2011

Comparing L-Moments and Conventional Moments to Model Current Speeds in the North Sea

Cameron A. MacKenzie, *iowa*
Steven R. Winterstein

Proceedings of the 2011 Industrial Engineering Research Conference
T. Doolen and E. Van Aken, eds.

Comparing L-Moments and Conventional Moments to Model Current Speeds in the North Sea

Cameron A. MacKenzie
CMackenzie@ou.edu

Steven R. Winterstein
SteveWinterstein@alum.mit.edu

Abstract

Accurately modeling current speeds at different depths is important for designing deep-water structures such as oil production platforms. These platforms are typically designed to withstand T-year loads, where the return period T may range from T=100 to T=10,000 years. We focus here on estimating T=100-year current speeds at a North Sea site. Conventional moment-fit results are compared with alternative models that use the data's L-moments. Different probability distribution models are studied. Two separate approaches are also compared: (1) a "vertical window" approach, in which peak data above a threshold speed are used, and (2) a "horizontal window" approach, which considers peak data over regular windows of time (here, monthly maxima).

Keywords
probability, extreme events, distribution fitting, moments, L-moments

1. Introduction

Accurately estimating the tails of a distribution is important for many risk-based applications. In finance, environmental sciences, and engineering, calculating values at the 99th or higher percentile can help decision makers determine the maximum level of risk against which they should protect. For example, offshore oil production platforms are typically designed to withstand T-year loads, where the return period T may range from T=100 to T=10,000 years. Because currents and waves generally show non-Gaussian features, estimating their T-year extremes is particularly challenging.

This paper models the extreme current speeds of the Ormen Lange site in the North Sea, of interest for oil and natural gas production. The data of current speeds at this site cover a little more than a year and a half. We extrapolate from that data to estimate the 100-year current speeds at various depths. Several other papers have explored this question by examining correlations between speeds at different depths and using regression to solve for distribution parameters [1, 2]. This paper builds upon those studies and uses some of the same models. We also bring into this modeling approach a comparison of L-moments [3, 4] and conventional moments to model extreme events.

The second section introduces L-moments and explains how sample L-moments can be calculated from data. Two modeling approaches for predicting extreme events are used: (1) a "vertical window" approach, in which peak data above a threshold speed are used, and (2) a "horizontal window" approach, which considers peak data over regular windows of time (here, monthly maxima). We fit distributions to data from both approaches, and compare distributions matched to conventional moments with distributions matched to L-moments. We conclude by discussing the relative merits of the various approaches and distribution models.

2. L-Moments

L-moments are linear combinations of the elements of an ordered sample of a random variable. Consider an ordered sample of size n such that $(X_{1:n} \leq X_{2:n} \leq \ldots \leq X_{n:n})$ are drawn from the distribution of X. The nth L-moment, λ_n, is a linear combination of the order statistics, $E[X_{i:n}]$, where $1 \leq i \leq n$ [4]. Similar to most other works using L-moments [4]-[6], this paper will focus on the first four L-moments.

$$\lambda_1 = \mathrm{E}[X_{1:1}]\ ; \quad \lambda_2 = \frac{1}{2}\mathrm{E}[X_{2:2} - X_{1:2}]\ ; \quad \lambda_3 = \frac{1}{3}\mathrm{E}[X_{3:3} - 2X_{2:3} + X_{1:2}]\ ; \quad \lambda_4 = \frac{1}{4}\mathrm{E}[X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}] \quad (1)$$

Rather than using the third and fourth L-moments directly, it is more convenient to use the L-moment ratios, L-skewness and L-kurtosis.

$$\tau_3 = \frac{\lambda_3}{\lambda_2}\ ; \quad \tau_4 = \frac{\lambda_4}{\lambda_2} \quad (2)$$

L-moments are similar to conventional moments, and the first L-moment is equivalent to the mean. The second L-moment, L-scale, measures a random variable's dispersion or the expected difference between two randomly drawn samples. The third L-moment is a measure of skewness. If the distribution is skewed to the right, then we expect $X_{3:3} - X_{2:3}$ to be greater than $X_{2:3} - X_{1:3}$ and that $\tau_3 > 0$. If the distribution is skewed to the left, we expect that $\tau_3 < 0$. The fourth L-moment measures kurtosis, or the peakedness of a distribution. A distribution with broader tails should have $\tau_4 > 0$, which implies that the difference between the extreme values, $X_{4:4} - X_{1:4}$, is more than three times greater than the difference between the central values, $X_{3:4} - X_{2:4}$ [4].

