

Teacher Supplement to Operation Comics, Issue #6

The purpose of this supplement is to provide content support for the mathematics embedded into the sixth issue of Operation Comics, and to show how the mathematics addresses the content standards provided by the National Council of Teachers of Mathematics (NCTM) for grades 3–5 and 6–8. This issue addresses the arithmetic mean of two numbers, the additive inverse of an integer, the sign of the product of integers. The mathematics used in this issue addresses both the NCTM standard for problem solving and the NCTM standard for reasoning and proof, for both grade ranges, in addition to the standards addressed below.

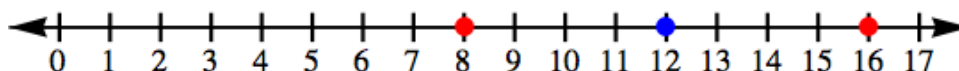
The Arithmetic Mean

In the story, we learn that the jet packs have separate fuel tanks to the jets for each hand, and that the pressure of the fuel in each tank is not necessarily the same all of the time. Wonderguy has a switch that he can activate to equalized the pressure in the tanks. Since the tanks are the same size, the pressure in both tanks after hitting the switch is the mean of the pressures of the two tanks before pushing the button.

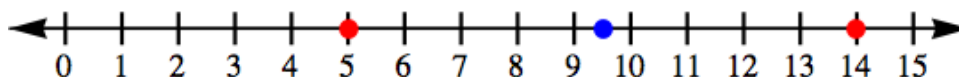
Finding Average Values

The first student worksheet has students finding the arithmetic mean of two numbers, and plotting the answer on a number line. It is important to note that the mean (blue) always falls halfway between the two numbers (red) on the number line. Solutions are given below.

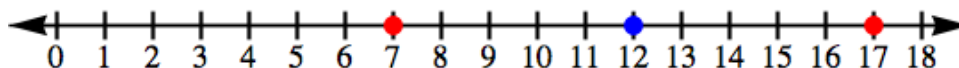
- 1) 8 and 16. The mean of the two numbers is $\frac{8+16}{2} = \frac{24}{2} = 12$. The results on the number line are shown below.

