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Geometry Made Manifest: Reorienting the Historiography of Ornament on the Iranian Plateau and Beyond

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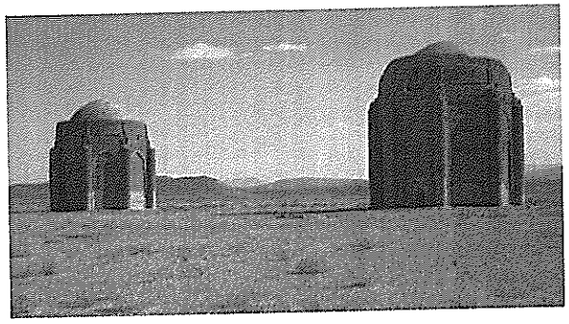
Reorienting the historiography of ornament on the Iranian plateau and beyond

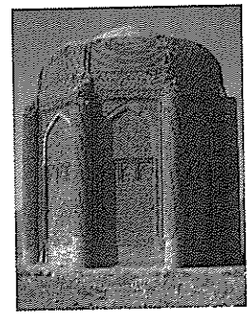
Carol Bier

From Owen Jones' *Grammar of Ornament* compiled in the middle of the nineteenth century to the discovery of quasicrystals in the late twentieth century and Lu and Steinhardt's and others' explorations of decagonal and quasiperiodic tilings in Islamic monuments of Iran, the interpretation of ornament in Islamic architecture and the arts has undergone profound reassessment during the past 150 years.[1] Broad-ranging questions concerning the universality of geometry that Oleg Grabar raised in his *Mediation of Ornament* and the probing studies of Alpay Ozdural in the 1990s on architectural ornament, as well as Gulru Necipoglu's historical exposition of *The Topkapt Scroll*, have contributed to a shift from assuming that ornament is ornamental to questioning and considering the underlying mathematical nature of ornament as well as the cultural specificity of its uses in Islamic art.[2]

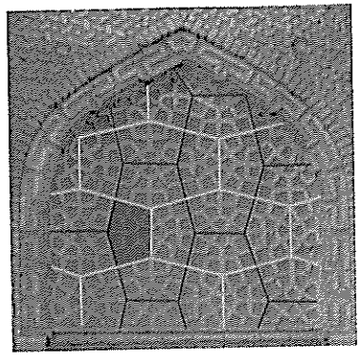
In Iran, Afghanistan, and Central Asia during the eleventh and twelfth centuries, the brick decoration of numerous architectural monuments dramatically expanded the repertory of geometric ornament. Among many monuments on the Iranian plateau, four tomb towers exemplify this new geometric style, characterized by patterns in the plane (illustrated in Figures 3.1–3.3); coincidentally, these four monuments also document the introduction and increasing use of glazed ceramics to highlight and articulate particular aspects of ornament (Figure 3.2). Mounting evidence and arguments suggest that these and other monuments with their exuberant geometric ornament reflect contemporary cultural values and represent points of intersection in the histories of architecture and mathematics. Contributing to a heuristic and discursive methodology to approach interpretations of meaning, case studies of these four monuments can serve as focal points to reorient the historiography of ornament in Islamic art of Iran and neighboring regions during a formative period in the eleventh and twelfth centuries.

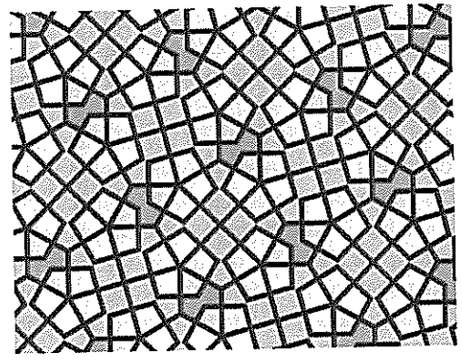
Considering surviving brick architecture of the Seljuq and post-Seljuq (that is, pre-Mongol) periods, and drawing on diverse textual sources, this chapter proposes a cultural milieu in which mathematics, architecture, poetry, philosophy, and theology each play an integrative role in the production of ornament and warrant consideration in its study. This interpretation renders literal meaning from the expression 'geometry made manifest' from a passage in the *Tabaqat-e Naseri* (*Naseri Generations*) by the thirteenth-century historian Juzjani (b. 1193). Juzjani speaks of a palace at the Ghurid capital of Firuzkuh, 'the like of which is not to be





a. (left) View of tomb towers at Kharraqan, Iran; later (western) tower at right (© Bernard O'Kane, 1974)
b. (right) Later (western) tower, rear facades (© Bernard O'Kane, 1974)





c. (left) Later tower, analysis indicating underlying pentagonal tessellation within a dual hexagonal grid in tympanum (© Carol Bier)
d. (right) Later tower, pattern of tympanum (© Brian Wichmann and A. J. Lee (http://tilingsearch.org/contents/data186/KH2.pdf))

Figure 3.1 Tomb towers, Kharraqan, dated by historical inscriptions to 460H/1067–1068 CE (eastern tower) and 486H/1093 CE (western tower). First published in 1966 by Stronach and Young, 'Three Seljuq Tomb Towers'

found in any country or in any capital . . . and of such configuration as no geometrician hath made manifest'.[3] Given existing paradigms for the interpretation of ornament, this might be considered hyperbole, not to be taken literally, offering an exaggerated statement of magnificence. Recent studies, however, from the end of the twentieth century and the beginning of the twenty-first, lead to new understandings of relationships between architecture and mathematics.[4]

The evidence to justify a shift in the historiography of our understanding of ornament and its meaning comes from standing architectural monuments and their

inscriptions, as well as from textual sources pertaining to the history of mathematics, philosophy, theology, and poetry. Consideration of both monuments and contexts suggests that a literal interpretation of Juzjani's comment may indeed be correct, with broader application to the intricate and complex patterns in brick and glazed ceramic that appear on pre-Mongol monumental buildings of the Iranian plateau, Khorasan, and elsewhere in the Islamic world of the eleventh and twelfth centuries.

The study of geometry in Persian architecture

Within the rubric of historiography, geometry has rarely received adequate attention. When it has, studies have often addressed geometry in terms of analysis and construction with particular relevance for contemporary design and training in traditional methods of using a pair of compasses and a straightedge for geometric constructions.[5] At levels of interpretation, geometry has generally been treated as cosmological and universal, rather than having temporal cultural specificity.[6] During the Soviet period, M. S. Bulatov documented numerous monuments in Central Asia, seeking to understand geometric systems of proportion.[7] Subsequently, Oleg Grabar raised numerous questions concerning why geometry, if universally accessible, had such a profound and particular presence in Islamic art.[8] The most profound advancement in addressing these questions came with Gulru Necipoglu's 1995 publication of a facsimile edition, reproduced to one-half scale, of an anonymous assemblage of geometric designs comprising a scroll preserved in the Topkapi Palace Museum.[9] The scroll contains no text. In an effort to date and localize the work, Necipoglu (who was Grabar's student and became his successor) gathered pertinent references to geometry from numerous Islamic historical sources and provided extensive commentary on interpretations of the past century, framing the discussion of geometric patterns within the discourses of ornament and Orientalism.[10] She attributed the scroll to the fifteenth century from within the Timurid/Turkmen realm and introduced the term *girih* to describe the particular form of grid-based geometric patterns, a term drawn from later use among architects in Central Asia.[11] Although the Topkapi Scroll has generated considerable interest within scientific and mathematical communities regarding contemporary twenty-first-century concerns, studies with this focus have not sought to elucidate original local historical or cultural contexts. For example, Peter Lu, as a graduate student in physics at Harvard University, collaborated with both Necipoglu and his undergraduate advisor at Princeton, who was at the forefront of the discovery and study of quasicrystals, to bring contemporary scientific understanding to the tilings in medieval Islamic architecture.[12] And the works of Peter Cromwell, a mathematician, advance current mathematical thinking that he argues may already be expressed diagrammatically in the Topkapi Scroll.[13]

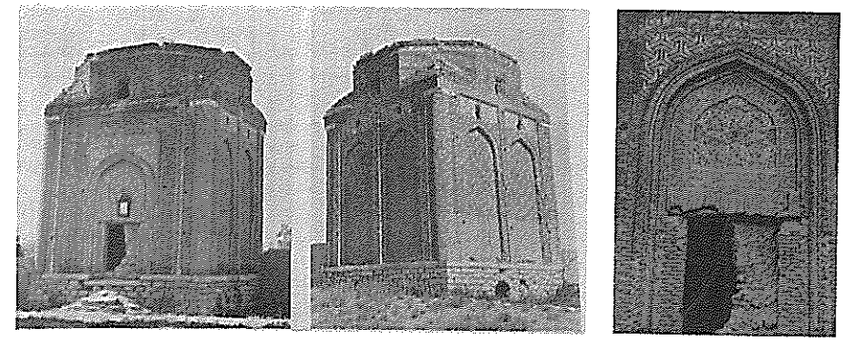
Geometry as ornament in Persian architecture: the cultural context

In 1259 CE, the year following the siege of Baghdad, Hulegu Khan appointed Nasir al-Din Tusi as chief astronomer to establish an observatory at Maragha.

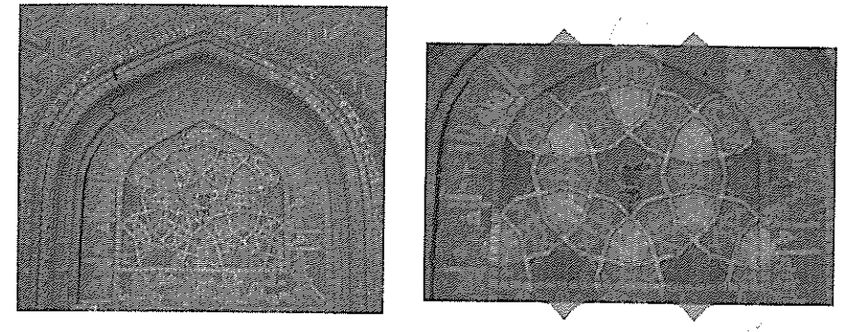
Although Baghdad, Mosul, Damascus, and Aleppo were known as centers for research and innovation in mathematics, during the previous century Maragha had witnessed the construction of monuments with the most elaborate geometry yet seen in Islamic architecture: cubic and cylindrical structures and decagonal prisms with polyhedral roofing arrangements; projecting cornices and arches with muqarnas; patterns in the plane with double hexagonal grids, intersecting nonagons, and overlapping dodecagons (Figure 3.2c–d), as well as tessellations involving five-pointed stars, decagons, and local pentagonal symmetries (Figure 3.3c–d). Such geometric forms and patterns appear at Maragha on tomb towers made of clay – clay formed into bricks, dried and then fired, cut to create patterns, and set with mortar. In the sequence of built monuments, one may note the progressive use of glazed elements. Some bricks were coated using a copper-based formula that resulted in a bright turquoise glaze when fired.[14] And blue-glazed bricks were used extensively to cover the long narrow triangular facets of the tall pyramidal roofs with polygonal bases, which reflected sunlight from every direction. These could be seen from miles around, as Yaqut commented on.[15] The visual complexity of intricately designed cornices, recessed niches, and arches caught the light and captured attention, with beauty perceived but resistant to easy analysis. Numerous points, lines, angles, and planes appear on revetments of cut and glazed brick, creating complicated two-dimensional spaces and three-dimensional forms covered with patterns that please the eye but that are difficult to apprehend. Without known architectural antecedents, it is conceivable that these buildings represent new forms of cultural expression.

Looking at Maragha from textual sources, its fame for mathematics was to come later; the 'school of Maragha' followed directly from the work of Nasir al-Din Tusi and the colleagues who comprised his team, which drew some of the best minds of his time from distant intellectual centers such as Fez, Cairo, and Bukhara.[16] But in the century before the Mongol conquests, a famous Jewish mathematician, Abu Nasr Samaw'al b. Yahya al-Maghribi, born in Baghdad (his father was from Spain), had retired to Maragha as the guest of the local ruler, Fakhr al-Din Abu'l-'Izz 'Abd al-'Aziz ibn Mahmud.[17] There, Samaw'al converted to Islam on Friday, 8 November 1163, as he reports in his autobiography, completed in 1167.[18] Perhaps not entirely by coincidence, his host, also known as Qawam-e Azarbayjan, was patron of the building of a cubic tomb structure in Maragha, Gonbad-e Sorkh, with a polyhedral dome and a unique tympanum above the entrance in which the arcuate surface exhibits three interlaced polygonal networks, with nonagons highlighted in turquoise glaze (Figure 3.2c). Taken together the three polygonal networks comprise a unique composition that likely reflects contemporary mathematical understanding of relationships among polygons and grids at both a theoretical and practical level. Surrounding the geometric composition, the historical inscription gives the name of the patron and the building's date of construction as 4 March 1148 – just fifteen years before Samaw'al's visit.

That geometry itself was important in this period is also suggested by references in the poetry of Sana'i, poet laureate of the Ghaznavid court (d. between



a. (left) Overviews (after Pope and Ackerman, *Survey of Persian Art* (1939), pl. 341)
b. (right) Front facade (after Godard, 'Notes Complémentaires' (1936), Figure 89)

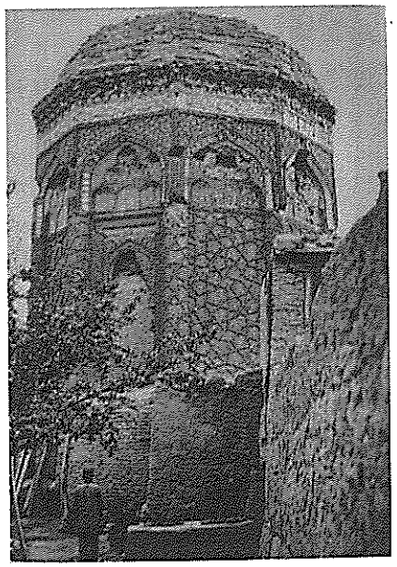


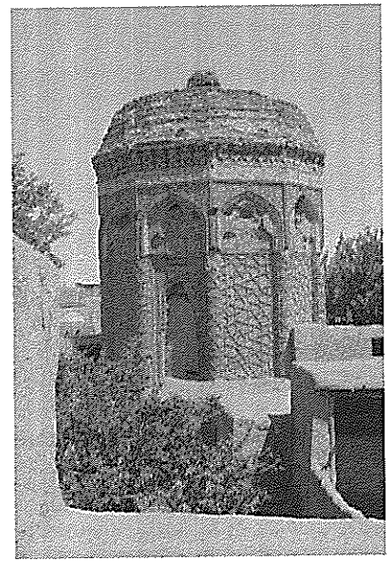
c. (left) Front facade detail, tympanum. Photo by Hans C. Seherr-Thoss (© Freer Gallery of Art, Arthur M. Sackler Gallery Archives, Smithsonian Institution, Washington, DC)
d. (right) detail of tympanum, analysis of modular construction (© Carol Bier).

