The purpose of this supplement is to provide content support for the mathematics embedded into the first issue of Operation Comics, and to show how the mathematics addresses the content standards provided by the National Council of Teachers of Mathematics (NCTM) for grades 3–5 and 6–8. The mathematics can be segmented into four blocks: the three problems worked by the team of Wonderguy, Claire, and Dillon, and the problem that Wonderguy poses to Captain Confusion as he is being led away by the police. The four problems and the content that they illustrate are given below, along with excerpts from the comic. All four problems address both the NCTM standard for problem solving and the NCTM standard for reasoning and proof, for both grade ranges, in addition to the standards addressed below.

**Number Sense and Divisibility Theorems**

The first problem can be solved by trial-and-error, but given the fact that time is ticking away rapidly, the better path is a reliance on divisibility theorems. When Claire and Dillon are “put into the game”, Wonderguy has already started working the problem by trial-and-error, inserting a “1” into the blank, and working the long division problem by hand.
To find the answer “7”, Claire and Dillon are using the divisibility theorem for 9 that states that a number is divisible by 9 if and only if the sum of the digits in the number is divisible by 9. The sum of the given digits is 20, and the next highest number divisible by 9 is 27. Hence, the missing digit must be a 7. Note that the leading digit cannot be a 0, so if the sum of the given numbers had actually been divisible by 9, the only solution would have been the answer “9”, to keep the sum a multiple of 9.

Wonderguy’s reluctance to believe their answer motivates them to show him why the theorem works, using a three-digit numeral as an example.
By writing the numeral in expanded form, and pulling a single digit-amount out of each place, they create pieces of the expanded form that are divisible by 9. For example, 9 times anything will be divisible by 9, and the same is true for 99 times anything. (The proof can actually be extended to a numeral with any finite number of digits, since any power of ten, minus 1, will always be divisible by 9.) Then, the divisibility of “ₐ b c” is equivalent to the divisibility of the sum a + b + c.
“Divisibility with Claire and Dillon”

The student supplement that accompanies this section of content has the students using the divisibility theorem for 9. Solutions are given below.

1)  4  2)  6  3)  3  4)  0 and 9  5)  7  6)  9  
Super-Tricky: #4 has two solutions, 0 and 9. #6 only has one, since the leading digit cannot be 0.

NCTM Standards

This problem addresses the NCTM standard for numbers and operations for grades 3–5, which says that “in grades 3–5, all students should understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals”, “recognize equivalent representations for the same number and generate them by decomposing and composing numbers”, and “describe classes of numbers according to characteristics such as the nature of their factors”.

Extensions to Other Divisibility Theorems

The work done proving the divisibility theorem for 9 leads to other results. Since 9, 99, 999, etc. are always also divisible by 3, then

**a number is divisible by 3 if and only if the sum of the digits is divisible by 3.**

For example, 111111 is divisible by 3, since the sum of its digits is 6, which is divisible by 3.

Without pulling out the singles, we have

\[
\text{all but the last digit last digit} = \text{all but the last digit} \times 10 + \text{last digit}.
\]

Since any multiple of 10 will be divisible by 10, then

**a number is divisible by 10 if and only if the last digit is divisible by 10, hence 0.**

Since any multiple of 10 will be divisible by 5, then

**a number is divisible by 5 if and only if the last digit is divisible by 5, hence 0 and 5.**

Since any multiple of 10 will be divisible by 2, then

**a number is divisible by 2 if and only the last digit is divisible by 2, hence 0, 2, 4, 6, and 8.**

Also, since

\[
\text{all but the last two digits last two digits} = \text{all but the last two digits} \times 100 + \text{last two digits},
\]

and since 100 is always divisible by 4, then

**a number is divisible by 4 if and only if the last two digits are divisible by 4.**
For example, 532840372 is divisible by 4, since 72 is divisible by 4.

Least Common Multiple

The second problem has a similar bend, in that it can be solved using brute force, but there is a more efficient way to find the solution. Wonderguy immediately goes to the brute force method, by suggesting that they list all factors of each number and compare lists until a common number is found on each list. Claire, however, has a better plan.

The prime factorization method has us decompose the two numbers into their prime factors. Primes are whole numbers greater than 1 that are only divisible by 1 and themselves. The least common multiple of two numbers is then the product of the highest occurrence of each prime in the two prime factorizations.
Dillon and Claire explain why this works in the comic.
There is one added piece of content at this point. Claire and Dillon show how it is faster to calculate $8 \times 3 \times 7$ mentally than using the standard algorithm. Using the commutative and associative properties of multiplication, they formulate that

$$(8 \times 3) \times 7 = 7 \times (8 \times 3) = (7 \times 8) \times 3.$$ 

Then, using the distributive property of multiplication over addition, they formulate that

$56 \times 3 = (50 + 6) \times 3 = (50 \times 3) + (6 \times 3).$
“Least Common Multiples with Claire and Dillon”

The student supplement that accompanies this section of content has the students using prime factorizations to find the least common multiple of two numbers. Solutions are given below.

1) 480  
2) 80  
3) 324

Super-Tricky: The larger of the two numbers, as in problem #2.

NCTM Standards

This problem addresses the NCTM standard for numbers and operations for grades 3–5, which says that “in grades 3–5, all students should develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as $30 \times 50$” and “select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tools”. It also addresses the standard for numbers and operations for grades 6–8, which says that “in grades 6–8, all students should use factors, multiples, prime factorization, and relatively prime numbers to solve problems”.

