Overturning Certainties in Near Eastern Archaeology

A Festschrift in Honor of K. Aslıhan Yener

Edited by

Çiğdem Maner
Mara T. Horowitz
Allan S. Gilbert

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CHAPTER 6

Alloys and Architecture: Periodic and Quasiperiodic Patterns in Sinan's Selimiye in Edirne

Carol Bier[1]

Abstract

The marble minbar of the Selimiye mosque in Edirne that was designed by the Ottoman architect, Sinan, and completed in 1575, bears a circular medallion of carved and pierced openwork in each of its triangular framing walls. The carved circular patterns are unusual in having radial symmetry with local five-fold and ten-fold rotations, but no periodic repeat. This contribution explores the relationship of this late 16th-century design to a similar array generated by X-ray diffraction of aluminum alloys, identified as a quasiperiodic pattern, which garnered the 2011 Nobel Prize in Chemistry. The 16th-century appearance of this pattern in an architectural context is attributed to the deliberate and conscientious attention to elements of geometry in the training of Ottoman architects, which drew upon a long tradition of geometric patterns in Islamic art. Given the intersections of Aslıhan Yener's and my life and interests in Turkey over four decades, the coincidence of this quasiperiodic pattern with long-range global order, not known or understood in either alloys or architecture before the 21st-century, seems an appropriate tribute to a long friendship and shared appreciation of overturning certainties.

According to the Ottoman architect Sinan, his greatest accomplishment was the Selimiye mosque in Edirne, for there at last he succeeded in building a dome exceeding that of the Greek Haghia Sophia, which had been built a thousand years earlier. Completed in 1575, the Selimiye's grand interior spatial configuration of domical hierarchies has generated considerable acclaim in the history of architecture and its practice, focused on Sinan's profound understanding of geometric forms. One design detail not previously recognized deserves special attention today in relation to contemporary mathematical interest in quasiperiodic patterns. Exhibiting local five-fold and ten-fold symmetries in a tiling

1 Carol Bier, Center for Islamic Studies, Graduate Theological Union, Berkeley, California, bier.carol@gmail.com.

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that covers the plane, it represents a pattern only recently identified as a decagonal array in an X-ray diffraction pattern of aluminum alloys, the discovery of which was awarded the Nobel Prize in Chemistry in 2011.

Given the particular circumstances of Aslihan Yener's life and interests, and the intersection over four decades of our lives in New York and Istanbul, an exploration of this topic linking alloys and architecture, seems an appropriate tribute to our friendship and a means of honoring both her Turkish heritage and her analytical interests in archaeological metallurgy.

All symmetry groups present in two-dimensional patterns exhibit periodicity, which is a function of translation that conforms to one of four basic symmetry operations (Stevens 1981; Grünbaum and Shepherd 1989; Abas *et al.* 1995; Bier 1997). Such periodic patterns are familiar in Islamic architecture as visually repeated designs that appear on stucco wall panels at Abbasid Samarra and inlaid wooden doors of Mamluk Cairo, glazed ceramic tiles of the Alhambra in Spain, and the marble jalis of Mughal monuments in India (Seherr-Thoss and Seherr-Thoss 1968; Blair and Bloom 1994; Ettinghausen *et al.* 2001; Broug 2013). All of these examples play with geometry in complicated and visually intriguing ways, and such patterns reached a peak of complexity in the interlacing ceramic mosaic designs and cut stone architectural ornament of the Seljuks of Rum in central and eastern Anatolia in the 13th-century (Schneider 1980; Bonner 2017).

Translational symmetry, often with internal axes of reflection, characterizes repeat patterns with intricate floral designs in the monumental compositions of glazed wall tiles made in Iznik, which became a defining feature of imperial Ottoman mosques, tombs, and palaces (Denny 2004). Yet in at least three monuments of the great Ottoman architect Sinan (d. 1588), patterns may be discerned in which translation is not present; without translation, they cannot be considered periodic. In today's mathematical vocabulary, they may be called quasiperiodic, based on newly discovered arrangements of atoms that resist any of the traditional classification of symmetry groups in the plane that describes the growth of crystals. One such pattern with five-fold and ten-fold axes of radial symmetry reached its apogee in Sinan's Selimiye in Edirne, where it appears in the two circular medallions of the framing walls of the minbar (Fig. 6.1). This unique expression in carved and pierced marble warrants close attention within the broader context of periodic and quasiperiodic patterns (Bier 2014).

