(j)}$ (type j bits) are transmitted over a BEC with erasure probability ϵ^j . Specifically, assume that $W^{(k_r)}$ is the set of parity nodes in the Tanner graph of the code. Let E be the set of edges in the graph and $E^{(j)}$ be the set of edges that are incident with a variable node of type j . Let also $E_i^{(j)}$ be the set of edges adjacent to the variable nodes of type j

and degree i and $q^{(j)}$ be the fraction of information bits of degree j . We define

$$\lambda^{(j)}(x) = \sum \lambda_i^{(j)} x^{i-1}, \quad (3)$$

where

$$\lambda_i^{(j)} = \frac{|E_i^{(j)}|}{|E^{(j)}|}. \quad (4)$$

Let $\Lambda = \{\lambda^{(j)}(x): j = 1, \dots, k_r\}$ and $\rho(x) = \sum \rho_i x^{i-1}$ be as defined in Ref. 15. Similar to Ref. 10 we also let $x_l^{(j)}$ be the probability of erasure on an edge from an information node of type j to a check node and y_l be the probability of erasure on an edge from a check node to a parity node. Similarly, z_l is the probability of erasure on an edge from a parity node to a check node and w_l is the probability of erasure on an edge from a check node to an information node. Then the initial values for the density evolution are

$$x_0^{(j)} = \epsilon^j, \quad z_0 = \epsilon^{k_r}. \quad (5)$$

The density evolution formulas can be stated as follows:

$$x_l = \sum_j q^{(j)} x_l^{(j)}, \quad (6)$$

$$y_l = 1 - (1 - z_{l-1})R(1 - x_{l-1}), \quad (7)$$

$$z_l = z_0 y_l, \quad (8)$$

$$w_l = 1 - (1 - z_{l-1})^2 \rho(1 - x_{l-1}), \quad (9)$$

$$x_l^{(j)} = \epsilon^j \lambda^{(j)}(w_l), \quad (10)$$

where $R(x)$ is defined as

$$R(x) = \frac{\int_0^x \rho(t) dt}{\int_0^1 \rho(t) dt}. \quad (11)$$

Using the above equations we can find the performance of an ensemble of the IRA codes over a nonuniform channel. This performance can be used to optimize the code parameters (the degree distribution) for a given channel.

B. Reducing the Error-Floor Effect

The error floor can be an obstacle to the application of IRA codes in VHM systems, because a BER of less than 10^{-12} is required. Conventional IRA codes may get an error floor at the BERs much higher than 10^{-12} . Therefore conventional IRA codes are not suitable for VHM systems. A standard solution to the error-floor problem is the use of concatenated codes; however, this is not a good solution. First, it adds complexity to the system. Second, if we use concatenated codes, the overall rate of the code decreases, resulting in a lower storage capacity. In this section we propose a method to circumvent the

error-floor problem without adding extra complexity or losing storage capacity.

The code parameters ($f_1, f_2, \dots, f_J; a$) (or, equivalently, the degree distribution of the code) play an important role in the error-floor problem. The error floor is usually caused by a small minimum distance. Therefore we may choose the degree distribution such that the resulting LDPC code has sufficiently large minimum distance.^{19,20} Unfortunately, IRA codes usually have a smaller minimum distance than these LDPC codes. In particular, the ratio of the minimum distance to the code length tends to zero as the code length tends to infinity. However, by properly choosing the code parameters we can significantly reduce the error floor effect. The more we avoid the degree-two variable nodes in the Tanner graph of the code, the more likely the error-floor effect is reduced. However, we note that this choice may result in capacity loss. Moreover, even with a proper choice of the code parameters, the minimum distance of the IRA ensemble may not be large enough.

The minimum distance of a binary code is the minimum number of the columns of the parity-check matrix that sum to a zero vector (in the binary field). We now propose a method that based on our experience can significantly decrease the error floor problem in VHM systems. The parity-check matrix of an IRA code has the following structure:

$$H = [H_1 | H_2], \quad (12)$$

where H_1 is a parity-check matrix of an ordinary LDPC code and H_2 has the following form:

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ & & & \ddots & & & \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}. \quad (13)$$

If the code has k information bits and r parity bits, then H_1 is an r by k matrix and H_2 is an r by r matrix.

In $H = [H_1 | H_2]$, we can simply choose H_1 so that the minimum distance of the code with the parity-check matrix H_1 is sufficiently high; for example, by satisfying the condition given in Ref. 19 or just by avoiding degree-two variable nodes. Thus it suffices to ensure that we do not create any problem by adding the matrix H_2 . This is achieved by considering the properties of the noise in the VHM systems.

Two properties of the noise in the VHM systems are utilized in our scheme. One is that the noise level is small with respect to the signal level. For example, a raw BER of 10^{-3} to 10^{-6} is typical. More importantly, the noise distribution is nonuniform, and the bits at the center of the page can have much lower raw BERs than those at the corner. It is

Fig. 2. Reducing the error-floor effect of IRA codes in VHM systems.

worth noting that the matrix H_2 is associated with the parity bits. By choosing parity bits from the highly reliable pixels in the page (bits from the center of the page), we can decrease the error floor level significantly. This is because bits associated with H_2 will be less likely to be in error. Since we usually use high-rate codes in the VHM systems, the number of parity nodes is small compared with the code length. Thus there exist enough bits to choose all the parity bits from the center of the page. Our experimental results show that the above method works very well in practice.

In summary, the design of IRA codes for VHM systems is similar to the design of LDPC codes for these systems. However, the error problem needs more care. In LDPC codes it is usually enough to satisfy the condition of having a linear minimum distance.^{19,20} For IRA codes, the discussion in this section shows that although the minimum distance is usually small, using the nonuniformity of the raw BERs we can still reduce the error-floor effect.

