Cable connected active tuned mass dampers for control of in-plane vibrations of wind turbine blades

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Abstract
In-plane vibrations of wind turbine blades are of concern in modern multi-megawatt wind turbines. Today's turbines with capacities of up to 7.5 MW have very large, flexible blades. As blades have grown longer the increasing flexibility has led to vibration problems. Vibration of blades can reduce the power produced by the turbine and decrease the fatigue life of the turbine. In this paper a new active control strategy is designed and implemented to control the in-plane vibration of large wind turbine blades which in general is not aerodynamically damped. A cable connected active tuned mass damper (CCATMD) system is proposed for the mitigation of in-plane blade vibration. An Euler–Lagrangian wind turbine model based on energy formulation has been developed for this purpose which considers the structural dynamics of the system and the interaction between in-plane and out-of-plane vibrations and also the interaction between the blades and the tower including the CCATMDs. The CCATMDs are located inside the blades and are controlled by an LQR controller. The turbine is subject to turbulent aerodynamic loading simulated using a modification to the classic Blade Element Momentum (BEM) theory with turbulence generated from rotationally sampled spectra. The turbine is also subject to gravity loading. The effect of centrifugal stiffening of the rotating blades has also been considered. Results show that the use of the proposed new active control scheme significantly reduces the in-plane vibration of large, flexible wind turbine blades.

1. Introduction
The reduction of vibrations in wind turbine blades is of topical interest. Wind turbine blade lengths have increased exponentially in the past two decades in order to maximize power output under specified atmospheric conditions. Several researchers have attempted to mitigate blade vibrations [1,2]. Uncontrolled vibrations can lead to structural damage which could reduce the life of the blades and increase maintenance costs. Ahlstrom [3] has also shown that large blade vibrations have a major influence on power production. Therefore the reduction of the vibration of wind turbine blades has become an increasingly important area of research in the wind turbine industry.

The main modes of vibration for wind turbine blades are flapwise and edgewise. In this paper, the control of in-plane blade vibrations (which are predominantly edgewise with some flapwise contribution) is considered, i.e. reducing blade vibrations which occur in the blade rotation plane. It is well known that in the in-plane direction the modal damping is low.
due to low aerodynamic damping. Aerodynamic damping of the blade vibrations in the edgewise direction can become
negative and as a result the sum of the structural damping and the aerodynamic damping can be less than zero.

As wind turbines have become larger and more flexible edgewise vibrations have become a serious problem. Moeller [4]
has shown that theoretically edgewise vibrations can occur on all rotor blades. It was found that edgewise vibration
increases with blade size and wind strength. Therefore, with modern blades of over 60 m length operating in harsh
conditions (high wind speeds, high turbulence intensities, offshore, etc.) edgewise blade vibrations have become a real
problem for the wind energy industry. Moeller stated that the appearance of edgewise vibration problems: “can certainly be
interpreted as a warning that it is not possible to continuously scale up a design without facing and resolving new vibratory and
aeroelastic problems”. The severity of the edgewise blade vibration is influenced by the structural design of the blade i.e. the
blade stiffness and the mass distributions have an influence. Thus, the problem worsens as blades become more flexible,
lighter structures.

Aeroelastic stability problems arising from edgewise vibrations were first noticed in the early 1990s and have been
studied by several researchers since then. Thomsen et al. [5] investigated the problems caused by edgewise vibrations in
stall-regulated wind turbines. Hansen [6] showed that edgewise blade vibration could cause aeroelastic instability in
modern commercial wind turbines. Riziotis et al. [7] presented an aeroelastic stability tool to consider the complete wind
turbine configuration. The use of stall strips to reduce edgewise blade vibrations has also been investigated by Riziotis et al.
[7] and Petersen et al. [8]. Stall strips were shown to increase the aerodynamic damping and therefore reduce edgewise
vibrations. However, both studies found that a significant power penalty was incurred when stall strips were mounted on
the leading edge of the blades.

Although structural control has been an active area of research for several decades, applying structural control
techniques to wind turbines is a new and developing area of research. There are only a few papers published that focus
on this area of research. Furthermore, the papers that are published in the literature generally focus on passive structural
control techniques [1,2,9,10]. Lackner and Rotea [11] investigated the use of passive and active structural control techniques
with mass dampers for floating offshore wind turbines. Stewart and Lackner [12] investigated the causes and effects of
control–structure interaction with respect to an active structural controller installed on a large-scale floating wind turbine.
Several of the studies recommended further investigations into active control strategies in future studies [9,12,11,10]. Active
control of wind turbines has been studied by Staino et al. [13]. The authors developed an active actuator control algorithm to
reduce blade vibrations. Active control was shown to perform well at controlling blade vibration. The authors recommended
further investigation into the application of active control in wind turbines. Recently Fitzgerald et al. [14] investigated the
use of active tuned mass dampers (ATMDs) for the mitigation of in-plane vibrations in rotating wind turbine blades. It was
found that an active control strategy achieved greater response reductions than a passive control strategy. The study also
showed that an ATMD control strategy is feasible for a wind turbine blade particularly for higher turbulent loadings with
enough room for stroke of the ATMD within the blade.

