A Model of Legal Systems As Evolutionary Networks: Normative Complexity and Self-Organization of Clusters of Rules

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A MODEL OF LEGAL SYSTEMS AS EVOLUTIONARY NETWORKS: NORMATIVE COMPLEXITY AND SELF-ORGANIZATION OF CLUSTERS OF RULES

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The paper draws both on legal theory and network science to explain how legal systems are structured and evolve. The basic proposition is that legal systems have a structure identifiable through a model of them in terms of networks of rules, and that their evolution is a property of their network structure. The paper is based on a model of rules which relies on the tenets of the network theory to describe how legal change unfolds within the network structure of legal systems. Section 1 presents an outline of current literature on the application of network theory to legal systems. Section 2 describes the various types of rules and hierarchies within the network structure of legal systems. Section 3 then advances the view that the hierarchies of rules operate as chains of production which ultimately create rules regulating individual situations (“Rules of the Case”) and defines the concept of production links between rules represented as nodes of such networks. Section 4 describes multiple chains of production of rules and defines a network concept of normative complexity. Section 5 describes the structure of clusters of common law cases and provides a notion of clustering coefficients of these cases. Section 6 discusses the main network property of legal systems at the global and local level (the existence of nodes which operate as legal “connectors” and the self-organization of Rules of the Case), while section 7 shows that legal systems are evolutionary networks and discusses how their evolution is a network property. Finally section 8 concludes by discussing potential applications of network science in respect to the U.S. tax system.


1. The use of networks in modelling legal systems

This paper draws both on legal theory and network science to explain how legal systems are structured and evolve. The basic proposition on which the paper is based is twofold: that in a given moment in time legal systems have a structure identifiable through a model of them in terms of networks of rules, and that the evolution of legal systems is a property of their network structure. Thus I argue that legal systems have an “evolutionary structure” which is a network growth property1. I will advance a model of rules which relies on the tenets of the network theory to

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describe how legal change unfolds within such network-like structure of legal systems. The conclusions I reach here about legal systems in general can be extended to cross-sections of them, such as, for example tax systems or other areas of legal systems and can lead to practical applications of network theory in the legal field. It should be clarified from the outset that the model of legal systems provided here serves as an explicative framework but does not amount to a direct and detailed description of social reality.

The claim which is made in this paper is that the evolutionary structure of legal systems can be modeled by using the tools of network science, which has been developed recently shedding light on the importance of networks in studying the complexity of natural and social systems. Networks are entities in which different “nodes” are connected through “links”, and represent complex systems in the natural and well as social domain. In the last decades the network paradigm has been extended to the most diverse contexts, from communication systems to railroads, from the spread of viruses to social communication. As a result “network science”, the interconnected web of research on network structures in the natural and social communication, has gained the standing of an interdisciplinary explanatory tool showing underlying common properties of networks.

The paper is organized as follows. This section briefly presents an outline of current literature on the application of network theory to legal systems. Section 2 describes the various types of rules and hierarchies within legal systems by distinguishing between “singular/general rules” and “primary/secondary rules”, and provides a matrix in which these rules are combined to define the essential components of the network structure of legal systems. In particular the concept of “formal hierarchy” is defined as that which occurs between a producing rule and a produced rule according to a rule of production. Section 3 advances then the view that the hierarchies of rules operate as different types of chains of production which ultimately create “individualized” or “actualized” singular rules regulating individual situations. These rules are defined as “Rules of the Case”. In section 3 legal systems are thus viewed as complex structures generating a countable number of Rules of the Case.

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4 For reviews of these applications of network theory see: ALAIN BARRAT, MARK BARTHELEMY, ALESSANDRO VESPIGNANI DYNAMICAL PROCESSES ON COMPLEX NETWORKS 180 – 293 (2008); MARK E. J. NEWMAN, ALBERT LASZLO BARABASI, & DUNCAN J. WATTS. THE STRUCTURE AND DYNAMICS OF NETWORKS 415-552 (2006); MELANIE MITCHELL, COMPLEXITY: A GUIDED TOUR 247 – 258 (2009).
Section 3 also explicitly defines the concept of “production links” between rules represented as nodes of the network and discusses in more details how the Rules of the Case are produced through underlying “generative processes” represented as network diagrams. The diagrams are agent-based models and show how an administrative authority or a court apply a statute, how a court in a common law system creates Rules of the Case, and how individuals apply statutes through self-compliance Rules of the Case. These diagrams are further developed in Section 4 with more complex representations of multiple chains of production as they operate in actual segments of legal systems and normative complexity is defined as the outcome of the combination of proliferation of Rule of the Cases and branching-off of normative powers.

In Section 5 simple networks of common law cases are presented together with a discussion of the notion of clustering coefficients in common law according to recent literature. In Section 6 legal systems are then presented as networks that are a combination of intertwined web of normative powers leading to various clusters of rules connected by production links. It is thus suggested that a distinguishing network property of legal systems at the global level is the existence of a power-law feature according to which there is a relatively small number of main rules (nodes) which operate as legal “connectors” or “hubs” having a high density of production links leading to Rules of the Case. It is also submitted that the distinguishing network property of legal systems at the local level is that they exhibit, particularly in common law systems, a “complex behavior” meant as self-organization of rules represented by nodes connected by production links. Section 7 shows that there is an evolutionary structure of legal systems which can be represented as a property of the legal network in different moments of time. A distinction is made between normative and network evolution of legal systems: normative evolution is the growth of Rules of the Case regulating individual situations, while network evolution is the modification of all other rules (general primary rules and singular/general secondary rules) leading to a distribution of normative powers capable of creating Rules of the Case.

Section 8 concludes by discussing potential applications of network science in respect to the U.S. tax system in the following areas: 1) network analysis of the Internal Revenue Code and measurement of normative complexity; 2) measurement of enforcement and self-compliance of specific rules of the Internal Revenue Code and Regulations; 3) analysis of complex behaviour and self-organization of clusters of tax Rules of the Case; and 4) tax design and policy issues. A brief evaluation of the the impact of change of rules in the U.S. tax system represented as a network of rules is proposed, both with reference to “top-down” changes through amendments of the Internal Revenue Code or through Internal Revenue Service (“IRS”) agencies, and “bottom-up” changes prompted by case law. The applications can be extended to other tax systems.

As anticipated, the final part of this section is dedicated to a brief survey on the application of network theory to law, which has, so far, received less attention if compared to other fields. It is only in the last few years that scholars have begun to apply some of the techniques and insights of network science to law and this literature can be roughly divided in two areas: a rather general discussion on potential uses of network theory in the law, and the application of network theory to common law cases. The former type of approach of a more general nature has been initiated

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recently in two recent papers which provide a description of the main features of network science in view of its potential applications in the field of law.\textsuperscript{6} The latter type of approach about common law is of a more empirical nature and describes the network structure of cases in common law under the rule of the binding precedent; in these papers the common law cases are the nodes of the network and the citations between cases are the links of the networks, and the research is conducted using the computing capabilities of existing legal databases. This line of research shows that common law operates as a clustered network in which the nodes are not all uniformly linked to each other, but rather, some are densely connected to each other in clusters, and these clusters only more loosely connected to one another.

Drawing on these two strands of recent research it is possible to summarize here, by way of introduction, a few essential points on network science in view of its application to the law. First of all the definition of a “network”: a network is a set of items, termed “nodes” or “vertices”, with connections among them, termed “links” or “edges”. According to Smith, for example, the “Web of Law” is the network that consists of cases and other legal authorities, such as statutes, legal treatises, law review articles (the nodes), and the citations that connect them to one another (the links)\textsuperscript{8}. This paper will further clarify how exactly this network structure of the law unfolds by showing that links between rules are production links established by “rules of production”.

A second concept is the distinction between random networks and scale-free networks: a random network is one in which the number of links of each node is described by a normal distribution, while a scale-free network is one in which no node can be said to have a typical number of links\textsuperscript{9}. In scale-free networks the distribution of links is statistically represented by a power law in which few nodes have a very high number of links compared to other nodes\textsuperscript{10}. This paper will show that legal systems tend to be scale-free networks because only a few rules (defined as “legal connectors”) produce a large number of Rules of the Case, i.e. have a high number of production links connecting them to Rules of the Case,

A third concept is that according to which a relatively small number of links between nodes is needed to connect any two nodes in the network (so called “small world” properties of networks). In particular the small world properties may be related to the presence of “hubs” which connect otherwise widely separated nodes. I will discuss here the example of common law in which certain leading cases are the legal basis to a high number of subsequent decisions, as well as the example of statutory law where legislative rules are self-complied with by a high number of individual recipients. Small world properties can be important in determining important features of a given network - for example the rate at which a piece of information spreads across the network - and real sections of the legal systems, such as lineages of cases in common law, have these small world properties; for example each case in common law is directly linked to its precedents to form a “small world” of interconnected cases. This paper will evidence that important features are evidenced by coefficients which measure the clustering among cases by looking at the production links connecting them.

A fourth concept is that of “network evolution”, that is the analysis of how nodes get connected in processes of growth of networks. When a network evolves new links between nodes are created according to certain patterns and therefore one of the main issues of current research is to understand how this occurs; network scientists have used the concept of “preferential attachment” that can lead to a highly heterogeneous, scale-free, degree distribution in which some nodes eventually acquire a very large number of links while others remain relatively unconnected. This preferential attachment concept has also been supplemented by a “fitness concept” under which certain nodes have a higher likelihood of acquiring new links in an evolutionary process in which there is a selection process leading to a relatively higher fitness of certain nodes compared to others. This paper will discuss quite in detail the evolution of legal systems in view of these features of network growth and advance some suggestions in terms of design of rules when policies are implemented considering the network property of the legal system.

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2. Types of rules and hierarchies within legal systems

Although research has been initially developed on the network properties of case law precedents, what is still missing is the use of network theory to describe the basic underlying properties of legal systems as a whole, and in particular what is their structure and how they evolve in network terms. In an attempt to bridge the gap between current legal theory and network science, I use here a widely accepted model of legal systems which is based on hierarchies of rules, in light of the fact that hierarchies are nothing else but special types of network connections.

This model of legal systems as hierarchies of rules is based on the distinction between by primary rules and secondary rules. Primary rules are prescriptive statements directly aimed at individuals establishing obligations or duties and so on; these rules directly regulate behavior. By contrast, secondary rules confer normative powers (the power to create binding rules) to specific institutions and lead to the creation or variation of duties, obligations and other prescriptive qualifications through the subsequent creation of primary rules by those institutions. Among the different types of secondary rules presented in legal theory, I refer here to those specific types of “secondary rules” which are related to the production of other rules (so called “rules of production”). An example of rules of production are the constitutional rules establishing hierarchies between “higher” constitutional rules and lower statutes. Another example are those rules enabling judges to apply statutes to individual situations by issuing binding decisions, as they establish hierarchies between the “higher” statute and the lower “decisions”. Another conspicuous example of these secondary rules is the stare decisis rule of common law systems, according to which a subsequent court is bound to apply a precedent decision if certain requirements are met (basically same facts and analogy of the ratio decidendi).