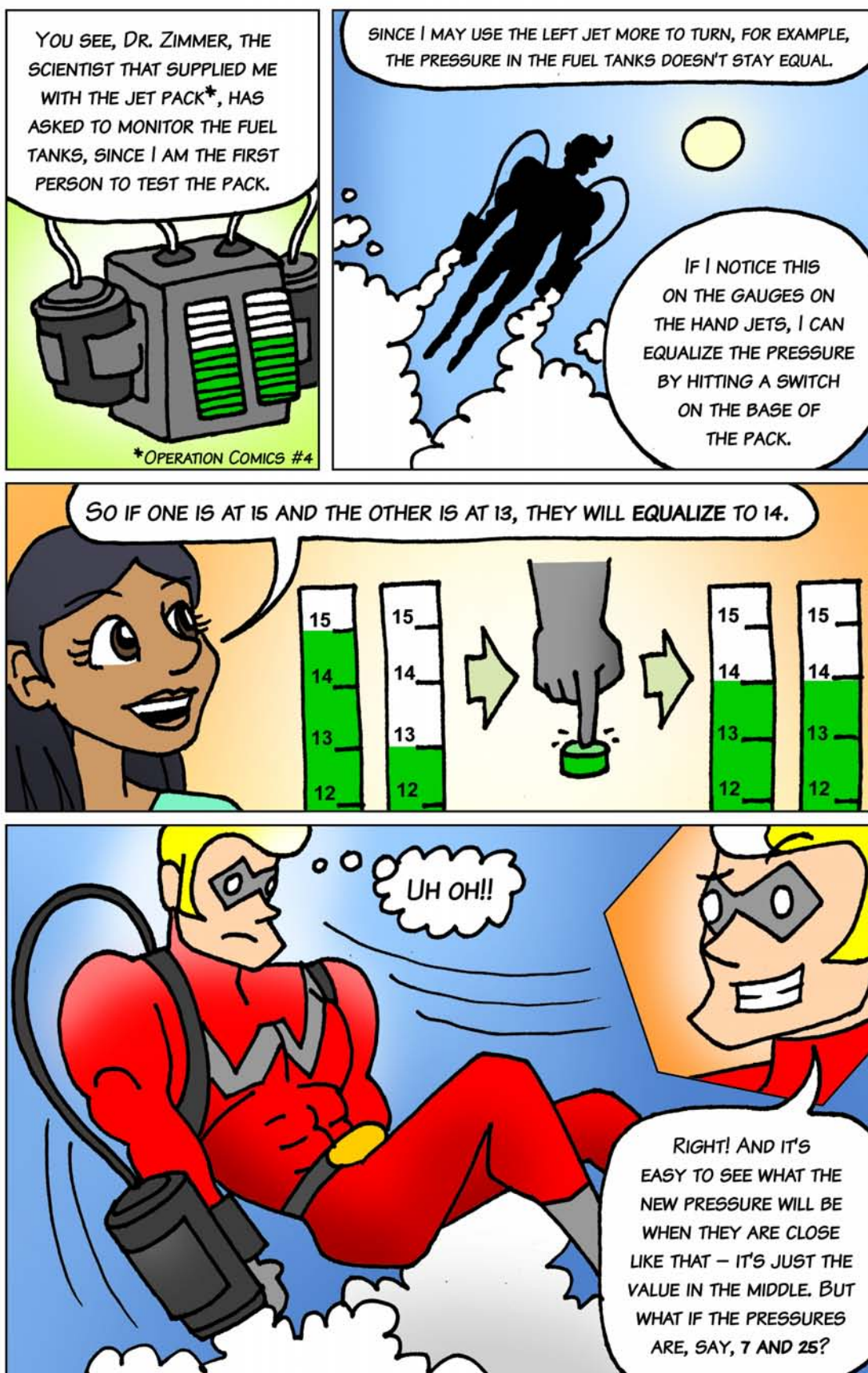


- 2) 5 and 14. The mean of the two numbers is $\frac{5+14}{2} = \frac{19}{2} = 9.5$. The results on the number line are shown below.

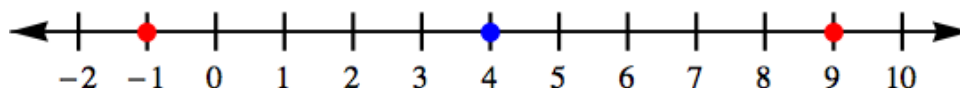


- 3) 7 and 17. The mean of the two numbers is $\frac{7+17}{2} = \frac{24}{2} = 12$. The results on the number line are shown below.

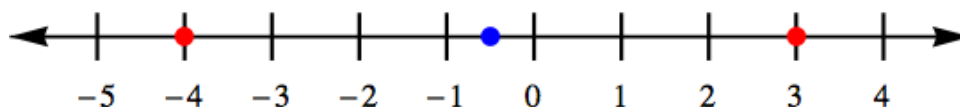




4) -1 and 9 . The mean of the two numbers is $\frac{-1+9}{2} = \frac{8}{2} = 4$. The results on the number line are shown below.



5) -4 and 3 . The mean of the two numbers is $\frac{-4+3}{2} = \frac{-1}{2} = -0.5$. The results on the number line are shown below.