Figure 3.2 Gonbad-e Sorkh, Maragha, dated by historical inscription to 542H/1148 CE

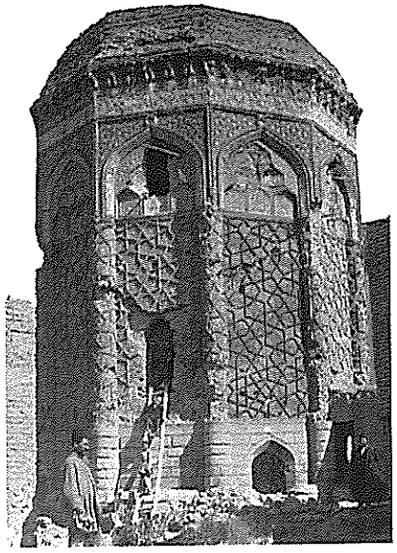
1131–1141). He likened 'the ability to behold the divine manifestation . . . to the intellectual way of perception of a geometrician': 'You only see with your imagination and your senses | when you have not learned about lines, planes, and points'.[19]

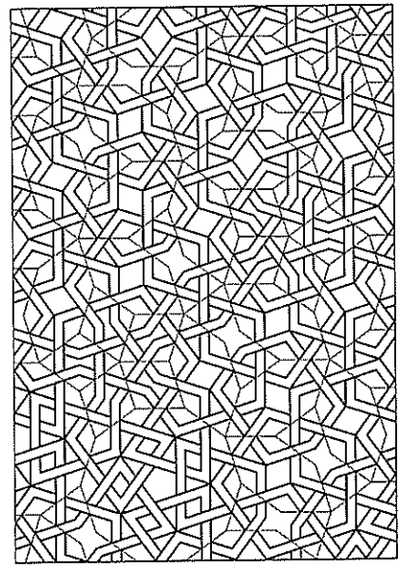
Initially centered in what is today Afghanistan, the Ghaznavids were a Turkic dynasty who promoted Persian culture through patronage of literature and monumental inscriptions. The Ghaznavids were succeeded by the Ghurids, who continued to promote Persian cultural traditions in language and literature. At their greatest extent, the Ghurids' rule extended from Khorasan in the west, where they were succeeded by the Khwarezmshahs, to Benares in the east. In northern India, they were succeeded by the Delhi sultanate, for whom Juzjani, quoted earlier in this chapter, was the principal historian.[20] He had been born in Firuzkuh, which





a. (left) Overview (after Godard, 'Notes complémentaires' (1936), Figure 96)
b. (right) View from WNW (Photo by Myron Bement Smith (1937) © Freer Gallery of Art and Arthur M. Sackler Gallery Archives, Smithsonian Institution, Washington, DC)





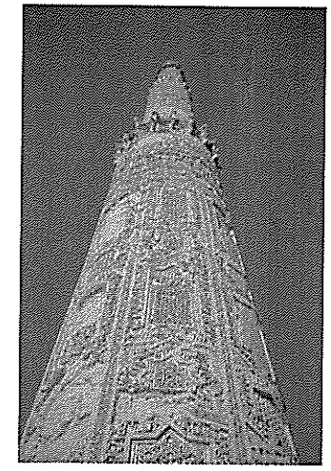
c. (left) Photograph by A. Sevruguin (ca. 1870s), overlaid with analysis of 'girih tiles', (after Lu and Steinhardt, 'Decagonal and Quasi-Crystalline Tilings', figure 2b)
d. (right) Analysis of panel (after Cromwell, 'The Search for Quasi-Periodicity in Islamic 5-fold Ornament', figure 9b)

Figure 3.3 Gonbad-e Kabud, Maragha, dated by historical inscription to 593H/1196–1197 CE

was by then the Ghurid summer capital.[21] Although the whereabouts of Firuzkuh are not definitively known, Jam is the likeliest candidate, located at the confluence of the Harirud and Jamrud Rivers deep in the province of Gur in central Afghanistan.[22] There, a lone tower remains standing (Figure 3.4) – a tapered cylindrical tower elaborately articulated with numerous geometric patterns and interlacing inscriptions – attesting to what must have been a much larger monumental presence.[23] Interpretations of the geometric patterns on the minaret of Jam, as well as those of earlier towers constructed in Ghazna, and many tomb towers and monuments in Iran and Central Asia, trace an arc that parallels that of archeological discovery in the mid-twentieth century coupled with the history of scholarship into the twenty-first century concerning the interpretation of Persian architecture.[24]

Ornament and interpretation in Persian architecture

To be able to link such biographical, mathematical, and architectural data today, one stands on shoulders of giants – those individuals who through doggedness and genius located monuments in situ and documented them, and those who translated texts, providing access to textual and non-textual primary sources. Even a century ago, these texts were not readily accessible to scholarship, and the accumulation of architectural evidence offered by the rich repository of monuments in situ awaited sequential discovery by individuals of British, French, German, Italian, and American origin, some trained as architects or draftsmen, others working in the field as archeologists, and a few adventurous travelers. It



(left) Jam minaret in the valley of Harirud (Photo by Andrea Bruno (1961) © Aga Khan Award for Architecture, www.archnet.org)
(right) Jam minaret, looking up at tapered cylindrical shaft with geometric patterns and calligraphy (Photo by Norbert Fontaine (2003) © www.archnet.org)

Figure 3.4 Jam, Afghanistan (late twelfth century CE); first published in 1959 by Maricq and Wiet, Le minaret de Djam

was, to be sure, an Orientalist endeavor, and they relied on locals as guides and assistants. Their willingness to risk travel in remote regions, to undertake excavations, as well as to prepare careful descriptions and accurate documentation of extant monumental architecture continues to serve as a basis for the study of these monuments, coupled with late twentieth-century and ongoing documentation for purposes of historic preservation and conservation efforts to protect cultural heritage.[25]

Looking back to the late nineteenth and early twentieth centuries, itinerant and indefatigable European and American travelers and scholars ushered forth a dramatic and adventurous period during which the major architectural monuments of Iran were admirably documented with careful field measurements and drawings of plans and elevations.[26] These efforts relied on numerous unnamed local individuals who provided labor, transport, protection, and other sources of critical support for these prodigious undertakings. Extensive photographic documentation was also undertaken, using large format cameras, which required the processing of negatives in field laboratories. Over the years, many such projects led to systematic publication by French, British, and German archeological institutes established in Iran and Afghanistan.[27] And major initiatives funded by Arthur Upham Pope's Iranian Institute resulted in the publication of the multivolume *Survey of Persian Art*, extravagant for its time. This voluminous effort was undertaken with the indefatigable editorial support of Pope's wife and former student, Phyllis Ackerman, as well as the participation of other Americans working in Iran. Documentary drawings and photographs from this intensively productive period in the 1930s have found their way to archival repositories that preserve these valuable resources and facilitate scholarly research today.[28]

The historiography of Persian architecture from its formative period in the late nineteenth century into the twentieth century focused on the descriptive documentation of buildings by means of field photography and the preparation of measured field drawings. At times, this included the documentation of geometric forms and patterns. Efforts to understand historical inscriptions helped to place monuments chronologically and to identify local rulers, patrons, and builders. Interpretation was restricted to the establishment of typologies based on a building's function (such as palace, mosque complex, tomb tower, or mausoleum) and an historical chronological framework to address issues of development. More recent efforts have sought to delineate patterns of patronage and to offer broader interpretive cultural syntheses, sometimes based on interdisciplinary approaches.

In early twentieth-century studies, the role of geometry comprised part of a larger documentary effort to describe form and structure. Initial goals of these traditional paradigms in the study of Persian architecture sought to establish an historical chronology in which each monument could be situated according to dynastic context and with an absolute date. Historical inscriptions might provide dates of construction and the names of architects, builders, engineers, craftsmen, and patrons. Qur'anic inscriptions were treated so as to identify chapter and verse. Rarely if ever were efforts devoted to the interpretation of monuments based on relational aspects of ornament, architecture, and Qur'anic inscriptions.

The twentieth century brought a dramatic expansion of architectural documentation and an equally dramatic increase in the close study of textual sources now available for refinements in our interpretation of architectural monuments in their original cultural contexts. Equally important was the new 'discovery' of monuments previously unknown to European and American scholars, which were introduced to new generations of young scholars by means of description and photographic documentation in periodicals devoted to archaeological study. The minaret at Jam (Figure 3.4) and the tomb towers at Kharraqan (Figure 3.1) offer good examples.[29] Beyond documentation, the heuristic processes of interpretation necessarily carried with them contemporary Western cultural values and biases.[30] Such values incorporated, for example, the European predilection for figural imagery and its representational aspects.[31] Also widespread were misconceptions about prohibition of figural representation in Islamic art.[32] Correlated with these Western ideas were notions concerning geometric patterns as abstract, repetitive, and nonrepresentational, and therefore bearing no meaning, or notions of contemporary spirituality imposed upon what were considered symbolic forms.

Working from a different direction, and embodying different sets of assumptions, several practitioners advanced new understandings of geometric patterns in Islamic art, emphasizing the symbolic role of geometry as expressive of metaphysical values.[33] In contrast to the secularist methodologies that remained dominant in the practice of Islamic art history in the late twentieth century, Wendy Shaw characterizes this metaphysical approach as an attempt to 'reunite Islam and art through very generalized, dehistoricized understandings of Islam and Sufism'.[34] Emerging from the publication program of the World of Islam Festival Trust in 1976, Keith Critchlow's outstanding geometric analyses of patterns were overshadowed, in part, by his interpretive framework as suggested by the title of his work, *Islamic Patterns: An Analytical and Cosmological Approach*.[35] Prescient in this respect was the work of Nader Ardalan and Laleh Bakhtiar, *The Sense of Unity: The Sufi Tradition in Persian Architecture*, published in 1973.[36] In general, the exhibitions associated with, and many of the publications originating from, the 1976 festival generated criticism as ahistorical from within the field of art history.[37] Others characterized this work as essentialist, deriving from the symbolic interpretation of the twentieth-century school of Perennialist Philosophy.[38] Perennialist thinkers and authors (Titus Burckhardt, S. H. Nasr, Keith Critchlow, and Martin Lings, among others) are at one end of the spectrum of interpretation: they treat geometry and other aspects of Islamic art from an essentialist perspective, with analysis of the geometry linked to contemporary Sufi interpretations. Objections to the Sufi hermeneutics seem to override the exceptional contributions of the precise geometric analyses of patterns, which were in any case removed from historical context and not treated in terms of internal or relative chronological developments. Issam El-Said, a student of Critchlow, carefully analyzed the underlying geometry and mathematical principles of repeat patterns.[39] The three systems of proportion he identified and analyzed rely respectively on 'Root Two' ($1:\sqrt{2}$), 'Root Three' ($1:\sqrt{3}$), and 'Root Five' ($1:\sqrt{5}$), which correlate with Bulatov's study of the monuments of Central Asia.[40]

Academic art historians have shunned the work of the Perennialists, although I believe that several of their latter-day assertions can be pushed further while using a scholarly method. The challenge is to identify and locate a scholarship of spirituality (or Gnosticism or any belief system in which there is an esoteric hermeneutics) that does not require one to be a practitioner.[41] The interpretations of the Perennialists rest on the doctrine of *tawhid*, focused on the oneness of God; they incorporate concepts of unity and diversity, unity and multiplicity, circles and the infinite. What remains lacking in their approach to mystical understanding within a scholarly idiom is reference to then contemporary textual sources of the eleventh and twelfth centuries independent of twentieth-century hermeneutics. Primary philosophical texts include those of al-Farabi and Ibn Sina, as well as Plato and Aristotle interpreted through them, and Suhrawardi's works as well as those of Ibn 'Arabi, who uses *al-nukta* (centerpoint) and *al-muhit* (circumference) – terms of geometric relevance – as epithets of God.

Within the field of art history in the twentieth century the consideration of geometric patterns as ornament emerged as a standard paradigm, based no doubt in large part on the characterizations of ornament by Owen Jones in the nineteenth century.[42] Jones removed ornament from cultural context and treated it as part of an emerging design science. Furthermore, in contrast to the development of European art with its focus on pictorial narrative, abstract geometric patterns were considered decorative, falling into the category of 'nonrepresentational' art.[43] Ananda K. Coomaraswamy, an early twentieth-century theoretician concerning non-Western art, called this interpretation into question: 'There is a Persian theory or philosophy of beauty that goes deeper than any mere appreciation of the exquisite'.[44] And without providing any specific examples or documentation, Terry Allen presciently surmised that 'The nonfigural art of the Islamic world is much more than mere decoration, it is a serious and visually intriguing exploration of pattern'.[45]

Alas, geometry has too often been dismissed quite summarily as repetitive, a quality not generally appreciated within Western art until the advent of modernism.[46] In 1973 Grabar introduced a distinction between 'ornamental' and 'decorative'. He suggested that geometry fell into the former category, having no intellectual or cultural content, and that the latter category covered a broader range of themes, 'regardless of iconographical meanings'.[47] In his subsequent work he moved on from the notion that geometry might be devoid of specific meaning to advance a broad interpretation of ornament as intermediary.[48] More specifically, of the four categories Grabar identifies as intermediaries in Islamic art (writing/calligraphy, architecture, nature/vegetation/arabesque, geometry), he characterized writing as 'the most meaningful' and the others as 'ubiquitous'.[49] Before him, his friend and colleague E. H. Gombrich had published a study focused on the psychological dimension of ornament and its 'sense of order', in which he identified three roles: framing, linking, and filling.[50] But Gombrich never acknowledged ornament as a *subject* of artistic exploration or depiction. And he never related ornament to the expression of metaphysical ideas. But framing the discourse of ornament as 'ornamental' has precluded cultural understanding of geometric patterns within specific cultural contexts.

The perceptive questions Grabar raised in his seminal work, *Mediation of Ornament* (1992), can now be refined, and assumptions in some cases countered. The geometry as expressed in these monuments was not yet 'universally available', despite modern assumptions and generic attributions.[51] Nor, with reference to the 'visual order' within which Grabar situated ornament, can one arrive at an understanding of these patterns as intentional visual expressions of mathematical thinking – making mathematics visible. For Grabar, 'ornament is that aspect of decoration which appears not to have another purpose but to enhance its carrier'.[52]

Toward the end of the twentieth century, two authors in particular, Wasma'a Chorbachi and Ozdural, independently struggled to relate geometric patterns to contemporary texts in which similar mathematical problems were discussed and resolved.[53] In the case of Ozdural, he proposed that regular meetings of architects and mathematicians took place.[54] Renata Holod has also attempted to define models of practice.[55] And Jonathan Bloom has explored the prospective impact of the introduction of paper on the transmission of architectural designs, along with the early development of systems of notation, which could serve as means of communication across geographic space.[56] Necipoglu applied the term '*girih* mode' to describe the geometric patterns in the Topkapi Scroll, and ascribed its use to dynastic ambitions.[57] In a long article, Terry Allen refuted this attribution as lacking 'evidence to support the claim that geometric designs were . . . "emblematic" of Abbasid caliphate and Sunnism' and, from his perspective, 'The argument from academic geometry fails'.[58] Yasser Tabbaa elaborated on the dynastic association of geometric ornament with the establishment of Sunni orthodoxy, with particular relevance to the patronage of Nur al-Din (1146–1174), whom Tabbaa credits as being 'the motivating force behind the Sunni revival'.[59] With clear evidence from biographers, Tabbaa argues cogently that architects and designers were part of the educated and intellectual elite.[60] Nonetheless, questions remain concerning the relationship between then contemporary mathematics, the role of geometers as builders, and the knowledge of geometry among builders during this period.