It is clear therefore that there is presently a gap in the literature and studies reported on active control are sparse. This
paper aims to address that knowledge gap through investigations carried out into active structural control for vibration
control of wind turbine blades. Inspired by the success of Staino et al. [13] in active tendon actuator control and Fitzgerald
et al. [14] in ATMD based control, this paper proposes a new control hardware with a cable connected active tuned mass
damper (CCATMD) merging the benefits of the two.

In this paper the use of CCATMDs to reduce in-plane vibrations in wind turbine blades is investigated. An active
structural control strategy is employed to overcome some of the issues that render traditional passive control of large
engineering structures such as wind turbine blades ineffective.

Using a passive controller such as a tuned mass damper (TMD) in large structural systems with complicated system
dynamics like wind turbines leads to problems. The mass ratio is the governing parameter that affects the performance of a
passive TMD. If the primary structure is very large (e.g. a wind turbine blade) the TMD will inevitably have a small mass
ratio. Furthermore, if the mass ratio is small the auxiliary system response may become very large. Therefore, free vibration
of the TMD will continue for a long time after the excitation has been removed from the primary structure [15]. Moreover,
the main problem with using a passive TMD to damp wind turbine blade vibration is that there is no single appropriate
dominant resonant frequency present consistently for the entire time history of the response as they change with rotational
frequency or operating conditions. The response is dominated by the rotational frequency of the blades, known as the 1P
frequency. It would be ideal to tune the blade dampers to the rotational frequency of the blades. However, tuning a TMD on
a rotating blade to the frequency of rotation has been shown to introduce an instability into the system [16], see Eq. (57).
These problems can be mitigated by introducing an active control strategy.

ATMDs are well known and have been used classically to reduce the dynamic response of structures [17,18]. In this paper
a new variation on the classical ATMD is proposed. The new type of damper is called a CCATMD. The CCATMD consists of a
classical ATMD with a cable connection. An optimal LQR controller is used with this CCATMD. The time varying nature of the
wind turbine system makes it difficult to control with passive dampers. Although the 1P frequency dominates the response
it is not possible to tune a TMD to this frequency. There is also a stochastic component to the loading which is impossible to
predict and impossible to tune a passive TMD to. The rotating wind turbine blades with tower interaction represent time
varying dynamical systems with periodically varying mass, stiffness and damping matrices. This complex dynamic system
is subjected to turbulent aerodynamic loading and periodically varying gravity loading which makes controlling blade
vibrations a serious challenge.
Three multi degree of freedom (MDOF) models of wind turbines are developed in this paper. The first model is an uncontrolled coupled wind turbine vibration model, the second model is a passively controlled coupled wind turbine vibration model with TMDs and the third model is an actively controlled coupled wind turbine vibration model with CCATMDs. These models are developed using an Euler–Lagrangian approach which leads to time varying systems with the possibility of negative damping. Time domain simulations are performed on the models using turbulent aerodynamic loading and gravity loading. TMDs are inside the blades and at the top of the tower and active and passive control strategies are investigated. The results have indicated encouraging prospects for the use of CCATMDs in the control of in-plane vibrations of wind turbine blades.

2. Euler–Lagrangian models

Dynamic models of wind turbines are created using an Euler–Lagrangian formulation. This is a generalized energy formulation which is used to reveal any couplings that may exist between degrees of freedom.

Hansen [6] derived the equations of motion for a simplified flapwise model using the Lagrangian formulation. Similar types of models were employed by both Murtagh et al. [1] and Arrigan et al. [10].

Lagrange’s equation is shown in Eq. (1).

\[ \frac{\partial T}{\partial q_i} - \frac{\partial T}{\partial \dot{q}_i} + \frac{\partial V}{\partial q_i} = Q_i \]  

where \( T \) is the kinetic energy of the system, \( V \) is the potential energy of the system, \( q_i \) is the generalized coordinate for degree of freedom \( i \) and \( Q_i \) is the generalized loading for degree of freedom \( i \).

By outlining the kinetic and potential energies the equations of motion for the system can be calculated by substitution into Eq. (1). A major benefit of the Euler–Lagrangian formulation is that all the dynamic coupling present in the system is automatically included provided the kinetic and potential energies are fully specified.

A coupled vibration model is considered in this paper. Wind turbine blades are structurally twisted (implying the principal moments of inertia at different blade sections are not aligned) and this induces a flapwise-edgewise coupling. The blade experiences significant flapwise vibrations, which can add to the edgewise damping. In order to realistically model the turbine and account for any couplings which may exist between the flapwise and edgewise vibrations and couplings which may exist between the blades and the nacelle/tower a coupled model is derived.