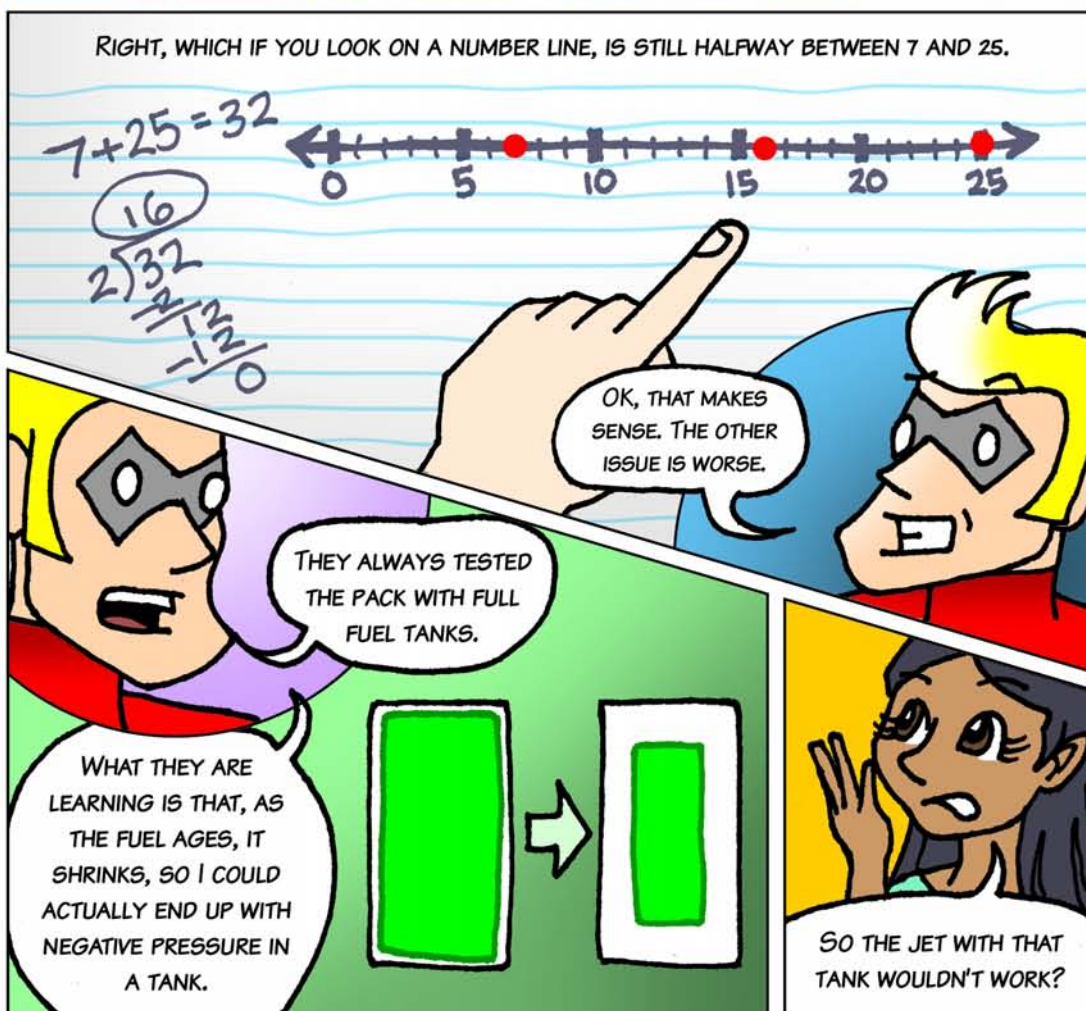
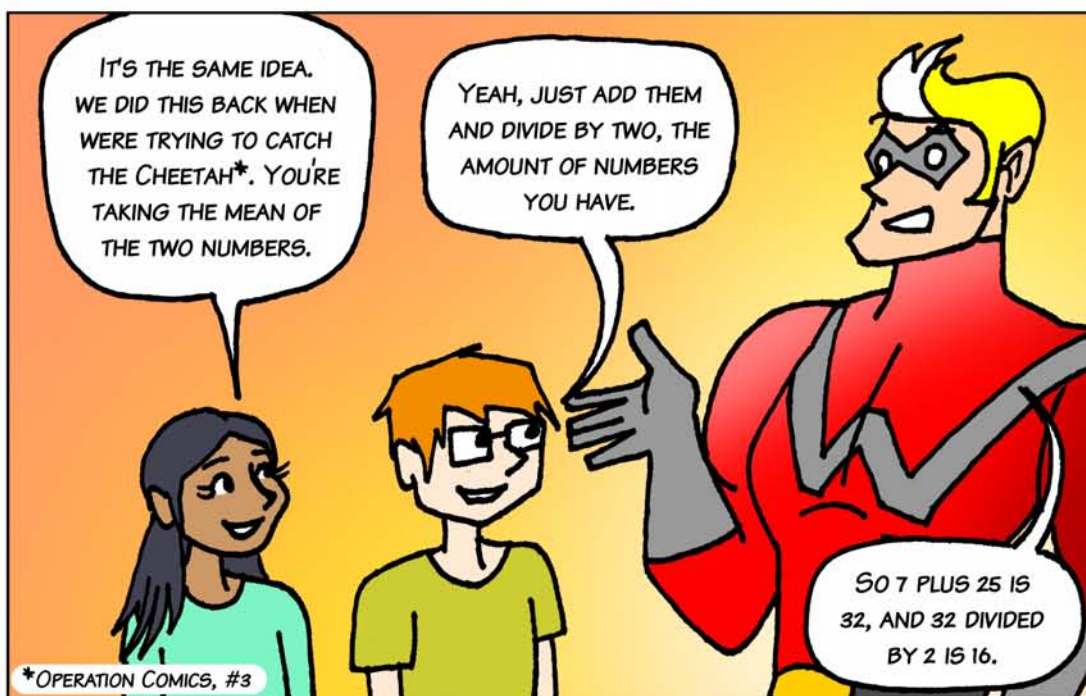
Super-Tricky! If the original two numbers are integers, and mean is also an integer, then the two numbers must have the same parity, meaning that they are either both even or both odd. If the mean ends in a “.5”, then the two numbers have different parity, meaning that one is even and the other is odd.