4. Simulation Results

Here we provide some simulation results to investigate the performance of the proposed coding schemes. We first studied the performance of the proposed method for reducing the error floor. We chose an IRA codes of length 1000 and rate $R = 0.75$, and evaluated its performance over a (uniform) Gaussian channel. We also evaluated the performance of the code over a nonuniform Gaussian channel. For the nonuniform channel, all of the parity nodes were chosen from reliable bits. Figure 2 shows the simulation results. For result comparison the figure shows the BER versus the channel capacity. Note that since the capacity is normalized, it is between zero and one. Total capacity in a VHM page is obtained by multiplying the normalized capacity by the total number of bits in the page. For a uniform channel,

we can simply find the normalized capacity of the channel C using¹⁴

$$C = - \int \phi(x) \log_2[\phi(x)] dx - \frac{1}{2} \log_2(2\pi e \sigma^2), \quad (14)$$

where

$$\begin{aligned} \phi(x) = & \frac{1}{\sqrt{8\pi\sigma^2}} (\exp\{-(x+1)^2/2\sigma^2\} \\ & + \exp\{-(x-1)^2/2\sigma^2\}), \end{aligned} \quad (15)$$

where σ is the variance of the noise. In the above formula the signal power is assumed to be $S = 1$. For the nonuniform channel the normalized capacity is the weighted average of the capacities for different SNRs. The noise variance for the uniform channel is in the range $0.5392 < \sigma < 0.388$. We see that the code over the nonuniform channel has much better performance in terms of the error floor than does the uniform channel.

To study the performance of the IRA codes in VHM systems, we implemented the designed IRA codes. For simulations, we chose the same system as Ref. 7. We assume the noise is additive white Gaussian whose variance is a function of the pixel location. The density evolution is quite general and can be applied to any symmetric¹⁴ noise distribution. The raw BER in a holographic storage depends on the position of the bit in the data page. We divide the page into different regions such that in each region pixels have almost the same probability of error.¹ In our simulations we divided a page into four regions. The system has a raw BER of roughly from 10^{-3} to 10^{-6} when 2000 pages are stored. This raw BER increases when the number of pages increases. The magnitudes of the systematic error and the thermal noise are assumed to remain unchanged with respect to M (the number of pages). The SNR per pixel can be computed by use of the scaling law that states the SNR is proportional to $1/M^2$ (Refs. 1 and 21). It is worth noting that the SNR is not always proportional to $1/M^2$. This is true if the pages are recorded by use of simple superposition, but methods have been described in the past for recording multiple pages with coherent refreshing or cascaded holograms that result in the SNR decreasing as $1/M$ (Refs. 22 and 23).

In our simulations we designed IRA codes for M different pages. For any value of M , we found an IRA code of length 10^4 bits with the BER of at most 10^{-8} . The designed codes do not show an error floor at least for the BERs higher than 10^{-8} . We performed the simulations for 10^{-8} , although the output BER of 10^{-12} is required for the holographic storage. This is because an enormous amount of computations are necessary to determine the code performance at the BER of 10^{-12} .

Figure 3 shows the storage capacity that is obtained by use of IRA, LDPC, and RS codes. The LDPC codes and IRA codes have lengths $n = 10^4$, and

Fig. 3. Comparison of different coding schemes for a VHM system.

the RS codes have length 512. The LDPC codes and the RS codes are the same codes used in Ref. 7. The IRA and LDPC codes are decoded by the standard iterative decoding with use of soft information; only hard decision decoding is considered for RS codes. We note that both LDPC and IRA codes result in a considerably higher maximum storage capacity (about 50% higher capacity) than the RS codes. Similar to the pages in Ref. 7 the optimum number of pages for the IRA codes is higher than that of the RS codes.

We now present one of the specific codes that we found. For the rate 0.9 the page was divided into four different regions (regions one to four), each with different noise power. The relative SNRs in the different regions are

$$\begin{aligned} \text{SNR}_2 - \text{SNR}_1 &= 1.61 \text{ dB}, \\ \text{SNR}_3 - \text{SNR}_1 &= 2.80 \text{ dB}, \\ \text{SNR}_4 - \text{SNR}_1 &= 3.74 \text{ dB}. \end{aligned} \quad (16)$$

For this data page, we found the following simple degree distribution for an IRA ensemble of rate 0.9. All check nodes have degree 38. Sixty percent of the bits in region 1 (bits in $W^{(1)}$) have degree 9, and other information bits have degree 3. The parity bits belong to region 2. Figure 4 shows the performance of this IRA code when the block length is $n = 10000$. It can be noticed from the figure that at the BER of 10^{-8} the gap from the capacity is less than 1 dB. We note that there are no degree-two information bits in the code. Thus we expect that the code should not suffer from the error-floor problem. As is clear from Fig. 4 the code does not present any error floor, at least for BERs higher than 10^{-8} . The figure also shows the performance of an optimized LDPC code from Ref. 7 with the same rate. We see that the performance of the LDPC code is slightly better than that of the IRA code (about 0.2 dB at a BER of 10^{-8}).

Fig. 4. Performance of an IRA code of length 10^4 and rate 0.9.

5. Conclusion

We studied the application of IRA codes in VHM systems. We verified that the use of carefully designed IRA codes provides several desirable features in the VHM systems. First, the IRA codes can approach the maximum theoretical storage capacity of the VHM systems. Second, these codes do not suffer from error floor if designed properly. Finally, both codes have efficient encoding and decoding that makes them more desirable than ordinary LDPC codes. Considering all these properties, we conclude that IRA codes are potentially suitable for the VHM systems.

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