Primary and secondary rules can either be general or singular rules. General rules are created ex ante by institutions with proper powers (law-makers or governmental agencies), take the form of statutes or regulations, and are related to transactions which occur after their enactment. Singular rules are created ex post by institutions with proper powers (administrative authorities and courts), are applicable to individual situations which occur before their enactment and take the form of judicial and administrative decisions (including settlements). One can combine in a matrix the distinction “singular/general rules” and the distinction “primary/secondary rules”, so that it is possible to establish a fourfold classification of rules, as shown at Table 1 below: general primary rules (“GPR”), Rules of the Cases or singular primary rules (“SPR”), singular secondary rules of production (“SSR”), general secondary rules of production (“GSR”).

General primary rules directly regulate ex ante classes of individual situations; an example of this type of rules is a statutory rule that generally imposes a duty on individuals with certain requirements. By contrast, singular primary rules directly regulate ex post “individual situations”, that is the behaviour of specified individuals; an example of this type of rules is an administrative or judicial decision that imposes a duty on a specific individual. General secondary rules of production directly regulate classes of individual acts of production of other rules; an example of this type of rules is the rule that generally empowers agencies (courts, administrative authorities or

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17 Law has an autopoietic character - AUTOPSYCETIC LAW: A NEW APPROACH TO LAW AND SOCIETY (Gunther Teubner Ed. 1988) - and there are many types of secondary rules concerning the application of law; these rules include rules that limit the content of other rules, rules that attribute normative powers, rules that define how normative powers should be exercised, rules that reserve certain normative powers to certain institutions, and so on.
individuals) to apply a statute or regulation to individual situations, or the *stare decisis* rule which generally empowers courts to apply a precedent decision to an individual situation if certain requirements are met. Finally *singular secondary rules* of production directly regulate a single individual act of production of other rules; an example of this type of rules is the individualized secondary rule according to which an agent (a court, an administrative authority, or an individual) applies to itself a general secondary rule, thereby issuing a Rule of the Case.

Thus a legal system in the synchronic plane (i.e. in a single moment of time) viewed in terms of production of rules can be seen as the combination of these four types of rules leading to hierarchical structures. Please note that rules do not exist in themselves, but are the result of the activities of those who apply and interpret the law.

<table>
<thead>
<tr>
<th>RULES</th>
<th>SINGULAR</th>
<th>GENERAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRIMARY</td>
<td>Rules of the Case (SPR)</td>
<td>general primary rules (GPR)</td>
</tr>
<tr>
<td>SECONDARY</td>
<td>singular secondary rules (SSR)</td>
<td>general secondary rules (GSR)</td>
</tr>
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### TABLE 1 – Types of rules.

I am specifically interested here to a behavioral and reductionist notion of “rules” as Rule of the Cases directly regulating individual situations (singular primary rules) and thus I distinguish these Rule of the Cases from other rules such as general primary rules (which potentially lead to Rules of the Case) and singular/general secondary rules (which are related to other rules and not directly to individual behavior). In this perspective it is acknowledged that general primary rules do have a prescriptive nature in relation to potential individual situations, and that singular/general secondary rules are essential in defining the structure of normative powers eventually leading to Rules of the Case, but it is also emphasized that general primary rules do not specifically regulate individual situations until they are “individualized” or “actualized” through a singular secondary rule of production leading to the creation of a Rule of the Case. Thus an individualized Rule of the Case is created by an agent in accordance with an individualized rule of production. I will discuss in section 3 how these Rule of the Cases are generated in legal systems.

Let us discuss now secondary rules of production: they establish hierarchical structures within legal systems. A hierarchy is a relation of two elements in which one element (at lower hierarchical level) depends on the other element (at higher hierarchical level). Hierarchies also operate in legal systems; among the different types of legal hierarchies I will discuss here formal and substantial hierarchies.

A “formal hierarchy” is defined here as one which is established by a rule of production (RP) and occurs between a producing rule (R1) and a produced rule (R2) when the RP establishes how R2 is

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19 In addition to substantial and formal hierarchies there are other types of hierarchies, such as so called logical hierarchies created by certain norm referring to other norms in various ways (such as norms abrogating other norms, legislative definitions, etc.), and axiological hierarchies established between two or more norms by the interpreter through value-judgements (such as hierarchies of legal principles).
produced from R1. A rule of production is a secondary rule because it is related to other rules, and can be a singular or a general secondary rule. General secondary rules of production stipulate *ex ante* about the hierarchical connection between producing and produced rules, for example that statutes are lower than constitutional rules. General secondary rules need however to be actualized by a specific individual agency (a legislator, an administrative authority, a court or an individual) for such an agency to create a rule. Such created rule can be a Rule of the Case regulating an individual situation, a general primary rule such as a statute, or another singular/general secondary rule of production.

Thus a singular rule of production can establish a formal hierarchy between any type of (primary or secondary) rule and therefore, down the chain of production, there can be several hierarchical connections between higher (singular or general) secondary rules and lower (singular/general) secondary rules before an actual Rule of the Case is created. These chains of production terminate when secondary rules ultimately lead to the creation of Rules of the Case (singular primary rules). For practical purposes I focus here on formal hierarchies between primary rules: a formal hierarchy is a relation between a higher (singular/general) primary rule R1 and a lower (singular/general) primary rule R2 which is established, each time, by a singular rule of production. Thus when there is a formal hierarchy one can say that, in accordance to the rule of production, the producing primary rule R1 “generates” or “produces” the produced primary rule R2. I provide herebelow some cases of formal hierarchies between primary rules in legal systems. More detailed descriptions will be provide at section 3 using network diagrams.

The first case is that of a legal system where there is a general rule of production (a secondary constitutional rule) according to which the legislator has the normative power to enact a general primary rule - such as one attributing rights or imposing duties - with a statute. When the legislator actualizes such general rule of production with an singular rule of production it creates a specific statute; in this situation the constitutional rule is the producing rule and the statute is the produced rule and thus one can say that the constitutional rule “generates” or “produces” the statute.

The second case of formal hierarchy between primary rules is that of a legal system where there is a general rule of production according to which administrative authorities have the normative power to implement a statutory rule - such as one attributing rights or imposing duties - with a regulation. When a specific administrative authority actualizes such general rule of production with a singular rule of production it creates a specific regulation; in this situation the statutory rule is the producing rule and the regulation is the produced rule and thus one can say that the statutory rule “generates” or “produces” the regulation.

The third case of formal hierarchy between primary rules is that of a legal system where there is a rule of production according to which courts or an administrative authority have a normative power to apply a statute with decisions regulating individual situations. When a specific court actualizes such general rule of production with a singular rule of production it creates a specific judicial decision, a Rule of the Case; in this situation the applied statute is the producing rule and the decision is the produced rule and thus one can say that the statute “generates” or “produces” the judicial or administrative decision.

The fourth case of formal hierarchy between primary rules is that of a common law system where there is a rule of production according to which courts have a normative power to apply previous

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20 Please note that in this case there is another secondary rule attributing general legislative powers to the legislator.

21 Please note that in this case there is another secondary rule attributing regulative powers to the administrative authority.
decisions in a subsequent case if certain requirements are met (*stare decisis*). When a specific court actualizes such general rule of production with a singular rule of production it creates a specific judicial decision, a Rule of the Case; in this situation the previous decision is the producing rule and the current decision is the produced rule and thus one can say that the precedent binding decision “generates” or “produces” the subsequent decision.

A formal hierarchy should be distinguished from a “substantial hierarchy”. A “substantial hierarchy” is one which occurs between two rules (R1 and R2) when a third validity rule (VR) establishes that the lower rule R2 is invalid if it does not meet certain validity requirements in respect to R1. The two rules R1 and R2 are linked by a substantial hierarchy created by a third rule VR which establishes the validity requirements (for example that R2 is invalid if it conflicts with R1). Thus a substantial hierarchy is a relation between a higher rule R1 and a lower rule R2 in which the lower rule R2 is subject to a certain validity requirements established by a third rule VR: in these cases one can say that the lower rule R2 is “subject” to the higher rule R1 via the operation of the third rule VR.

The first case of substantial hierarchy is that found in constitutional judicial review of legislation: in accordance to the constitutional rule (VR) that statutes (R2) cannot conflict with a rule of a written constitution (R1), a statute R2 enacted by the proper institution must not conflict with constitutional rule R1, which for example establishes the principle of equal treatment between similar situations (so called horizontal equity). In respect of these types of substantial hierarchy one can say that statutes are “subject” to constitutional rules.

The second case of substantial hierarchy is that found in judicial review of regulations in respect of statutes; in these cases the rule which establishes the substantial hierarchy (R3) provides that regulations (R2) must meet certain validity requirements (basically that the regulation R2 must not conflict with statutory rules R1 which limit or pre-determine the prescriptive content of the regulations). As a consequence, a regulation R2 enacted by the proper institution must not conflict with the statutory rule R1 which is implemented through that regulation R2. In these types of substantial hierarchy one can say that the regulations are “subject” to the statutes on the basis of which they are issued.

The third case of substantial hierarchy is that found in judicial review of decisions issued by authorities in respect of statutes and regulations; in these cases the rule which establishes the substantial hierarchy (VR) provides that the binding decisions issued by authorities (R2) must meet certain validity requirements (basically that the decision R2 must not conflict with statutory rules R1 which limit or pre-determine the prescriptive content of the same decision). As a consequence, a decision R2 enacted by the proper institution must not conflict with the statutory rule or regulation R1 which is being implemented through that decision R2. In these types of substantial hierarchy one can say that the decisions issued by authorities are “subject” to the statutes and regulations on the basis of which they are issued.

In a chain of substantial hierarchies statutes must conform with constitutional rules, regulations must conform with statutes and decisions issued by authorities must conform with statutes and regulations. There can be both formal and substantial hierarchies at each level of the legal system; for example a statute may be “produced” by a set of constitutional rules (those being implemented by the statute) as well as be “subject” to another set of constitutional rules (establishing validity requirements, such as those on equal treatment); a decision by authorities may be “produced” by a set of legislative rules (those being applied by such a decision) as well as be “subject” to another set of legislative rules or principle (those on the validity requirements of administrative decisions). On the basis of the concepts of formal and substantial hierarchy one can thus distinguish between
the *existence* and *validity* of rules: a rule is *existent* if it is produced by a higher rule pursuant to rule of production establishing a formal hierarchy (for example an administrative decision enacted in accordance to secondary rules that regulate the exercise of the normative powers concerning such a decision), while a rule is *valid* if it meets the validity requirements provided for by a third rule establishing the substantial hierarchy (for example an administrative decision lawfully implementing a statute). A rule, to be valid, must exist. An example of a both existent and valid general rule is a statute created in accordance with higher secondary rules (production rules) which does not conflict with the constitutional rules establishing validity requirements (such as, for example, equal treatment)\(^22\).