Given an ordered set of data of size m, $(x_{1:m} \leq x_{2:m} \leq \ldots \leq x_{m:m})$, the sample L-moments l_1, l_2, l_3 and l_4, the sample L-skewness, t_3, and the sample L-kurtosis, t_4, can be calculated.

$$\begin{gathered} l_1 = b_0\ ; \quad l_2 = 2b_1 - b_0\ ; \quad l_3 = 6b_2 - 6b_1 + b_0\ ; \quad l_4 = 20b_3 - 30b_2 + 12b_1 - b_0 \\ t_3 = \frac{l_3}{l_2}\ ; \quad t_4 = \frac{l_4}{l_2} \end{gathered} \quad (3)$$

where

$$b_r = m^{-1} \sum_{j=r+1}^{m} \frac{(j-1)(j-2)\ldots(j-r)}{(m-1)(m-2)\ldots(m-r)} x_{j:m} \quad (4)$$

Because sample L-moments are less influenced by outliers than conventional moments, Hosking [3, 7] and others [8]-[10] have used L-moments to fit distributions to a set of data. The parameters of several distributions, each with two or three unknown parameters, are calculated in order that the first two or three sample L-moments from the data equal the first two or three exact L-moments of the distributions. L-skewness and L-kurtosis can be used as goodness-of-fit measures to compare which distribution provides the best fit for the data. This process of using L-moments to fit a distribution to a set of data has been applied to problems in modeling wind velocities [8], the response of steel risers to extreme conditions [9], traffic densities [11], and hedge fund returns [12], among others.

However, precisely because L-moments are less sensitive to a distribution's tails, L-moments may not provide a good estimation of the probability of extreme events. As demonstrated in [13], distributions fitted to match L-moments may diverge from the true distribution much sooner than distributions fitted to match conventional moments. We compare distributions matched to L-moments with distributions matched to conventional moments for the specific application of modeling extreme current speeds in the Norwegian Sea. We deploy distributions frequently used to model extreme events in order to estimate the 100-year current speed at the Ormen Lange natural gas field in the Norwegian Sea.

3. Ormen Lange Current Data

Ormen Lange is located in the Norwegian Sea, about 140 kilometers off the coast of Norway, and is the largest natural gas field on the Norwegian continental shelf. The Norwegian energy company Statoil has been producing natural gas there since September 2007, and the company measures the current speeds at several different depths ranging from 20 to 750 meters. Calculating 100-year or even 1000-year current speeds can help Statoil properly design engineering structures that are resilient to weather conditions in the Norwegian Sea. The data set used in this study includes measurements at two different depths, 75 and 300 meters. The speed of the current is measured at each depth every 10 minutes. The data set for each depth consists of $97,024$ points, which corresponds to $97,024 \big/ (6 * 24 * 365) = 1.85$ years or approximately 22.5 months.

4. Peak-Over-Threshold Data

Because of our interest in extreme values, we first extract data points that are most relevant to modeling extreme current speeds. We use two different methods to extract the most relevant data: a peak-over-threshold model and a

Table 1: Summary statistics for peak-over-threshold [cm/s]

	75 meters	300 meters
Number of data points	163	156
Minimum	70.3	65.6
Maximum	99.7	106.2
Mean	75.4	72.0
Standard deviation	5.2	7.0
Skewness	2.2	2.3
Kurtosis	8.4	8.8
L-scale	2.5	3.3
L-skewness	0.39	0.44
L-kurtosis	0.22	0.23

monthly maxima model. We fit these new data sets separately to different distributions.

The peak-over-threshold model creates a more homogeneous sample by removing points that are irrelevant (i.e., those points that fall below the threshold) and correlated (by retaining only one point that exceeds a given threshold during a period of time) [2, 14]. Here we choose a threshold v_{TH} as the 99 percent fractile, leading to the values v_{TH}=70.2 cm/s at the 75-meter depth and v_{TH}=65.5 cm/s at the depth of 300 meters. Between every upcrossing of the threshold v_{TH} and its subsequent downcrossing, we retain only the peak (maximum) value. Table 1 displays the summary statistics resulting from this method.