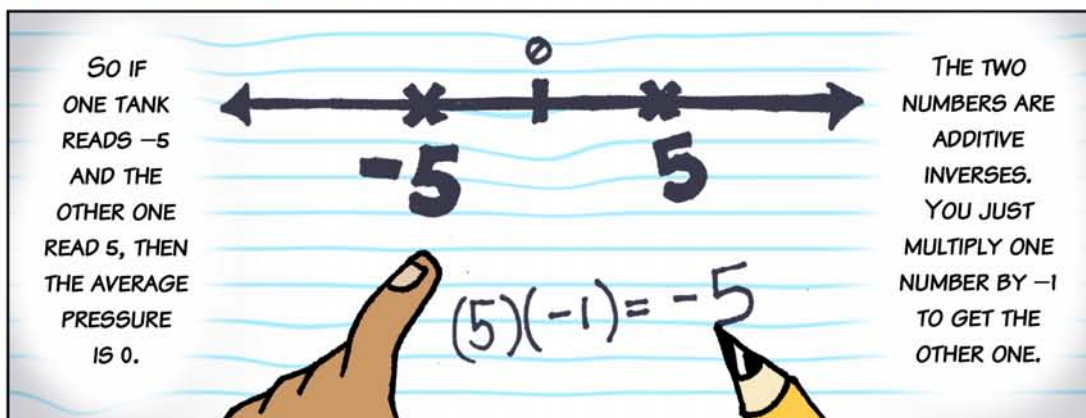
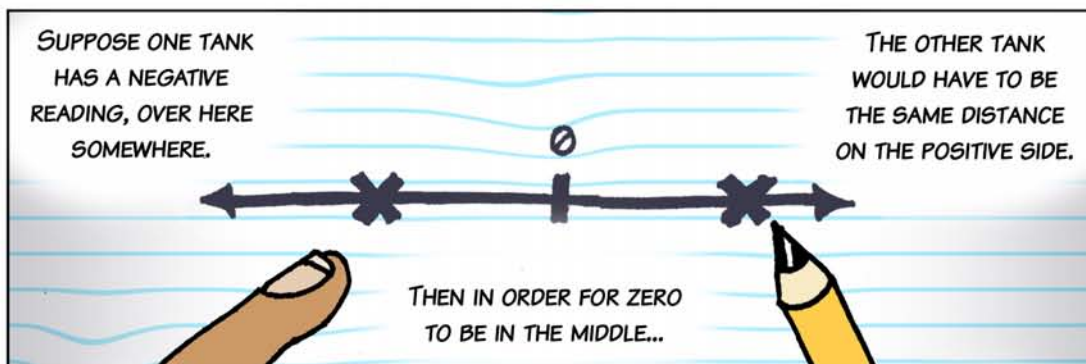
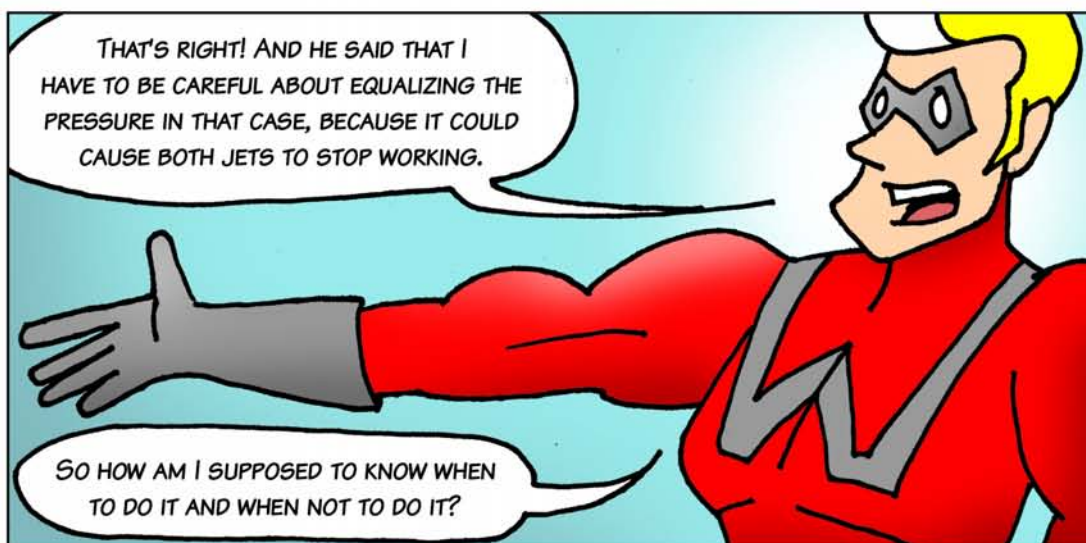
NCTM Standards