Several surveys of Islamic architecture and major reference works compiled at the end of the twentieth century synthesized the work of several academic generations in the fields of art and architectural history and Islamic studies.[61] Similar compilations document the trajectory of the history of mathematics.[62] The mass of raw data our forebears gathered in the twentieth century continues to sustain scholarship in the twenty-first century, offering great potential for reexamination that may lead us toward new paradigms for interpretation of architecture and ornament in cultural context. Sophisticated search engines have facilitated previously unimagined access to electronic media, including image databases, online encyclopedias, and dictionaries. The combination of increased resources and broad access now allows for the critical analysis and review of extensive documentation, both textual and visual, and the recombination of sources to consider new interpretive narratives. The full breadth of online resources is available through such search engines as Google Images and Google Scholar, and the Google Earth application. Websites and electronic databases continue to expand, such as ArchNet, 'an international online community for architects, planners, urban designers . . . with a focus on

Muslim cultures and civilizations'.[63] Three particularly useful Web sites for the study of Islamic patterns are 'Pattern in Islamic Art: The Wade Photo Archive', 'A Tiling Database', and 'Symmetry and Pattern: The Art of Oriental Carpets'.[64] Infinitely more resources are available today than at any time in the past, bringing to the fore new potential for reevaluation and reinterpretation of monuments in their original cultural contexts.

Ornament as geometry in Persian architecture

Analysis of four tomb towers on the Iranian plateau, constructed over a period of more than a century (1067–1197) and completed before the Mongol conquests of the early thirteenth century, provides the basis for this historiographic study. Examination of architectural ornament in relation to cultural context yields data that allow for a reinterpretation of ornament and the consideration of the expressive quality of geometry as ornament. The two tomb towers at Kharraqan (Figure 3.1) and Gonbad-e Sorkh (Figure 3.2) and Gonbad-e Kabud (Figure 3.3) at Maragha serve as focal points for this study; their decorative programs, executed in cut brick, rely exclusively on many different geometric patterns. And each of these buildings bears Qur'anic and historical inscriptions in Arabic (discussed later in this chapter). Among many other extant brick monuments of pre-Mongol Iran, Azerbaijan, Iraq, Afghanistan, and Central Asia, these four together exemplify the use of ornament that articulates geometric patterns. Constructed of fired brick and with brick ornamentation, over time they also indicate the progressive inclusion of glazed ceramic elements, which continues to develop extensively in the following century.[65] The four buildings have various prismatic shapes and they each once had a polyhedral dome of a particular type – double shell, hemispherical with a circular base with an outer dome of triangular facets that form a pyramid with a polygonal base, exploring geometry in three dimensions. This type of roofing arrangement is visible today in the restored tomb tower of Yusuf ibn Kusayr at Nakhchevan in the Republic of Azerbaijan, although those in Iran give evidence of blue-glazed bricks used for the triangular facets.[66] The two tomb towers at Kharraqan (Figure 3.1) are situated in an out-of-the-way location, off the road between Hamadan and Qazvin. Early Arab geographers refer to the place as Kharraqanayn, using the dual form, and locate it in the province of Jibal.[67] Stern explains the duality as a reference to an eastern and western region; the two buildings stand in the western region as a pair of octagonal prisms – each tower has an octagonal plan that rises with eight rectangular faces, each face with a niche and a round buttress at each exterior angle.[68] Although they appear as a pair, according to their respective inscriptions they were built twenty-five years apart (460H/1067–1068 CE; 486H/1093 CE) in the second half of the eleventh century (fifth century *hijri*). The historical inscriptions also give the names of individuals. The builder of the earlier tower named in the inscription is Muhammad ibn Makki al-Zanjani; in the later tower he is referred to as Abu al-Ma'ali ibn Makki al-Zanjani, both names presumably signifying the same person; neither name is

known from a textual source. Names of the deceased seem Turkic, but they bear no titles and have not yet been conclusively identified.[69]

The tomb towers at Maragha are both later, located further north in Iranian Azerbaijan; they are two among several in that town that remain standing. Gonbad-e Sorkh (Figure 3.2) is cubic in structure with an octagonal transitional zone; it dates from the middle of the twelfth century and is dated by historical inscription to 542H/1142 CE. Gonbad-e Kabud (Figure 3.3) was constructed as a decagonal prism (often incorrectly published as octagonal) and dates from the late sixth/twelfth century and is dated by historical inscription to 593H/1196–1997 CE.[70] The decagonal prism of Gonbad-e Kabud was built just a couple of decades before the first conquests of the Mongols, when Atabeks of the Ahmadili family ruled the town and its hinterland.[71] Another decagonal prism was erected a decade earlier in Nakhchivan, the so-called tomb of Mo'mine Khatun, with a different array of geometric patterns on the facets of the monument.[72]

What visually links these brick structures is the extensive and intriguing use of geometric patterns in their ornamentation, as well as the exceptional geometric forms of their structure with polyhedral roofing. Each of the four monuments (as with Mo'mine Khatun) was built as a tomb tower, prismatic in form with flat exterior faces and buttresses at each exterior angle; the entire structure was capped by a polyhedral dome, composed of triangular faces, the number of which varies with the number of faces on the prismatic structure (in the case of Gonbad-e Sorkh, with the number of faces on the zone of transition). Each of the towers bears monumental Arabic inscriptions in prominent locations, encircling the building's facades high up beneath the cornice, above and surrounding the entrance, and on interior surfaces.[73] The inscriptions include excerpts from the Qur'an (see later in this chapter).

To frame an interpretation of these individually unique monuments, which nonetheless bear many similarities as detailed earlier in this chapter, two factors need to be countered, each of which has much to do with the historiography of ornament and twentieth-century cultural responses within the field of Islamic art history as studied in Europe and the U.S. The first is the paradigmatic treatment of geometric patterns as normative; the second is the all too typical dismissal of Qur'anic inscriptions as formulaic. These concerns are addressed later in this chapter.

Tomb towers at Kharraqan (460H/1067–1068 CE; 486H/1093 CE)

Every one of the eight facades on each of the tomb towers at Kharraqan exhibits geometric ornament executed in cut brick.[74] Each of the two towers bears a particular historical inscription on the front facade (that is, the face in which there is an entryway) and the same Qur'anic excerpt (chapter 59, verses 21–24, discussed later in this chapter), which encircles the building beneath the cornice. Contrary to the notion that the excerpt is repeated, and therefore repetitive, what I find particularly interesting is that the inscriptions are the same, reflecting a conscious choice

on the part of the patron or builder. Despite the extraordinary similarities, the buildings are distinct from one another: the second tower has many more patterns, and an additional Qur'anic inscription (chapter 23, verse 115, discussed later in this chapter) on the front face above the entryway.

The panels above the encircling Qur'anic inscription contain overlapping polygons with an illusion of interlace constructed of cut bricks that delineate straight lines meeting at angles of 90°, 45°, or 135° (90° + 45°). Elsewhere, variations in the size, placement, and orientation of bricks (horizontal, vertical, oblique; horizontal + vertical), and the intervals between bricks, create different patterns of light and shade, articulating the visual organization of the facade and enveloping the round buttresses that abut each exterior angle of the octagonal plan. In addition, square bricks are laid flat face to the exterior, and nested in three layers, to delineate an arch, creating the effect of an arched niche. The two towers, built twenty-five years apart, are comparable in size (twelve meters to the springing of the dome); each has eight sides or facets entirely constructed of fired brick, and each is covered on all facets with geometric ornament with considerable variety. The later tower has many more patterns because each arch of the facade is divided into three smaller arched panels, each of which bears a pattern. In both towers, each geometric pattern is created by a design algorithm of straight lines intersecting at different angles, forming a variety of shapes that combine to form overlapping or intersecting polygons (Figure 3.1c).

Geometric patterns cover each face of the two towers, each face treated equally (except for the entrance). In each tower the entrance facade bears an unelaborated entryway, above which is located a rectangular panel capped by tympanum, recessed space defined by an arch, and additional inscriptions, and it is the tympanum that has received special treatment, exhibiting the most highly complex pattern of all (Figure 3.1c–d). On the earlier tower, the infinitely repeating interlacing pattern bounded by the arcuate outline of the tympanum, straight lines intersect at intervals to create nine twelve-pointed stars (a tenth would have been at the apex of the arch). In the negative space at the center of each star, carved in brick, is the name of God, Allah. The elongated letters, with an ornamental decorative element placed between the two *lam*s, are slightly more elaborate than those of the Qur'anic and historical inscriptions on the monument. But they are not atypical of calligraphy on inscriptions and coins for this period.[75] The repetition of the name of God, achieved here through a design algorithm with symmetry-breaking at its center, acts as a form of visual *dhikr*, a repetition and remembrance in front of the building above the entrance. The pattern itself exhibits both linearity and interdimensionality by virtue of its illusionary interlacing. In each spandrel, to the right and left of the arch, is a roundel; one bears an octogram (eight-pointed star), the other a hexagram, or six-pointed calligraphic star with the repeated name of Mohammad.[76]

In the later tower at Kharraqan the periodic pattern in the tympanum is also unique, but it does not contain the name of God. It bears what at first glance is a simple pattern, composed of the repetition of triangles, squares, and pentagons. Despite its apparent simplicity, it is uniquely complex because of its many

simultaneous symmetries. This must have been an amazing achievement for its time, and the pattern has not been identified on any other monument to date. Taken together these simple geometric forms create an intricate tessellation within which the repetition of a pattern can be read in several ways (Figure 3.1c–d). The unit that is repeated to form the tessellation is composed of a square that is divided into four; a pair of triangles, each divided into three parts; and a pentagon that is divided into five parts. Several underlying grids can be identified, yielding different readings and inherent ambiguity: one reading yields a pattern composed of squares and pentagons and triangles; another yields a tessellation of squares; a third reveals a tessellation of hexagons aligned horizontally; a fourth reveals a tessellation of hexagons aligned vertically; by overlapping these two analyses a tessellation of pentagons is revealed (Figure 3.1c).[77] This is exceptional in that pentagons do not normally tessellate; this pattern represents a rare instance based on length of sides and angles that allows for a repetition in which pentagons contribute to a tessellation. Furthermore, dividing each pentagon of this tessellation in half reveals a tessellation of quadrilaterals. As in the case of the tympanum on the earlier tower with its twelve-pointed stars each surrounding the name of God, that this pattern was reserved for the tympanum above the entry would seem to elevate it to a special status. All of these factors contribute to the need for a historiographic reorientation that places ornament squarely in the philosophical and mathematical realms of number, shape, and nature of space.[78]

Maragha: Gonbad-e Sorkh (542H/1148 CE)

Consideration of the geometric aspects of Gonbad-e Sorkh at Maragha reaffirms the need for a reorientation in our thinking. Constructed nearly half a century after the later tower at Kharraqan, it is a cubic structure with an octagonal zone of transition between the square chamber below and the domed roof above. The vertical planes of the facade have arched niches arranged in pairs, which exhibit varying patterns of bricks laid horizontally and vertically. The elaborated entry facade has a single niche, capped by a projecting arched tympanum articulated with several recessed ribs of brick; it bears a finely detailed inscription in an elegant style, and a recessed panel of geometric ornament that exhibits a high degree of ambiguity and complexity. As at Kharraqan, the tympanum has received the most extravagant attention artistically. Especially notable is the extensive use of turquoise glaze that forms the background of a pattern with threefold symmetry in the spandrels and delineates overlapping nonagons in the tympanum (Figure 3.2c). The complex ornament within the tympanum articulates three sets of polygonal networks that overlap and intersect with one another: nonagons, hexagons, and dodecagons. The overlapping intersecting dodecagons form a central six-pointed star in negative space, in which Milwright noted the remains of a ceramic boss with dark blue glaze.[79] This marks a center, simultaneously, of three polygonal networks (Figure 3.2c–d) – the center of the central hexagon, and the center of the six-pointed star that is the negative space left within each of the sets of overlapping and intersecting nonagons and dodecagons. Analysis of the mode

of construction of the tympanum suggests what may perhaps be the earliest use of modularity – three modules may be identified that combine different shapes; the space between juxtaposed modules is effectively concealed by the higher relief outlines of the dodecagons, applied last (Figure 3.2d).[80]

Maragha: Gonbad-e Kabud (593H/1196–1197 CE)

The fourth monument considered in this historiographic context that relates the articulation of ornament to a mathematical form of expression that is distinctly geometric is the Gonbad-e Kabud at Maragha, a decagonal prismatic structure that once bore a polyhedral dome with ten triangular faces and a decagonal base. The main inscription surrounds the building in a register beneath the cornice; here the lettering is more elaborate than at Kharraqan, with foliated serifs and occasional flourishes. The cornice is articulated with three tiers of muqarnas. Below this, the entire building is covered by geometric patterns arranged in registers. Each of the ten faces bears a niche with an arch and tympanum that is divided into a sequence of smaller arches, each element of which contains a different pattern. In the upper spandrels, the design shows a progression from five- to six- to seven-pointed stars, articulated by two interlaced polygonal networks, one that is curvilinear, and one that is angular.[81] The lower spandrels exhibit an interlaced pattern of rayed stars that show a progression from eight- to nine- to ten-pointed stars, some of which exhibit petal-sharing. This pattern is discussed later in this chapter in relation to an accompanying Qur'anic inscription. As at Kharraqan, round buttresses are constructed at each exterior angle, but here, the visual composition of a single major pattern encompasses both the flat rectangular sides of the prismatic structure, as well as the cylindrical buttresses. This pattern is arranged in nine rectangular panels repeated in pairs surrounding the entry facade and extending to cover the buttresses; an article in *Science* (2007) drew considerable attention to the tower's surrounding two-dimensional pattern that exhibits local fivefold and tenfold symmetries but nonetheless has periodicity based on translational symmetry.[82] In 2011 Dan Shechtman received the Nobel Prize in Chemistry for his discovery of a strangely similar pattern with local pentagonal and decagonal symmetries, recognized in the diffraction of a natural alloy.[83] The patterns Shechtman studied were characterized as 'quasi-crystalline' on the basis of their not fitting into known and acknowledged categories of crystalline structures exhibiting symmetry of the plane.[84] The announcement of the Nobel award notes that similar patterns appear in 'medieval Islamic mosaics'.[85] That such patterns were depicted on Islamic architectural monuments in the fifteenth century remains undisputed; however, the execution of the pattern that is repeated nine times around the shaft of the Gonbad-e Kabud in Maragha in the late twelfth century with characteristics that challenge scientific thinking today affirms the notion that this achievement was acknowledged in its own time. For the unique pattern on this monument, a drawing of its two-dimensional generative design occurs in the Topkapi Scroll.[86]

In 1979 the Nobel Prize in Physics was awarded to its first Muslim recipient, Dr. Abdus Salam, of the Ahmedi community in Pakistan, who saw religion as integral to his scientific research. He stated, 'The Holy Quran enjoins us to reflect on the verities of Allah's created laws of nature; however, that our generation has been privileged to glimpse a part of His design is a bounty and a grace for which I render thanks with a humble heart'.[87] In his acceptance speech he quoted two verses from *Surat al-Mulk* (chapter 67, verses 3–4):

> Thou seest not, in the creation of the All-merciful any imperfection, Return thy gaze, seest thou any fissure. Then Return thy gaze, again and again. Thy gaze, Comes back to thee dazzled, aweary.