We use two distributions to model the peak-over-threshold data. The first model assumes that $W = V - \nu_{TH}$ follows a Weibull distribution, where V is the "peak" for every upcrossing of the threshold ν_{TH} [15]. The random variable W is the difference between the peak and the threshold. Because the Weibull has two parameters, a scale and a shape parameter, we match the first two moments (both the conventional and the L-moments) to derive the two parameters.

4.1 Weibull Model

A random variable W is said to follow a Weibull distribution if its cumulative distribution function (CDF) is as follows:

$$F_W(x) = 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^{(1/p)}\right] \; ; \quad x \geq 0 \tag{5}$$

Its two parameters, α and p, both positive in value, are related to the mean, μ, and variance, σ^2, of W as follows:

$$\mu = \alpha\Gamma(1+p) \; ; \quad \sigma^2 = \alpha^2\Gamma(1+2p) - \mu^2 \tag{6}$$

in which $\Gamma(\cdot)$ is the gamma function. Our 2-moment Weibull fit follows the standard practice of estimating μ and σ^2 by the sample moments of the data, and inferring consistent α and w values.

The L-moments can also be expressed as functions of the Weibull's two parameters.

$$\lambda_1 = \alpha\Gamma(1+p) \; ; \quad \frac{\lambda_2}{\lambda_1} = 1 - \frac{1}{2^p} \; ; \quad \frac{\lambda_3}{\lambda_1} = 1 - \frac{3}{2^p} + \frac{2}{3^p} \; ; \quad \frac{\lambda_4}{\lambda_1} = 1 - \frac{6}{2^p} + \frac{10}{3^p} - \frac{5}{4^p} \tag{7}$$

The Weibull fit to L-moments means solving for α and w so that λ_1 and λ_2 match the sample L-moments of the data.

4.2 Quadratic Weibull Model

To account for skewness in the data, we use a Quadratic Weibull distribution for the peak-over-threshold data [16]. Conventionally, the Quadratic Weibull model begins with W, a Weibull variable fit to the first two moments of the data (as described above). It then adds a quadratic correction:

$$Z = z_{min} + \kappa(W + \varepsilon W^2) \tag{8}$$

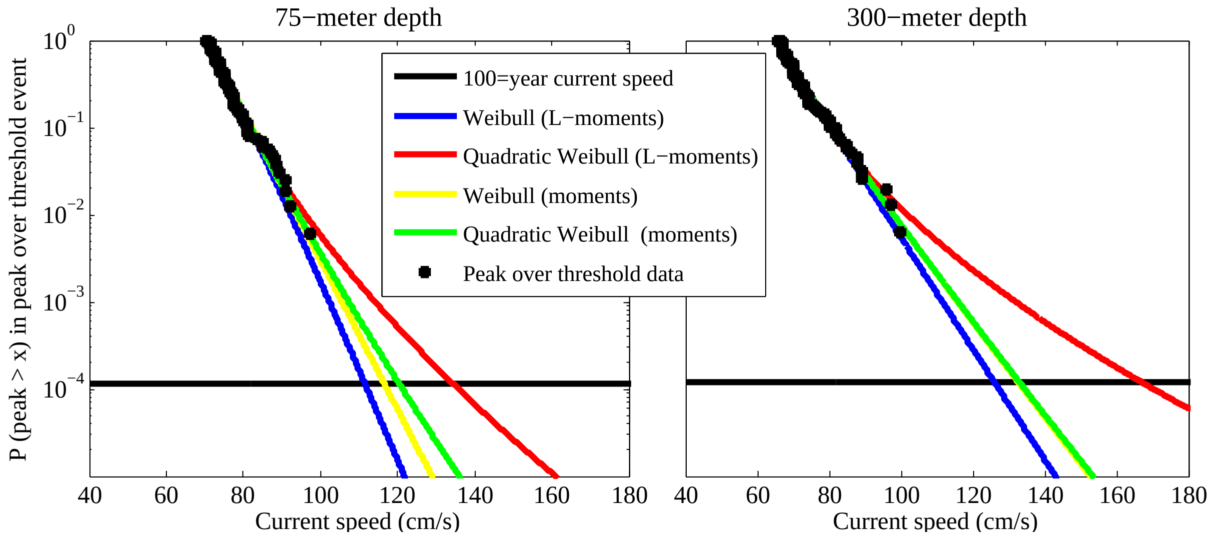

Figure 1: Probability distributions for peak-over-threshold data

The three new parameters—z_{min}, κ, and ε—are found sequentially: (1) ε is first found to preserve the skewness of the data, (2) the scaling factor κ is used to preserve the variance, and (3) the shift z_{min} is finally introduced to preserve the mean value [17].