Finding the mean addresses the NCTM standard for data analysis and probability for grades 6–8, which says that “in grades 6–8, all students should find, us, and interpret measures of center and spread, including mean and interquartile range.” It also addresses the NCTM standard for number and operations for grades 3–5, which says that “in grades 3–5, all students should develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers,” and should “develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results,” the latter of which is also included in the grades 6–8 standards.

The Additive Inverse

In the story, we learn that the jet pack fuel tanks can actually have negative pressure, and that equalizing the pressure in the tanks could have the positive effect of making both hand-jets work again, or the negative effect of causing both to stop working. In the course explaining to Wonderguy when he should or should not equalize the pressure in the tanks, Claire and Dillon introduce the concept of the additive inverse of a number.

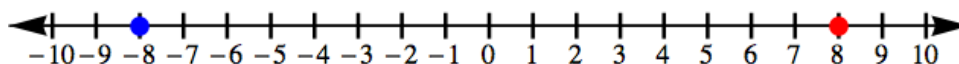




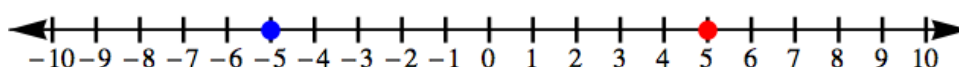
Finding Additive Inverses

The second student worksheet has students finding the additive inverse of some integers, and plotting the answer on a number line. It is important to note that the additive inverse (blue) is always equidistant from 0 to the original number (red), just on the opposite side of 0. It is also important to note that the additive inverse of an additive inverse is the original number (important to the story). Solutions are given below.

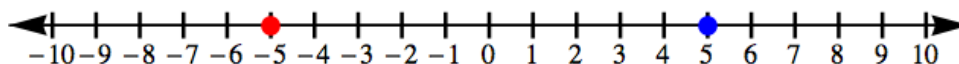
- 1) 8. The additive inverse of 8 is -8 . The result is shown on the number line below.



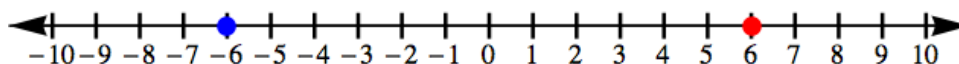
- 2) 5. The additive inverse of 5 is -5 . The result is shown on the number line below.



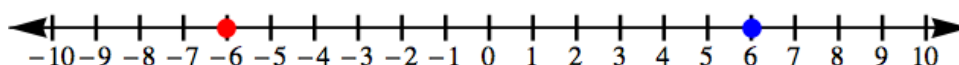
- 3) -5 . The additive inverse of -5 is 5. The result is shown on the number line below.



- 4) 6. The additive inverse of 6 is -6 . The result is shown on the number line below.