In this paper I focus on the existence of rules rather than on their validity; in fact I use the concept of formal hierarchy to discuss the structure of legal systems, as the lower rules are created by the higher rules as a result of the operation of rules of production, while the validity of rules is subject to different kinds of judicial review at each level of the legal system which are not discussed here in detail\(^23\). Although according to Kelsen\(^24\) and Raz\(^25\) hierarchies among rules are commonly worded in terms of “*chains of validity*”, I argue here that in most cases a “chain of validity” is in reality a “*chain of existence*”, i.e. a hierarchical structure – defined here as a “chain of production” - relating to the production of different levels of rules.

3. **Production links within networks of Rules of the Case**

In this section I discuss in more detail how (potentially unlimited) sets of Rules of the Case are produced through underlying “generative processes” represented as network diagrams, drawing on existing literature on legal theory. Different types of chains of production ultimately create Rules of the Case, each of them regulating an individual situation. This perspective allows me to view legal systems as complex structures generating a countable number of “individualized” or “actualized” Rule of the Cases that result from chains of production and which exert their prescriptive effects in respect to individual situations. General primary rules (statutes and regulations) are “individualized” or “actualized” by Rules of the Case created by administrative authorities, courts and self-compliant...

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\(^22\) In legal systems in which constitutional or judicial review is subsequent to the enactment of rules, there can exist rules (both singular or general) that are produced pursuant to a formal hierarchy but that are in conflict with an a higher rule pursuant to a substantial hierarchy; these rules exist until they become invalid when they are declared so by an institution with proper normative powers; for example a decision by administrative authorities is binding until eliminated by a judge with proper powers or by the same administrative authorities.

\(^23\) I have briefly discussed above a chain of substantial hierarchies in which statutes must conform with constitutional rules, regulations must conform with statutes, and decisions issued by administrative authorities must conform with statutes and regulations, but the validity requirements are defined by complex sets of rules and principles and require a thorough knowledge of law-in-action of each legal system.

\(^24\) Kelsen for example bases the chain of validity concept on the so called “basic norm” and traces back the validity of any norm to the validity of such a basic norm. For example he notes that an individual norm is *valid* as part of a definite legal order because it has been created in conformity with a statute; he adds “this statute, finally, receives its validity from the constitution, since it has been established by the competent organ in the way the constitution prescribes. If we ask why the constitution is *valid*, perhaps we come upon an older constitution. Ultimately we reach some constitution that is the first historically and that was laid down by an individual usurper or by some kind of assembly (…). It is postulated that one ought to behave as the individual, or the individuals, who laid down the first constitution have ordained. This is the basic norm of the legal order under consideration” (HANS KELSEN, GENERAL THEORY OF LAW AND STATE 115 (1961), cited by JOSEPH RAZ, THE CONCEPT OF A LEGAL SYSTEM 97 (2003)).

\(^25\) Raz defines the concept of chain of validity as a concept of all those norms (primary and secondary) such that (1) each of them *authorizes* the creation of just one of the others of the set, except at most one which does not *authorize* the creation of any norm (a singular primary rule); and (2) the creation of each of them is *authorized* by just one norm in the set (a rule of production), except for one norm, the creation of which is not authorized by any norm in the chain. Raz uses of a concept of *authority* (Italics added) which more explicitly corresponds to a concept of “existence” of norms; JOSEPH RAZ, THE CONCEPT OF A LEGAL SYSTEM 97 (2003).
individuals. As each “individualized” or “actualized” Rule of the Case regulates a specific individual situation, there is a unique combination between each Rule of the Case and the individual fact pattern regulated by that rule.

When there are formal hierarchies, rules are created and exist in the legal system because of the existence of rules of production, which are singular/general secondary rules. I can therefore define here a key-concept of “chain of production” of rules by a bijection: whenever a produced primary rule (R2) is derived from a producing primary rule (R1) according to a singular secondary rule on production (RP) there is a “chain of production” of primary rules. The term “chain of production” is used in the sense that of a chain of rules that are existent because are produced through production links by higher rules (a “chain of existence”).

The defining characteristic of legal systems is that there are rules of production establishing connections between primary rules which determine the existence of the chains of production of Rules of the Case. These connections are defined here as “production links”. Production links between rules operate because there are secondary rules which ensure that primary (singular and general rules) be applied by proper agents using normative powers. For example a decision made by an administrative authority or a court (a produced rule) is connected to a statutory rule (a producing rule) by a production link established by a singular rule of production that actualizes a general rule of production. In those cases the general rule of production - according to which agents such as administrative authorities or courts apply statutes to the facts covered by such statutes - is each time individualized by a specific agent rendering a decision (a Rule of the Case). Similarly a decision made by a court in a common law system (a produced rule) is connected to a precedent decision (a producing rule) by a production link established by a singular rule of production which actualizes a general rule of production (stare decisis). In these cases the general stare decisis rule is individualized each time by a specific court rendering a decision (a Rule of the Case).

In order to highlight the operation of production links within legal systems, I will present in this section a set of network diagrams adopting a notation drawn from graph theory and network science\(^26\). A graph or a network is a topological object consisting of points called nodes, and lines, called links, which abstract away from the details of the situation which is represented except for its connectivity. A set of rules can therefore be represented using a network notation, in which the nodes represent rules and the links represent production links between rules\(^27\). More specifically the links are directed links as they go only in one direction indicated by the arrow.

There is a specific reason why the links are directed links in these network representations of the legal system: the rule of production establishing a formal hierarchy exists before the lower primary rule is enacted on the basis of such a rule of production, so that a precedent producing rule leads to a subsequent produced rule and not the other way around.\(^28\) For example a decision by an administrative authority enforcing a statute is “lower” than such a statute in a chain of production, so that the node representing this decision is linked to the node representing the statutory rule by a (production) directed link. Thus the Rule of the Case (the decision) is “lower” than the statutory rule in the chain of production represented as a network. In network diagrams “lower” means

\(^{26}\) An approach using diagrams has been already developed by Joseph Raz, The Concept Of A Legal System 97 (2003), who uses tree diagrams made by circles and lines. In these diagrams every circle represents a normative power (conferred by a secondary rule), while every line immediately below a circle represents a rule (either a primary rule regulating behavior, or a secondary rule conferring normative powers) which is created through the actual exercise of such normative power. This paper adopts a different notation drawn from network science, see next footnote.


\(^{28}\) A "formal hierarchy", as defined in this paper, is one that occurs between a rule on production (R1) and a produced rule (R2) when R1 establishes how R2 is produced from R1; in this situation R1 is superior to R2.
“subsequent”, and “higher” means “precedent”: a decision by an administrative authority enforcing a statute is by definition subsequent to such a statute, and production links between precedent and subsequent rules in the chain of production are expressly represented by directed link.

Fig. 1 below represents in network terms the concepts of rule of production, producing/produced rule, and production link.

![Diagram of rule of production, producing/produced rule, and production link](image-url)

In Fig. 1 I represent production links created by rules of production using the network notation in which the nodes are rules and the links are production links between rules. Rules of productions and links connecting them in this diagram (as well as in the following diagrams) are represented using dotted lines to distinguish them from primary rules and links connecting them. According to the model of “individualized rules” a general secondary rule of production must be individualized as a singular secondary rule of production by the agent creating a singular primary rule. For example the general secondary rule of production according to which administrative authorities have the power to apply general primary rules to individual situations, leads to a singular rule of production applied by a specific administrative authority when such an authority applies a general primary rule to an individual situation and creates a Rule of the Case. Another example is found in common law, where the general secondary rule of production according to which the precedent is binding if certain requirements are met (stare decisis rule), leads to a singular rule of production applied by court when such a court applies a precedent decision to regulate an individual situation, thereby creating a Rule of the Case. A third example is that in which there is a general secondary rule of production according to which individuals have the power to apply general primary rules to their own individual situations, as that rule leads to a singular rule of production applied by an individual when such an individual self-enforces a general primary rule to her/his own individual situation creating a Rule of the Case.

In Fig. 1 the general secondary rule of production RP1 produces a (potentially unlimited) set of individualized singular secondary rules of production (RP2, RP3, RPn). Among these rules, RP2 is the singular rule of production establishing the formal hierarchy represented by a directed link between a higher Rule 1 and a lower Rule 2; this directed link is a production link which indicates that Rule 1 is the producing rule (plus sign at the beginning of the link) and Rule 2 in the produced rule (minus sign at the end of the link). Thus the network representation shows that, in accordance
A rule of production (node) is a secondary rule as it refers to other rules, the producing rule R1 and the produced rule R2, that can be both singular/general primary rules or singular/general secondary rules. Thus the produced rule can be another secondary rule which eventually leads to the creation of primary rules. For clarity sake, I limit the analysis of the concept of formal hierarchy to two types of relationships between primary rules. The first type is represented by a directed link between a higher general primary rule R1 and a lower singular primary rule R2, and occurs when general primary rules are applied by agents (courts, administrative authorities, or individuals) that create Rules of the Case. The second type of relationship between primary rules is represented by a directed link between a higher singular primary rule R1 and a lower singular primary rule R2 and occurs when in common law precedent Rules of the Case are applied by courts following the *stare decisis* rule and creating new Rules of the Case. Thus the network representation shows that, in accordance to the rule of production, the producing primary rule R1 “generates“ or “produces” the produced primary rule R2.

In the following Fig. 2a, 2b, and 2c the nodes represent rules (which can be primary/secondary and singular/general, depending on the case) and the directed links represent production links between primary rules. Fig 2a, 2b, and 2c below represent chains of production leading to a Rule of the Case, which is either created (*i*) by administrative authorities and courts (Fig. 2a and Fig. 2b), or (*ii*) directly by an individual in the cases of conscious self-compliance (Fig. 2c). For practical reasons in the following diagrams I will represent the singular rules of production just as nodes without representing the underlying general rule of production as I did in Fig. 1 above; it is however understood that in each case such a higher general production rule exist.

In the following situations at Fig. 2a, 2b, and 2c there are several common elements: first, there is an underlying general secondary rule of production attributing normative powers to agents (administrative authorities, courts or individuals); second, there is a singular rule of production establishing a formal hierarchy between a producing rule (a statute/regulation in Fig. 1a and 1c, a binding precedent in Fig. 1c), and the primary produced rule (a court/administrative decision in Fig. 1a, a common law case in Fig. 1b, a self-compliance rule in Fig. 1c); third, the formal hierarchy between the producing rule and the produced rule is represented as a production link (a directed link) between those rules; fourth all diagrams result in a Rule of the Case.
Fig. 2a above represents the creation of a Rule of the Case by an administrative authority or by a court regulating an individual situation on the basis of the general primary rule. All links represented at Fig. 2a above are directed links (links with an arrow) as they only go in one direction from one node (a rule) to another node (another rule in the chain of production). There is a dot at the end of the node which terminates the diagram representing the individual situation regulated by the Rule of the Case. In Fig. 2a above on the basis of a rule of production (RP1, node) from a constitutional rule (CR) a general statutory rule is created (Rule 1, node). On the basis of a singular rule of production (RP2, node) Rule 1 is implemented by an administrative authority or a court which issues a binding Decision 1 to apply the general primary rule to an individual situation.