We also fit an alternative Quadratic Weibull model, based on the first 3 L-moments of the data. In this case, W is first fit to λ_1 and λ_2, and then ε, κ, and z_{min} are sought to preserve the L-skewness, L-scale, and mean values respectively.

4.4 Numerical Results

The peak-over-threshold model and the distributions matching to two and three moments are shown in Figure 1. The distribution of the data (the plotted points) is determined by assuming $F(x_{i:m}) = \frac{i}{m}$.

We focus first on the Weibull models, fit to the first two conventional moments or L-moments. In such cases the next highest moment of the fitted model—here, the skewness, α_3, or L-skewness, τ_3—can be used to test its goodness-of-fit. In this case, we find the conventional two-moment Weibull fit to be more consistent with the data than the corresponding L-moment fit. For example, at depth d=300m the fitted Weibull distribution has α_3=2.4, while the sample data show α_3=2.3. In contrast, for the Weibull fits based on L-moments, the fitted models predict the L-skewness, τ_3, to have the values (0.30, 0.34) at d=75m and d=300m. These differ markedly from τ_3 results of the sample data, (0.39, 0.44), as cited in Table 1.

We use simulation to assess the statistical significance of these differences. Specifically, we use the parameters of the fitted Weibull distributions to randomly generate new samples of the same size (163 data for 75 meters and 156 data for 300 meters). If the new data has a mean and standard deviation or L-scale within 5 percent of the sample statistics of the original data, we calculate the skewness or L-skewness of this new random sample. Using this simulated set, we can estimate the likelihood of getting the skewness or L-skewness from the original data set given the parameters of the Weibull distribution.

The simulation results support the superiority of the moment-fit Weibull over its L-moment counterpart. For the Weibull model fit to L-moments, only a few out of over 3000 sets of simulated data points have L-skewness values as large as (.39, .44), as found for the data. When the Weibull is instead matched to conventional moments, 15 percent of the data sets have a skewness at least as large as 2.2 at 75 meters, and 24 percent have a skewness at least as large as 2.3 at 300 meters.

Similarly, the fourth moment can be used as a goodness-of-fit measure for the Quadratic Weibull distribution. In simulating from this distribution, we only keep a set of data points if the skewness of those points is within 5 percent of the

Table 2: Summary statistics for monthly maxima [cm/s]

	75 meters	300 meters
Number of data points	23	23
Minimum	49.8	48.3
Maximum	99.7	106.2
Mean	80.5	77.7
Standard deviation	12.0	13.7
Skewness	-0.81	0.09
Kurtosis	3.5	2.8
L-scale	6.8	7.8
L-skewness	-0.15	0.03
L-kurtosis	0.20	0.16

sample skewness of the original data. In this case, our simulations show that *both* Quadratic Weibull models—based on moments or L-moments—are statistically acceptable. Each is quite quite likely to generate the corresponding sample kurtosis or L-kurtosis at both water depths.

Nonetheless, the Quadratic Weibull model based on L-moments appears somewhat unsatisfactory here as a model of extremes. Recall that it is based on a Weibull variable W in Equation 8 fit to the first two L-moments of the data. As our preceding results showed, this model of W is narrower in its tail than the data, yielding improbably small τ_3 estimates. To compensate, the Quadratic Weibull must use a correction term (εW^2 in Equation 8) that is relatively large, and hence it predicts much broader distribution tails and 100-year speeds. It may well be that this volatility of extremes based on L-moment fits arises from the general insensitivity of these moments to distribution tail characteristics [13].

5. Monthly Maxima

Another method for retaining the most important points for modeling extreme events is to select the most extreme point during a given time period [1]. We select the maximum current speed for each "month" in the original data set of $97,024$ speeds. We approximate a month by assuming that each month is 30 days, so the data set is divided into 22 sets of 30 days each (or $30*6*24 = 4320$ points) and a 23rd set that only includes 1984 points. From each set, we extract the maximum current speed. Table 2 shows summary statistics.

