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Young's modulus and internal damping in a vibrating rod

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A high- Q mechanical oscillator in the form of a metallic rod with an eddy current driver is described. With this oscillator and a simple crystal transducer the characteristics of a mechanical oscillator can be easily studied and Young's modulus for the metal can be found. With an inexpensive FM transducer system the internal damping of the metal can also be measured.

In a recent paper¹ Wilson and Lord described an apparatus for a dynamic measurement of Young's modulus. This article describes an apparatus developed in our laboratory that allows the student to determine Young's modulus for a material from longitudinal wave motion in a rod. The system is a high- Q mechanical oscillator, which clearly illustrates the resonance characteristics of such a system. Since resonant mechanical systems are studied in most mechanics courses, the student can observe and use the oscillator characteristics to determine Young's modulus and the internal damping of the sample.

A metal rod is clamped securely at its center to a rugged metal frame by three set screws located 120° apart (Fig. 1). The rod is driven at one end by an eddy current driver, and the oscillations are picked up by a transducer at the opposite end. The driving mechanism consists of a

coil, which encircles but does not touch the rod, and a permanent magnet, which is mounted with one pole near the end of the rod. The coil is driven by an audio amplifier fed by an audio oscillator. Eddy currents induced in the rod interact with the magnet to drive it at the audio oscillator frequency.²

The output of the transducer system is monitored with an oscilloscope or an ac voltmeter. A frequency counter is connected to the audio oscillator to give a precise frequency value. In practice, the period can be timed more precisely than the frequency can be counted (± 1 count), so the period is used to calculate the frequency. By measuring the amplitude as a function of frequency, while maintaining a constant driving force, the student obtains a sharp resonance curve (Fig. 2).

For a copper rod 21.9 cm long the fundamental frequency is 8672 Hz. The third and fifth harmonics can also be observed. The velocity for longitudinal waves in a solid rod³ is $v = (Y/\rho)^{1/2}$ where Y is Young's modulus and ρ is the density of the material.

The velocity of the waves can be determined from the resonant frequency times the wavelength. The wavelength for the fundamental vibration of a rod clamped at its center is twice the length of the rod. Since the density is easily measured, the student can calculate Young's modulus.

The amplitude of oscillation is

$$A = \frac{A_0 \omega_0 / \omega}{[(\omega_0 / \omega - \omega / \omega_0)^2 + Q^{-2}]^{1/2}},$$

where ω is the driving frequency, ω_0 is the natural frequency of the equivalent undamped oscillator, and Q is the quality factor of the oscillator.⁴ The frequencies at which the amplitude of oscillation is one-half the amplitude at resonance can be calculated in the limit of high Q . This leads to the relation

Fig. 1. Schematic drawing of the apparatus.

Fig. 2. Resonance curve for a 21.9-cm-long copper bar.

$$Q = \sqrt{3} f_0 / \Delta f,$$

where Δf is the fullwidth of the resonance at half the maximum amplitude and f_0 is the resonant frequency.

With a storage oscilloscope or by photographing the trace, the free decay of the oscillation can be observed when the driving force is cut off. The logarithmic decrement, δ , is defined as the natural logarithm of the amplitude ratio of successive oscillations. This is readily determined from a semilog plot of the amplitude as a function of time since $\ln A = -\delta(f_0 t) + \text{const.}$ The logarithmic decrement equals π/Q , so that an independent measurement of Q is obtained.⁵

The apparatus may be used with rod-shaped samples of nonconducting materials such as bone or rock drilling cores by cementing the driving coil to the specimen. A core drilling of a common metamorphic rock, gneiss, provided a very sharp resonance, comparable to that of the copper rod.

We have used two different pick-up transducers on the apparatus: a piezoelectric crystal and a capacitive-controlled FM oscillator. The piezoelectric transducer was made by removing the cover and aluminum diaphragm from a crystal microphone (Radio Shack No. 33-100, \$2.00). The point from the piezoelectric element was mounted in light contact with the end of the rod. This method has the advantages of low cost, simple construction, and sufficient signal level to drive the counter. The disadvantage of the piezoelectric transducer is that it introduces damping into the system, which decreases Q .