In 2011, Dan Shechtman received the Nobel Prize in Chemistry for his discovery of quasicrystals identified in the electron diffraction patterns of an aluminum alloy (Fig. 6.2). What distinguishes this and related aluminum alloys is a crystal structure that counters standards of crystallography as developed in the late 19th-century and accepted by the International Union of Crystallographers. In the original publication of this discovery, Shechtman and his colleagues describe, “a metallic solid … with long-range orientational order [that is] inconsistent with lattice translations”, acknowledging that, “elemental crystallography indicates that fivefold axes are inconsistent with translational order” (Shechtman *et al.* 1984), emphasizing the lack of any periodic repeat. The Nobel announcement begins with the statement, “In quasicrystals we find the fascinating mosaics of the Arabic world reproduced at the level of atoms” (Royal Swedish Academy of Sciences 2011a). It goes on to explain that the, “medieval Islamic mosaics of the Alhambra Palace in Spain and the Darb-e Imam Shrine in Iran have helped scientists understand what quasicrystals look like at the atomic level. In those mosaics, as in quasicrystals, the patterns are regular—they follow mathematical rules—but they never repeat themselves”. This statement likely echoed the broad popular and scholarly attention of mathematicians and scientists garnered by an article published in *Science* in 2007, which focused on two monuments in Iran, namely, the 15th-century Darb-e Imam in Isfahan and the late 12th-century decagonal Gonbad-e Kabud in Maragha (Lu and Steinhardt 2007a and 2007b; Makovicky 2007; Bier 2011; see also Makovicky 1992 and Bonner 2003).

Shechtman’s initial discovery took place in 1982 when he was at the National Bureau of Standards (now the National Institute of Standards and Technology, or NIST) in Gaithersburg, Maryland. His notebooks from that time document his startling observations of diffraction patterns with local five-fold and ten-fold rotational symmetries that do not occur in any of the symmetry groups then acknowledged by crystallographers. The research was first published in Shechtman *et al.* (1984). The two-dimensional diffraction patterns, result from the projection of a three-dimensional structure. Similar tilings of the plane were first discovered and explored by Roger Penrose in the 1970s (Senechal 1995). They are known today as Penrose tilings, and they can cover the plane using only two elemental shapes, a pair of rhombuses (one fat, one thin), or a related pair of kites and darts, each tiling mapping onto the other. The operative angles of 36° and 72° relate these tilings to pentagons and a proportional system based on the $\sqrt{5}$. Wasma’a Chorbachi (1989) had earlier linked this quasiperiodic plane geometry to patterns in Islamic architecture, recognizing the explanatory constructions she found in an anonymous undated treatise appended to a Persian translation of an earlier work by Abu’l Wafa’ Buzjani (d. 998), *Kitāb fī mā yaḥtāj ilayh al-ṣāniʿ min al-aʿmāl al-handasiyya* [*A Book on those Geometric Constructions which are Necessary for a Craftsman*]. The manuscript is in the Bibliothèque National, Paris (ms. persane 169) (for translation and commentary, now see Necipoğlu, ed. 2017). A reconstruction of the five-fold pattern by means of overlapping decagons (10-sided polygons) to create a five-pointed star appeared in Bulatov (1978) and was also among the patterns Hankin (1925) documented in India on Islamic monuments. The construction was once again studied by Chorbachi and Loeb (1992).

In the case of metallic alloys as studied through electron microscopy, diffraction patterns in two dimensions reflect three-dimensional structures of atoms arranged without periodicity (that is, there is no translational symmetry present), although they do exhibit long-range order and manifold symmetries in an icosahedral phase, in which pentagonal symmetry is inherent. [This is because an icosahedron, with 20 equilateral triangular faces, is composed of two pentagonal pyramids (five equilateral triangles meeting at a vertex), above and below a central band of ten equilateral triangles that complete the 20-sided polyhedron]. The carved marble openwork of the medallions on the Selimiye minbar (Fig. 6.1) visually relate more closely to the diffraction pattern of these alloys than to either of the designs at Darb-e Imam or that repeated around the shaft of the monument at Gonbad-e Kabud.

The openwork of the Selimiye minbar also shares a visual affinity with an orthogonal projection in a Coxeter plane of a regular polyhedron, H_3, or an omnitruncated 120-cell, in which there are multiple symmetries (two-fold, three-fold, five-fold, and ten-fold) in three-dimensions without any translational symmetry (Fig. 6.3).[2] The question of projection from a higher dimension (as addressed for example by Banchoff 1990, Robbin 2006, and Robbin 2015) is yet to be explored in relation to Sinan's monuments, but recent research into this mathematical realm may offer a prospective means for a better understanding of Sinan's methods. As illustrated in Fig. 6.3, when viewed as a projection in two dimensions from a particular vantage point, the omnitruncated 120-cell exhibits five pairs of parallel lines that extend as axes from a central decagon. The organization of this system corresponds to that of Sinan's medallions, one on either side of the Selimiye minbar (Fig. 6.1). Although a decagram, or ten-pointed star, appears at the center of each medallion, it resides within an implied decagon, geometrically dividing the central circle into ten equal segments.

Al Ajlouni's study (2012) of the long-range global order of quasiperiodic patterns in Islamic architecture (Fig. 6.4) suggests a direct relationship of the pattern repeated around the shaft of the decagonal Gonbad-e Kabud in

2 I observed the construction of an omnitruncated 120-cell uniform polytope in August 2014, while attending the Bridges Conference (Connections in Mathematics, Music, Art, Architecture, Culture) at the Gwacheon National Science Museum in Seoul, Korea; the process was documented in a video: https://www.youtube.com/watch?v=HOEcz9T-WCs.