This chain of production is made by a set of three production links between rules: there is a link of production respectively between the current constitution (CR, node), the general rule (Rule 1, node), and the binding decision which applies the general primary rule (Decision 1, node). CR, Rule 1 and Decision 1, are connected by two formal hierarchies established by rules of productions and represented by two production links; more specifically, according to the relevant rule of production, Rule 1 is the producing rule and Decision 1 is the produced rule.29

Fig. 2b below represents another situation of creation of Rules of the Case, but in this case I factor in common law constraints. In this situation, for clarity sake, it is assumed that there is no statute (general primary rule) to be applied by a court (by contrast this occurs in Fig.2a), and that two courts decide two novel cases not relying on precedents (Decisions 1 and 2), while other courts rely exclusively on precedent cases (Decisions 3, 4 and 5). It is also assumed the existence of an implicit constitutional rule on which the rule of precedent as well as judicial power of courts are based. This is clearly an over-stylized situation which eschews all the subtleties of the rule of precedent in common law systems, but it is used here to model the operation of production links in case law. This is a chain of production in which a higher Rule of the Case (the precedent decision) produces a lower Rule of the Case (the subsequent decision), in accordance with the rule of production (the *stare decisis* rule).

More precisely here the *stare decisis* is a general secondary rule of production which attributes to judges the power to enact binding decisions on the basis of previous cases if certain conditions are met; *stare decisis* also creates a duty for future courts to adjudicate on the basis of the binding precedent. In network notation the *stare decisis* is a general rule of production relating to a binding precedent (producing rule) and a current decision (produced rule). In Fig. 2b the general *stare decisis* rule is actually represented by singular rules of production RP2, RP3 and RP4 derived from an underlying general secondary rule of production dictating the binding precedent (*stare decisis*).

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29 Similarly according to the relevant rule of productions, CR is the producing rule and Rule 1 is the produced rule.
Fig. 2b. Creation of a Rule of the Case by a court in a common law system.

All links represented at Fig. 2b above are directed links and there is a dot at the end of each node representing the individual situations regulated by the Rules of the Case. In Fig. 2b a rule of the current constitution is created (CR, node) on the basis of an underlying rule of production (RP1); in turn on the basis of CR several Rules of the Case are created by courts (Decisions 1 through 5, nodes), each of them applying a singular primary rule to an individual situation.

Let us assume that initially (when no precedent is available) two decisions by two different courts are issued regulating two different individual situations represented by dots (Decision 1 and Decision 2, nodes). On the basis of the stare decisis (RP2, RP3 and RP4) Decisions 1 and 2, in addition to regulating individual situations also constitute, in a potential hierarchy, the producing rule (a Rule of the Case) for future courts to create a produced rule established by a stare decisis rule of production. Let us now assume that subsequently, on the basis such a rule of production RP2, RP3 and RP4 (the stare decisis rule individualized by each court when deciding a case), three other decisions are issued by three courts (Decision 3, 4, and 5, nodes) regulating other individual situations. Decisions 3 relies on Decision 1, while Decisions 4 and 5 rely on Decision 2. In turn Decisions 3, 4, and 5 each constitutes a producing singular primary rule (a Rule of the Case) for future judges under the stare decisis rule.

This chain of production is made by a set of different production links between rules represented by nodes: there is a link of production respectively between the current constitution (CR, node), Decision 1 and Decision 3, Decision 2 and Decision 4, Decision 2 and Decision 5 (nodes). These rules are connected by production links established by the stare decisis rule individualized by each court; more specifically, according to such a rule of production. Decision 1 is a producing rule in respect to Decision 3 which is a produced rule, and Decision 2 is a producing rule in respect to Decision 4 and Decision 5 which are produced rules.

In Fig. 2b the stare decisis rule individualized by each court has created formal hierarchies between previous cases and subsequent cases. More precisely here there is a production link connecting two decisions (nodes), for example a connection between the “higher” Decision 1 and the “lower” Decision 3. Please note that Decision 3 is “lower” in respect to Decision 1 just because of the structure of the formal hierarchy, and not in some other normative sense. The same can be said for Decision 2 in respect to Decisions 4 and 5.

Thus Fig. 2b represents a chain of precedents in a common law system among decisions rendered by different judges in which each previous decision is the higher-precedent rule in respect to the lower-subsequent rule. Hence one can say that all these decisions rendered under the binding precedent (i) are directly “generated” or “produced” by the precedent decisions and (ii) are
ultimately “generated” or “produced” by the \textit{stare decisis} rule individualized by each court. As a result the binding force of precedents expands its normative impact within the system by linking previous Rules of the Case to new Rules of the Case with formal hierarchies in a sort of continuous branching-off\textsuperscript{30}. Please also note that Decisions 1 through 5 do not “individualize” or “actualize” a general primary rule but each time create a new Rule of the Case regulating an individual situation.

The last of the three diagrams - Fig 2c below - represents the creation of a Rule of the Case directly by an individual (self-compliance rule). In these situations an individual imposes to herself/himself to do something through a conscious act of compliance with a general primary rule that is addressed to all potential recipients; this typically occurs when such an individual is aware of a prohibition or an obligation provided for by a statute or regulation and acts consequently. A very good example of this situation is found in tax systems, in which all individuals are requested each year to file a tax return and pay their own due taxes spontaneously on the basis of tax returns, subject to potential audits\textsuperscript{31}. I will use this tax example here to represent self-compliance rules.

![Diagram](image)

\textit{Fig 2c. Creation of a self-compliance Rule of the Case directly by individuals.}

All links represented at Fig. 2c above are directed links and there is a dot at the end of the node which terminates the diagram representing the individual situation regulated by the Rule of the Case. In Fig. 2c above a rule of the current constitution is created (CR, node) on the basis of an underlying rule of production (RP1); in turn on the basis of the singular rule of production RP2 a general primary rule is created by the legislator (Rule 1, node, for example a tax statutory rule). Rule 1 is implemented by the individual who creates a Rule of the Case (Decision 1, node, for example a filed tax return) which regulates her or his own individual situation on the basis of the general primary rule (Rule 1). Here there is a Rule of the Case even if there is no exercise of normative powers by administrative authorities or courts. In Fig. 2c there is a chain of production


that generates a final node representing a self-compliance singular rule related to an individual situation (Decision 1).

In Fig. 2c there is a link of production respectively between the current constitution (CR, node) the general rule (Rule 1, node), and the self-compliance rule (Decision 1, node). In the chain of production a Rule of the Case is created by the individual (Decision 1, node), which, in this example, is a Rule of the Case regulating an individual situation on the basis of the general primary rule (rule 1).

Please note that when an individual recipient acts in line with a general primary rule without consciously creating for herself or himself a self-compliance singular rule, there is no self-compliance rule and, in a network diagram representation there is no node representing a Rule of the Case regulating an individual situation. If an individual by happenstance complies with an unknown rule there is no creation of a self-compliance rule, but there is simply a fact. Self-compliance rules are typical of complex normative systems in which individuals are expected to know the rules in order to comply with them. As a result, for example in tax systems, compliance without awareness is unlikely to occur.

The introduction of the requirement of “conscious” self-compliance rules demands a simple definition of a psychological model of rule-observance. Self-compliance rules are found in “nomopoiesis”, a situation in which an individual imposes to herself/himself to behave in compliance with a general primary rule. Nomopoiesis is opposed to “nomothesis”, a situation in which a person with proper normative powers (for example an administrative authority or a court orders somebody else (an individual) to do something by enacting a binding singular primary rule specifically addressed to such an individual. I argue here that both consciousness and individual action are required for a self-compliance rule to exist (a self-compliance rule is a specific nomopoietic instance of a singular rule). For example a demand of payment by administrative authorities to an individual is a singular rule even if there is no awareness of it by the recipient but such a singular rule is not a self-compliance rule. A corollary is that there is no self-compliance rule when there is a consciously non-compliant individual in respect to a general or singular rule (for example a tax evader who willingly does not comply with a demand of payment from tax authorities). If a individual does not comply there is a singular primary rule (the demand of payment) but no self-compliance rule.

4. A network definition of normative complexity

The three diagrams at section 3 above (Fig. 2a, 2b, and 2c) are a first set of simplified situations of creation of Rules of the Case using a network representations of production links (directed links) between rules (nodes). Fig 2a, 2b, and 2c above show that when there is a chain of production leading to a Rule of the Case, this rule can be either created (i) by administrative authorities and courts (Decision 1 in Fig. 2a and Fig. 2b), or (ii) directly by an individual recipient in the cases of conscious self-compliance (Decision 1 in Fig. 2c). In Fig 2a, 2b, and 2c whenever there is a Rule of the Case (represented by a node), there is a corresponding individual situation regulated by such a

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32 For example the recipient may not be aware of the primary rule specifically directed to him; legal systems have developed strict legal presumptions as to the knowledge by the recipient of singular rules in order to limit litigation on that that specific issue.

33 One, in theory, could say that in that situation the individual creates for herself/himself a “non-self-compliance rule”, an act of negative nomopoiesis leading to tax evasion.
Rule of the Case (represented by a dot). Altogether these diagrams show that chains of production, in order to regulate individual situations, must lead to Rules of the Case.

In this paragraph I address a second set of more complex situations. Fig. 3 and 4 represent multiple chains of production of Rules of the Case which “individualize” or “actualize” primary rules and show that this process leads to the “proliferation of Rules of the Case” combined with the “branching-off of normative powers”. These two concepts are used to define a concept of normative complexity which relies on network theory and which views legal systems as a collection “individualized” Rules of the Case resulting from chains of production.

In a network diagram when a rule is produced by an agent (an administrative authority, a court or an individual) this occurs because there is a singular rule of production (derived from a general rule of production) attributing to such an agent the power apply a primary general rule with a binding Rule of the Case. Whenever Rules of the Case are created there is a singular rule of production establishing a production link between the producing rule (the general primary rule) and the produced rule (the Rule of the Case). These rules of production operate in respect to decisions by administrative authorities or by court, as well as to self-compliance decisions (fig. 2a, 2b, 2c). It is therefore appropriate to represent the operation of secondary rules with the specific network diagram which is presented below at Fig. 3.

![Diagram](image-url)

**Fig 3. “Proliferation of Rules of the Case” in statutory law.**

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34 The are various concepts of “complexity” that are found mainly in the political economy literature; see: George Warskett et al., The Complexity of Tax Structure in Competitive Political Systems, 5 INTERNATIONAL TAX AND PUBLIC FINANCE 127 (1998); DEMOCRATIC CHOICE AND TAXATION (Walter Hettich & Stanley Winer eds., 1999); Louis Kaplow, A Model of the Optimal Complexity of Legal Rules, 11 J.L. ECON. & ORG. 150 (1995); Louis Kaplow, How Tax Complexity and Enforcement Affect the Equity and Efficiency of Income Tax, in TAX POLICY IN THE REAL WORLD (Joel Slemrod ed., 1998).
In Fig. 3 above on the basis of rule of production RP1 a rule of the current constitution is created (CR, node); in turn on the basis of RP2, a general secondary rule of production (RP3) is created according to which general primary rules (statutes and regulations) can be applied by agencies (administrative authorities, courts, and individuals) through the creation of Rules of the Case regulating individual situations. The general rule of production RP3 is then individualized in a (potentially unlimited) set of singular secondary rules of production (RP4 through RPn) by each agent that applies a statute (Rule 1) to an individual situation. Each individualized rule of production (RP4 through RPn) establishes a production link between a producing rule (Rule 1) and a produced rule (Decisions 1 through n, Rules of the Case).