The coupled wind turbine model consists of rotating pre-twisted blades modeled as continuous beams of variable mass and stiffness. The blades are coupled in the in-plane and out-of-plane directions. The blades are attached at the root to a large mass representing the nacelle/tower of the turbine. The model therefore accounts for coupling in the two directions of blade vibration and accounts for dynamic interaction between the blade and the tower in the fore-aft and side-to-side directions. A schematic representation of the blade vibration model is shown in Fig. 1.

The in-plane and out-of-plane vibrations of the \( i \)th blade are modeled by two generalized degrees of freedom, \( q_{i,\text{in}}(t) \) and \( q_{i,\text{out}}(t) \). The related in-plane and out-of-plane mode shapes, \( \phi_{i,\text{in}}(x) \) and \( \phi_{i,\text{out}}(x) \), have been normalized at the tip so that \( q_{i,\text{in}}(t) \) and \( q_{i,\text{out}}(t) \)
represents the in-plane tip displacement and $q_{i,\text{out}}(t)$ represents the out-of-plane tip displacement. The structural twist of the blade is accounted for in the calculation of the in-plane and out-of-plane mode shapes. Therefore these mode shapes each have both edgewise and flapwise components. The mode shapes can be obtained by carrying out a finite element analysis of the blade.

In the following formulation it has been assumed that the displacement at any point $x$ along the $i$th blade is given in terms of the fundamental mode shapes and generalized coordinates such that

$$u_{i,\text{in}}(x, t) = \phi_{i,\text{in}}(x)q_{i,\text{in}}(t)$$

$$u_{i,\text{out}}(x, t) = \phi_{i,\text{out}}(x)q_{i,\text{out}}(t)$$ (2)

The displacement of the tower in the rotor plane (side-to-side) is modeled by a single degree of freedom, $q_{n,\text{in}}(t)$, and the displacement of the tower out of the rotor plane (fore-aft) is modeled by a single degree of freedom, $q_{n,\text{out}}(t)$. The Euler–Lagrange equation is shown in Eq. (1).

The kinetic and potential energies were calculated and substituted into Eq. (1) to obtain the equations of motion for the system. Eq. (3) shows the potential energy term,

$$V_{\text{coupled}} = \frac{1}{2} \sum_{i=1}^{3} \int_{0}^{L} \left[ EI_{\text{in}}(x) \left( \frac{\partial \phi_{i,\text{in}}}{\partial x} \right)^2 + EI_{\text{out}}(x) \left( \frac{\partial \phi_{i,\text{out}}}{\partial x} \right)^2 ight]$$

$$+ 2EI_{\text{out}}(x) \left( \frac{\partial \phi_{i,\text{in}}}{\partial x} \right) \left( \frac{\partial \phi_{i,\text{out}}}{\partial x} \right) + N(x) \left( \frac{\partial u_{i,\text{in}}}{\partial x} \right)^2 + N(x) \left( \frac{\partial u_{i,\text{out}}}{\partial x} \right)^2$$

$$+ G(x) \left( \frac{\partial u_{i,\text{in}}}{\partial x} \right)^2 + G(x) \left( \frac{\partial u_{i,\text{out}}}{\partial x} \right)^2 \right] \, dx + \frac{1}{2} k_{n,\text{in}} q_{n,\text{in}}^2 + \frac{1}{2} k_{n,\text{out}} q_{n,\text{out}}^2$$ (3)

where $L$ is the length of the blade, $E$ is the modulus of elasticity of the blade. The rotational speed of the rotor is denoted by $\Omega$ and is assumed to be constant throughout this paper. The term $k_{n,\text{in}}$ is the in-plane stiffness of the tower/nacelle and $k_{n,\text{out}}$ is the out-of-plane stiffness of the tower/nacelle. The rotation of the blade leads to a centrifugal stiffening. The term $N(x)$ denotes the centrifugal force that gives rise to this stiffening and is given in the following equation:

$$N(x) = \Omega^2 \int_{0}^{L} \mu(\xi) \xi \, d\xi$$ (4)

where $\mu$ denotes the mass distribution per unit length. The gravity force acting on the blade will also contribute to the stiffness of the blade. The term $G(x)$ denotes the gravity force acting on the blade. The origin of this force is shown in Fig. 2. The term $G(x)$ is defined in the following equation:

$$G(x) = -\frac{1}{2} g \cos\Psi_{i} \int_{0}^{L} \mu(\xi) \, d\xi$$ (5)

where $\Psi_{i}$ is the azimuthal angle of the $i$th blade and is given by

$$\Psi_{i} = \Omega t + \frac{2\pi}{3}(i-1)$$ (6)