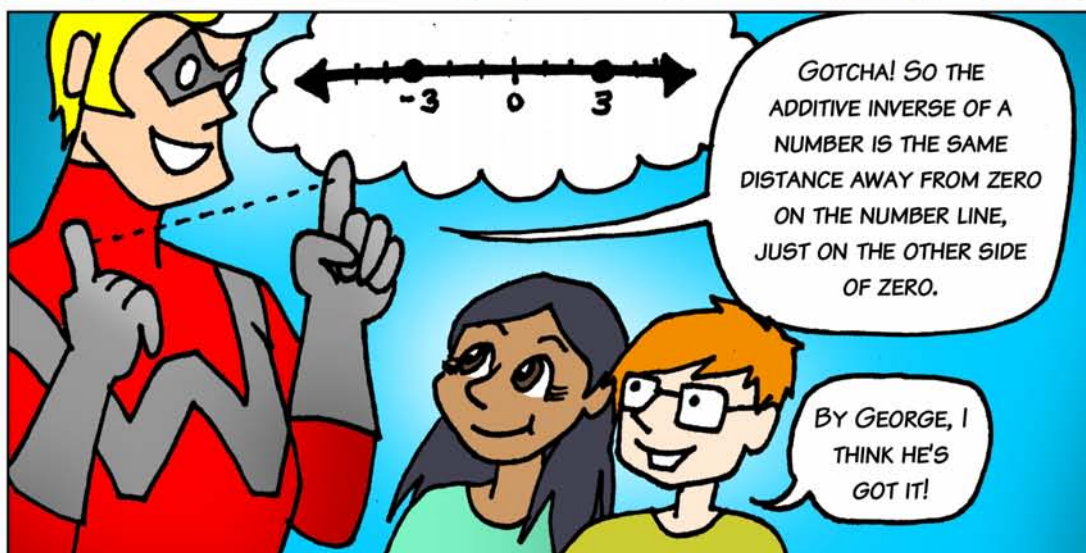
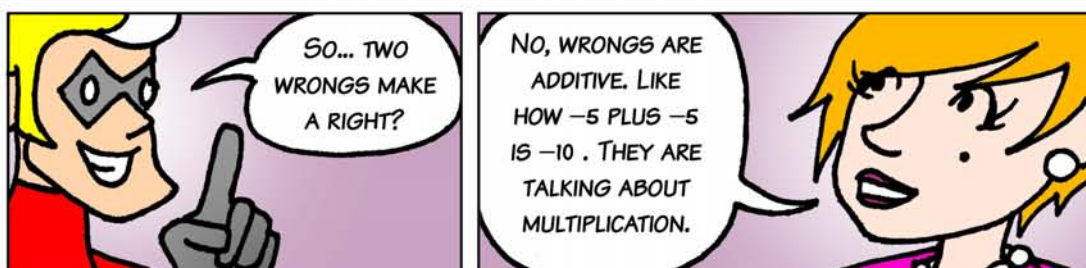
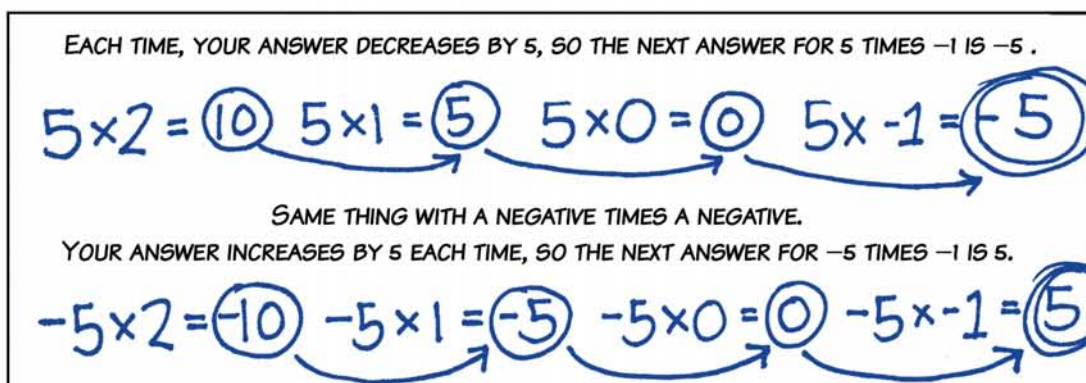
- 5) -6 . The additive inverse of -6 is 6. The result is shown on the number line below.