Thus statutes are implemented by different agencies that issue a set of Rules of the Case (Decisions 1 through n) to apply the general primary rule to a (potentially unlimited) set of individual situations. These agencies, each time create a Rule of the Case, do so on the basis of a specific individualized singular secondary rule of production (RP4 through RPn). Therefore there is a bijection between each individualized singular rule of production and each resulting Rule of the Case created by an agency. On the basis of the general rule of production, a (potentially unlimited) set of Rules of the Case is created (Decisions 1 through Decision n, nodes) applying the general primary rule to a (potentially unlimited) set of individual situations.

This chain of production is made by a set of production links between rules: there is a link of production respectively between, the current constitution (CR, node), the general rule of production (RP3), the individualized rules of production (RP4 through RPn), the general rule (Rule 1, node), and the Rules of the Case which apply the general primary rule (Decisions 1 through n, node). In particular Rule 1 is connected to Decisions 1 through n by formal hierarchies established by the individualized rules of production and represented by production links; more specifically, according to each individualized rule of production, the general primary rule is the producing rule and Decision 1 through n are the produced rules (Rules of the Case).

While in the diagrams at Fig. 2a and 2b there is only one chain of production with a defined number of production links, in the diagram at Fig. 3 above there is a number of different chains of production which is equal to the number of Rules of the Case (decision 1 through n); in other words each Rule of the Case is generated by a single chain of production composed by different production links. In Fig. 3 above there are n chains of production: each chain of production results in a final Rule of the Case (Decisions 1 through n) regulating individual situations. Each of these Rules of the Case shares a common underlying set of rules connected by production links (CR, RP3, one of the individualized RP 4 through RPn, Rule 1) which is replicated each time an individual Rule of the Case is issued. Please also note when there are multiple chains of production the number of Rules of the Case exceeds the number of general primary rules being applied; this means, for example, that a single statutory rule can generate a multitude of Rules of the Case through a multitude of production links.

The feature of this diagram at Fig. 3 representing multiple chains of production can thus be generalized as follows: two or more chains of production may have all their underlying rules in common except one or more, and this depends on the number of Rules of the Case ultimately created. In Fig. 3, for example, there is one general primary rule (Rule 1) and n Rules of the Case (Decision 1 through n). This phenomenon is defined as “proliferation of Rules of the Case” and occurs when general primary rules (such as statutes and regulations) are applied to individual situations either through decisions by administrative authorities or courts, or through self-

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35 The diagram in Fig. 3 above is the same as diagrams at Fig. 1a, 1b and 1c down to the node representing the general primary rule
compliance by individuals. The Rules of the Case (Decisions 1 through n, in Fig. 3) which result from multiple chains of production, are represented in the diagram by terminal nodes as they terminate multiple chains of production with multiple Rules of the Case regulating individual situations. This diagram thus represents the enforcement of statutes and regulations through administrative agencies, courts and self-compliant individuals.

Proliferation of Rules of the Case occurs also in common law systems and can be represented by network structures. These structures are similar to those representing the proliferation of Rules of the Case derived from statute (see above Fig. 3). In common law the network of Fig. 3 above should however be adapted by replacing the general rule of production RP3 (according to which general primary rules can be applied by different agencies) with *stare decisis*, another general rule of production according to which singular primary rules (precedent binding decisions) are applied by courts in certain requirements are met. The network of Fig. 3 above should also be adapted by replacing the single general statutory rule being implemented by agencies (Rule 1) with various binding precedents (Precedents 1 through n) leading to Rules of the case (Decisions 1 through n).

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Fig 4. “Proliferation of Rules of the Case” in common law.

In Fig. 4 above on the basis of rule of production RP1 a rule of the current constitution is created (CR, node); in turn on the basis of RP2, a general secondary rule of production (RP3) of *stare decisis* is created according to which binding precedents can be applied by courts through the creation of Rules of the Case regulating individual situations. The general rule of production RP3 is then individualized in singular secondary rules of production (RP4 through RPn) by each court which applies precedents (Precedent 1 through n) to an individual situation. Each individualized

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Please note that a general primary rule triggers the creation of a (potentially unlimited) set of Rule of the Cases; a general primary rule which triggers the creation of just one Rule of the Cases is, in practical terms, a Rule of the Case.
rule of production (RP4 through RPn) establishes a production link between a producing rule (Precedent 1 through n) and a produced rule (Decisions 1 through n, Rules of the Case).

Thus precedents are implemented by different courts that issue a set of Rules of the Case (Decisions 1 through n) applied to a (potentially unlimited) set of individual situations. These courts, each time they create a Rule of the Case, they do so on the basis of a specific individualized singular secondary rule of production (RP4 1 through RPn). Therefore there is a bijection between each individualized singular rule of production and each resulting Rule of the Case created by a court, and, on the basis of the general rule of production, a (potentially unlimited) set of Rules of the Case is created (Decisions 1 through decision n, nodes) applying the precedents to a (potentially unlimited) set of individual situations.

This chain of production is made by a set of production links between rules: there is a link of production respectively between the current constitution (CR, node), the general rule of production (RP3), the individualized rules of production (RP4 1 through RPn), the precedents (Precedent 1 through n), and the Rules of the Case which apply the general primary rule (Decisions 1 through n, node). In particular each precedent is connected to Decisions 1 through n by formal hierarchies established by the individualized rules of productions and represented by production links; more specifically, according to each individualized rule of production, each precedent is the producing rule and Decision 1 through n are the produced rules (Rules of the Case).

An additional feature of both Fig. 3 and Fig. 4 is that the rules of production presuppose that each individual agency is endowed with proper normative powers\(^\text{37}\), so that the more the agencies the more the normative powers that are exercised. Thus the diagrams at Fig. 3 and Fig. 4 represent, in addition to the proliferation of Rules of the Case, a phenomenon defined as "multiple branching-off of normative powers", which occurs when different agencies are given normative powers to deliver Rules of the Case. The existence of different agencies (including individuals to whom the rules are addressed) with autonomous normative powers clearly increases the likelihood of creation of subsequent Rules of the Case, so that the proliferation of Rules of the Case and the branching-off of normative powers operate in combination triggering an increasing complexity of the legal system. Thus two striking features of legal systems become apparent:

1) the proliferation of Rules of the Case: certain primary rules (basic statutory rules, regulations, binding precedents) directly give rise to a great number of Rules of the Case regulating individual situations (Decisions 1 through n, Fig. 3 and 4 above), because rules of production create a (potentially) unlimited set of production links leading to Rules of the Case, and

2) the branching-off of normative powers: the different normative powers conferred by secondary rules lead to a potential exponential growth of Rules of the Case because they multiply the number of agencies capable of creating Rules of the Case (Decisions 1 through n, Fig. 3 and 4 above).

The combination of proliferation of Rule of the Cases and branching-off of normative powers creates a situation which is defined here as “normative complexity”. In this situation a legal system generate Rules of the Case (decision n, nodes) through different levels of normative powers, each of them regulating an individual situation, so that legal systems are the final collection of Rules of the Case generated through chains of production. In the diagram represented at Fig. 3 and 4 above each individual situation is regulated by a specific Rule of the Case which is generated by an underlying complex process.

\(^{37}\) Specific general secondary rules attribute normative powers to the agencies; these secondary rules should be distinguished from the rules of production, another type of secondary rules that create production links within the system.
5. Legal systems as clustered networks of rules

Using this network notation let us take three decisions of a court that operates in a common law system under the rule of binding precedent and assume that Decision 1 (binding precedent) is cited and confirmed by both Decision 2 and 3 as, and Decision 2 (binding precedent) is cited and confirmed by Decision 3. The network representation on this situation is the following.

![Network Diagram of Rules of the Case in common law]

*Fig. 5. A simple network of Rules of the Case in common law.*

The Rules of the Case of the court in a common law system are connected by production links in a network representation in which such links are directed links. In networks representing common law subsequent cases are linked to previous cases, like in citation networks: the subsequent-citing Decision 2 cites and confirms the cited Decision 1, but not the other way around, because Decision 2 comes after Decision 1 in time. The link between Decision 1 and Decision 2 in the model adopted here is a production link, so that the direct link represents the production of Rules of the Case typical of common law.

In Fig. 5 it is assumed that there is no statute (general primary rule) to be applied by a court and that a court decides a case not relying on precedents (Decision 1), while other two courts rely on such Decision 1 as a binding precedent (Decisions 2 and 3). On the basis the general rule of production of *stare decisis* (SD) two individualized rules of production (RP2 and RP3) lead to Decisions 1 and 2. In Fig. 5 the directed outgoing links connecting the different Rules of the Case represent production links and form a closed network which has a certain density, measured by a clustering coefficient, obtained by dividing the number of links between the singular rules by the number of all possible links.

In general the local clustering coefficient in a graph with n nodes is the fraction between the number of the existing connections between nodes and the number of all possible connections between nodes; as a result the clustering coefficient for a fully connected graph is one. For example the clustering coefficient of the cluster made by Decision 1, Decision 2 and Decision 3 in Fig. 5 is one

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because the number of links between these singular rules (3) is equal to the number of all possible links (3). The clustering coefficient among common law cases of Fig. 5 shows the level of interconnectivity between them and implies an underlying legal semantics in terms of robustness of a common law trend based on precedents. In qualitative terms the high clustering coefficient of Decision 1, Decision 2 and Decision 3 represents the fact that, in the example, these decisions form a dense and homogeneous cluster of case law in which each previous decision is cited by subsequent decisions as legal precedent. The directed links among the Rules of the Case (Decision 1, Decision 2 and Decision 3) are production links which represent formal hierarchies between them (chains of production).

Recent literature concerning applications of network theories to common law has developed various measures to represent the cohesion of series of cases rendered under the *stare decisis* rule. Smith for example uses regression measures to examine the determinants of cohesion in the Supreme Court’s reliance on precedent in the network of U.S. Supreme Court precedents and highlights that the magnitude of ideological decision-making is consistently associated with a reduction in network cohesion. In another paper Seth Chandler has specifically analyzed the network structure of Supreme Court jurisprudence in the United States. The paper is mainly focused on methodologies of data collection and tool building and complements the results obtained by Thomas Smith. In particular the paper identifies four measure of “linkedness” of Supreme Court cases by presenting four different kinds of rankings based on number and type of links, the most cited cases, the most connected cases, the most “between” cases and the cases belonging to the “main core”. The most cited cases ranking is a measure of importance which looks at the number of times a case has been cited by other Supreme Court cases. The most connected cases ranking is based on the “geodesic”, which the shortest path between two nodes i.e. the number of citation between cases. The ranking of most “between” cases focuses on the nodes that lie on the highest proportion of network geodesics, based on the intuition that these serve as a communications hub that facilitate the transmission of ideas. Finally the cases belonging to the “main core” are identified through a procedure which remove the nodes that have fewer than a set number of connections to other nodes in the network.

