Valuing Land in Dispute Resolution: Using Coefficient of Variation to Determine Unit of Measurement

Bryan Younge
VALUING LAND IN DISPUTE RESOLUTION
USING COEFFICIENT OF VARIATION TO DETERMINE UNIT OF MEASUREMENT
BY BRYAN YOUNGE, MAI, ASA

ABSTRACT
By quantifying and comparing the CV of at least two unit price results in a sample set (namely PSF and FAR for land analyses in dense urban markets), a major debate can be settled well in advance of the point in time when either party in a dispute begins to hazard speculative adjustments.

ARTICLE
Investors of commercial land who are engaged in a legal dispute often find themselves in irreconcilable positions when trying to determine a fair price for real estate. The most common approach over the past several decades to the valuation of land by valuation experts has been the sales comparison approach. This approach is a method of developing an opinion of market value in which a property is compared to other similar properties that have been recently sold. While parties with various interests in the same tract of land might arrive at their price positions using a similar set of data—or in some circumstances the exact same set—the arbitration or dispute settlement process is often compromised in the analysis phase because the methodology used in the adjustment process among the parties involved can be appreciably dissimilar.

When employing the sales comparison approach to land value, it is foremost preferable that all of the properties being analyzed are in the same geographic area and of similar potential uses. One premise of the sales comparison approach is that the market will establish a price for the subject property in the same manner that the prices of comparable or competitive properties are established. Accordingly, the sales prices of the properties deemed most comparable to the subject property tend to set the range in which the value of the subject property will fall.

But oftentimes this range of value on a per-square-foot of land area is very wide, particularly in central business districts of major cities where zoning and various use restrictions are highly regimented about what can be built on the land. A substantial reason behind this wide range is the notion that one or more of the comparable properties could have a significantly higher potential building-to-land area ratio (Floor Area Ratio or FAR) than the other tract, thereby potentially generating a greater return. An investor of land in an area of high density would be compelled to analyze the land not only on its physical characteristics, but also on the permitted uses, the highest and best use and, ultimately, the potential size of the proposed improvements. In addition, there are highly complex legal instruments in existence involving—for example—ground leases of urban assets that require the thoughtful development of procedures and framework within periodic arbitration events during the term of the lease to keep all parties in equitable positions. Accordingly, we are seeing that the analysis of land sales on the basis of FAR in dense urban areas is a useful tool that is used by market participants, and that lately there has been a proliferation of appraisal-related coursework and core publications that teach the statistical implications of selecting units of measurement such as FAR as part of standard valuation techniques. Such techniques are particularly useful in resolving disputes both within and outside of the courtroom.
In this case study, we examined prices that buyers have recently paid for 26 land tracts in two separate submarkets (13 sales within each submarket) within one central business district of a major Midwestern city. From this information, we show how to employ coefficient of variation (CV) as a statistical tool to ascertain the appropriate unit of comparison. The following paragraphs summarize the instruments that are used to develop CV.

**Floor Area Ratio as a Unit of Measure**

Floor area ratio (FAR) is a term for the ratio of a building's total floor area to the size of the plot of land upon which it is built. The terms can also refer to limits—governmental or otherwise—imposed on such a ratio. The formula for FAR is as follows:

$$\text{Floor area ratio} = \frac{\text{Total covered area on all floors of all buildings on a certain tract}}{\text{Area of the tract}}$$

In commercial transactions, particularly in dense urban areas, using price per square foot of land as the unit of comparison is often not appropriate. As discussed, this is because land value, particularly within central business districts, is almost entirely driven by the site's highest and best use as well as its propensity to generate future economic benefits. As will be demonstrated, without accounting for the varying FARs that are permitted within this analysis, one might be making inappropriate comparisons.

**Coefficient of Variation: Price per Land Sq. Ft. vs. Price per Maximum FAR**

**Overview**

In probability theory and statistics, the sample set that produces the lowest coefficient of variation suggests the lowest degree of uncertainty, variability and/or error, and the highest level of confidence. According to Peter Klibanoff, Professor of Managerial Economics and Decision Sciences at the Kellogg School of Management, Northwestern University, variance is "a measure of the spread around the mean determined by averaging the squared deviations from the mean."

Similarly, CV is recognized in the statistics world as a normalized measure of dispersion of a probability distribution or frequency distribution. The CV is useful in analyzing unit measures (such as land sales) because the standard deviation of data is expressed in the context of the mean of the data. For comparison between data sets with different units (i.e. price per land square-foot vs. price per maximum FAR) or widely different means (such as the case with our comparable land sales when considering the context of these two units of measure), one should use the CV to make a quantifiable determination about which unit of measure is most applicable.