These Qur'anic verses are among the texts that appear on Gonbad-e Kabud at Maragha, as is the Qur'anic verse that refers to 'bounty and grace' (chapter 55, verse 27, discussed later in this chapter). The coincidental and parallel use of these texts by a twentieth-century physicist and a twelfth-century monument highlights the prospective relationship of text and image at Gonbad-e Kabud. The exceptional significance of the pattern that surrounds the shaft of the monument could conceivably have been accentuated by the selection of this passage, the choice of which might suggest the theological usefulness of the inscription in relation to scientific endeavors and the exceptional prescience of the pattern in the late twelfth century.

This relational approach to architecture and text offers great potential for refining our understanding of both buildings and inscriptions, creating a new arc for the study of Islamic architecture in Iran and elsewhere. Before turning our attention to the Qur'anic inscriptions in more detail, let us emphasize several points relating to the use of geometry in these four monuments, which offers an interesting balance between the universal and the particular. It also expresses something that is somewhere between the 'seen' that is visible, and the 'unseen' that is invisible, namely the lines of construction of the patterns (the dichotomy is present in the Qur'anic inscription, for which see later in this chapter). The patterns themselves bear infinite extension; they are not themselves bounded, except arbitrarily by the delimiting of space, as in a rectangular panel, a niche, or an arch. Most of the overlapping and intersecting polygons exhibit a subtle feature that suggests interlace, but this is illusionary (that is, the interlace doesn't exist in real space).

Such geometric patterns proliferate on the Iranian plateau in architectural monuments of the eleventh and twelfth centuries, and they are not restricted to public buildings of a particular function.[88] They also appear on monuments built with Mongol patronage in the following decades.[89] One would like to interpret their appearance in the eleventh and twelfth centuries as formative, and those of subsequent generations as normative within the confines of Islamic art and architectural ornament. If this is, indeed, the case, it is no wonder that Qur'anic passages were selected to highlight these new and exciting developments, and utilize the mathematical patterns in a manner that was culturally relevant within their original contexts.

Qur'anic inscriptions in (Persian) architecture

As mentioned earlier in this chapter, an obstacle to the study of Qur'anic inscriptions in Persian architecture is the Western predilection for treating these inscriptions as standardized and formulaic. Furthermore, early European and American studies of monumental Qur'anic inscriptions in Arabic required decipherment. Initially, it seemed sufficient to identify chapter and verse; beyond that, the relationship of text to architectural form and function was considered. Studies corroborated particular selections of Qur'anic passages for use in buildings such as mosques, schools, and fountains. Tombstones and funerary monuments, proscribed in the Qur'an, required more complicated explanation justified and substantiated by practice, but Qur'anic references to paradise and visual representations of paradise are not uncommon.[90] Not until 1981 was an index of monumental Qur'anic inscriptions on Islamic monuments published.[91] This seminal volume included four case studies (Dome of the Rock in Jerusalem; Nilometer in Cairo; Maristan of Nur al-Din in Damascus; Madrasa of Sultan Hassan in Cairo), which demonstrated the cultural significance and historical particularity of Qur'anic material. The importance of this work cannot be overstated, although Dodd and Khairallah included only those inscriptions published up to 1974, and relied primarily on the *Répertoire chronologique d'épigraphie arab* and the compilations of Max van Berchem, neither of which focused on Iran or lands further east.[92] Many historical monuments on the Iranian plateau and beyond are therefore not included, despite extensive documentary research in the 1970s before the Islamic Revolution.

The two more recent compilations by Sheila Blair, *The Monumental Inscriptions from Early Islamic Iran and Transoxiana* (1992) and *Islamic Inscriptions* (1998), correct this lack to a certain extent.[93] Although the earlier work provides a catalog of texts, the later work is narrative in format, offering a synthesis of approaches to the study of inscriptions as a guide for students undertaking such research. Blair's interests are directed toward historical inscriptions. The table of contents of *Islamic Inscriptions* reveals this slant, addressing Qur'anic inscriptions in the section on sources and methods, rather than as a separate subject parallel to that of foundation texts with historical content. Blair's work exemplifies the predilection in Western scholarship for historical inscriptions, which were considered to offer more substance for art-historical concerns, providing such information as the names of builders and patrons with dates of construction. With few exceptions, rarely has the content of inscriptions been treated with respect to the identification of motives that underlie the selection and placement of inscriptions on particular monuments.[94] Details of calligraphy and calligraphic styles, however, have been considered as expressive of contemporary aesthetic values, traditions, or dynastic imprints.[95] Closer studies of Qur'anic inscriptions on major architectural monuments such as the Taj Mahal in Agra, India, and the Dome of the Rock in Jerusalem indeed suggest the careful and critical selection of excerpts were at times used to define an iconographic program that informs a building's cultural meaning and intentionality at the time of its construction.[96]

The importance of cultural context took on new meaning in studies determining which Qur'anic passages were selected and why in relation to individual monuments; the choice reflected local cultural responses to both the Qur'an and current political, social, and theological concerns. European and American scholars began to raise questions concerning the manifestation of cultural interactions, seeking to understand local meanings. In the later twentieth century, as scholarly interests expanded to encompass patronage and local histories, more intensive efforts were directed toward understanding architectural responses to local cultural concerns. And recent scholarship strongly reinforces the importance of the relational study of text and architecture as potential means for interpretation, allowing local choices for specific Qur'anic excerpts to inform our prospective understanding of a building's program.

Linking geometric ornament and Qur'anic inscriptions at Kharraqan and Maragha

Situating these monuments with their Qur'anic inscriptions and geometric patterns is not easy within existing paradigms without a shift in historiographic concerns. By using a relational approach to compare similarities and differences in Qur'anic inscriptions, and assuming a relationship between the selected excerpts and the diversity of geometric patterns that appear on the two tomb towers at Kharraqan and Gonbad-e Kabud in Maragha, we may gain further insight into the meanings of these monuments in relation to then contemporary mathematical concerns.[97]

The two tomb towers at Kharraqan bear the same Qur'anic inscription (chapter 59, verses 21–24), disposed around all eight sides of the building in a band beneath the cornice and the springing of the dome.[98] Spanning the facets and the adjoining buttresses, the inscription on both towers is executed in cut brick, laid and measured carefully so as to circumscribe the entire structure, uniting all sides of the octagonal prism. Above the inscription on each face (of both towers) is a rectangular panel with a geometric pattern, also of cut brick. Verse 21 begins with a statement: 'Such are the similitudes (*amthal*) which We propound to men that they may reflect'. The term *amthal* is a plural form of two Qur'anic terms, *mathal* and *mithal*, both having to do with likeness and similarity. This duality has caused confusion over the centuries. Rumi attempted to explain the differences between the two words and their meanings.[99] Further clarification is needed.[100] In classical Islamic mathematical texts, *mithal* (with a long *a*) is the word mathematicians use for 'example', as in the case of a 'geometric problem'.[101] In each of the towers at Kharraqan, the geometric patterns on each face are different, and the sets of patterns on each building are different, amounting altogether to more than 100 distinct patterns.[102] As Dominique Raynaud reminds us, with reference to the work of a tenth-century mathematician, each 'geometric construction is basically the solution to a given problem'.[103] For every geometric pattern that appears on a monument, it is a visual solution to a problem yet to be identified.

It is conceivable, therefore, that the passage in which the term *amthal* occurs was selected specifically to signify the multiplicities of geometric patterns that appear

on both of these monuments, which would render them meaningful in a religious context. The specificity of this vocabulary suggests that this text may actually refer to the patterns, serving as verbal analogs to the associated visual images of abstract patterns, helping to inform our understanding of these two monuments in an effort to contextualize their meaning at the time of their construction. If so interpreted, the patterns become visual renderings of the Qur'anic text and relate to what is provided to mankind for reflection. This passage also makes reference to 'the unseen' (*al-ghayb*) and 'the seen' (*al-shahadat*), which might also be taken as alluding to the visible geometric patterns, for which the lines of construction are not visible. The patterns might then be perceived as being expressive of, or related to, God's capabilities as Creator, much like the ninety-nine beautiful names, which are also mentioned in this passage (verses 22–23), of which one is 'Bestower of Forms' (*al-musawwir*). The selected passage concludes with 'Whatever is in the heavens and on earth', that is, all that is created, 'declares God's praises and glory' (verse 24), which is not inconsistent with this interpretation. That the same passage (chapter 59, verses 21–24) was chosen for the later tower reinforces the interpretation of the plural, *amthal*, as signifying the many patterns on each monument, and rendering it a particularly noteworthy selection.[104] The selection of verses in each case refers to the ninety-nine names of God, and singles out three, *al-khaliq* (Creator), *al-bari'* (Producer of Forms), and *al-musawwir* (Bestower of Forms), which al-Ghazali (d. 1111) associated with the actions of 'making', specifically in relation to architecture and beauty of forms:[105]

> *Al-khaliq, al-bari', al-musawwir*: it might be thought that these are synonymous names and that they all refer back to creation and innovation, but this is not so. Whatever is brought out from nonexistence to existence requires first, design (*taqdir*), second, bringing into existence (*ijad*) in accordance with the design, and third, form giving (*taswir*) after being brought into existence. God, praised and exalted be he, is a creator (*khaliq*), in that he is a designer (*muqaddir*); a producer (*bari'*) in that he is an inventor (*mukhtari'*) able to bring things into existence (*mujid*); and a form giver (*musawwir*) in that he arranges the forms of his inventions in the best (*ahsan*) order. This is like a building. . . . This is the custom in designing, building, and form-giving.

On the later (western) tower is an additional, short, Qur'anic inscription (chapter 23, verse 115), executed in cut brick on the front facade, above the entryway and below the tympanum. This text, too, may carry a specific reference to geometric patterns on the monument:

> Did ye then think
> That We had created you
> In jest, and that ye
> Would not be brought back
> To Us (for account)?

This inscription asks two rhetorical questions. The second question, '[Did you think] that you would not be brought back to us?' may relate to the soul and

its resurrection at the time of death, which is eminently suitable for a funerary monument. While the literal meaning of this verse relates this question to life and God's creation, suitable for a tomb tower (although it is not recorded on other tomb towers), it seems reasonable to interpret this in the specific context of this elaborately ambiguous geometric pattern – the pattern of the tympanum above the entryway, with its tessellation of pentagons, squares, and triangles – which would have been amazing in its time. Because the possibilities for a pentagon to tessellate are rare, we might infer referential meaning to the question, 'Did you then think we created you in jest?' If this inference is correct, 'you' would refer here to the tessellation itself.

As mentioned earlier in this chapter, this peculiar arrangement of shapes is unique for this period in arts and architecture. In this specific instance, I propose that the rhetorical question of the Qur'anic quotation relates locally to the endeavor involved in creating the visually confounding pattern of pentagons and squares.[106] This would also yield a double meaning to the question that follows, 'And that you would not be brought [back for account]?' at the levels of both a theological accounting of the soul on the day of judgment and a mathematical accounting regarding the uniqueness of this pattern.

In spite of the replication of one passage on the two monuments at Kharraqan, the diversity of Qur'anic passages among these tomb towers is striking because the four buildings share the same function. The Qur'anic texts should not be treated in isolation from one another, but rather relationally in the context of each building in consideration of both architectural form and ornamentation. Local cultural responses to the Qur'an at the time would have involved the selection of the specific passages to be recorded on each monument, although we do not know who made these selections, nor at what stage in the process of design and construction.

Turning to Gonbad-e Kabud in Maragha, several Qur'anic passages appearing there may also relate to geometric patterns.[107] As noted earlier in this chapter, not all of the passages have been definitively read, in some cases because of damage and loss.[108] In the crypt is a passage (chapter 55, verses 26–27) in a *naskhi* (cursive) style, carved in plaster on a wall panel. According to Godard, this passage is often found on Muslim tombs, but here 'Bounty and Honor' (*al-ikram*) may make reference to the ornate decoration of the tomb tower:[109]

> 26 All that is on earth will perish:
> 27 But will abide (for ever) the Face of thy Lord full of Majesty, Bounty and Honour.

Indeed, Godard translates verse 27 as 'surrounded by majesty and glory', making this relationship even more evocative of the external ornament of extraordinary geometric patterns.[110]

One source identifies the Throne Verse (chapter 2, verse 255) as that which is inscribed around the exterior beneath the cornice in cut brick and mosaic faience, which would be appropriate for a funerary location and appears on other funerary monuments.[111] Another source, however, identifies this Kufic inscription as Qur'an chapter 2, verse 26:[112]

> God disdains not to use the similitude (*mathal*) of things, lowest as well as highest. Those who believe know that it is truth from their Lord; but those who reject Faith say: 'What means God by this similitude (*mathal*)?' By it He causes many to stray, and many He leads into the right path; But he causes not to stray, Except those who forsake (the path).

The word 'similitude' (*mathal*) occurs twice in this passage and may again refer to the decagonal geometric problem expressed by the pattern around the shaft of this decagonal tower at Maragha.

The excerpt selected for the interior chamber, however, may have the most direct relevance to this discussion of the decagonal tower with its extraordinary pattern of pentagonal and decagonal symmetries (chapter 67, verses 1–4):

> In the name of God, most gracious, most merciful!
> Blessed be He in Whose hands Is Dominion; And He over all things hath Power;
> He Who created Death and Life, that He may try which of you is best in deed: And he is the Exalted in Might, Oft-Forgiving.
> He Who created the seven heavens one above another; No want of proportion wilt thou see in the Creation of (God) Most Gracious. So turn thy vision again: Seest thou any flaw?
> Again turn thy vision a second time: [it] will come back to thee dull and discomfited, in a state worn out.

Note that this is the same passage that Abdus Salam quoted (with a different translation), which he described as 'the verse of a physicist', mentioned earlier in this chapter. While the reference to judgment at the time of death would not be unusual for a funerary monument, a portion of the third verse may have specificity to the geometric patterns on the exterior facade. For there the artisans have taken liberties with the patterns in the spandrels of arches, rendering progressive sequences of five-, six-, and seven-pointed stars, and eight-, nine-, and (half) ten-pointed stars, adjusting lines and angles of the star polygons to allow for their diminution or expansion, and by 'petal-sharing' of the nine- and (half) ten-pointed stars. And the pattern with local pentagonal and decagonal symmetries is confounding to both the trained and untrained eye.[113] This Qur'anic text may also be interpreted as referring specifically to the visual complexities of the architectural decoration, the adjustments of which could be considered flaws, although difficult to see.[114] The prescience of this inscription, if so interpreted, lies in the fourth verse, suggesting that by looking again, one might find these flaws, but the process itself will be difficult, causing discomfort. To judge from scholarly controversies in recent years, indeed it has.[115]

For the most part the Qur'anic passages selected for use on these buildings are not standard for funerary monuments. Relating these inscriptions to the geometric patterns that appear on these buildings reveals interconnections that may advance our understanding as to why these particular Qur'anic passages were excerpted for use on these monuments, and why such geometric patterns became so prolific at this time. Reading geometric patterns as visual commentary relates the practice of pattern-making in the eleventh and twelfth centuries to then contemporary discourse concerning mathematics, philosophy, and the Islamic sciences in Iran.[116]

Taking a course on philosophical allegory (*mithal*) with Mohammad Azadpour at Johns Hopkins several years ago, I had my first inkling of this prospective relationship – it became apparent when I considered the possibility that the Qur'anic term, *amthal*, referred to in the key inscriptions at Kharraqan, which is repeated on both the earlier and later towers, might actually relate visually to the geometric patterns themselves – not only the *mithal* as used in philosophical discourse, but also the *mithal* as used by mathematicians for 'illustration', 'figure', or 'example' in discussions of geometry at the time. It is, however, the term *mathal* that appears in discussions of poetics (see later in this chapter).