This area of research on the authority of the Supreme Court precedents is completed by two recent papers by James Fowler et al. The two papers analyze two slight different data set containing the majority opinions written by the U.S. Supreme Court and the cases they cite from 1754 to 2002 in the *United States Reports* and describe a method for creating authority scores using the network data. The second paper, in particular, shows that cases that overrule a previous case quickly become and remain even more important as the reversed decisions decline, while the reversed cases tend to be much more important than other decisions confirming precedents.

The shortcoming of this line of research on links among common law cases is that it refers to links representing simple citation among cases, while such link are not evidence of actual production of rules through cases and therefore do no amount to production links in the sense adopted in this paper. Thus an important clarification must be made: network evolutionary analysis of cases cannot be generically based on citations between cases, but must be specifically based on the retention by

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subsequent cases of the holding of the previous case/cases. Evolutionary analysis of cases must be
context-oriented to define exactly which cases are confirmed in the holding of subsequent cases.
Not all citations represent a reliance on authority: courts cite precedents just to mention them in
passing or because they disagree; often cases are cited to build an argument leading to a decision
which is not similar or related to such cited precedents. In other situations precedent cases are cited
expressly to overrule them or to distinguish the facts, thereby eliminating rather than maintaining
production links. In general, precedents are cited within elaborate legal reasoning geared or based
on such cases, but without any evident link between those precedents and the decision in the case at
hand. In other situations precedents are cited in respect to their obiter dicta rather than in respect to
their ratio decidendi, and in some situations very remote and important cases on which a decision is
based are not even cited explicitly because it is obvious that they constitute a binding precedent45.

6. Network properties of legal systems: legal “connectors” and self-organization of Rules of the
Case

In previous sections 3 and 4 I described how the Rules of the Case are produced using production
links in network diagrams and I defined normative complexity as proliferation of Rule of the Cases
combined with branching-off of normative powers, while in section 5 I showed that normative
complexity implies the existence of networks of rules with varying cluster coefficients measuring
different levels of internal cohesion. Thus legal systems viewed as networks are a combination of
intertwined web of rules produced by a vast array of institutions; these institutions create a huge
number of Rules of the Case knitted together by numerous production links, leading to various
clusters of highly connected rules. The rules represented by nodes are not all uniformly linked to
each other, but rather, some nodes are densely connected by production links to other nodes in
clusters, and these clusters may be connected to one another at varying degrees.

This suggests that legal systems should be viewed by looking at their network properties both at
global and local level. Here the adjectives “global” and “local” do not refer to territorial aspects of
the legal system (such as distinctions between federal and state level) but to network properties
which have a topological nature. The network properties at global level are the underlying
hierarchical constraints imposed by formal hierarchies between rules (described above at section 3)
and which are the outcome of the operation of rules of production. The network properties at local
level are the features of the resulting structure of clusters of Rules of the Case created through
production links (decisions by administrative authorities, decisions by courts, private self-
compliance rules) with varying levels of clustering coefficients. Global and local network properties
are interdependent and intertwined as specific local relationships may affect collective behaviour
and the other way around. There are no empirical studies yet on the correlation between these
clusters, but it is reasonable to argue that Rules of the Case in the same legal cluster (for example a
lineage of cases in common law) are likely to be related to each other in terms of meaning and
subject matter, so that there these topological links are probably determined by an underlying legal
semantics.

The network properties of the legal system at the global level are all-pervasive and dictate how the
system works, while the network properties of the legal system at the local level define the
distributed characteristics of the system. Thus the legal system can be represented as a set of
clusters of Rules of the Case (for example lineages of cases in common law, coherent sets of
decisions by administrative authorities implementing statutes, sets of self-compliance rules by
individuals) determined by the rules of production. As a consequence a legal system as a whole

45 On implicit or mute law, see Gerald Postema, Implicit Law, 13 LAW AND PHILOSOPHY 361 (1994); Rodolfo Sacco,
would possibly be represented by a network in which at global level, there would be normative complexity (proliferation of Rules of the Cases and branching-off of normative powers), and at local level, there would be sections (represented by complete or quasi-complete graphs in which Rules of the Case are connected by production links) that have differing clustering coefficients.

The distinguishing network property of legal systems at the global level is the existence of a power-law feature because in actual legal systems there is a high number of Rules of the Case and a relatively small number of main nodes generating such Rules of the Case, as well as a high density of production links around these main nodes, which thus operate as “connectors”\(^{46}\). By transposing the network concept of connector into the legal context one can say that a connector is a node (a rule) which has a number of production links (which creates a number of other rules) exceeding the average number of rules which are produced by the other nodes (the other rules). In actual legal systems only a few nodes (rules) – like for example a leading case or a widely respected statute - have a very high number of production links (i.e. create a high number of Rules of the Case), while most of the nodes (rules) have a very limited amount of production links (i.e. create a limited number of Rules of the Case). Thus legal systems exhibit the striking feature of including nodes (rules) to which a very high number of production links (i.e. Rules of the Case) is connected.

Recent studies have specifically showed, for example, that there is a power law distribution of links among cases in common law with the result that few important cases operate as connectors. Further empirical research should be able to prove that this power law typical of complex networks operates also in respect to other aspects of the legal systems such as the proliferation Rules of the Case. At this stage of research empirical data are still lacking and thus it can only be inferred that there are two main different types of connectors in legal systems: statutes and regulations and secondary general rules of production establishing production links including the *stare decisis* rule in common law countries.

Statutes and regulations are big connectors in current legal systems in which legislative production is massive because they lead to a huge number of Rules of the Case through decisions by administrative authorities or courts and self-compliance by individuals. Change of law through statutes and regulations triggers multiple sets of Rules of the Case that are generated subsequently in chains of production from the newly introduced legislation, either through subsequent decisions of administrative agencies and case law, or through self-compliance by individuals. A statute with a low level of self-compliance by recipient individuals tends to trigger a high enforcement by administrative agencies; in other words a low number of self-compliance rules requires enforcement through Rules of the Cases issued by administrative agencies\(^{47}\). By contrast a statute with a high level of self-compliance by recipient individuals does not require significant enforcement by administrative agencies; in other words a high number of self-compliance rules ensures enforcement without the need of Rules of the cases issued by administrative authorities. These different types of proliferation of Rule of the Cases are associated with different costs: litigation and transaction costs for judicial decisions, operative costs for administrative decisions, compliance cost for self-compliance rules, and therefore the production of rules through rules in complex legal systems is a costly phenomenon. Thus by associating for example the enforcement costs to the number of


production links leading to the Rules of the Cases it is possible to determine the implementation cost of a rule, particularly of a statutory rule.

Also leading cases in common law are big connectors because numerous Rules of the Case are created on the basis of the binding precedent in a strikingly important path-dependence pattern. This is clearly showed by using the notation technique of Fig. 5 above to represent a network of cases in common law. The binding precedent rule of common law implies that each case consists of backward-looking constraints to connect the case to previous cases in order to reproduce in the ruling of those previous cases (previous nodes of the network), as well as forward-looking effects which connect that case to future cases which shall be decided (future nodes of the network).

The fact that a network property at local level is that only a few nodes have a very high number of production links, while most of the nodes have a very limited amount of them, significantly affects the operation of legal systems: a modification of a connector (for example the repeal of a statute which is massively enforced or self-complied, or the overruling of a leading case) has a direct impact on the creation of Rules of the Case and is more relevant than the change of a node which is not a connector (for example a general primary rule which is seldom enforced or self-complied, or the repeal of a minor decision that confirms only a few precedents). Fowler for example has evidenced that the cases of the Supreme Court in the United States that overrule previous cases increase their relevance because lead to an increased subsequent cases of the Supreme Court itself and that the reversed cases tend to be much more important than other decisions confirming precedents; this clearly shows the impact of overruling in the case law of the Supreme Court and the relevance of previous cases as rules that attribute normative power to future judges.

The distinguishing network properties of legal systems at the local level is that they exhibit “complex behavior”. Complex behaviour is a term which generally identifies the outcome of the operation of self-organization of systems resulting from the interdependence of their constituent elements. In respect to legal systems, complex behaviour specifically identifies the outcome of the operation of self-organization of rules (nodes) connected by production links. When there is normative complexity (proliferation of Rule of the Cases combined with the branching-off of normative powers) there is generation of rules irrespectively from the intentionality of individual actors. This is a subtle point: intentionality of actors is not denied by the network properties of legal systems, yet it is constrained by them. Each individual agent exercises a form of discretion, but the legal effects of the actions taken by these actors in the aggregate amount to properties of the network. As Kathrine Strandberg correctly notes, an important lesson from network science is “the importance of network structure in determining collective behaviors, such as social norms and behavioral regularities, and flows of information, influence and other goods. Legal analysis often conceptualizes individual legal actors as responding independently to legal rules in the context of global average social forces. Network science demonstrates that collective behavior may be determined not only by the average impact of global social forces but also by the specific network structures by which these social forces are mediated. Social network studies demonstrate, for


example, that access to information depends on one’s position in the network.”

Simple interactions can generate an emerging complex behaviour; this occurs in legal systems where actors with complete intentionality operate within network constraints defined by the chains of productions of rules described in previous paragraphs, and the result is an emerging normative complexity of the system.

There are basically three different types of self-organization at local level in legal systems: the behavior of court under the *stare decisis* rule, self-compliance by individuals and enforcement by agencies. Self-organization is particularly evident in common law when there is a high clustering coefficient among precedent and subsequent decisions (Rules of the Case, each regulating specific fact situations), as in these occurrences subsequent judges follow previous decisions; this phenomenon is captured in network terms by links representing formal hierarchies between cases. Self-organization is also evident when there is a high level of spontaneous compliance of statutes by individuals: in this situation a general primary rule leads to a kind of synchronized behavior of individuals, each self-enforcing with proper Rules of the Case the general primary rules of the statutes. Self-organization is finally evident in bureaucratic action enforcing statutes, as in a network of rules this action is represented as decisions (Rules of the Case) connected by production links in which previous decisions by agencies influence their following decisions and form coherent policies that lead to a situation a path dependency in which agencies follow their own previous policies.

Because of these self-organization properties, actual legal systems, viewed “bottom-up” in their constituent elements evidenced by network diagrams, are not centrally designed, even if in the higher echelons they consist of statutes devised “top-down” to address policy issues. The network diagrams of legal systems represent a multitude of Rules of the Case generated through production links (formal hierarchies) and each of them regulating an individual situation. In conclusion legal systems represented in network diagrams have a twofold property: (i) they generate a huge number of Rules of the Case exhibiting self-organization properties, and (ii) they appear as an interlocked and interdependent web of Rules of the Case with varying clustering coefficients. Although few data are available as to the number of Rules of the Case of actual legal systems as a whole, when legal systems are viewed “bottom-up” in this network perspective their most prominent features are that they operate as networks and that they exhibit normative complexity together with self-organization.