Super-Tricky! The sum of a number and its additive inverse is 0. The product of a number and its additive inverse is nonpositive. Since 0 times its additive inverse 0 is 0, it is not entirely correct to say that the product is negative.

NCTM Standards

Finding the additive inverse addresses the NCTM standard for number and operations for grades 3–5, which says that “in grades 3–5, all students should explore numbers less than 0 by extending the number line and through familiar applications,” and should “identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems.” It also addresses the NCTM standard for number and operations for grades 6–8, which says that “in grades 6–8, all students should develop meaning for integers and represent and compare quantities with them,” and “understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.”



Multiplying Positive and Negative Numbers

In the course of explaining how to find the additive inverse of a number, Claire and Dillon mention multiplying by -1 , a concept with which Wonderguy is not familiar. They take the time to explain the notion of multiplying by a negative number by showing Wonderguy how the negative multiplication rules just extend the pattern of regular multiplication.

Multiplying Integers

The third student worksheet has students multiplying positive and negative integers, extending the pattern started with the positive integer multiplication to the negative integer multiplication. Solutions are given below.

- 1) Since the solutions to $3 \times 3 = 9$, $3 \times 2 = 6$, $3 \times 1 = 3$, and $3 \times 0 = 0$ are decreasing by 3 each time, the natural extension is for $3 \times -1 = -3$, $3 \times -2 = -6$, and $3 \times -3 = -9$.
- 2) Since the solutions to $4 \times 3 = 12$, $4 \times 2 = 8$, $4 \times 1 = 4$, and $4 \times 0 = 0$ are decreasing by 4 each time, the natural extension is for $4 \times -1 = -4$, $4 \times -2 = -8$, and $4 \times -3 = -12$.
- 3) Since the solutions to $-3 \times 3 = -9$, $-3 \times 2 = -6$, $-3 \times 1 = -3$, and $-3 \times 0 = 0$ are increasing by 3 each time, the natural extension is for $-3 \times -1 = 3$, $-3 \times -2 = 6$, and $-3 \times -3 = 9$.
- 4) Since the solutions to $-4 \times 3 = -12$, $-4 \times 2 = -8$, $-4 \times 1 = -4$, and $-4 \times 0 = 0$ are increasing by 4 each time, the natural extension is for $-4 \times -1 = 4$, $-4 \times -2 = 8$, and $-4 \times -3 = 12$.

Super-Tricky! The product of a positive number and a negative number is a negative number. The product of two negative numbers is a positive number.

NCTM Standards

Finding the additive inverse addresses the NCTM standard for number and operations for grades 3–5, which says that “in grades 3–5, all students should explore numbers less than 0 by extending the number line and through familiar applications,” and should “identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems.” It also addresses the NCTM standard for number and operations for grades 6–8, which says that “in grades 6–8, all students should develop meaning for integers and represent and compare quantities with them,” and “understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.”

One last note . . .

While my purpose here is to produce a comic book with embedded mathematical content, and mathematics is my background, I did endeavor to write a good story, and hopefully, the comics can be used for their literary elements as well. The following are a few examples of subtle things at work in the story.

- Students familiar with the origin of Captain Confusion (given in *Operation Comics #1: Captain Confusion's Revenge*) will know that his initial crime was committed as a result of his many years of agonizing over a failure he had at an academic team meet when he was an elementary student. By the end of this story, he is helping Dr. Zimmer with using his gene-confusion ray as a tool for doing good. One could ask the question whether handling that initial failure better could have led to a different life for "Rupert".
- Wonderguy does not seem angry with Miss Garmon when she reveals that she is Captain Confusion's sister and that she may have inadvertently aided Captain Confusion in turning Wonderguy into his recessive-gene alter ego. Should he have been more angry? A curious discussion point would be whether or not Wonderguy would still want her as his secretary after this story.
- The story ends with Captain Confusion deciding to work for good. Another curious discussion point would be whether or not he will continue to do so.

This document, as with the student worksheets, is a work in progress. Please contact me with corrections or suggestions, and I will make the needed changes. Thanks for inviting Wonderguy into your classrooms, and please encourage your students to contact me with their comments and suggestions for future episodes.