5.1 Gumbel Model

Extreme values of environmental parameters such as monthly maxima are often modeled with a Gumbel distribution [18] with the distribution function $F_G(x)$.

$$F_G(x) = \exp\left[-e^{-(x-\xi)/\omega}\right]\;;\qquad -\infty < x < \infty \tag{9}$$

where ξ is the location parameter and $\omega > 0$ is the scale parameter. As with the Weibull distribution, we solve for these two parameters so that the Gumbel distribution matches the first two conventional moments or the first two L-moments:

$$\mu = \lambda_1 = \xi + \omega\gamma\,;\quad \sigma^2 = \frac{\pi^2}{6}\omega^2\,;\quad \lambda_2 = \omega\log(2) \tag{10}$$

where γ is Euler's constant, 0.5772 [4].

5.2 Generalized Extreme Value Model

In order to incorporate skewness or L-skewness, we use the generalization of the Gumbel distribution called the Generalized Extreme Value (GEV) distribution, $F_{GEV}(x)$.

$$F_{GEV}(x) = \exp\left[-\left(1-\frac{k(x-\xi)}{\omega}\right)^{1/k}\right]\;;\qquad 1-\frac{k(x-\xi)}{\omega} \geq 0 \tag{11}$$

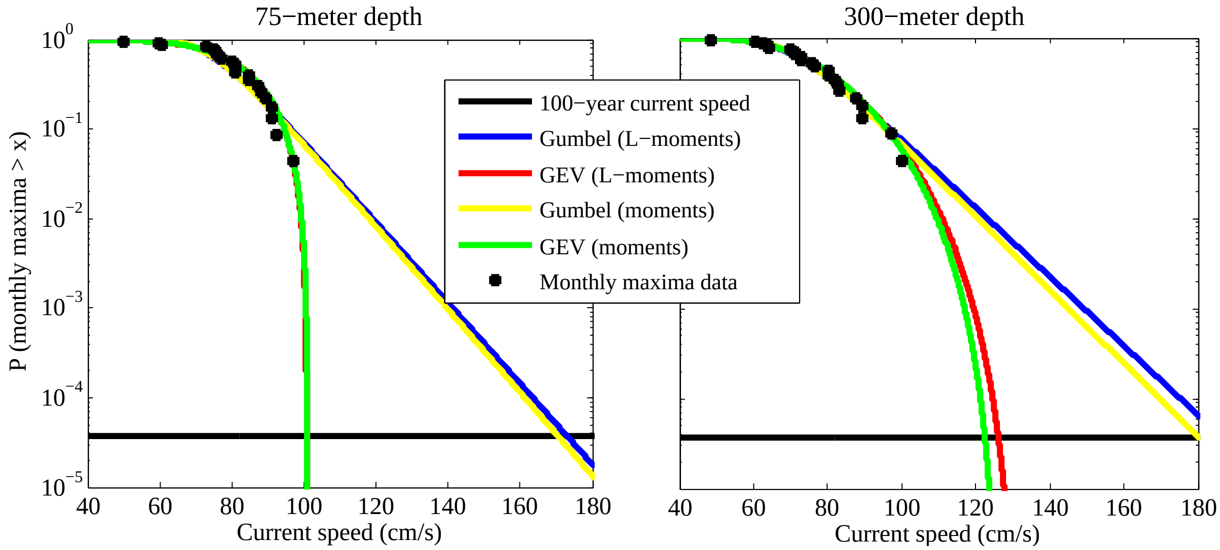

Figure 2: Probability distributions for monthly maxima data

where k is a shape parameter. For the special case when k=0, this result returns a Gumbel distribution. If $k > 0$, Equation 11 shows that the GEV distribution has an upper bound maximum at

$$x_{max} = \xi + \frac{\omega}{k} \tag{12}$$

The first three conventional moments and the first three L-moments can be expressed as functions of the location, scale, and shape parameters.

$$\begin{gathered} \mu = \lambda_1 = \xi + \frac{\omega[1-\Gamma(1+k)]}{k}\ ; \quad \sigma^2 = \frac{\omega^2(g_2 - g_1^2)}{k^2}\ ; \quad \alpha_3 = \frac{-g_3 + 3g_1 g_2 - 2g_1^3}{(g_2 - g_1^2)^{3/2}} \\ \lambda_2 = \frac{\omega(1-2^{-k})\Gamma(1+k)}{k}\ ; \quad \tau_3 = \frac{2(1-3^{-k})}{1-2^{-k}} - 3 \end{gathered} \tag{13}$$

where $g_j = \Gamma(1+kj)$. We use Matlab to solve these equations so that the parameters in the GEV distribution match either the conventional moments or L-moments.