**7. Network growth and evolution legal systems.**

In the previous sections 2 through 6 I described the *structure* of legal systems which underlie their operation in the synchronic plane (i.e. in a single moment of time), while in section 6 I introduced the concepts of complex behavior and self-organization which occur in the diachronic plane (i.e. in different moments of time). In this section I continue to discuss how legal systems operate in the diachronic plane by showing that there is an evolutionary structure of legal systems which can be represented as a property of the network in different moments of time.

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Evolution of a legal system occurs between an initial moment \( t_1 \) and subsequent moments \( t_2, t_3, \ldots, t_n \). In a given initial moment of time \( t_1 \) (in the synchronic plane), the legal system is composed by (i) one single set of general rules, and (ii) different sets of singular rules. The single set of general rules is determined by the combination of formal and substantial hierarchies, as enacted general rules continue to exist until they are abrogated or declared invalid\(^{53}\). The different sets of Rule of the Cases include the Rules of the Case existing in moment \( t_1 \) within the legal system, i.e. singular rules created by agencies with proper powers (administrative authorities, courts, or individuals). In subsequent moments \( t_2, t_3, \ldots, t_n \) (in the diachronic plane) legal systems evolve because their structure at moment \( t_1 \) is modified. If one considers a legal system in subsequent moments of time, \( t_1, t_2, t_3, \ldots, t_n \), such legal system is seen as being made by different sets of rules (set1, set2, set3, setn) which follow one another. Thus one can identify the evolution occurring in moment \( t_2, t_3, \ldots, t_n \) if one has first identified the structure of the legal system at moment \( t_1 \).

Evolution of legal systems can be basically of two types: normative evolution when Rules of the Case directly regulating individual situations (Rule of the Cases) are added or eliminated, or network evolution when all other kinds or rules which potentially lead to the creation of new Rules of the Case are added or eliminated (these rules are general primary rules and singular/general secondary rules of production)\(^{54}\). There is normative additive evolution when Rules of the Case directly regulating individual situations are added, and network additive evolution when general primary rules and singular/general secondary rules of production are added. There is normative diminutive evolution when Rules of the Case directly regulating individual situations are eliminated, and network diminutive evolution when general primary rules and singular/general secondary rules of production are eliminated.

In practical terms with normative additive evolution the number of regulated individual situations actually increases depending on the number of Rules of the Cases which are added. By contrast, with network additive evolution the number of regulated individual situations changes only potentially but the normative complexity (as defined in this paper) is increased because singular/general secondary rules of production introduce new production links, and new general primary rules newly identify situations which are potentially subject to be individually regulated by Rules of the Case.

With normative diminutive evolution the number of regulated individual situations actually decreases depending on the number of Rules of the Cases which are eliminated\(^{55}\). By contrast, with network diminutive evolution the number of regulated individual situations changes only potentially but the normative complexity (as defined in this paper) of the legal systems is reduced because the elimination of singular/general secondary rules scales down the distribution of normative powers, and the elimination of general primary rules reduces the number of situations which are potentially subject to be individually regulated by Rules of the Case.

Thus considering both normative and network evolution and creation and elimination of rules, the evolution of legal systems can be of the following four types (shown in Table 4 below):

1. normative additive evolution: new Rules of the Case are added;
2. normative diminutive evolution: existing Rules of the Case are eliminated;

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\(^{53}\) Please note that those general rules which become invalid by operation of judicial or constitutional review, originally are existent.

\(^{54}\) Among secondary rules I consider here only rules of production in terms of creation of hierarchical structures through production links.

\(^{55}\) This situation occurs specifically when administrative decisions are revoked/annulled or judicial decisions appealed/overturned; therefore in legal systems normative diminutive evolution is a less massive phenomenon than normative additive evolution.
3. *network additive evolution*: new general primary rules and new singular/general secondary rules of production are added;

4. *network diminutive evolution*: existing general primary rules and existing singular/general secondary rules of production are eliminated.

<table>
<thead>
<tr>
<th>EVOLUTION</th>
<th>NORMATIVE</th>
<th>NETWORK</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADDITIVE</td>
<td>1. Type 1</td>
<td>3. Type 3</td>
</tr>
<tr>
<td></td>
<td>New Rules of the Case are added</td>
<td>New general primary rules and new singular/general secondary rules of production are added</td>
</tr>
<tr>
<td>DIMINUTIVE</td>
<td>2. Type 2</td>
<td>4. Type 4</td>
</tr>
<tr>
<td></td>
<td>Existing Rules of the Case are eliminated</td>
<td>Existing general primary rules and existing singular/general secondary rules of production are eliminated</td>
</tr>
</tbody>
</table>

Table 2 – *Types of evolution of legal systems*

The four types of evolution of Table 4 can thus be conceived as properties of a network representing the legal system in different moments of time in the following way:

1. Rule of the Cases which are added are represented by new terminal nodes of the network which directly regulate individual situations (*normative additive evolution*);
2. existing Rule of the Cases which are eliminated are represented by the elimination of those existing nodes of the network which directly regulate individual situations (*normative diminutive evolution*);
3. general primary rules and singular/general secondary rules which are added are represented by new nodes of the network which potentially create new production links leading to Rules of the Case\(^\text{56}\) (*network additive evolution*);
4. existing general primary rules and singular/general secondary rules which are eliminated are represented by the elimination of existing nodes of the network which potentially create new production links leading to potential Rules of the Case (*network diminutive evolution*).

As I have shown in section 3 discussing the chains of production of legal systems, there is a continuous *process of creation of norms through norms*: statutes are enacted pursuant to constitutional rules, regulations are issued pursuant to statutory rules, binding administrative and judicial decisions and self-compliance rules by individuals are created because of the existence of legislative rules and generate new Rules of the Case; various other rules (general and singular) are created as a result of delegation of powers within administrative authorities and because a limited number of legal connectors leads to a situation in which the Rules of the Case greatly outnumber the general primary rules found in the statutes.

When this process is represented by a network diagram a kind of “*network growth*” emerges as a property of the network showing that legal systems evolve increasing or decreasing the number of nodes and links. Growth is a fundamental feature of networks in general, as they change

\(^{56}\) All these types of rules do not directly regulate individual situations: (i) general primary rules may lead to Rules of the Case through decisions by administrative authorities or courts; (ii) general secondary rules establish formal hierarchies between rules; (iii) singular secondary rules are general secondary rules individualized by specific agencies creating new rules.
continuously by increasing the number of nodes and links and this feature is found in legal systems as well when represented as networks. Network dynamic representation of legal systems shows that as new nodes and links are added, normative complexity increases (defined above at section 4 as a combination of proliferation of Rule of the Cases and branching-off of normative powers). By contrast network representation shows that as existing nodes and links are eliminated, normative complexity decreases. As additive evolution usually exceeds diminutive evolution, normative complexity is a property of the evolving network.\textsuperscript{57}

The increase of normative complexity is highlighted by three aspects of evolution of legal systems. First, when there is normative additive evolution complexity increases because of the increase of the sheer number of Rules of the Case generated by underlying generative hierarchies; for example, common law undergoes a process in which the lineages of cases grow and branch-off while the cluster coefficients among connected cases are modified, or bureaucratic policies are subject to path dependency constraints. Second, when there is normative network evolution, the new general primary rules introduced in the legal system through legislative reforms potentially lead to the proliferation of Rule of the Cases (subsequent decisions by administrative authorities and courts, self-compliance rules) to ensure application of legislation. Third, when there is normative network evolution, new singular/general secondary rules of production are introduced in the legal system, triggering the branching-off of production links.

The connection between structure and evolution of legal systems which emerges as a network property is particularly evident when one looks at the change of secondary rules, as these rules attribute normative powers to specific institutions, which in turn lead to the exponential growth of Rules of the Case. As I have highlighted at section 4, in this process there is an increase of multiple chains of production of secondary rules (branching-off of normative powers) and this, in turn, leads to the exponential growth of Rules of the Case, as each new chain of production leads to a (potentially unlimited) set of Rules of the Case resulting from a general primary rule (proliferation of Rule of the Cases). Another network property of evolution of legal systems is that the modifications of big connectors have a greater impact than other modifications. A few rules (connectors) create a high number of singular rules (i.e. have a very high amount of production links), while most of the rules create a limited number of singular rules (i.e. have a very limited amount of production links). As a result, a change of one of the connectors dramatically impacts on the structure of the legal systems with a ripple effect in terms of potential production links (likelihood of creation of new Rules of the Case).

8. A case for the application of network theory to the U.S. tax system

The analysis of legal systems as networks of rules developed in this paper constitutes the groundwork for practical applications of the network approach, particularly in respect to complex aggregates of rules. The U.S. tax system is suited for these kinds of applications, as tax rules are contained in a hierarchically ordered corpus composed by the Internal Revenue Code (the “IRC”) and the Regulations (the “Regs”). In the U.S. a complex net of binding rules is in place, with the result that after the IRC, the most important source of tax law is the Regulations promulgated by the Treasury Department. The Code sections are numbered and the Regulations have a prefix “1” and

\textsuperscript{57} Normative complexity within the network would be reduced only if the modifications brought forth by diminutive evolution exceeded those brought forth by additive evolution, in other terms if the system were simplified by actually reducing its constituent elements.
the number following the prefix is the number of the section of the Code under consideration; this clearly indicates a formal hierarchy between IRC and regulations.

Most of current research on networks has been developed relying on data available on the Internet or in other databases; likewise it is possible to collect empirical data on the U.S. tax system by using existing database information (such as Westlaw or Nexis) relating to specific numbered Sections (§) of the IRC and Regs in order to retrieve the cases relating to such Sections. A practical concept of “rule” should be used here as an actual set of regulatory arrangements concerning a specific tax policy issue, for example a specific Section (or part of a Section) under Subpart F rules concerning Controlled Foreign Corporations, or under any other Part or Subpart of the IRC.

The applications of the network approach to the U.S. tax system envisaged here can be summarized in the following areas: 1) network analysis of the U.S. tax system and measurement of normative complexity; 2) measurement of enforcement and self-compliance of specific rules of the IRC and Regs; 3) analysis of complex behaviour and self-organization of clusters of tax Rules of the Case; 4) tax design and policy issues.

The first kind of practical applications of network theory is structural analysis of segments of the IRC, for example when devising legislative changes, or for general purposes of maintenance of the normative organization of the Code. By network analysis I mean here the analysis of the hierarchical structures among different types of rules resulting from production links (see Table 1 above). One set of research questions posed by this network analysis are those relating to the level of branching-off of normative powers (see above section 4 on normative complexity) within segments of the IRC and can be phrased in network terms by looking at how long are the chains of productions made by secondary rules eventually leading to Rules of the Case, and by asking whether all these secondary rules are necessary and/or appropriate. A second set of research questions is related to general primary rules, i.e. those rules of the IRC and Regs that lead to the creation of Rules of the Case through the production links described above at section 3 (IRS agencies, tax courts, self-compliant taxpayers). The measurement of Rules of the Case is aimed at defining the scope of actual proliferation of rules, which, together with the branching-off of normative powers, is a dimension of normative complexity of the IRC.