Consideration of the Qur'anic passages on these four monuments suggests that the terms *mithal* (s.) and *amthal* (pl.), and possibly *mathal* (s.), refer directly to the geometric patterns, 'upon which we should reflect'. The geometric patterns would then visually express the conceptual connection that contemporary texts make so explicit. Such an interpretation further links the Qur'anic term *mithal* (in the singular) to the *mithal* of the mathematician – the geometric problem as illustration, which in turn leads us to the prospective recognition that such geometric patterns fit neatly – like a jigsaw puzzle – into the dynamic cultural matrix of the time, in which *mithal*, and specifically *'alam-e mithal*, implied a nexus of relationships among discourses in philosophy, theology, poetry, and poetics, a cultural nexus in which Qur'an and calligraphy, mathematics (through geometry), art, and architecture could each play an integrative and expressive role in articulating a deep affinity and affiliation, relating natural wonders to God's creation.

The Qur'anic passages selected for these monuments seem to reinforce programmatic links between text and image. And the Qur'anic excerpts inscribed are exceptional in their use of terminology that deals literally with vision and perception (*al-basar*; *al-futur*; *'abathan*; *ra'a*), each of which also has counterparts in philosophical discourse of the conceptual realm.

Aesthetics and poetics

Several other fields of textual study remain largely untapped in relation to the interpretation of the geometric ornament of these four monuments and their architectural and cultural contexts. One such field is that of Qur'anic exegesis, or *tafsir*, which for this period is historically complicated by the early developments of Sufism and Shi'ism, including the Isma'ili movement; study of then contemporary *tafsir* is beyond the scope of this chapter. A second field is the study of Persian poetry, in which the works of Nezami Ganjavi (1141–1209) are particularly noteworthy for their extensive reference to both geometry and architecture. A third relevant area of textual study and intellectual history is philosophy, in which *mithal* is treated as allegory.[117] The intersections of poetry and philosophy

with art and mathematics, respectively, result in aesthetics and poetics, the latter treated within the subject of logic in the Aristotelian tradition.[118]

Geometry is especially evident in Nezami's romantic epic, *Haft Paykar*, of which one of two surviving manuscript traditions is dedicated to the ruler of Maragha, 'Ala'-al-Din Korpe Arslan b. Aq Sonqor.[119] The date of composition, as surmised from a reference within the poetic text, is 1197, the very same year that the Gonbad-e Kabud was under construction. Several scholars familiar with the text and manuscripts have drawn attention to the extensive references to geometry within the text of this long poem, in which seven architectural pavilions provide a narrative setting.[120] Although no manuscripts survive from the time of its composition, later illustrated manuscripts distinctly show geometric patterns in the decoration and detail of the seven pavilions in which the stories take place. Meisami has translated *Haft Paykar* in verse, attempting to preserve the intricacy of the original text, in which the poet himself describes the design and ornament of his work. Meisami provides an introductory commentary that addresses Nezami's use of rich and vivid imagery: 'Design and number are, indeed, the principles on which the poem is based'.[121] Points, lines, planes, and circles structure the narrative; Meisami offers interpretation of some of the number symbolism in this allegory of spiritual growth. Of particular interest are her concerns regarding the meaning of 'seven lines . . . fixed my gaze', in a passage that she suggests may refer to geometric constructions.[122] Surely, the 'six directions' (north, south, east, west, up, and down) Nezami mentions describe three-dimensional space and define the spatial dimension.[123] Soucek, in discussing a manuscript of Nezami's *Khamseh* of 1524–1525, also raises questions concerning mathematical meanings. She suggests that patterns of poetic language are 'carefully organized but the intricacy of the whole discourages mathematical analysis'. She continues, 'Instead, the eye is led from curve to counter-curve and the circles appear magically interwoven with one another, inspiring contemplative viewing'.[124] Her description sounds very much like the construction of a geometric pattern in which the first step is the generation of overlapping circles that creates an array of points of intersection, a stage in the process of design that is usually absent in the final product. Such overlapping circles comprise the underlying two-dimensional structure, for example, of the three polygonal networks that are simultaneously present on the tympanum of Gonbad-e Sorkh (Figure 3.2c–d). Soucek also queries the relationship between six and seven, drawing attention to an apparent contradiction between 'the sixes and hexagons' and the 'symbolism of seven'.[125] But perhaps this can also be explained by the fact that six circles of equal diameter surround a seventh in the center, and this entire composition can be circumscribed by a hexagon, a feature that may also be explored in the geometry of the tympanum of Gonbad-e Sorkh (Figure 3.2) and that of the earlier tower at Kharraqan (Figure 3.1). The charm of this sort of mathematical play is captured by Chelkowski's comment on Nezami's brilliant literary achievement as a storyteller: '[He] was not only a painter with words, but a sculptor or architect, who, using simple bricks as his medium, builds palaces of breath-taking color, form, and intricacy'.[126] He further states, 'Nizami firmly believed as well that the unity of the world could be perceived through

arithmetical, geometrical, and musical relations. Numbers were the key to the one interconnected universe, for through number multiplicity becomes unity, and discordance, harmony'.[127] Such concerns with interrelationships of numbers, shapes, harmonies, and the universe have a long history in Pythagorean thought and were also treated in the tenth-century epistles of the Ikhwan al-Safa'.[128]

When physicist Abdus Salam said that 'The deeper we seek, the more is our wonder excited, the more is the dazzlement for our gaze', he alluded to the quest of the poet. In her dissertation on Arabic poetry Lara Harb explores the concept of wonder, *ta'ajjub*.[129] Harb proposes a shift in medieval Arabic literary criticism of the late tenth-early eleventh century from an appreciation of the truthfulness and naturalism that characterizes the classical style of pre-Islamic poets to the ability of the poet to evoke wonder. Stimulating the effect of wonder remains a key literary component that extends to the visual, as Persis Berlekamp examined in both illustrations and narrative of the fourteenth century with particular reference to Qazwini's *The Wonders of Creation and the Oddities of Existence*.[130] Although her focus is on figural images, she addresses the broader implications of talismanic images and how they mediate relationships between humans and the cosmos. The word used by medieval authors writing in both Arabic and Persian is *tilsam* (or *tilasm*), which might be many things, including 'a monumental statue, an engraved ring, a written tablet or scroll, or an inscribed shirt'.[131] The complexity of geometric patterns in Islamic art similarly may evoke wonder, yet the relationship is not yet fully articulated and deserves much closer study, as does the relationship between geometry and poetry.

To frame such a study, one might begin by considering the pursuit of wonder by geometers through their use of points, circles, lines, angles, and planes, as parallel to the pursuits of poets through words and philosophers through discourse. Ibn Sina's commentary on Aristotle's *Poetics* appears at the end of the section on logic in his *Kitab al-Shifa'*.[132] Ibn Sina describes the need for imaginative assent (*takhyil*) to a poetic statement. It is 'compliance due to wonder and pleasure'. Azadpur explores what he calls the 'disciplining of the imagination' as a spiritual practice and explains how Ibn Sina distinguishes the imaginative representation (*takhyil*) in philosophy as expressed through discourse and in poetry as expressed through words. In philosophy, the 'imaginative representation is an acceptance of the astonishment (*ta'ajjub*) and delight in discourse itself'.[133]

Handasa and *muhandis*

Arabic vocabulary associated with geometry is also relevant to the discussion of making geometry manifest visually in architectural monuments of the Iranian plateau and beyond. The Arabic word *muhandis*, used in Arabic, Persian, and Ottoman Turkish texts, came to refer both to architect and to engineer, but its quadriliteral root, *handasa*, referred specifically to geometry in the ninth and tenth centuries. *'Ilm al-handasa* was both 'knowledge of geometry' and 'science of geometry'.[134] As with many other Arabic words having a quadriliteral root, *handasa* is a loan word from Middle Persian. According to the *Encyclopaedia of*

Islam (revised edition), the Arabs were introduced to geometry through translation of Greek sources and referred to this subject as *jumatriya*, a term, deriving from Greek, used in the *Fihrist* of Ibn al-Nadim of the tenth century.[135] But the term *handasa* for 'geometry' had become standardized in tenth-century mathematical texts.[136] Suter, in the first edition of the *Encyclopaedia of Islam* states, 'the geometrician is called *al-muhandis* in Arabic'.[137] The term *muhandis*, although derived from the quadriliteral root *h-n-d-s*, is not a loan word. Rather it represents an Arabic grammatical construction (that is, not a borrowing). As explained in an early Ottoman architectural treatise of the seventeenth century, *muhandis* is the active participle of *handasa*.[138] With a *mim*-prefix in this grammatical form, the meaning is transformed and associated with a person or agent, one doing or having to do with, *handasa*, today an architect or engineer. In the tenth century, however, it referred more specifically to someone who worked with geometry (an agent), and is so named in building inscriptions as *muhandis*. Although the terms *mi'mar*, *banna'*, and *muhandis* are differentiated nomenclature in genealogical names, in L. A. Mayer's compilation of architects' names, he finds no easily made distinctions among them.[139] The Ottoman treatise emphasizes that geometry, *handasa*, was a science, having to do with volume and form. Although it states that the term was no longer in use, this text reaffirms the evidence of earlier Arabic lexicography, which explains that *handasa* was of Persian origin and provides an explanation for the shift from the phoneme *za* to the letter *sin*.[140]

A unique inscription on a pair of wooden doors in the Bimaristan of Nur al-Din in Damascus ascribes the title *al-muhandis* to a woodworker, Abu'l-Fadl Ibn Abd al-Karim Muhammad al-Harithi (d. 1202–1203).[141] He was himself a native of Damascus, and is credited with having studied Euclid and Ptolemy.[142] Tabbaa uses this information, given by Damascus-born thirteenth-century biographer Ibn Abi Usaibi'ah to argue in favor of such capable craftsmen being among the intellectual elite.[143] But the relationship is even more specific: Abu'l-Fadl studied with Sharaf al-Din al-Tusi (ca. 1135–1213), one of the most prolific mathematicians writing in the twelfth century. Of the many works he wrote, only three survive, and the Arabic texts have been edited and translated.[144] He taught mathematics in Damascus, Aleppo, and Mosul, and he was the teacher of Kamal al-Din ibn Yunus (1156–1242) in Mosul, who taught Nasir al-Din al-Tusi (1201–1274). In his work on algebra, a subject considered the invention of an early predecessor, al-Khwarezmi (ca. 780–850), advanced by Abu Kamil, and further developed by Omar Khayyam (1048–1131), Sharaf al-Din al-Tusi was particularly interested in finding ways to solve equations with polynomials using numerical methods and geometric proofs. Through the manipulation of squares and rectangles al-Tusi extracted the roots of cubic equations as positive integers.[145] Although only three of his works survive, they provide ample evidence for the thrust of his mathematical research, and we know a few details of his life.[146] According to Ibn Abi Usaibi'ah, Sharaf al-Din al-Tusi was 'outstanding in geometry and the mathematical sciences, having no equal in his time'.[147] With regard to al-Tusi's work on geometrical dissections Berggren speculates, 'The relevance of this kind of problem to problems that a craftsman would encounter in doing medieval Islamic tilings of plane surfaces is clear'.[148] And Rashed suggests that al-Tusi's work represents 'an essential contribution' to an algebra aimed at studying curves 'by means of equations, thus inaugurating the beginning of algebraic geometry'.[149]

Taking a long historical view, Rashed characterized the mathematical endeavors of this period in the following manner:[150]

> Al-Khwarizmi's successors undertook a systematic application of arithmetic to algebra, algebra to arithmetic, both of them to trigonometry, algebra to the Euclidean theory of numbers, algebra to geometry, and geometry to algebra.

From Rashed's perspective based on textual sources, this was an exceptionally inventive period of mathematics that prefigured what was to develop in Europe in the seventeenth century with the work of Descartes and Fermat, who also studied algebra and its relationship with geometry. A direct relationship between the visual arts of geometric patterns, both in the so-called decorative arts such as woodwork, and in monumental architecture, executed in Iran and Iraq using cut bricks, and carved in stone in Syria and Anatolia, may thus be identified with this fertile period of major advances in mathematical thinking. With respect to the historiography of Islamic mathematics, however, Rashed has pointedly called into question current systems of classification that reveal distinct biases of the European tradition and he calls for a new periodization that is respectful of classical mathematics as it developed in Islamic lands with court patronage.[151] Such a vision finds parallels within the current historiographic assessment of Islamic art, for which Shaw has offered a critical perspective to counter what she identifies as 'epistemological structures grounded in Western modes of perception'.[152]

In the form of glazed bricks and ceramic revetments, the trajectory of geometric ornament of the late twelfth century demonstrated an increase in the use of glazed ceramic elements, which persisted in architectural expressions of the following centuries despite the major disruptions of the Mongol conquests. Its impact is directly felt in thirteenth-century Seljuq architecture in Anatolia.[153] But its impact may also be tracked in the following centuries across the Islamic world from Nasrid Spain to Mughal India. In contrast to its instantiation in architecture, however, the trajectory of the history of mathematics may have been more majorly interrupted, whether by the Mongol conquests or other factors. Many of the mathematical subjects that were being explored in the late twelfth century seem to have emerged once more in the minds of mathematicians in the seventeenth century in Western Europe.[154]

To return to the questions concerning the relationship of *handasa* to *muhandis*, it was not the builder, *al-banna'*, nor the architect, *al-mi'mar*, but the *muhandis*, who, working with *handasa*, literally made geometry manifest.

Conclusion

In contrast to the traditional twentieth-century paradigms that treat ornament as ornamental, decorative, and nonrepresentational, we may now summarize the

basis for a new paradigm in which ornament is explicitly mathematical and visually expressive of new ideas in mathematical thinking. The formative period for this development would seem to be the Iranian plateau and neighboring lands in the eleventh and twelfth centuries. Based on an integrated reading of historical and Qur'anic inscriptions, poetic references, and written historical sources, geometric patterns in monuments of Iran dating from the eleventh and twelfth centuries do seem to exemplify a cultural specificity: they express mathematical concerns at a defining moment in time and space, a moment at which architecture and its ornamentation map equivalent developments in the history of mathematical thinking and ideas, rendering geometry manifest. In this sense, the monuments studied seem to represent clear intentionality to express mathematics visually – not just any mathematics that is inherent in processes of pattern-making, but the then current mathematics that was specifically embedded in a distinct cultural context in which such mathematical forms of expression were just beginning to be understood, representing contemporary breakthroughs in mathematical thinking.