Maragha with those that she has generated using a basic underlying grid of nested decagrams. She has developed what she calls a “global multi-level hierarchical framework model” (HFM) that is able to describe the long-range order of quasiperiodic formations in Islamic architecture. Her method involves building a progression of multi-level hierarchical formations that grow based on the Fibonacci sequence of numbers (0, 1, 1, 2, 3, 5, 8, 13, 21 …) in which each new number is the sum of the two previous numbers, and the ratio of each number divided by the previous number verges on the so-called Golden Ratio (ϕ = 1.618 …, which is irrational). She surmises that historical antecedents in Islamic architecture may reflect the use of sequences of simple consecutive geometry, and that by such means, designers and artisans would have been able to resolve the complicated long-range principles of quasiperiodic formations (Al Ajlouni 2012). Her generation of such quasiperiodic patterns thus can be seen to relate directly to the geometric patterns in cut brick of Gonbad-e Kabud (Fig. 4a–f) and in ceramic mosaic at Darb-e Imam. Her analysis and generation of these designs as quasi-periodic patterns with long-range global order, but without translational symmetry (as illustrated in Fig. 4f), also demonstrates in the central portion with center point and ten-fold radial symmetry a distinct relationship to the radial geometry of the carved marble screens on the minbar in Sinan’s Selimiye, which also have local ten-fold symmetry but no translation (as is evident in Fig 6.1) (see also Cromwell 2009: 45 and Fig. 13).

Apart from visual affinity, however, what if any relationship may be ascertained between Sinan’s radial ten-fold pattern in the medallions of the minbar at Selimiye and contemporary mathematical concerns of the 21st - century? This article explores two aspects of this relationship. The first offers a brief exploration of Sinan’s knowledge and use of geometry, which places him in a direct lineage with his forbears in the lands of the Ottoman Empire and the greater Islamic world, as well as in relation to Johannes Kepler’s studies of polygons in the following decades. Kepler is often credited with scientific advances of the 17th-century, particularly with reference to planetary orbits and the laws of planetary motion, whereas Sinan’s library of books on geometry was passed along by Imperial decree in 1578 to the newly established astronomical observatory of Takiyüddin in Galata, across the Golden Horn from the Ottoman imperial palace at Topkapı (Necipoğlu 2011: 149–150). The second noteworthy aspect is the evident distinction between the quasiperiodic pattern of the minbar at Selimiye as well as its direct antecedents, and the *periodic* patterns of marble openwork panels that may be found in many of the monuments in which Sinan himself claims a role in design and construction (Crane and Akin 2006).

We know of Sinan's interest in and penchant for geometry first and foremost from the plethora of buildings in which he took a leading part in design, oversight, and construction, and second from a unique surviving manuscript by Cafer Efendi, *Risāle-i mi'mariyye*, written in the early 17th-century, which describes in great detail the "science of geometry" that was paramount in the training of Ottoman architects (Crane 1987: 32; Necipoğlu 2011: 132). This treatise concerns the life of Sedefkar Mehmet Agha, who was among the royal architects trained by Sinan.

Sinan worked successively for three Ottoman sultans. His greatest patron was Süleyman, who ruled for 46 years and died in 1566. Süleyman was succeeded by Selim II, who died in 1574, followed by Murad III, who ruled until he died in 1595, several years after the death of Sinan. Spanning five decades contemporaneous with the height of Europe's Renaissance, Sinan's work as chief architect included grand imperial commissions that transformed the landscape of Istanbul, which had become capital of the Ottoman Empire after its conquest from the Byzantines in 1453. He also had commissions for palaces and public works from viziers. Great building complexes (*külliye*), such as the imperial Süleymaniye mosque complex and the Selimiye in Edirne, were conceived in their entirety and designed down to every last detail. As chief architect, he was responsible not only for design and oversight of construction throughout the Ottoman Empire but also for the training of the corps of royal architects in what was at the time called "the science of geometry".

Sinan is best known for his genius in combining architecture and engineering to open up interior space by creatively utilizing piers, columns, and arches to support a succession of domes and half-domes arranged above square, hexagonal, or octagonal ground plans. He is also credited with the innovative use of arcades that effectively served to visually lighten the assumed weight of architectural façades by introducing deep shade beneath the mass of cascading domes (Kuban 2010: 289). Contrasts of light and shade also provided important aesthetic elements affecting the perception of interior spaces, in which the visual effect of mass is diminished by the use of windows, galleries, and low screening walls.

Born near Kayseri in central Anatolia, Sinan was the son of a stonemason. He was accepted into the Janissary corps at a young age and began his training as an apprentice in carpentry. He progressed to engineering, accompanied the sultan on numerous military campaigns, and was accepted into the corps of royal architects where he was recognized for his skill in many endeavors. With training in stone masonry and carpentry, he was surely well-versed from the start in the uses of geometry in two- and three-dimensions, and this is clearly reflected in his early architectural accomplishments, most notably the Şehzade Mosque complex in Istanbul, which was his first imperial commission.