In order to calculate CV, we must first determine the standard deviation of our selection of land sales on the basis of:

1. Price per Land Square Foot (Price Land PSF)
2. Price per Maximum Potential Floor Area Ratio (Price Max FAR)

**Calculation of Standard Deviation**

In statistics and probability theory, the standard deviation (usually expressed as SD or $\sigma$) shows how much dispersion or variation exists from the mean figure within a set of figures. A low standard deviation

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1 Also cited in his textbook: Klibanoff, Peter; Sandroni, Alvaro; Moselle, Boaz; Saraniti, Brett. Managerial Statistics: A Case-Based Approach. Thomson South-Western, 2006, p. 27.
indicates that the data points tend to be very close to the mean (also called expected value). A high standard deviation indicates that, particularly in urban settings, the data points are spread out over a large range of values.

The standard deviation of a random variable, statistical population, data set or probability distribution is the square root of its variance. A useful property of the standard deviation is that, unlike the variance, it is expressed in the same units as the data, in this case, price per square foot.

Calculation of CV

In order to prepare the landscape for the argument, we have performed a statistical analysis of the sale prices of numerous land sales that have been consummated in two distinct submarkets of one major central business district located in the Midwest. The first sample set includes 13 sales that are located in a perimeter submarket of a downtown venue where the density is moderately high, and the second set includes 13 sales that are located in a CBD submarket where the density is very high. In both markets, due to the density and the vertical tenancy of the surrounding neighborhoods, it is reasonable to assume that price per maximum FAR could be the unit of choice.

The table below summarizes the price of each sale (all of which have been confirmed and have occurred within the past four-year period), its value expressed on a per-unit basis (both on a price per square-foot of land basis and on a price per maximum FAR) and the resulting CV for both sample sets:

<table>
<thead>
<tr>
<th>SALE NO.</th>
<th>SALE PRICE</th>
<th>SITE SIZE (SF)</th>
<th>POTENTIAL BUILDING AREA (ABOVE GRADE)</th>
<th>FAR</th>
<th>PRICE: PSF LAND AREA</th>
<th>PRICE: PSF MAX FAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,900,000</td>
<td>11,543</td>
<td>50,000</td>
<td>4.33</td>
<td>$164.60</td>
<td>$38.00</td>
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<tr>
<td>2</td>
<td>3,000,000</td>
<td>16,552</td>
<td>49,656</td>
<td>3.00</td>
<td>$181.25</td>
<td>$60.42</td>
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<tr>
<td>3</td>
<td>300,000</td>
<td>4,830</td>
<td>9,660</td>
<td>2.00</td>
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<td>$82.82</td>
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<td>$117.98</td>
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<tr>
<td>5</td>
<td>595,000</td>
<td>3,101</td>
<td>3,721</td>
<td>1.20</td>
<td>$191.87</td>
<td>$159.89</td>
</tr>
<tr>
<td>6</td>
<td>2,909,671</td>
<td>11,543</td>
<td>50,000</td>
<td>4.33</td>
<td>$252.07</td>
<td>$58.19</td>
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<td>7</td>
<td>1,850,000</td>
<td>7,492</td>
<td>14,984</td>
<td>2.00</td>
<td>$246.93</td>
<td>$123.47</td>
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<tr>
<td>8</td>
<td>550,000</td>
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<td>1.00</td>
<td>$150.01</td>
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<tr>
<td>11</td>
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<td>$162.95</td>
<td>$135.79</td>
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<tr>
<td>13</td>
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<td>7,492</td>
<td>14,984</td>
<td>2.00</td>
<td>$186.87</td>
<td>$93.43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STATISTICAL ANALYSIS</th>
<th>PSF LAND AREA</th>
<th>PSF MAX FAR</th>
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</thead>
<tbody>
<tr>
<td>Standard Deviation:</td>
<td>$34.56</td>
<td>$51.42</td>
</tr>
<tr>
<td>Population Mean:</td>
<td>$188.75</td>
<td>$103.10</td>
</tr>
</tbody>
</table>
| Coefficient of Variation | 0.183  | 0.499
| Difference:          | 172%          |             |
While the CV metric produces a statistical figure that can easily be compared on a quantitative basis, it is also helpful to have a visual aid to represent the results. The following is a side-by-side graphic representation that illustrates the data being analyzed in each of the two submarkets.
COEFFICIENT OF VARIATION: PRICE PSF LAND VS. PRICE MAX FAR