Looking back on the past 150 years of the historiography of ornament in Iran and neighboring lands, with few exceptions most art historians tended to describe the presence of geometry in works of art, referring generically to 'geometry', 'interlace', 'arabesque', or 'ornament', without further elaboration beyond basic attributes, such as 'complicated', 'intricate', or 'complex'. The construction of geometry, however, is necessarily respectful of the laws of nature; in Islamic art this is primarily but not exclusively through the inherent presence of symmetry. This restricts the possibilities for the construction of patterns through repetition of a design to reflection across an axis, translation along a vector, glide reflection along and across an axis, or rotation around a point.[155] Artisans in the Islamic world traditionally played with repetition in the formation of patterns in different media. Artisanal choice is indicated in the combination of symmetry with symmetry-breaking, which demonstrates a playfulness in the construction of patterns with inherent ambiguities. Islamic art tends to express an aesthetic of ambiguity with visual delight in the manifold possibilities that pertain to the processes of pattern-making – defining a design unit that is repeated according to an organizing principle. That the results both please the eye and tease the mind can be seen in all media of Islamic art, and can lead to a nuanced appreciation of geometry in relation to ambiguity and aesthetics. The linking of perception and visual delight is pervasive, from Spain and North Africa to India and beyond, and may have roots in scientific literature of the period as proposed in the recent work of Hans Belting.[156] In assessing the universality of geometry, however, Grabar asked why it seemed to have such particular importance in Islamic art, raising the paradox of universality in relation to particularity.[157]

Based on his interpretation of textual sources concerning Islamic mathematics emerging in Baghdad from the synthesis of Greek, Babylonian, and Indian traditions, historian of Near Eastern mathematics Jens Hoyrup argues (without reference to visual sources) that the period of Abbasid suzerainty and patronage witnessed a unique integration of the theoretical and the practical in mathematics, indeed with a theological dimension.[158] The precise connections are yet to be explored, but that there are specific cultural connotations to the visual expression of mathematics yields an implication for a radical shift in the historiography of ornament, the basis for which is laid out in this chapter.

As evidence is gathered (see Figures 3.1–3.4), it is becoming clear that the presence of geometry on the Iranian plateau and beyond in the eleventh and twelfth centuries is neither by happenstance nor merely decorative.[159] Close study of these geometric patterns collectively gives new meaning to Juzjani's comment, as stated in the introduction. The extraordinary playfulness and experimentation in the use of bricks, and the introduction of glazed elements in pre-Mongol monuments on the Iranian plateau, established trajectories in styles of architecture that persisted into the thirteenth century. With regard specifically to the profusion of geometric patterns in the eleventh and twelfth centuries, these initiatives visually express new advances in mathematics, which included the geometric expression of algebraic problems. If this is indeed the case, this brickwork represents 'geometry made manifest', and these buildings would therefore be aptly characterized by the implication of Juzjani's statement. It behooves us to take quite literally that geometers were indeed 'making geometry manifest' through architecture and ornament. Far from being merely decorative or ornamental, the geometric patterns on monumental architecture of the eleventh and twelfth centuries would then designate contemporary ideas that were culturally significant, fully illuminating a temporal and spatial specificity of meaning for Islamic geometric ornament: making new advances in mathematics visible, making geometry manifest.

The relational study of geometric ornament within particular cultural contexts can chart a new course for the study and interpretation of cultural ideas. The forms and patterns that appear on architectural monuments in the eleventh and twelfth centuries on the Iranian plateau and lands beyond visually express what were then contemporary mathematical ideas. This ornament is mute if studied in isolation from contemporary cultural values. It can be given voice if examined critically in relation to associated textual references from the Qur'an and other cultural forms of expression. We must continue to look critically at both ornament and geometry to discern this visual discourse and to understand its meaning.

On the Iranian plateau and lands beyond in the eleventh and twelfth centuries, as highlighted in this chapter, lines of mathematical thought found expression not only in fired clay and cut brick, but also in poetry and poetics, in philosophy and theology, and in the adaptive use of Qur'anic inscriptions in architecture. By utilizing a relational approach and a heuristic method, we may thus develop understanding of an Islamic aesthetic not based on a European paradigm of painting and sculpture, mimesis and representation, landscapes, portraiture, and linear perspective, which delineate the study of art history. Geometric patterns demonstrate an algorithmic aesthetic that is culturally rooted. In the late eleventh and twelfth centuries geometry itself formed the basis for major cultural forms of expression in visual media and may have had a much broader impact than previously acknowledged. During this extraordinary period of cultural flourishing, geometry is not drawn on as universally available, but rather as a particular cultural manifestation.

This conclusion has profound implications for the study and understanding of the appearance of geometric patterns elsewhere in Islamic cultures, requiring a reorientation of the historiography of ornament.

Notes

1 Owen Jones, *Grammar of Ornament* (London: Day & Son, 1856); P. J. Lu and P. J. Steinhardt, 'Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture', *Science* 315 (22 February 2007): 1106–1110, Supplementary Online Materials, Figures S1–S8, www.sciencemag.org/cgi/content/full/315/5815/[page] DC1; Emil Makovicky, '800-Year-Old Pentagonal Tiling from Maragha, Iran, and the New Varieties of Aperiodic Tiling It Inspired', in *Fivefold Symmetry*, ed. I. Hargittai (Singapore: World Scientific, 1992), 67–86; Jay Bonner, 'Three Traditions of Self-Similarity in Fourteenth and Fifteenth Century Islamic Geometric Ornament', *Meeting Alhambra: ISAMA-Bridges Conference Proceedings*, ed. J. Barrallo et al. (Granada: University of Granada, 2003), 1–12.

2 Oleg Grabar, *The Mediation of Ornament* (Princeton, NJ: Princeton University Press, 1992); Alpay Ozdural, 'Omar Khayyam, Mathematicians, and Conversations with Artisans', *Journal of the Society of Architectural Historians* 54, no. 1 (March 1995): 54–71; Ozdural, 'On Interlocking Similar or Corresponding Figures and Ornamental Patterns of Cubic Equations', *Muqarnas* 13 (1996): 191–211; Ozdural, 'A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan', *Technology and Culture* 39, no. 4 (1998): 699–715; Ozdural, 'Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World', *Historia Mathematica* 27, no. 2 (2000): 171–201; Ozdural, 'The Use of Cubic Equations in Islamic Art and Architecture', *Nexus: Architecture and Mathematics* IV (2002): 165–179; Gulru Necipoglu, *The Topkapı Scroll: Geometry and Ornament in Islamic Architecture* (Santa Monica, CA: Getty Center for the History of Art and the Humanities, 1995). See also Carol Bier, 'The Decagonal Tomb Tower at Maragha and Its Architectural Context: Lines of Mathematical Thought', *Nexus Network Journal* 14, no. 2 (2012): 251–273.

3 H. G. Raverty, trans. *Tabaqat-e Naseri* (London, 1881), vol. 1, 403–404, online at http://catalog.hathitrust.org/Record/000672608 (accessed 30 August 2013). The passage is also translated in D. Fairchild Ruggles, ed., *Islamic Art & Visual Culture: An Anthology of Sources* (Malden, MA: Wiley-Blackwell, 2011), 151.

4 In addition to works cited in notes 1 and 2, see Wasma'a K. Chorbachi, 'In the Tower of Babel: Beyond Symmetry in Islamic Design', *Computers and Mathematics with Applications* 17: 751–789 [repr. in *Symmetry* 2: Unifying Human Understanding, ed. I. Hargittai (New York: Pergamon, 1989)]; Wasma'a Chorbachi and Arthur D. Loeb, 'An Islamic Pentagonal Seal (from Scientific Manuscripts of the Geometry of Design)', in Hargittai, *Fivefold Symmetry*, 283–305; Carol Bier, 'Geometry and the Interpretation of Meaning: Two Monuments in Iran', in *Bridges: Mathematical Connections in Art, Music, and Science*, ed. R. Sarhangi (Winfield, KS: Southwestern College, 2002), 67–78; Carol Bier, 'Art and Mithal: Reading Geometry as Visual Commentary', *Iranian Studies* 41, no. 4 (2008): 491–509; Carol Bier, 'Number, Shape, and the Nature of Space: Thinking through Islamic Art', *Oxford Handbook for the History of Mathematics*, eds. E. Robson and J. Stedall (Oxford: Oxford University Press, 2009), 827–851. See also outstanding analyses of Islamic geometric ornament using traditional methods of construction with compass and straightedge in Eric Broug, *Islamic Geometric Patterns* (London: Thames & Hudson, 2008), amplified with an excellent compilation of photographs of architectural monuments and analytical diagrams in Eric Broug, *Islamic Geometric Design* (New York and London: Thames & Hudson, 2013).

5 Keith Critchlow, *Islamic Patterns: An Analytical and Cosmological Approach* (London: Thames & Hudson, 1976); Issam El-Said and Ayse Parman, *Geometric Concepts in Islamic Art* (London: World of Islam Festival Publishing Company, 1976); Issam El-Said, *Islamic Art and Architecture: The System of Geometric Design*, eds. Tarek El-Bouri and Keith Critchlow (Reading, UK: Garnet Publishing, 1993). Analysis and construction have also been used for the reconstruction of geometric patterns for historic preservation: R. Ajlouni and P. Justa, 'Reconstruction of Eroded and Visually Complicated Archaeological Geometric Patterns: Minaret Choli, Iraq' (Prague, 2011), http://cipa2011.cz/proceedings/index.htm.

6 Critchlow, *Islamic Patterns*; see also Samer Akkach, *Cosmology and Architecture in Premodern Islam: An Architectural Reading of Mystical Ideas* (Albany: State University of New York, 2005).

7 M. S. Bulatov, 'Geometricheskaia garmonizatsiia v arkhitekture Srednei Asii, IX–XV vv.', istoriko-teoreticheskoe issledovaniya (in Russian) (Moscow: Nauka, 1988).

8 Grabar, *Mediation of Ornament*, ch. III.

9 Necipoglu, *Topkapi Scroll*.

10 Ibid., see esp. ch. 4.

11 Ibid., x.

12 See Lu and Steinhardt, 'Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture'.

13 Peter R. Cromwell, 'Islamic Geometric Designs from the Topkapi Scroll I: Unusual Arrangements of Stars', *Journal of Mathematics and the Arts* 4, no. 2 (2010): 73–85, and Peter R. Cromwell, 'Islamic Geometric Designs from the Topkapi Scroll II: A Modular Design System', *Journal of Mathematics and the Arts* 4, no. 3 (2010): 119–136.

14 Donald N. Wilber, 'The Development of Mosaic Faience in Islamic Architecture in Iran', *Ars Islamica* 6, no. 1 (1939): 16–47; Douglas Pickett, *Early Persian Tilework: The Medieval Flowering of Kashi* (London: Associated University Presses, 1997).

15 Quoted in Wilber, 'Development of Mosaic Faience', 23, n. 25.

16 Fuat Sezgin, ed. *The School of Maragha and Its Achievements, Islamic Mathematics and Astronomy*, vol. 50–51 (Frankfurt 1998) and personal conversation, Roshdi Rashed (June 2013).

17 Sabine Schmidtke, 'Samaw'al al-Maghribi, al-', in *Encyclopaedia of Jews in the Islamic World*, ed. Norman A. Stillman. Brill Online (2013), http://referenceworks.brillonline.com/entries/encyclopedia-of-jews-in-the-islamic-world/samawal-al-maghribi-al-COM_0001760 (accessed 1 February 2014).

18 Ibid.

19 Tr. T.J.P. de Bruijn, *Of Piety and Poetry: The Interaction of Religion and Literature in the Life and Works of Hakim Sana'i of Ghazna* (Leiden, 1983), 238: Tu nabini juz az xiyal-u havass/cun na-i xatt-u sath-u nuqta-sinas (BV, fol. 4a, 12).

20 For details of his life, see entries by C. E. Bosworth for 'Tabaqat-e Naseri' and 'Menhaj-e Seraj', *Encyclopaedia Iranica* (2010), http://iranicaonline.org/articles/tabaqat-naseri and http://iranicaonline.org/articles/menhaj-seraj (accessed 31 December 2013).

21 C. E. Bosworth, 'Firuzkuh: i. The Ghurid Capital', *Encyclopaedia Iranica* (1999), www.iranicaonline.org/articles/firuzkuh (accessed 1 August 2013).

22 See M. Keyvani, 'Jam', *Encyclopaedia Iranica*, vol. XIV, fasc. 4 (2008), 430–432, www.iranicaonline.org/articles/jam-1 (accessed 25 January 2013) and Bosworth, 'Firuzkuh, i. The Ghurid Capital'.

23 F. B. Flood, 'Jam Minaret', *Encyclopaedia Iranica*, v. XIV, fasc. 4 (2008): 432–436, www.iranicaonline.org/articles/jam-minaret (accessed 22 January 2013), offers a synthesis of what is known to date from historical sources and the archeological record; see also F. B. Flood, 'Ghurid Monuments and Muslim Identities: Epigraphy and Exegesis in Twelfth Century Afghanistan', *Indian Economic and Social History Review* 42, no. 3 (2005): 263–294.

24 A. Maricq and G. Wiet, *Le minaret de Djam: La découverte de la capitale des sultans Ghourides XIIe-XIIIe siècles, Mémoires de la Délégation archéologique française en Afghanistan* 16 (Paris, 1959); J. Moline, 'The Minaret of Gam (Afghanistan)', *Kunst*

des Orients 9 (1973): 131–148; R. Pinder-Wilson, 'The Minaret of Mas'ud III at Ghazni', in *Studies in Islamic Art* (London: Pindar Press, 1985), 89–102 and 'Ghaznavid and Ghurid Minarets', *Iran* 39 (2001): 155–186.

25 Several recent publications reexamine early field photographs and the contexts in which they were taken; see for example, Ayse Gursan-Salzmann, *Exploring Iran: The Photography of Erich F. Schmidt, 1930–1940* (Philadelphia: University of Pennsylvania Museum of Archaeology and Anthropology, 2007) and James Bamberg and Charles Melville, *Images of Persia: Photographs of Laurence Lockhart 1920s–1950s* (Cambridge: Center of Middle Eastern and Islamic Studies, University of Cambridge, 2002). With reference to the historiography of similar circumstances of early scholarship and exploration in Turkey, see Patricia Blessing, 'Friedrich Sarre and the Discovery of Seljuk Anatolia', *Journal of Art Historiography* 11 (2014). Iranians as scholars emerged as leaders in the field in the late twentieth century, often having studied in Europe or the U.S.