In studies of Sinan's architectural works, mention is often made of domes, vaults, arches, and piers, and the creative opening up of interior space (Aslanapa 1971; Kuran 1987; Goodwin 2003; Kuban 2010; Necipoğlu 2011). Güney (2009: 189) emphasizes that Sinan was particularly skillful at composing the window arrangements between arches, and that walls with windows had the effect of hiding the thickness of both arches and buttresses, but there is no mention of the presence of the low screening walls with two-dimensional patterns that also contribute to the expanse of interior space. Such walls, with pierced panels, are a feature in nearly all of Sinan's buildings except his public works (such as bridges and aqueducts). Carved of white marble, the panels filter the light, creating ever-changing pattern effects by means of projection onto the large open spaces beneath Sinan's domes. They also allowed light to pass from second floor gallery windows to the main sanctuary and domed areas. Similar panels, carved with openwork, are designed to serve a screening function in a variety of architectural contexts, none of which are load-bearing: upper level galleries and balconies, the parapets of minarets from where the muezzin issued the call to prayer, windows, fountains, and open windows in free-standing walls. Even in the case of windows, lunettes, and tympana, an arch diverts the weight of the wall above. The use of such pierced panels delineates space, all the while allowing the flow of air and light, and introducing ambiguities of partitions that both divide and unite.

Periodic Patterns in Pierced Openwork Screens

The pierced openwork screens of carved marble exhibit polygonal networks and various configurations of intersecting polygons, almost always with periodic patterns that incorporate translational symmetry (Bier 2014). The vast majority of these screens are of square or rectangular shape; balustrades may have vertical edges with parallel diagonal lengths, and windows may be arcuate in form. Periodic patterns define the pierced marble windows of the exterior enclosure walls of Sinan's tomb, which he designed, situated within a triangular plot across from the Süleymaniye complex. With a pivotal *sebil* (public fountain) at the acute juncture of two enclosing walls, the plan of the tomb (Necipoğlu 2011: Fig. 124) has been likened to a pair of compasses, perhaps giving further indication of Sinan's passion for, and fascination with, geometry. Within the architectural contexts he designed are large furnishings such as carved stone or wood minbars, which are also fitted with large flat panels that combine to form three-dimensional compositions; for example, low screens are used to frame the upper parts of the stone and wood structures that serve as loges for royalty and religious leaders (*maqsura* and the muezzin's *mahfil*), and chests and stands for housing the Qur'an (*kursi*).

Aesthetically, all of these openwork panels with periodic patterns play with light and dark, solid and void. The effects of sunlight, constantly changing throughout the day and in different seasons, create projections of shadows that contribute visual interest to the articulated ambiguities of interior and exterior space that is mediated by the screens of intersecting polygons. By examining vertices, edges, faces, and angles, the screens may be classified according to their compositions. All of the tessellations of regular polygons (triangles, squares, hexagons having equal sides and equal angles) have been identified; as underlying grids to structure patterns, these tessellations are often but not always articulated in the carved marble.

The following distinct categories of intersecting polygons and polygon networks are evident: (1) intersecting dodecagons (in carved marble, forged iron, or woodwork) without an articulated grid, or with a hexagonal grid, or a dual tessellation of hexagons and equilateral triangles; (2) dual tessellations of hexagons and equilateral triangles, yielding a tessellation of kites (examples in carved marble, forged iron, woodwork); (3) networks of adjacent dodecagons in a square grid; (4) intersecting nonagons; and (5) intersecting nonagons with a hexagonal grid. While dodecagons occur in a variety of configurations, intersecting one another, or with a triangular and/or hexagonal grid, or forming a network within a square grid, nonagons occur only as intersecting polygons or forming a network with a hexagonal grid. Interestingly, although the nonagon is considered to be not constructible using compass and straightedge, intersecting nonagons have historical precedent in the 12th-century Gonbad-e Sorkh at Maragha in Iran (Bier 2012; Buitraga and Huylebrouck 2015: Figs. 13–14).

These screens explicitly convey the essence of geometry through points, lines, and planes, with specific relevance to vertices, edges, and faces, articulating relationships among intersecting polygons, star polygons, and polygonal networks. An Arabic word, *shabbaka*, is used in Ottoman Turkish and translated in Cafer Efendi's *Risāle* (Crane 1987: 88) as, "lattice, grillwork, screen". The term for the openwork stone panels, based on a combination of Persian and Arabic grammatical constructions (not at all unusual in Ottoman contexts) would seem to be *aḥcār-i müşebbeke,* which translates literally to "lattice-worked stones", meaning "stone networks", or "reticulated stones". This is the term used to describe the walled enclosure of a primary school, designed by Sinan, which was "surrounded by a most beautifully constructed *aḥcār-i* *müşebbeke*" (Necipoğlu 2011: 149). It is this form of wall pierced with reticulated windows that Sinan selected for his own tomb enclosure. All such reticulated panels with periodic patterns bear polygonal networks and intersecting polygons in which translational symmetry effects repetition to cover the plane.