5.3 Numerical Results

The monthly maxima current speed and the distributions matching to two and three moments are shown in Figure 2. At d=75m, the distributions matched to conventional moments are virtually identical to the corresponding distributions matched to L-moments. At d=300m, the distributions are very similar but the distributions matched to L-moments have slightly broader tails than the distributions matched to conventional moments.

A Gumbel distribution's skewness is $\alpha_3 = 1.1$ and its L-skewness is $\tau_3 = 0.17$. Because the sample α_3 and τ_3 values are negative at d=75m and very close to zero at d=300m, the GEV distributions matched to three moments require $k > 0$ to achieve a smaller skewness. Because $k > 0$, Equation 12 shows that the GEV distribution predicts an upper bound maximum on the current speed: x_{max}=101 cm/s at d=75m for moments and L-moments, x_{max}=127 cm/s at d=300m for conventional moments, and x_{max}=132 cm/s for L-moments. These truncation levels seem rather implausible, and likely to yield unconservative estimates of T-year loads (e.g., for T=100 years and beyond).

Regarding the Gumbel model, it appears to substantially overestimate extremes. We may again quantify this goodness-of-fit through the first moment not used in the fitting; here, the skewness. At the 75-meter depth, simulating the Gumbel distributions reveals only one set of 23 data points out of more than 3000 sets with $\tau_3 \leq -0.15$, and no data sets with $\alpha_3 \leq -0.81$. At 300 meters, only 10 percent of the simulated data sets have $\alpha_3 \leq 0.09$, and only 5 percent have $\tau_3 \leq 0.03$. These results suggest that the Gumbel distribution is also a rather implausible model for this data set, regardless of whether it is fit to L-moments or conventional moments.

6 Conclusion

Four different distributions were used to model the extreme current speeds at Ormen Lange: Weibull and Quadratic Weibull distributions for peak-over-threshold data, and Gumbel and GEV distributions for monthly maxima data. For each of these four distributions, we solved for parameters to match conventional moments and to match L-moments. Higher-order moments were used to measure how well the distributions matched the data.

Estimating T=100-year conditions from a database several orders of magnitude shorter—here, T=1.85 years—is a daunting task, and any estimate of 100-year current speed must contain significant uncertainty. Nonetheless, we believe some general conclusions are warranted. First, we find neither the Gumbel nor the GEV distribution to yield a satisfactory model of monthly maxima. The Gumbel model is found overbroad, predicting implausibly large 100-year speeds (more about this below). To compensate, the GEV model predicts—as it must when any narrower-than-Gumbel data set is encountered—an upper-bound threshold x_{max} (Equation 12). While it may be plausible to infer an upper-bound speed from physical principles, it seems quite tenuous to do so based on the statistics of a T=1.85 year database. We therefore caution against the use of GEV fits to predict extremes whenever "sub-Gumbel" conditions (i.e., $k > 0$ cases) are encountered. In our experience, these cases seem to arise rather often.

Models based on peak-over-threshold events appear more promising. Here we have fit both Weibull and Quadratic Weibull models, using both conventional moments and L-moments. The conventional two-moment Weibull fit agrees with its three-moment extension, suggesting that it is fairly well-supported by the data. The L-moment counterparts are plausible but somewhat volatile. The Weibull fit to two L-moments has a rather narrow tail (and improbably low τ_3). To compensate, including τ_3 in a Quadratic Weibull fit yields a rather broad-tailed model. This may reflect the deficiency of L-moments, relative to ordinary moments, in accurately conveying information about the distribution tails.

Finally, it is interesting to speculate as to why the Gumbel model fails in this case. If the individual peak-over-threshold events occur in Poisson fashion and if $V - v_{TH}$ is exponentially distributed, the resulting extremes over a regular interval should precisely follow the Gumbel model. The exceedance probabilities of $V - v_{TH}$ in Figure 1 do appear to fall in roughly log-linear fashion, consistent with an exponential model. Why, then, does the Gumbel model fail? One suggestion lies in the relatively short time interval (1 month) over which maxima were chosen. This choice, while required by the relatively short database available, can lead to a rather heterogeneous sample; e.g., due to seasonal variations. To support this, note that the smallest of the monthly maxima (Figure 2) are less than v_{TH}, the 99% fractile threshold we assign to identify peak-over-treshold events. Thus, we have a more heterogeneous sample when considering monthly maxima than peak-over-threshold events. The net result of this is to favor "vertical window" peak-over-threshold approaches, which are likely to identify all extreme events of interest, over "horizontal window" approaches which consider maxima over regular periods of time, which may be heterogeneous especially if the periods are less than one year.