FIGURE 2-A – PERIMETER SET

$ Land SF

$300.00

$250.00

$200.00

$150.00

$100.00

$50.00

$0.00

$ F AR

$220.00

$200.00

$180.00

$160.00

$140.00

$120.00

$100.00

$80.00

$60.00

$40.00

$20.00

$0.00

Sale Number

1 2 3 4 5 6 7 8 9 10 11 12 13

Price PSF Land

Price Max FAR
As shown in the previous table, the CV using price per square foot of maximum FAR as a measure of unit in the CBD sample set is significantly lower than the metric produced using price per square foot of land. Specifically, the CV using Max FAR is 0.291, while the metric using PSF of Land Area is 0.448. This means that the amount of variation using Price PSF Land as a unit of measure is 54 percent higher than the variation using Price Max FAR as a unit of measure. In other words, there is a much higher level of confidence in analyzing the sales using FAR, allowing for a cleaner platform from which to administer adjustments. Those adjustments will also be smaller, thereby naturally narrowing the gap between the strike points for the buyer and seller. This also likely proves that buyers were basing their investment decisions using this metric. The above right graphic shows that the peaks and valleys of the Price PSF Land category (blue line) are more dramatic than the results produced when analyzing the same sales on a Price Max FAR basis.

Conversely, the CV readings for the Perimeter Sales are such that there is a significantly lower degree of variation when analyzing the 13 sales on the basis of price per square foot of land area. This can be confirmed both by the lower CV produced by Price PSF Land, as well as the smoother blue line in the above left graphic. Therefore, buyer and seller will more likely establish a negotiated price if the land is analyzed this way since unit price adjustments would be less drastic compared to using FAR.

It should be noted that the 13-property sample size in each case analysis represents is relatively large for appraisal standards but might be relatively small on the basis of developing a study with a high degree of confidence. The CV result is useful in providing a numerical solution to relationships in units of measurement no matter how large the sample size is. The purpose of comparing these two data sets is to highlight the inference that FAR, for example, is supported by statistical analysis as being the proper unit of measurement. Of course, there are likely CBD markets that have land transactional characteristics
which do not lend credence to using FAR as a basis of measurement. Again, this can be simply supported by the development of CV.

**SETTLING THE ARGUMENT**

As a further analysis of the data above, we performed a demographic snapshot of each of the sales to understand the population densities in the immediate vicinity. The higher a population density is, the higher the propensity for that market to champion vertical construction (thereby indicating higher FAR potential). Accordingly, one can infer that, the higher the population density, the higher likelihood that a wider range of FAR zones exist. The average population density within one mile from each of the sales is approximately 65,000 for the perimeter set and 90,000 for the CBD set. We also note that the FAR for the CBD sales is much higher compared to the Perimeter Sales (14.64 versus 2.43, respectively). From this information, it can be inferred that using price per maximum FAR as a unit of measurement might be more applicable in areas of significant urban density with high maximum FAR metrics.

Investors and related parties alike often dispute about certain adjustments to the unit prices of comparable land sales when determining the value of land in urban markets. Can we use comparables with different zoning designations? Different intended uses? What about land that has a unique shape? Frontage? Access? Visibility? These are all pertinent questions when engaging in the adjustment process of the comparable sales and are customary topics that are addressed in an arbitration or dispute resolution process. The good news is that, by quantifying and comparing the CV of at least two unit price results in a sample set (namely PSF and FAR for land analyses in dense urban markets), a major debate can be settled well in advance of the point in time when either party begins to hazard speculative adjustments. Equally important, the parties can be aligned where the valuation methodology is concerned.

**ABOUT THE AUTHOR**

Mr. Younge is a Managing Director with Cushman & Wakefield of Illinois, Inc., the National Practice Leader of the Sports and Entertainment group and a senior member of the Dispute Analysis and Litigation Support group. He has been active in real estate valuation and consulting since 1998. Prior to his employment with Cushman & Wakefield, Mr. Younge worked at Arthur Andersen and Deloitte & Touche in the Valuation Services groups in Chicago as a Managing Senior Consultant administering several high-profile hospitality real estate valuation assignments. At PricewaterhouseCoopers, Mr. Younge worked with the Hospitality and Leisure Consulting group in Los Angeles performing valuation, consulting, feasibility, and litigation-related assignments for hotel and leisure-related assets. Mr. Younge has also performed numerous appraisals on hotels and casino facilities with HVS International in the San Francisco and Boulder offices, and was an interim acquisition, development and investment analyst for Sage Hospitality Resources in Denver.