Quasiperiodic Patterns in Carved Openwork

In relation to the articulation of plane geometry and the study of pattern in monuments designed by Sinan, pride of place seems to have been reserved for the screening walls that frame the stairways of minbars of imperial mosques (Figs. 6.1 and 6.5). These sidewalls are triangular in shape, departing from the other basic shapes of the architectural screens described above. Placed to the right of the mihrab, the triangular walls may also bear carved openwork panels. With historical antecedents, for example, in Ayyubid and Mamluk carved wood minbars, in the Ottoman period these tend to be composed of multiple panels of different shapes—a parallelogram forms the balustrade framing the stairs, which serves as the hypotenuse of a right triangle, the vertical and horizontal edges of which comprise a sequence of rectangles and squares. The enclosed triangular area often contains a circular medallion set within an arabesque floral design. Within the medallion, the pattern often appears to emanate from a central point, articulated or implied, with radial symmetry. In cases with six-fold symmetry, this yields a periodic pattern. But in other very particular examples—limited to the circular compositions of several of Sinan's major mosques—five- and ten-fold divisions of the circular medallion yield patterns with pentagons and decagons that are arranged radially without periodicity (Figs. 6.1 and 6.5). That is, the patterns if extended could not cover the plane with translational symmetry, comprising what we now know to be quasi-periodic patterns.

With regard to the central medallions of the minbar at Selimiye in Edirne, despite the absence of periodicity in the radial arrangement of the design, the medallions share many features that relate them to the tracery and grillwork that appear in many of Sinan's other monuments as low pierced walls, windows, balustrades, and parapets. All of these serve a screening function, allowing for the circulation of air as well as the penetration and projection of light. The minbar at Selimiye also bears periodic patterns within the triangular format of its framing walls, in the horizontal, vertical, and diagonal borders of the framing wall, in the diagonal screening wall of the balustrade, and in the square and rectangular panels on the sides of its vertical back wall (as illustrated in Kuban 2010: 310). It is only the circular medallion on the side of each framing sidewall that bears a non-periodic pattern. With the exception of the circular medallions of several other minbars, all of these architectural features exhibit periodic patterns in the plane. And all of these patterns play with relationships among polygons, forming polygonal networks and intersecting polygonal grids (Bier 2014), sometimes cut apart and rearranged as if dissections of a geometric shape.

To date, quasi-periodic patterns may be discerned in three monuments in which Sinan was directly involved in the design: the minbar at Selimiye in Edirne (Fig. 6.1), earlier in the minbar of his first imperial commission of Süleyman, the complex of Şehzade (Fig. 6.5), and in the complex of Azapkapı, commissioned by Sokollu Mehmet Paşa. Analysis of the pattern within the medallions of the minbar at Şehzade as demonstrated by Majewski (2011: Fig. 91) shows a relationship of underlying triangles that characterize pentagonal constructions, but the quasi-periodic nature of this pattern is not mentioned. The absence of translational symmetry hinges upon the shared use of radial symmetry and angles in multiples of 18° (36° and 72°), properties of the angles and composite triangles of pentagons (Fig. 6.5). Further research may yield similar five-fold patterns in the minbar medallions of later commissions that indicate a sustained interest in exploring the geometry of this pattern.

The absence of reference to these forms in scholarly literature on Ottoman architecture may be attributable to the greater architectural interest today attached to Sinan's innovative and exceptionally creative approach to three-dimensional forms and the formal methods he used to shape the spatial dimension. A review of the design, use, and function of these panels is long overdue; their forms are not only ideally suited to their screening function, but they contribute overall to the effects of lightness in larger interior spaces and introduce ambiguities to the division of interior and exterior space when in the architectural context of porches, balconies, and fountains. Relative to other architectural features, these panels are modest in scale, but significantly, they illustrate archetypes of plane geometry in a manner rarely seen in the annals of architecture. In Sinan's monuments they are particularly expressive of an abstract and formal elegance of design that we more often associate with the minimalist aesthetics of modernism. And, although underappreciated as a category of architectural production, they document a sustained concern and applied methodology in exploring the geometry of polygonal networks half a century before the publication of Kepler's *Harmonices Mundi* [*Harmonies of the World*] (1619). Their potential importance as studies in stone may be quite significant, contributing a substantive means for architectural training in the "science of geometry" that is so often repeated throughout Cafer Efendi's *Risāle* (Crane 1987).