Extensions to Other Uses for Prime Factorizations

This same method may be used to find the least common multiple of more than two numbers. For example, consider the least common multiple of 24, 30, and 42. Since

\[ 24 = 2^3 \times 3, \]
\[ 30 = 2 \times 3 \times 5, \] and
\[ 42 = 2 \times 3 \times 7, \]
then the least common multiple will be \( 2^3 \times 3 \times 5 \times 7 = 840. \)

Prime factorizations can also be used to find the greatest common factor of two numbers, just like with the least common multiple except that it is found by taking the product of the lowest occurrence of each prime. For example, consider the greatest common factor of 24 and 28. Since

\[ 24 = 2^3 \times 3 \text{ and} \]
\[ 28 = 2^2 \times 7, \]
then the greatest common factor will be \( 2^2 \times 3^0 \times 7^0 = 4. \) This method can also be extended to more than two numbers.

Common Difference

The last problem is an inverse problem, in that we are given a “black box” that outputs a number for each one that they enter, and they have to deduce what the formula is. Wonderguy can enter values into the blank besides the “\( x = \)” and the computer will give him an associated value for \( y. \)
Actually, Wonderguy’s assessment is accurate. Inverse problems are notoriously difficult, because often times the answer “could be anything”. However, if we know in advance that the formula is limited to using just the types of formulas that would be used in elementary school, then we do not have to worry about the possibility of exponential functions, logarithmic functions, etc.

Claire and Dillon correctly have Wonderguy enter consecutive whole numbers, to see if they can spot the pattern.

They are able to detect the common difference between the results from consecutive whole number inputs, each time increasing by 2. They also correctly determine that this is linear growth, so the formula must include the term “$2x$”. However, Wonderguy is astute in pointing out that this is not the correct formula.
Having correctly determined that the formula is linear, Claire and Dillon then determine that the only unknown left in the formula is the constant term, which is most easily determined by finding the associated $y$-value for $x = 0$.

“Finding Formulas with Claire and Dillon”

The student supplement that accompanies this section of content has the students finding linear formulas that describe the given $x$-, $y$-values. Solutions are given below.

1) $y = 3x + 4$  
2) $y = 4x - 2$  
3) $y = -2x + 11$

Super-Tricky: Students will discover that the graphs of these formulas are all lines.
This problem addresses the NCTM standard for algebra for grades 3–5, which says that “in grades 3–5, all students should describe, extend, and make generalizations about geometric and numeric patterns”, “represent the idea of a variable as an unknown quantity using a letter or a symbol”, “model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions”, and “investigate how a change in one variable relates to a change in a second variable”. It also addresses the standard for data analysis and probability for grades 3–5, which says that “in grades 3–5, all students should collect data using observations, surveys, and experiments” and “propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions”. It also addresses the algebra standard for grades 6–8, which says that “in grades 6–8, all students should represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules” and “use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships".
Captain Confusion’s Quick Recall Question

Having been tipped off by Principal Willoughby, Wonderguy poses the very problem that Captain Confusion had missed during the academic meet 20 years ago. He once again incorrectly interprets the problem to be

$$\frac{1}{2a} = 24.$$
Captain Confusion’s solution to the problem is a solution to the equation that he interpreted, since

\[
\frac{1}{2a} = 24 \Rightarrow 48a = 1 \Rightarrow a = \frac{1}{48}.
\]

However, taking the stated problem and interpreting it using the order of operations means that first you would divide 1 by 2 and then you would multiply by \(a\), so the correct equation would be

\[
\frac{1}{2}a = 24,
\]

which Wonderguy correctly notes has the solution \(a = 48\).

I did not generate a student supplement for this content, since algebra examples are abundant and easy to generate, and also since Claire and Dillon were not involved with the solution of the problem. I can generate a supplement if desired.
NCTM Standards

This problem addresses the NCTM standard for algebra for grades 6–8, which says that “in grades 6–8, all students should use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships”.

One last note . . .

While my purpose here is to produce a comic book with embedded mathematical content, and mathematics is my background, I did endeavor to write a good story, and hopefully, the comics can be used for their literary elements as well. The following are a few examples of subtle things at work in the story.

• There are several contextual clues in the story that Wonderguy and Principal Willoughby know each other outside of this one meeting. One could ask students if they think Wonderguy and Principal Willoughby know each other before this adventure, and ask them to support that statement with reasons.
• The female student on the academic team with the younger Captain Confusion is never explicitly identified, although it is implied that it was the principal. One could ask students who they think the young girl is, and ask them to support that statement with reasons.
• We purposefully drew Wonderguy so that it would be almost impossible for him to really have a “secret identity” (huge muscles, almost florescent blonde hair, and the strip of white hair in the middle of his head), although at least the principal’s assistant is not able to connect him to Wonderguy. One could ask students how likely it would be that he could keep his “secret identity” a secret, and ask them to support that statement with reasons.
• We purposefully drew Captain Confusion to look older than the principal, although conceivably they would be about the same age. One could ask students for reasons why this would be the case, anticipating comments on the effects of stress and anger on aging.

This document, as with the student supplements, is a work in progress. Please contact me with corrections or suggestions, and I will make the needed changes. Thanks for inviting Wonderguy into your classrooms.