26 Documentation of monuments exists in the following early publications: Marcel Dieulafoy, *L'art antique de la Perse: Achemenids, Parthes, Sassanides* (Paris: Librairie centrale d'architecture, 1884[–1889]), 5 vols.; Ernst Diez, *Persien: Islamische Baukunst in Churasan* (Hagen: Volkwang-Verlag, 1923); Arthur Upham Pope and Phyllis Ackerman, eds., *A Survey of Persian Art from Prehistoric Times to the Present* (London and New York: Oxford University Press, 1938–1939), 6 vols. Early on, Wilber traced the development of the use of glazed ceramics (glazed brick, mosaic, faience, tile) in 'Development of Mosaic Faience'. The impact of the Persian cultural sphere on architecture is also evident in the work of Ernst Herzfeld, 'Damascus: Studies in Architecture – I', *Ars Islamica* 9 (1942): 1–53; 'Damascus: Studies in Architecture – II', *Ars Islamica* 10 (1943): 13–70; and 'Damascus: Studies in Architecture – III', *Ars Islamica* 11/12 (1946): 1–71. Several works published recently critically examine these early endeavors and assess their broad impact on collecting and exhibiting Islamic art; see especially, the special thematic issue edited by Linda Komaroff, *Exhibiting the Middle East: Collections and Perceptions of Islamic Art*, *Ars Orientalis* 30 (2000) and Andrea Lermer and Avinoam Shalem, eds., *After One Hundred Years: The 1910 Exhibition, 'Meisterwerke muhammadanischer Kunst', Reconsidered* (Leiden and Boston: E. J. Brill, 2010). See also Jay Gluck, Noel Siver, and Sumi Hiramoto Gluck, eds., *Surveyors of Persian Art: A Biography of Arthur Upham Pope and Phyllis Ackerman* (Ashiya, Japan: Survey of Persian Art, 1996). Erich F. Schmidt, *Flights over Ancient Cities of Iran* (Chicago, IL: University of Chicago Press, 1940) provides documentation of a different sort (aerial photography), using what was then a new technology, literally offering new perspectives on archeological sites and monuments. More recent aerial photographs provide comparative perspectives with critical commentary: David Stronach and Ali Mousavi, eds., *Irans Erbe: In Flugbildern von Georg Gerster* ([Darmstadt]: Philipp Von Zabern, 2009).

27 *Athar-e Iran: Annales du Service Archéologique de l'Iran*, vols. 1–4 (1936–1949); *Archaeologische Mitteilungen aus Iran* (published by the German Archeological Institute, Tehran section), vols. 1–9 (1929–1938), with publication suspended from 1938 to 1968, when a new series began; *Iran: Journal of the British Institute of Persian Studies*, which began publication in 1963. For architectural documentation of Islamic sites and monuments (not specifically Iran), *Muqarnas: An Annual on the Visual Cultures of the Islamic World*, sponsored by the Aga Khan Program for Islamic Architecture at Harvard University and the Massachusetts Institute of Technology, has assumed a central role since 1983.

28 Photographic archives include those held in the U.S. in the collections of Harvard University, Cambridge, MA; Smithsonian Institution, Washington, DC; University of Michigan, Ann Arbor; and in the UK, the Creswell Archive held at the Ashmolean Museum in Oxford. Access to many special collections is facilitated through the ArchNet Digital Library at http://archnet.org/library/ (accessed 2 January 2014).

29 For Jam, see notes 22, 23, and 24 of this chapter; for Kharraqan, see David Stronach and T. Cuyler Young Jr. 'Three Seljuq Tomb Towers from Iran', *Iran* 4 (1966): 1–20.

30 Initially explored by Edward Said, *Orientalism* (New York: Pantheon Books, 1978), who characterized the cultural dominance of the West; several critiques of this seminal work have since appeared – see, for example, Daniel W. Varisco, *Reading Orientalism: Said and the Unsaid* (Seattle: University of Washington Press, 2006).

31 This prejudicial attitude in European scholarship and its impact on the study of Islamic art is eloquently discussed in David Freedberg, *The Power of Images: Studies in the History and Theory of Response* (Chicago and London: University of Chicago Press, 1989).

32 Cf. Terry Allen, 'Aniconism and Figural Representation in Islamic Art', in Terry Allen, *Five Essays on Islamic Art* (Sebastopol, CA: Solipsist Press, 1988), 17–37.

33 Titus Burckhardt, *Art of Islam: Language and Meaning* (London: World of Islam Festival Publishing Company, 1976); Seyyed Hossein Nasr, *Islamic Science: An Illustrated Study* (London: World of Islam Festival Publishing Company, 1976); Martin Lings, *The Quranic Art of Calligraphy and Illumination* (London: World of Islam Festival Trust, 1976).

34 Wendy M. K. Shaw, 'The Islam in Islamic Art History: Secularism and Public Discourse', *Journal of Art Historiography* 6 (2012).

35 Critchlow, *Islamic Patterns*.

36 Nader Ardalan and Laleh Bakhtiar, *The Sense of Unity: The Sufi Tradition in Persian Architecture* (Chicago and London: University of Chicago Press, 1973), acknowledge the influence of Seyyid Hossein Nasr on their interpretations. See also the work of Laleh Bakhtiar, *Sufi: Expressions of the Mystical Quest* (London: Thames & Hudson, 1976).

37 Grabar was a strong proponent of this critical stance. See his 'Geometry and Ideology: The Festival of Islam and the Study of Islamic Art', chapter V, first published in *A Way Prepared, Essays on Islamic Culture in Honor of Richard Bayly Winder*, eds. Farhad Kazemi and R. D. McChesney (New York and London: New York University Press, 1988), 145–152. See also Shaw, 'The Islam in Islamic Art History'.

38 Yasser Tabbaa provides an excellent critique in the introduction to his book, *The Transformation of Islamic Art during the Sunni Revival* (Seattle and London: University of Washington Press, 2001), 4–6. Samer Akkach accepts the idea that 'cosmological thinking mediates human acts of making and space ordering' (p. iv), but challenges the symbolic interpretive paradigm of the Sufi hermeneutics of the Perennialists in his study, *Cosmology and Architecture in Premodern Islam*, 4–9. See also Necipoglu, *Topkapi Scroll*, 76–82.

39 El-Said and Parman, *Geometric Concepts*; El-Said, *Islamic Art and Architecture*.

40 Bulatov, 'Geometricheskaia garmonizatsiia v arkhitekture Srednei Asii'.

41 Avinoam Shalem in 'What Do We Mean When We Say "Islamic Art"? A plea for a critical rewriting of the history of the arts of Islam', *Journal of Art Historiography* 6 (June 2012) argues cogently for the development of schema based on internal cultural evidence rather than established European paradigms. Such an endeavor is earlier reflected in Bier, 'Geometry and the Interpretation of Meaning'; Bier, 'Art and *Mithal*'; Carol Bier, 'Taking Sides but Who's Counting? The Decagonal Tomb Tower at Maragha', in *Bridges Conference* (Mathematical Connections in Art, Music, Science), eds. R. Sarhangi and C. Sequin (Phoenix, AZ: Tessellations Publishing, 2011), 497–500; Bier, 'The Decagonal Tomb Tower at Maragha and Its Architectural Context'.

42 Jones, *Grammar of Ornament*.

43 See, for example, Eric Schroeder's study of brickwork, 'Islamic Architecture F. The Seljuq Period', in Pope and Ackerman, *Survey of Persian Art*, vol. 2, 981–1045, where he extols the beauty of 'naked brick' and sincerely appreciates its structural manifestations; see especially the section 'The Character of the Seljuq Achievement'. See also

S. P. Seherr-Thoss and H. C. Seherr-Thoss, *Design and Color in Islamic Architecture: Afghanistan, Iran, Turkey* (Washington, DC: Smithsonian Institution Press, 1968).

44 A. K. Coomaraswamy, 'Review of Eric Schroeder's Persian Miniatures in the Fogg Museum of Art (Cambridge 1942)', in *The Art Bulletin* XXV (1943): 381.

45 Allen, 'Aniconism', 37.

46 Curiously, while modernism eschewed ornament, it embraced geometry, which flourished even in periods of minimalism; see James Trilling, *Ornament: A Modern Perspective* (Seattle and London: University of Washington Press, 2003).

47 Oleg Grabar, *The Formation of Islamic Art* (New Haven and London: Yale University Press, 1973), 189.

48 Grabar, *Mediation of Ornament*, ch. 1, 'A Theory of Intermediaries in Art', 9–46.

49 Ibid., 46.

50 E. H. Gombrich, *The Sense of Order: A Study in the Psychology of Decorative Art* (Ithaca, NY: Cornell University Press, 1979).

51 Grabar, *Mediation of Ornament*; see review by Bier, 'Ornament and Islamic Art', in *Middle East Studies Association Bulletin* 28 (1994): 28–30.

52 Grabar, *Mediation of Ornament*, 5.

53 Chorbachi, 'In the Tower of Babel: Beyond Symmetry in Islamic Design', and Chorbachi and Loeb, 'An Islamic Pentagonal Seal'; Ozdural, 'Omar Khayyam, Mathematicians, and Conversations'; Ozdural, 'On Interlocking Similar or Corresponding Figures'; Ozdural, 'A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan'; Ozdural, "Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World'; Ozdural, 'The Use of Cubic Equations in Islamic Art'.

54 Ozdural, 'Omar Khayyam, Mathematicians, and Conversazioni'.

55 Renata Holod, 'Text, Plan and Building: On the Transmission of Architectural Knowledge', in *Theories and Principles of Design in the Architecture of Islamic Societies*, ed. Margaret Sevcenko (Cambridge: Aga Khan Program for Architecture at Harvard University and the Massachusetts Institute of Technology, 1988), 1–12.

56 Jonathan M. Bloom, 'On the Transmission of Designs in Early Islamic Architecture', *Muqarnas* 10 (1993): 21–28.

57 Necipoglu, *Topkapi Scroll*, pp. 9, 92, and *passim*.

58 Terry Allen, *Islamic Art and the Argument from Academic Geometry* (Occidental, CA: Solipsist Press, 2004), 24.

59 Tabbaa, *Sunni Revival*, 3.

60 Ibid., 88.

61 Richard Ettinghausen, Oleg Grabar, and Marilyn Jenkins-Madina, *Islamic Art and Architecture, 650–1250* (New Haven, CT: Yale University Press, 2001); Sheila S. Blair and Jonathan M. Bloom, *The Art and Architecture of Islam 1250–1850* (New Haven, CT: Yale University Press, 1994); Robert Hillenbrand, *Islamic Architecture: Form, Function, and Meaning* (New York: Columbia University Press, 1994); Jonathan M. Bloom and Sheila S. Blair, eds., *The Grove Encyclopedia of Islamic Art and Architecture* (Oxford and New York: Oxford University Press, 2009), a compilation of entries authored by individual scholars, originally published in Jane Turner, ed., *The Dictionary of Art*, 10 vols. (New York: Grove, 1996); E. Yarshater, ed., *Encyclopaedia Iranica* (London and Boston: Routledge & Kegan Paul, 1982–2012), www.iranicaonline.org/ (accessed 31 December 2013); H.A.R. Gibb et al., *Encyclopaedia of Islam*, rev. edn (Leiden: Brill, 1960–), http://referenceworks.brillonline.com/browse/encyclopaedia-of-islam-2 (accessed 31 August 2013).

62 Victor J. Katz, *A History of Mathematics: An Introduction*, 3rd edn (Boston, MA: Addison-Wesley, 2009); *Complete Dictionary of Scientific Biography* (electronic resource) (Detroit: Charles Scribner's Sons, 2008); Roshdi Rashed, ed., *Encyclopedia of the History of Arabic Science* (London: Routledge, 1996).

63 ArchNet Digital Library at https://archnet.org/library (accessed 14 August 2013).

64 David Wade, 'Pattern in Islamic Art: The WADE Photo-Archive', 2nd edn, www.patterninislamicart.com/ (accessed 31 December 2013); Brian Wichmann and Tony Lee, 'A Tiling Database', http://tilingsearch.org/ (accessed 4 February 2013); and Carol Bier, 'Symmetry and Pattern: The Art of Oriental Carpets', http://mathforum.org/geometry/rugs/ (accessed 31 December 2013).

65 Wilber, 'Development of Mosaic Faience', and Pickett, *Early Persian Tilework*.

66 Noted by Myron Bement Smith with reference to the 'Blue Tomb' at Maragha: Myron Bement Smith Collection, Freer Gallery of Art and Arthur M. Sackler Gallery Archives, Smithsonian Institution, Washington, DC.

67 Guy Le Strange, *The Lands of the Eastern Caliphate* (London: Frank Cass, 1966), 196.

68 Stronach and Young, 'Three Seljuq Tomb Towers'.

69 S. M. Stern, 'The Inscriptions of the Kharraqan Mausoleums', *Iran* IV (1966): 21–27; Sheila S. Blair, ed., *The Monumental Inscriptions from Early Islamic Iran and Transoxiana* (New York and Leiden: E. J. Brill, 1992), 135.

70 Andre Godard, 'Notes Complementaires sur les Tombeaux de Maragha', *Athar-e- Iran* 1 (1936): 125–160.

71 V. Minorsky, 'Maragha', *Encyclopaedia of Islam*, 2nd edn, eds. P. Beam, Th. Bianquis, C. E. Bosworth, E. van Donzel, and W. P. Heinrich (Brill, 2011) http://brillonline.nl/subscriber/entry?entry=Islam_COM-0676 (accessed 29 March 2011, University of California, Berkeley); V. Minorsky, 'Ahmadilis', *Encyclopaedia of Islam*, 2nd edn, 300–301; and K. A. Luther, 'Atabakan-e Maraga', *Encyclopaedia Iranica*, iranica.com (accessed 31 December 2013).

72 Pope and Ackerman, *Survey of Persian Art*, vol. 4, pl. 345.

73 Gonbad-e Kabud in Maragha bears many additional inscriptions, much damaged, surrounding each of the arches of the facade. To my knowledge, these have not been read or published. See also, Daneshvari, 'Complementary Notes'.

74 For detailed photographs, see Stronach and Young, 'Three Seljuq Tomb Towers'.

75 Stern, 'The Inscriptions on the Kharraqan Mausoleums', 23.

76 Comparing photographs from the mid-1960s, one of these roundels must have been removed soon after its 'discovery' by Western scholars; more recent photographs indicate that the second roundel has also been removed.

77 Illustrated in Bier, 'Decagonal Tower at Maragha and Its Architectural Context', fig. 9a–f.

78 Bier, 'Number, Shape, and the Nature of Space'.

79 Marcus Milwright, 'Gonbad-e Sork', *Encyclopaedia Iranica* (2002), www.iranicaonline.org/articles/gonbad-e-sork (accessed 2 August 2013).

80 My analysis was initially presented in a paper, 'Persian Monuments of Practical Geometry', at the 9th Biennial Meeting of the International Society for Iranian Studies held in Istanbul 1–5 August 2012. See also Reza Sarhangi, Slavik Jablan, and Radmila Sazdanovic, 'Modularity in Medieval Persian Mosaics: Textual, Empirical, Analytical, and Theoretical Considerations', *Visual Mathematics Journal* 7, no. 1 (2005): 281–292. See also Peter R. Cromwell, 'Analysis of a Multi-Layered Geometric Pattern from the Friday Mosque in Yazd', *Journal of Mathematics and the Arts* 6, no. 4 (2012): 159–168.