The "Science of Geometry" and the Study of Architecture

The science of geometry (*'ilm al-handasa*) is credited as being central to the training of Ottoman architects. We know from biographical sources that other precious containers, some made with inlaid mother-of-pearl (*sedefkar*), tortoise shell, and rare woods, exquisite in every detail, were used for the training of architects on the grounds of the gardens at Topkapı Palace (Crane 1987: 32). Many such examples exist today and can be seen among the doors, window frames, chests, and cabinets that remain as original furnishings in Topkapı Palace, or as objects in museums in the imperial capital as well as in European collections with Ottoman holdings. The carved and inlaid objects with mother-of-pearl do not have openwork, although they may exhibit elaborately complicated geometric patterns set with juxtaposition of diverse materials, often with five- and ten-fold symmetries that are sometimes arranged in a periodic pattern based on a rhombic grid (Broug 2013: figs. 5.92 and 5.93).

In contrast with the mother-of-pearl work, most of the screens are carved of a single material—a high quality pure white marble, which corresponds in the *Risāle - imi'mariyya* to either *maliki mermeri* or *Marmara mermeri*, which are described as jewel-like. *Marmara mermeri* is quarried from Marmara Island in the Sea of Marmara, the Proconnesus marble of antiquity (Crane 1987: 71–72; Asgarı 1978). The use of a darker marble, as on the minaret of Mihrimah Sultan Mosque in Üsküdar (1543–48), may correspond to the description of a locally quarried "black marble", called *Üsküdar mermeri* (Crane 1987: 72 and no. 31). Screens that are higher up (as the balconies of minarets) sometimes show less elegant details of construction; this and other qualitative differences also lead to the hypothesis that such panels may have served as *études* in the training of young architects. Each panel was designed to fit a specific space; the proportional system for each polygonal network would have remained the same, but the scale would have needed adjustment to fit each designated space. Articulated moldings often frame the pierced area. It is indeed conceivable that these panels, which exhibit so many different polygonal networks, would have been eminently suitable for training young architects in the subject of plane geometry.

The fact that in all of the pierced openwork panels the edges of polygons are straight lines may also lend credence to the effectiveness of these slabs for training. A curvilinear line might be more difficult to chisel, but would also contribute a different aesthetic effect. Polygonal networks delineate two-dimensional space, but they also are key to the development of an understanding of geometry in two dimensions. Although this is elementary, it is nonetheless necessary for an understanding of space in three dimensions.

According to the *Risāle-i mi'mariyya*, the new Janissary interns assigned to the Corps of Royal Architects were first trained in carpentry and wood-carving; those who worked with mother-of-pearl inlay (*sedefkār*) achieved the highest acclaim (Crane 1987). The geometric patterns seen on the carved wooden doors of mosques, tombs, and palaces, and the extensive mother-of-pearl work in palaces and pavilions for doors, cabinets, and niches, as well as benches for scribes and boxes and stands for the Qur'an, all give evidence of sustained concern and aesthetic interest in polygonal networks.

All of the periodic patterns described above have historical antecedents within the Islamic architecture of Iran, Uzbekistan, Syria, Egypt, and pre-Ottoman Turkey, in works created, often with royal patronage in a variety of media—including wood, metal, glazed ceramic mosaic and tile, and paper (Broug 2008; Bonner forthcoming). Sinan's genius isolated these forms and reduced apparent complexity to its geometric essence, establishing an elegant minimalism in white marble that emphasizes dark and light in the articulation of form; his screens even removed the illusionary interlace that is otherwise so characteristic of geometric patterns in Islamic art elsewhere. But the formal arrangements of radial symmetry without translation that appear in the central medallions of the minbars of several of his major monuments do not appear to have historical antecedents, except for their mathematical relationship to Gonbad-e Kabud in Maragha and Darb-e Imam in Isfahan, and may well indicate new advances in the understanding of two-dimensional geometric forms expressing what today is beginning to be understood as quasi periodic patterns.

Conclusions

Al Ajlouni (2012) summarizes the discoveries concerning quasiperiodicity of the 1980s and the relationship to Islamic architecture, emphasizing the global long-range order combined with a radial symmetry and the absence of translation. Makovicky, a crystallographer, has identified the presence of quasiperiodicity in eight-fold, ten-fold, and twelve-fold patterns in Andalusia and the Maghreb (Makovicky and Hach-Alí 1996; Makovicky *et al.* 1998; Makovicky and Makovicky 2011). Whether or not such architectural endeavors in the production of ornament reflect advances in mathematical understanding is yet open to discussion. But the newly identified presence of such patterns in Ottoman architecture offers an opportunity for renewed consideration of the active participation of the junior corps of royal architects in studying geometry and the prospective role of Sinan, in particular, contributing to the advancement of mathematical knowledge through practical studies in stone, for which we have no textual counterparts.

Ottoman mathematical enterprise in the 16th-century tended to focus on applications including astronomy and the arts of warfare (Aslan 2014). In contrast, the contemporary architectural enterprise focused on expanding and extending engineering capabilities and may have involved both applied mathematics and experimentation that advanced mathematical understanding. The periodic and quasiperiodic patterns of carved and pierced marble screens demonstrate a clear and consistent aesthetic intent to achieve formal elegance. As a category of architectural production, carved marble slabs with intersecting polygons fit into a trajectory that addresses the geometric representation of patterns in the plane. But the two-dimensionality in reality is only conceptual; the screens themselves are three-dimensional, an aspect that is aesthetically highlighted by the subtlety of beveled edges of intersecting polygons.