References

[1] Winterstein, S.R., and Haver, S., 2009, "Turkstra Models of Current Profiles," Proc. of the 28th International Conference on Ocean, Offshore and Arctic Engineering, May 31-June 5, Honolulu, Hawaii.

[2] Winterstein, S.R., Haver, S., Kvingedal, B., and Nygaard, E., 2011, "Turkstra Models of North Sea Currents: Wider Than You'd Think," Proc. of the 30th International Conference on Ocean, Offshore and Arctic Engineering, June 19-24, Rotterdam, Netherlands (submitted).

[3] Hosking, J.R.M, 1990, "L-moments: Analysis and Estimation of Distributions Using Linear Combinations of Order Statistics," Journal of the Royal Statistical Society, Series B, 52(1), 105-124.

[4] Hosking, J.R.M., and Wallis, J.R., 1997, Regional Frequency Analysis: An Approach Based on *L*-Moments, Cambridge University Press, Cambridge.

[5] Guttman, N.B., 1993, "The Use of L-Moments in the Determination of Regional Precipitation Climates," Journal of Climate, 6, 2309-2325.

[6] Pandey, M.D., van Gelder, P.H.A.J.M., and Vrijling, J.K., 2001a, "Assessment of an L-Kurtosis-Based Criterion for Quantile Estimation," Journal of Hydrologic Engineering, 6(4), 284-292.

[7] Hosking, J.R.M., 1992, "Moments or L Moments? An Example Comparing Two Measures of Distributional Shape," The American Statistician, 46(3), 186-189.

[8] Pandey, M.D., van Gelder, P.H.A.J.M., and Vrijling, J.K., 2001b, "The Estimation of Extreme Quantiles of Wind Velocity Using L-Moments in the Peaks-Over-Threshold Approach," Structural Safety, 23, 179-192.

[9] Cheng, J., Spillane, M., Bian, S., and Cao, P., 2010, "Extreme Response Estimate of Steel Catenary Risers Using L-Moments," Proc. of the 29th International Conference on Ocean, Offshore and Arctic Engineering, June 6-11, Shanghai, China.

[10] Najafian, G., 2010, "Comparison of Three Different Methods of Moments for Derivation of Probability Distribution Parameters," Applied Ocean Research, 32(3), 298-307.

[11] Hosking, J.R.M., 2007, "Some Theory and practical Uses of Trimmed L-Moments," Journal of Statistical Planning and Inference, 137, 3024-3039.

[12] Darolles, S., Gourieroux, C., and Jasiak, J., 2009, "L-Performance with an Application to Hedge Funds," Journal of Empirical Finance, 16(4), 671-685.

[13] Winterstein, S.R., and MacKenzie, C.A., 2011, "Extremes of Nonlinear Vibration: Models Based on Moments, L-Moments, and Maximum Entropy," Proc. of the 30th International Conference on Ocean, Offshore and Arctic Engineering, June 19-24, Rotterdam, Netherlands (submitted).

[14] Smith, R.L., 1984, "Threshold Methods for Sample Extremes," appears in Statistical Extremes and Applications, Tiago de Oliveira, J. (ed.), Reidel, Dordrecht, 621-638.

[15] Lechner, J.A., Simiu, E., and Heckert, N.A, 1993, "Assessment of 'Peaks Over Threshold' Methods for Estimating Extreme Value Distribution Tails," Structural Safety, 12, 305-314.

[16] Kashef, T., and Winterstein, S.R., 1998, "Moment-Based Probability Modelling and Extreme Response Estimation: The FITS Routine," Rept. RMS 31, Reliability of Marine Structures Program, Department of Civil & Environmental Engineering, Stanford University.

[17] Winterstein, S.R., and Kashef, T., 1999, "Moment-Based Load and Response Models with Wind Engineering Applications," Journal of Solar Energy Engineering, 122(3), 122-128.

[18] Carter, D.J.T., and Challenor, P.G., 1981, "Estimating Return Values of Environmental Parameters," Quarterly Journal of the Royal Meteorological Society, 197, 259-266.