In the FM method the transducer consists of a capacitor formed by a 0.01-cm gap between the end of the rod and a fixed metal plate. The capacitor is part of the tank circuit of an FM oscillator shown in Fig. 3. The natural frequency of the tank circuit is

$$f_t = 1/2\pi(LC)^{1/2},$$

where L is the inductance of the tank circuit and C is the total capacitance. The diameter of the rod (0.95 cm) and a gap of 0.01 cm produce a capacitance on the order of 6 pF. The inductor is described later in this paper. The fre-

Fig. 3. Circuit diagram of the FM oscillator. The capacitance is in units of picofarads.

quency of the oscillator must fall within the FM broadcast band (88–108 MHz). A change in C (i.e., motion of the bar) gives a change in the carrier frequency to a good approximation as

$$\delta f_t = \frac{1}{2} f_{t0} \delta C_s / C_s,$$

where C_s is the capacitance of the sample. For small displacements

$$\delta C_s = C_s \delta d / d_0,$$

where d_0 is the equilibrium gap separation and δd is the displacement from equilibrium. Thus we see that the frequency of the oscillation (≈ 90 MHz) is modulated at the vibrational frequency of the rod and that the maximum modulation is proportional to the displacement of the rod. The output of the oscillator is coupled into an FM radio by wrapping three turns of the wire from the output around the antenna and grounding the end. The ac voltmeter and oscilloscope were connected to the earphone jack or across the speaker leads. This method is more complex than the crystal transducer, but it produced a negligible amount of damping on the rod.

We use both methods. First, the crystal transducer is installed and the resonant frequency is found by slowly sweeping the audio oscillator through the expected range. The width of the resonance will be great enough so that it can be found. All of the parameters of interest for the oscillator can be found, but the Q obtained is dependent on the damping of the crystal and not merely the rod.

To obtain a more precise measure of the frequency and the internal damping of the rod, the FM transducer is installed. The resonance is so narrow that it is difficult to find it with the FM method alone. The bar is set on the resonant frequency determined by the crystal transducer. The FM tuner is then tuned to the carrier frequency of the FM oscillator and the oscilloscope will show the modulation wave form (i.e., the oscillations of the bar). Tuning is considerably simplified if the FM tuner has a tuning indicator. With the FM tuner properly set for the oscillator carrier frequency, the audio oscillator can be stepped across the resonance to give a very narrow resonance curve (Fig. 2). For the copper rod, Q was 1800. We know that this resonance width is characteristic of the copper rod and not the rest of the system because with a similar brass rod the system had Q equal to nearly 7000.

The audio oscillator, oscilloscope, and frequency counter can be any common laboratory instruments. We used an Amperex PCA-1-9 audioamplifier (\$8.00). The driving mechanism was constructed from a 2.75-in. 12- Ω speaker (Newark No. 56F250, \$2.50). The speaker was disassembled, and the coil and permanent magnet were used for the driver. The 12- Ω speaker coil provided a readily available coil that was well matched in impedance to the audio amplifier. The apparatus frame was made in the physics shop from aluminum and wood.

The circuit for the FM oscillator is shown in Fig. 3. The oscillator should be built into a rigid metal box with a half-wavelength coaxial cable going to the modulating capacitor. The inductor in the tank circuit consists of a loose coil 0.5 cm in diameter with 4–6 turns of No. 24 copper wire. The output coil consists of two turns of wire

placed about 1 cm from the first coil. A metal shield was clamped around the capacitor end of the rod holder to eliminate extraneous electrical signals.

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¹F. Wilson and A. E. Lord, Jr., *Am. J. Phys.* **41**, 653 (1973).

²A number of research-grade apparatuses are described by A. S. Nowick and B. S. Berry, in *Anelastic Relaxation in Crystalline Solids* (Academic, New York, 1972).

³A. P. French, *Vibrations and Waves* (Norton, New York, 1971), p. 170.

⁴Reference 3, p. 89.

⁵Reference 3, p. 68.

ROBERT FROST ON NUCLEAR ENERGY

With things like the atomic bomb I pride myself on putting them in their relative place a priori without ever having been in a physics laboratory. I said right off uranium was only a new kind of fuel. Wood will rot down to ashes in a few years. It can be touched off and reduced to ashes in a few minutes. It can be touched off and let down with a bump—exploded. Uranium will rot down to radium in a few million years if left to itself, and from radium to lead in a few thousand more. Lead is the ashes, the dead end. It can no more be touched off than the ashes of wood. Most of the matter we live with on and among has long since spent its radioactiveness. We don't have to be afraid of anybody's setting it going; any radiomaniac. Uranium is a strange rare survival from a world once generally active with alpha beta gamma delta and other rays. We live comfortably with practically nothing but the end products. Now Einstein speaks up and tells the hysterical; no new principle is involved. It is just a bigger explosive we have got. We have found a way to let uranium down all those million years with a bang. Of course very magnificent on the part of man. The mind keeps penetrating deeper into matter. I knew at once that the important question was how much uranium there was in the world, and how many elements there were still left that hadn't been levelled off. Einstein says not many. Maybe two or three of those in very small quantities like uranium. So the imagination can rest from getting up doomsdays. The new explosive can be bad for us. But it can't get rid of the human race for there would always be left, after the last bomb, the people who fired it—enough for seed and probably with the same old incentive to sow it. There's a lot of fun in such considerations.

—Robert Frost, circa 1945, in a letter to his daughter Lesley.
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