In Sinan's buildings, these panels illustrate archetypes of plane geometry in a manner rarely seen in the annals of architecture, expressive of a minimalism we usually associate with modernism. Although modest in scale, these carved panels are exceptional in their elegant simplicity and abstraction. Comprising a series of architectural panels with pierced openwork or tracery to let in light and allow for air circulation, the form of these screens is ideally suited to their function. But the attention to detail and diversity in the patterns carved leads to speculation as to their role in the training of architects in the science of geometry and relates craft and technology to the production of knowledge as studies in geometry in stone. Such knowledge of geometry as is evinced in Sinan's architectural screens long pre-dates the scientific understanding of quasiperiodicity as revealed in recent analyses of aluminum alloys.

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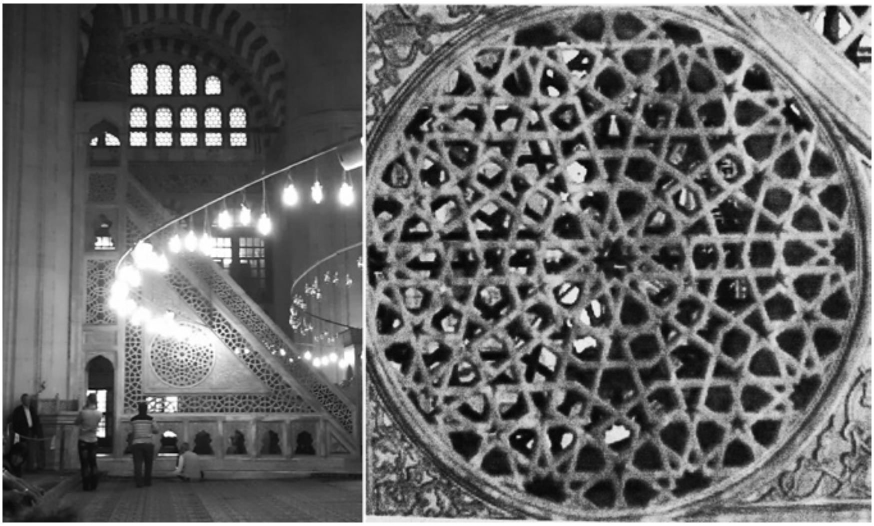

FIGURE 6.1 *Edirne, Selimiye Mosque (1568–74). Interior view (left) with minbar. Detail (right) showing circular medallion on framing wall of right face of minbar.*
© C. BIER, 2013.

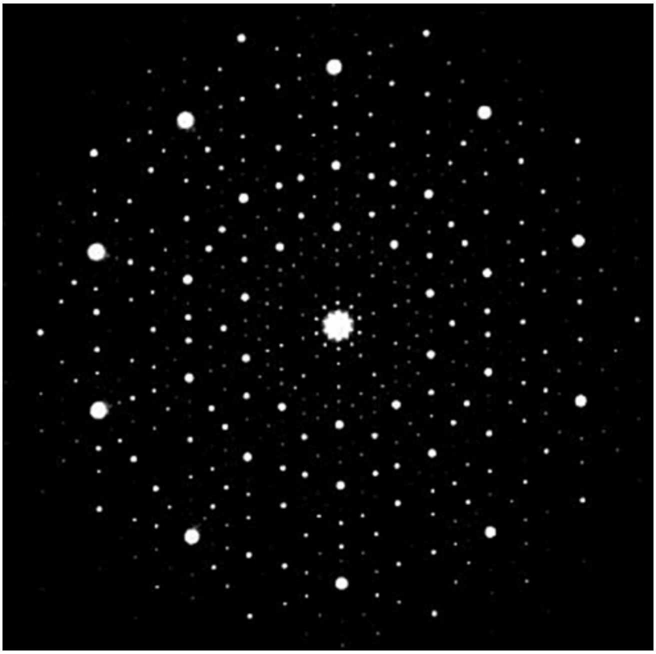

FIGURE 6.2 *Electron diffraction pattern from an icosahedral quasicrystal.*
ROYAL SWEDISH ACADEMY OF SCIENCES (2011B: FIG. 6.2).