81 Bier, 'The Decagonal Tower at Maragha and Its Architectural Context', fig. 4a–c, illustrates this sequence, based on field drawings executed by Wilber, 'Development of Mosaic Faience', fig. 17.

82 Lu and Steinhardt, 'Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture', emphasize the periodicity of this pattern around the shaft of the monument.

83 The Royal Swedish Academy of Sciences, 'Press Release: A Remarkable Mosaic of Atoms', www.nobelprize.org/nobel_prizes/chemistry/laureates/2011/press.html (accessed 26 February 2013).
84 The Royal Swedish Academy of Sciences, 'Scientific Background on the Nobel Prize in Chemistry 2011: The Discovery of Quasicrystals', www.nobelprize.org/nobel_prizes/chemistry/laureates/2011/advanced-chemistryprize2011.pdf (accessed 26 February 2013).
85 See note 83.
86 Necipoglu, *Topkapi Scroll*, 313 (panel 50); see also several articles by Cromwell in *Journal of Mathematics and the Arts* (London, 2010).
87 'Abdus Salam-Biographical', www.nobelprize.org/nobel_prizes/physics/laureates/1979/salam-bio.html (accessed 1 January 2014).
88 L. Korn, 'Saljuqs vi. Art and Architecture', *Encyclopaedia Iranica* (2010), www.iranicaonline.org/articles/saljuqs-vi (accessed 14 August 2013).
89 S. Blair, 'Il-Khanids ii. Architecture', *Encyclopaedia Iranica* (2012), www.iranicaonline.org/articles/il-khanids-ii-architecture (accessed 14 August 2013); Donald N. Wilber, *The Architecture of Islamic Iran: The Il-Khanid Period* (Princeton, NJ: Princeton University Press, 1955).
90 Abbas Daneshvari, *Medieval Tomb Towers of Iran: An Iconographical Study* (Lexington, KY: Mazda Publishers in association with Undena Press, 1986).
91 Erica Cruikshank Dodd and Shereen Khairallah, *The Image of the Word: A Study of Quranic Verses in Islamic Architecture*, vol. 1 (text and photographs), vol. 2 (indexes) (Beirut: American University of Beirut, 1981).
92 *Répertoire chronologique d'épigraphie arab* (Cairo: L'Institut français d'archaeologie orientale, 1931) and *Matériaux pour un Corpus inscriptionum Arabicarum* (Paris: E. Laroux, 1894–).
93 Blair, *Monumental Inscriptions from Early Islamic Iran and Transoxiana*, and Sheila S. Blair, *Islamic Inscriptions* (New York: New York University Press, 1998).
94 Exceptions include Holly Edwards, 'The Ribat of 'Alib. Karmakh', *Iran* 29 (1991): 85–94; Flood, 'Ghurid Monuments and Muslim Identities'; and Rana Shani, *A Monumental Manifestation of the Shi'ite Faith in Late Twelfth-Century Iran: The Case of the Gunbad-i 'Alawiyan, Hamadan*, Oxford Studies in Islamic Art XI (Oxford: Oxford University Press, 1996).
95 Yasser Tabbaa, 'The Transformation of Arabic Writing: Part 2, The Public Text', *Ars Orientalis* XXIV (1994): 119–147; and Blair, *Islamic Inscriptions*.
96 Wayne E. Begley, 'The Myth of the Taj Mahal and a New Theory of Its Symbolic Meaning', The Art Bulletin LXI, no. 1 (1979): 7–37; Oleg Grabar, *The Dome of the Rock* (Cambridge MA: Belknap Press of Harvard University, 2006). Cf. F. B. Flood, 'Light in Stone: The Commemoration of the Prophet in Umayyad Architecture', in *Bayt al-Maqdis*, eds. Julian Raby and Jeremy Johns (London and New York: Oxford University Press, 1992); see also Sheila Blair and Jonathan Bloom, 'Inscriptions in Art and Architecture', in *The Cambridge Companion to the Qur'an*, ed. Jane Dammen McAuliffe (Cambridge and New York: Cambridge University Press, 2006), 163–178.
97 Bier, 'Taking Sides'; Bier, 'Geometry and the Interpretation of Meaning'; Bier, 'Art and *Mithal*'.
98 Stronach and Young, 'Three Seljuq Tomb Towers'.
99 See Mahdi Tourage, 'Metonymy and Metaphor, Mithal and Mathal', in *Rumi and the Hermeneutics of Eroticism* (Leiden: E. J. Brill, 2007), 1–82.
100 See also Akkach, *Cosmology and Architecture in Premodern Islam*, 28–29.
101 Elaheh Kheirandish, 'Science and *Mithal*: Demonstrations in Arabic and Persian Scientific Traditions', *Iranian Studies* 41, no. 4 (2008): 465–489. See also Roshdi Rashed, *Geometry and Dioptrics in Classical Islam* (London: Al-Furqaan Islamic Heritage Association, 2005), 1126–1127.

102 Bier, 'Geometry and the Interpretation of Meaning', 67.
103 Dominique Raynaud, 'Abu al-Wafa' Latinus? A Study of Method', *Historia Mathematica* 39 (2012): 39.
104 Another plural of *mithal* is *muthul*, derived from the same triliteral root, and this is the term used in Islamic sources for Platonic forms *(muthul al-eflatun)*. According to Maria Subtelny (personal communication, 9 March 2003), the terms *mithl* and *mithal* are carefully distinguished by the mystical poets of medieval Iran. But without specific reference to the monument, it is difficult to substantiate a direct relationship.
105 Quotation is from Al-Ghazali's *Al-Maqsad al-Asna*, as translated by Akkach, *Cosmology*, 51.
106 This verse (23:115) is attested in only one other location in architecture, the much later Bara Gombad Mosque in Delhi, dated to the late fifteenth century (Dodd and Khairallah, *Image of the Word*, vol. 2, 83).
107 Godard, 'Notes Complementaires'. See also Myron Bement Smith Collection, Freer Gallery of Art and Arthur M. Sackler Gallery Archives, Smithsonian Institution, Washington, DC.
108 Abbas Daneshvari, 'Complementary Notes on the Tomb Towers of Medieval Iran. 1: The Gunbad-i Kabud at Maraghe 593/1197', *Art et Sociéte dans le Monde Iranien*, ed. C. Adle (Paris: Editions Recherche sur les Civilizations, 1982), 287–295; ArchNet http://archnet.org ('an international web-based community based at MIT's Design Lab in the School of Architecture and Planning, with a focus on Muslim cultures and civilizations').
109 Godard, 'Notes Complementaires'.
110 Ibid., 138–140.
111 Daneshvari, 'Complementary Notes', 294.
112 ArchNet http://archnet.org. Note that I have not been able to determine which is the correct identification.
113 E. Makovicky, 'Comment on "Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture,"' *Science* 318 (30 November 2007): 1383, fig. 2a.
114 Bier, 'Taking Sides'.
115 Makovicky, '800-Year-Old Pentagonal Tiling from Maragha'; Lu and Steinhardt, 'Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture'; Makovicky, 'Comment on "Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture"'; P. J. Lu and P. J. Steinhardt, 'Response to "Comment on 'Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture,'" *Science* 318 (30 November 2007): 1383; Peter Saltzman, 'Quasi-Periodicity in Islamic Geometric Design', *Nexus: Architecture and Mathematics* VII (2008): 153–168; E. Makovicky, 'Another Look at the Blue Tomb of Maragha, A Site of the First Quasicrystalline Islamic Pattern', *Symmetry: Culture and Science* 19, no. 2–3 (2008): 127–151; Peter R. Cromwell, 'The Search for Quasi-Periodicity in Islamic 5-Fold Ornament', *Mathematical Intelligencer* 31, no. 1 (2009): 36–56; Bier, 'Taking Sides'.
116 Bier, 'Art and *Mithal*'.
117 Mohammad Azadpur, *Reason Unbound: On Spiritual Practice in Islamic Peripatetic Philosophy* (Albany: State University of New York, 2011).
118 Lara Harb, 'Wonder in Medieval Arabic Aristotelian Poetics', paper presented at the 222nd Meeting of the American Oriental Society, Boston, MA, 18 March 2012.
119 François de Blois, 'Haft Peykar', *Encyclopaedia Iranica* (2002), online at www.iranicaonline.org/articles/haft-peykar (accessed 31 December 2013), who discusses the manuscript traditions as well as translation history.
120 Nizami, *The Haft Paykar: A Medieval Persian Romance*, trans. with an introduction and notes by Julie Scott Meisami (Oxford and New York: Oxford University Press, 1995); Peter J. Chelkowski, *Mirror of the Invisible World: Tales from the Khamseh of Nizami* (New York: Metropolitan Museum of Art, 1975); P. P. Soucek, 'The Khamseh of 1524/25', in Chelkowski, *Mirror of the Invisible World*, 11–20.

121 Meisami, *Haft Paykar*, xxv.
122 Ibid., 276, offers an explanatory note to section 4, lines 36–40.
123 Ibid., 273, offers an explanatory note to section 3, lines 8–9; see also Julie Scott Meisami, *Medieval Persian Court Poetry* (Princeton, NJ: Princeton University Press, 1987), 299–304.
124 Soucek, 'The Khamseh of 1524/25', 11–12.
125 Ibid., 18.
126 Chelkowski, *Mirror of the Invisible World*, 2.
127 Ibid., 113.
128 Ian Netton, *Muslim Neoplatonists: An Introduction to the Thought of the Brethren of Purity* (Ikhwan al-Safa') (London and Boston: Allen Unwin, 1982); Seyyed Hossein Nasr, *An Introduction to Islamic Cosmological Doctrines* (Cambridge, MA: Belknap Press of Harvard University, 1964); Nader El-Bizri, ed., *Epistles of the Brethren of Purity (Ikhwan al-Safa') and Their Rasa'il: An Introduction* (Oxford: Oxford University Press, 2008).
129 Lara Harb, 'Poetic Marvels: Wonder and Aesthetic Experience in Medieval Arabic Literary Theory', PhD dissertation (New York University, 2013).
130 Persis Berlekamp, *Wonder, Image, & Cosmos in Medieval Islam* (New Haven and London: Yale University Press, 2011).
131 Ibid., 119–120.
132 Harb, 'Wonder'.
133 Ibn Sina, *al-Shifa'*, quoted in Azadpur, *Reason Unbound*, 77.
134 See H. Suter, 'Handasa', *Encyclopaedia of Islam*, 1st edn (1913–1936) (Brill Online, 2013), http://referenceworks.brillonline.com/entries/encyclopaedia-of-islam-1/handasa-SIM_2686?s.num=8 (accessed 24 February 2013) and 'Ilm al-Handasa', *Encyclopaedia of Islam*, 2nd edn (Brill Online 2013), http://referenceworks.brillonline.com/entries/encyclopaedia-of-islam-2/ilm-al-handasa-COM_1408 (accessed 31 December 2013). See also Edward William Lane, *An Arabic-English Lexicon* (Beirut: Librairie du Liban, 1968), vol. 8, 2904. According to Roshdi Rashed (personal communication, 11 June 2013), al-Kindi refers to al-ṣanat al-handasa, 'art of geometry'.
135 'Ilm al-Handasa'; *Ibn al-Nadim, The Fihrist of al-Nadim: A Tenth-Century Survey of Muslim Culture*, trans. Bayard Dodge (New York: Columbia University Press, 1970), 634.
136 Jan P. Hogendijk (personal communication, 14 December 2012).
137 Suter, 'Handasa'.
138 Howard Crane, trans. *Risale-i Mi'mariyye: An Early-Seventeenth-Century Ottoman Treatise on Architecture by Cafer Efendi* (Leiden and New York: E. J. Brill, 1987), 30.
139 L. A. Mayer, *Islamic Architects and Their Works* (Geneva: A. Kundig, 1956), 25; see also Blair, *Islamic Inscriptions*, 49–51.
140 Crane, *Risale-i Mi'mariyye*, p. 30 and n. 29. The spelling is also addressed by Lane, *Arabic-English Lexicon*, 2904.
141 Tabbaa, *Sunni Revival*, 88.
142 L. A. Mayer, *Islamic Woodcarvers and Their Works* (Geneva: A. Kundig, 1958), 53; see also J. J. O'Conner and E. F. Robertson, 'Sharaf al-Din al-Muzaffar al-Tusi', www-history.mcs.st-andrews.ac.uk/Biographies/Al-Tusi_Sharaf.html (accessed 24 February 2013).
143 Tabbaa, *Sunni Revival*, 88.
144 Roshdi Rashed, *Oeuvres mathématiques: algèbre et géométrie au XIIe siècle* (Paris: Belles Lettres, 1986), 2 vols.
145 Jan P. Hogendijk, 'Sharaf al-Din al-Tusi on the Number of Positive Roots of Cubic Equations', *Historia Mathematica* 16, no. 1 (1989): 69–85, www.sciencedirect.com/science/article/pii/0315086089900992 (accessed 24 February 2013). See also J. L. Berggren, 'Innovation and Tradition in Sharaf al-Din al-Tusi's al-Mu'adalat', *Journal of the American Oriental Society* 110, no. 2 (1990): 304–309.

146 O'Conner and Robertson, 'Sharaf al-Din al-Muzaffar al-Tusi'.
147 J. L. Berggren, 'Al-Tusi, Sharaf Al-Din Al-Muzaffar Ibn Muhammad Ibn Al-Muzaffar', *Dictionary of Scientific Biography* 13 (New York: Charles Scribner & Sons, 1980), www.encyclopedia.com/doc/1G2–2830906159.html (accessed 31 December 2013).
148 Ibid.
149 Roshdi Rashed, *The Development of Arabic Mathematics: Between Arithmetic and Algebra* (Dordrecht/Boston: Kluwer Academic Publishers, 1994), quoted in www-history.mcs.st-andrews.ac.uk/Biographies/Al-Tusi_Sharaf.html (accessed 2 January 2014).
150 Rashed, *Between Arithmetic and Algebra*, 352.
151 Rashed, *Between Arithmetic and Algebra*, Appendix 1, 'The Notion of Western Science: "Science as a Western Phenomenon"', and Appendix 2, 'Periodization in Classical Mathematics'.
152 Shaw, 'The Islam in Islamic Art History'.
153 Gerd Schneider, *Geometrische Bauornamente der Seldschuken in Kleinasien* (Wiesbaden: Reichert, 1980).
154 Rashed, *Between Arithmetic and Algebra*; see also 'The Mathematics of Islam', in Katz, *A History of Mathematics*, ch. 9.
155 See Carol Bier, *Symmetry and Pattern: The Art of Oriental Carpets* [1997] at http://mathforum.org/geometry/rugs/ (accessed 24 February 2013).
156 See Hans Belting, *Florence and Baghdad: Renaissance Art and Arab Science* (Cambridge, MA: Belknap Press of Harvard University, 2011).
157 Grabar, *Mediation of Ornament*, 128–129.
158 Jens Hoyrup, 'The Formation of "Islamic Mathematics": Sources and Conditions', *Science in Context* 1/2 (1987): 281–329.
159 See Bier, 'Decagonal Tomb Tower at Maragha', with references to earlier studies.