The second kind of practical applications of network theory to the U.S. tax system is the analysis of enforcement and self-compliance of specific rules of IRC and Regs. In network terms, the measurement of enforcement of rules of IRC and Regs is the number of individual applications of these rule by IRS agencies or tax courts. While tax cases applying specific rules of the IRC are published and available on databases, individual audits are not. Therefore to develop a network description of segments of the IRC it would be necessary to collect also data on applications through Rules of the Case (decisions by IRS agencies) of specific rules of the IRC. That could be

Local tax doctrine and case law has clarified in details how the IRC and Regulations interact and this has to do also with the validity problem. The legislative regulations are given “controlling weight unless they are arbitrary, capricious, or manifestly contrary to the statute” (National Muffler Dealers Ass’n v. U.S.), and “only if the code has a meaning that is clear, unambiguous and in conflict with a regulation does a court have the authority to reject the Commissioner’s reasoned interpretation and invalidate the regulation” (Redlark v. Commissioner). Interpretive regulations are entitled to great weight, but may be held invalid by a court when they are inconsistent with the statutory language or when there is no statutory to support the regulatory provision. When a statute may reasonably be construed to have any one of several alternative meanings, the selection of one such meanings in the Regulations will be conclusive even if one or more of the alternative constructions were preferable (Fulman v. U.S.). A court will invalidate a Regulation only if its construction is “unreasonable and inconsistent with the language, history and purpose of the statute” (Estate of Bullard v. Commissioner).

done internally at IRS level where data on audits are available. In practice, in network terms, the research question would be how many Rules of the Case are produced as individual applications of a specific general primary tax rule of the IRC and Regs and this would be measured by constructing networks of links between these primary rules and the related Rules of the Case (see Fig. 3 above for a simplified description).

Each application of a general primary rule of the IRC and Regs leading to a Rule of the Case is a production link in the network representing a segment of the IRC, and when a single general primary rule leads to many Rules of the Case, that general primary rule is a legal connector. Therefore this kind of analysis would be aimed at finding whether specific rules of the IRC and Regs are relevant in network terms or not. This assessment has an impact in terms of tax reform, as the repeal of a connector which may be found within the IRC has a greater impact than the repeal of other rules. Application of general primary rules of IRC and Regs can be a result of enforcement by the IRS, but also a result of litigation ensuing from tax audits. Network analysis therefore should also be based on the production links that a general primary rule of the IRC and Regs can generate in terms of cases decided as a result of litigation. In practical terms the network approach should also address the question as to how many enforced cases are litigated within segments of the U.S. tax system.

Direct enforcement and litigation are also connected to self-compliance, considering that self-compliance Rules of the Case are those found in duly filed tax returns that are not challenged by IRS and/or litigated. The tax returns that are not audited or litigated constitute applications of general primary rules of IRC and Regs and therefore are Rules of the Case in network terms. These self-compliance Rules of the Case are neither published (like tax cases) nor traceable (like tax audits), but take the form of tax returns (specifically those parts of tax returns which are self-applications by taxpayers of specific rules of IRC and Regs). Hence it is possible to have an approximate measure the production links of a general primary rule that leads to self-compliance Rules of the Case by adding to the number of tax returns audited by the IRS (decisions) the number of tax returns not audited by the IRS (filed tax returns, the vast majority of cases). If the analysis also encompasses self-compliance rules, a likely result of network approach to tax enforcement would then be that those provisions of the IRC and Regs that are widely self-complied with and that do not trigger audits or litigation operate as connectors within the network, according to power law distributions.

In conclusion the measurement of Rules of the Case resulting from direct enforcement by IRS, litigation and self-compliance may identify the rules of the IRC and Regs that generate the highest number of production links, thereby operating as legal connectors. Furthermore, by associating costs to each production link (administrative costs and resources for enforcement, litigation costs for cases, compliance costs for self-compliance by taxpayers) it would be possible to gauge segments of the IRC in terms of distributed costs along the network; for example an IRC rule that is heavily audited would be associated with costs borne by the IRS (production links connecting such a rule with each IRS decision), while an IRC rule that is widely self-enforced would be associated with compliance costs that are shifted to individual taxpayers.

The third kind of practical applications of network theory to the U.S. tax system is the analysis of complex behavior and self-organization of clusters of tax Rules of the Case; this application has been already proved relevant in the analysis of evolution of common law cases, and research has already been conducted in this area by looking at the citations among Supreme Court case. This research could be further improved by looking at the retention by subsequent tax cases of the holding of previous case/cases (see Fig. 5 above) in respect to specific issues within the IRC. This
would imply, for example, the analysis of production links between precedent and subsequent tax cases in specific areas and could lead to measures of evolutionary fitness of lineages of cases.

A direct measure of the evolutionary fitness of a case is the number of production links generated by such case because, as noted above at Section 5, every time a precedent case is binding on a subsequent case there is a production link between the precedent case and the subsequent case. The mapping of links among tax cases (Rules of the Case) should identify which cases are connectors, (i.e. which are the cases that have a higher number of links than other cases), as well measure the clustering coefficient of groups of cases. A similar kind of analysis could also be developed in respect to enforcement policies by looking at the production links connecting previous decisions to subsequent decisions taken by IRS agencies.

The fourth kind of practical applications of network theory to the U.S. tax system are those relating to tax design and policy issues which involve the impact of change of rules on segments of the IRC represented as a network of rules. A change of U.S. tax system viewed from a network perspective can in theory occur through policy arrangements that vary along a continuum from a “top-down pattern” to a “bottom-up pattern” of tax design, a summarized below at Table 3. In a “top-down pattern” a change of the U.S. tax system would be introduced either (i) by amendments to the IRC bound to be subsequently interpreted and implemented by IRS guidelines and/or case law, or (ii) by guidelines issued by the IRS in conjunction with case law without recourse to legislation. In a “bottom-up pattern” a change of the U.S. tax system would be triggered by case law.

<table>
<thead>
<tr>
<th>CHANGE IMPLEMENTED THROUGH IRC AND REGS</th>
<th>CHAINS OF PRODUCTION</th>
<th>TYPE OF ADDITIVE EVOLUTION</th>
<th>POLICY PATTERN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amendments to IRC and Regs directly regulating behavior combined with subsequent</td>
<td>Network evolution New general primary rules</td>
<td>Top-down (legislative discretionary powers)</td>
<td></td>
</tr>
<tr>
<td>Decisions by IRS agencies</td>
<td>Normative evolution New Rules of the Case</td>
<td></td>
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<tr>
<td>Decisions by tax courts</td>
<td></td>
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<tr>
<td>Private self-compliance</td>
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<tr>
<th>CHANGE IMPLEMENTED THROUGH IRS AGENCIES</th>
<th>CHAINS OF PRODUCTION</th>
<th>TYPE OF ADDITIVE EVOLUTION</th>
<th>POLICY PATTERN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amendments to IRC and Regs that distribute normative powers among agencies Decisions by IRS agencies regulating individual situations</td>
<td>Network evolution New general/singular secondary rules of production</td>
<td>Top-down (administrative discretionary powers)</td>
<td></td>
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<td></td>
<td>Normative evolution New Rules of the Case</td>
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<tr>
<th>CHANGE WHICH OCCURS THROUGH CASE LAW</th>
<th>CHAINS OF PRODUCTION</th>
<th>TYPE OF ADDITIVE EVOLUTION</th>
<th>POLICY PATTERN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decisions by tax courts</td>
<td>Network evolution New singular secondary rules of production</td>
<td>Bottom-up (judicial discretionary powers and binding precedent in common law systems)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Normative evolution New Rules of the Case</td>
<td></td>
<td></td>
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</tbody>
</table>

TABLE 3 – Network impact of change in the U.S. tax system.
A summarized view of the possible types of change of the U.S. tax system and their impact in network terms is presented at Table 2 above, where each type of tax change is considered in respect to various factors: the underlying chain of production (first column), the type of additive evolution of the tax system (second column), and the type of policy pattern (third column). Table 2 shows that tax change can be implemented through amendments to IRC and Regs, through IRS agencies or can occur spontaneously through case law. Table 2 also shows the possible impact of legal change on the network structure of the U.S. tax system by looking at the various types of rules of the legal systems described at section 2 of the paper.

The first kind of modification in network terms of the U.S. tax system is that in which the change would be implemented through amendments to IRC and Regs. This is a “top-down pattern” of tax design in which a tax issue is solved by statutes and Regs which may be subsequently implemented by decisions by IRS agencies regulating individual situations and/or case law. The amendments to IRC and Regs lead to new Rules of the Case regulating individual situations and therefore they trigger network additive evolution. Therefore one should also look at the multiple sets of Rules of the Case (decisions by IRS agencies, case law, or self-compliance rules by individuals) within segments of the IRC that are the outcome of the IRC changes. The impact of the legislative change therefore should be assessed in respect of these Rules of the Case generated within the U.S. tax system. For example an IRC section with a low level of spontaneous self-compliance by recipients may trigger a high enforcement through Rules of the Cases issued by IRS (or possibly by courts in ensuing litigation) and therefore may imply significant enforcement costs.

The second kind of possible modification of the U.S. tax system is that in which the change would be implemented through IRS agencies, i.e. decisions taken by those IRS agencies that regulate ex post individual situations. This is also a “top-down pattern” which is however more flexible than simple legislative change, as it usually operates through the exercise of administrative discretionary powers rather than through the enactment of detailed statutory rules. This kind of tax change could also be prompted by legislation that re-distributes normative powers of IRS agencies. When this occurs new production links are added to the network leading to potential Rules of the Case. The result of the tax change in these cases is therefore a reorganization of segments of the U.S. tax system through a replacement of secondary rules resulting in a different distribution of normative powers. These kinds of change increases the normative complexity of the U.S. tax system as new chains of production are added generating (potentially unlimited) multiple sets of subsequent Rules of the Case. These situations occur when the adopted policy solution requires procedural arrangements to ensure compliance, for example the setting up of proper offices and the monitoring of the rulings issued over time. The impact of these changes of the U.S. tax system should therefore be assessed in respect to the increased normative complexity created by new branching-off of normative powers, as well as in respect to the new Rules of the Case which can be created as a result of these normative powers. By contrast, when IRS directly issues decisions regulating ex post individual situations there is normative additive evolution as new Rules of the Cases are introduced in the tax system through administrative action.

The third kind of modification of the U.S. tax system is that in which the change occurs through case law, i.e. decisions by tax courts. This kind of change constitutes a “bottom-up” pattern and cannot be directly controlled by policy-makers (legislators or IRS agencies). In a common law system, however, these case law trends have a kind of self-organization and complex behavior which is caused by the high clustering coefficient among cases and can lead to stable situations.
which can be described by network analysis by looking at the clustering coefficients between such cases.