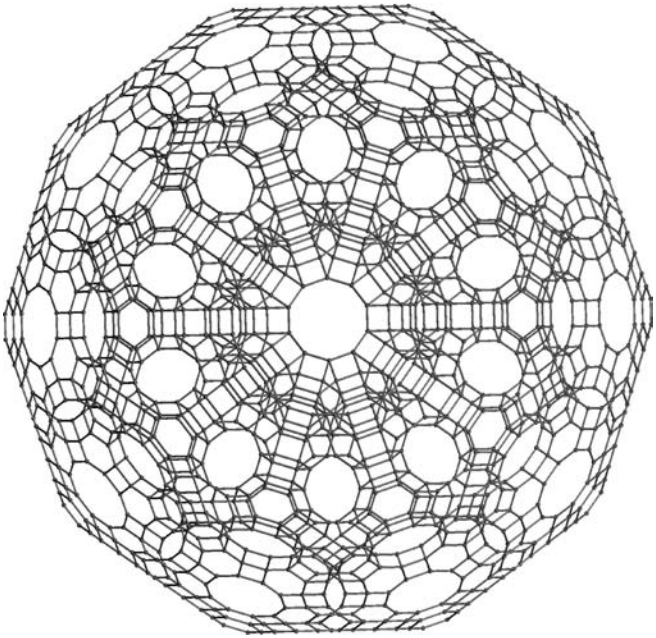

FIGURE 6.3 *Omnitruncated 120-cell, orthogonal view centered on decagonal prism cells.*
© ROBERT WEBB, 2011. IMAGE CREATED USING STELLA SOFTWARE, HTTP://WWW.SOFTWARE3D.COM/STELLA.PHP.

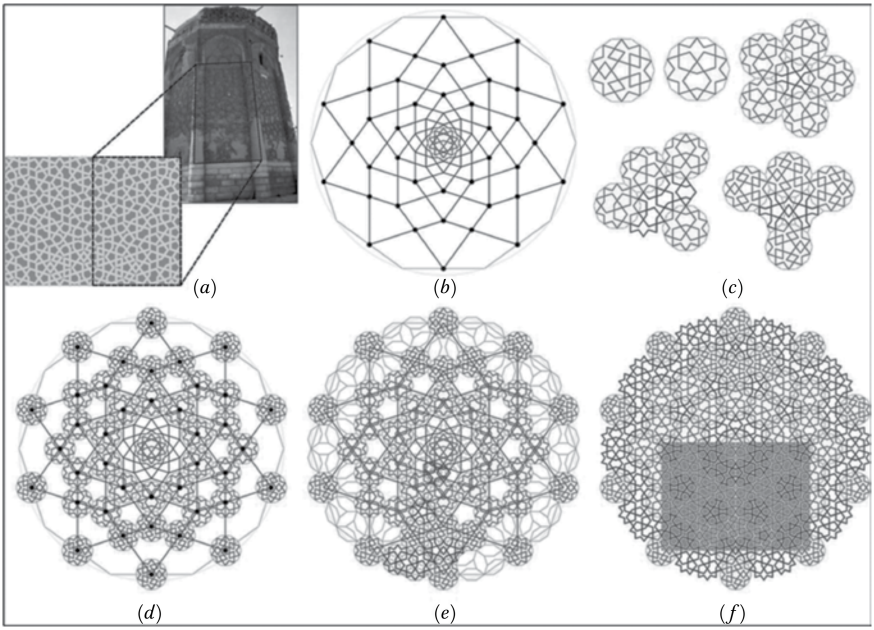

FIGURE 6.4 *Al Ajlouni's sequential construction of a long-range quasi periodic pattern with local five-fold and global ten-fold radial symmetry to demonstrate possible generation of pattern that appears on each of nine facades of the decagonal Gonbad-e Kabud in Maragha, dated 1193 CE by historical inscription.*
(AL AJLOUNI 2012: FIG. 7).

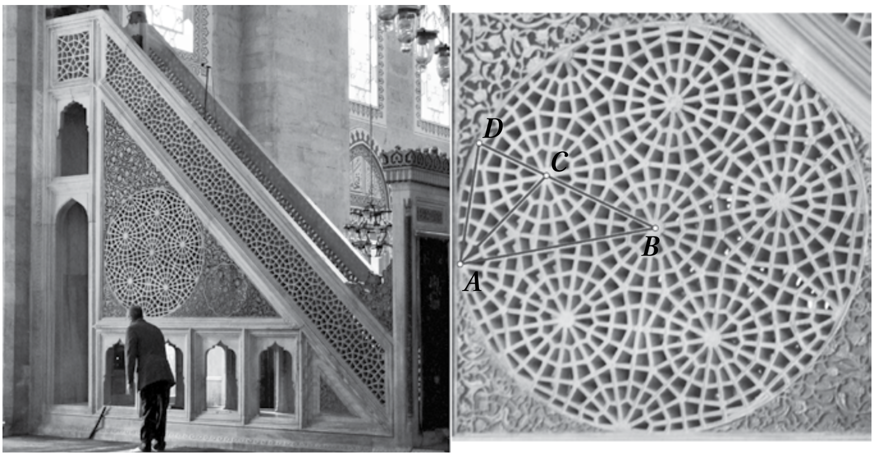

FIGURE 6.5 *Istanbul, Şehzade Mosque (1543–1548). Interior view (left) with minbar; (right) detail, circular medallion on left face of minbar with five-fold symmetry.*
© M. MAJEWSKI (2011: FIG. 91), REPRODUCED WITH PERMISSION.