![Fig. 2. Gravity acting on blade.](image)
In Eq. (3),

\[
\theta_{\text{lin}} = \frac{\partial u_{\text{lin}}}{\partial x} \\
\theta_{\text{lout}} = \frac{\partial u_{\text{lout}}}{\partial x}
\]  

(7)

The parameters \(I_{\text{in}}(x), I_{\text{out}}(x),\) and \(I_{\text{inout}}(x)\) are the second area moments of inertia and the second area products of inertia of a cross section of the blade. By using \(I_{\text{in}}(x)^*\) and \(I_{\text{out}}(x)^*\), the principal second area moments of the cross-section, \(I_{\text{in}}(x), I_{\text{out}}(x),\) and \(I_{\text{inout}}(x)\) can be calculated by the following equation:

\[
\begin{align*}
I_{\text{in}}(x) &= \frac{I_{\text{in}}(x)^* + I_{\text{out}}(x)^* + I_{\text{in}}(x)^* - I_{\text{out}}(x)^*}{2} \cos (2\beta) \\
I_{\text{out}}(x) &= \frac{I_{\text{in}}(x)^* + I_{\text{out}}(x)^* - I_{\text{in}}(x)^* - I_{\text{out}}(x)^*}{2} \cos (2\beta) \\
I_{\text{inout}}(x) &= \frac{I_{\text{in}}(x)^* - I_{\text{out}}(x)^*}{2} \sin (2\beta)
\end{align*}
\]  

(8)

where \(\beta\) is the pre-twist angle of a cross section of the blade with respect to the hub. Thus \(\beta\) is zero at the tip and \(\beta\) is at a maximum value at the hub. The effect of the blade structural pre-twist is therefore accounted for in this model following an approach developed by Zhu [19].

The terms in Eq. (3) can alternatively be written as

\[
V_{\text{coupled}} = \frac{1}{2} \sum_{i=1}^{3} (k_{e,\text{in}} + k_{g,\text{in}} + k_{g,\text{in}}) q_{i,\text{in}}^2 + \frac{1}{2} \sum_{i=1}^{3} (k_{e,\text{out}} + k_{g,\text{out}} + k_{g,\text{out}}) q_{i,\text{out}}^2
\]

\[
+ \sum_{i=1}^{3} k_{q,\text{in}} q_{i,\text{in}} q_{i,\text{in}} + \frac{1}{2} k_{d,\text{in}} q_{i,\text{in}}^2 + \frac{1}{2} k_{d,\text{out}} q_{i,\text{out}}^2
\]  

(9)

where \(k_{e,\text{in}}\) and \(k_{e,\text{out}}\) represent the elastic stiffness in the in-plane and out-of-plane directions respectively and are expressed as

\[
k_{e,\text{in}} = \int_0^L E I_{\text{in}}(x) \left( \frac{\partial^2 \phi_{\text{in}}(x)}{\partial x^2} \right)^2 dx
\]

\[
k_{e,\text{out}} = \int_0^L E I_{\text{out}}(x) \left( \frac{\partial^2 \phi_{\text{out}}(x)}{\partial x^2} \right)^2 dx
\]  

(10)

Similarly, \(k_{g,\text{in}}\) and \(k_{g,\text{out}}\) represent the geometric stiffness arising out of centrifugal stiffening in the in-plane and out-of-plane directions respectively and are given by

\[
k_{g,\text{in}} = \Omega^2 \int_0^L \left( \int_0^L \mu(\xi) \xi \frac{d\xi}{dx} \right) \left( \frac{\partial^2 \phi_{\text{in}}(x)}{\partial x^2} \right)^2 dx
\]

\[
k_{g,\text{out}} = \Omega^2 \int_0^L \left( \int_0^L \mu(\xi) \xi \frac{d\xi}{dx} \right) \left( \frac{\partial^2 \phi_{\text{out}}(x)}{\partial x^2} \right)^2 dx
\]  

(11)

while \(k_{g,\text{in}}\) and \(k_{g,\text{out}}\) are the stiffness terms arising from the gravity force in the in-plane and out-of-plane directions respectively and are given by

\[
k_{g,\text{in}} = -\frac{1}{2} \int_0^L \cos \Psi \int_0^L \mu(\xi) \frac{d\xi}{dx} \left( \frac{\partial^2 \phi_{\text{in}}(x)}{\partial x^2} \right)^2 dx
\]

\[
k_{g,\text{out}} = -\frac{1}{2} \int_0^L \cos \Psi \int_0^L \mu(\xi) \frac{d\xi}{dx} \left( \frac{\partial^2 \phi_{\text{out}}(x)}{\partial x^2} \right)^2 dx
\]  

(12)

The stiffness term arising from the coupling of the in-plane and out-of-plane degrees of freedom given by

\[
k_{c} = \int_0^L E I_{\text{inout}}(x) \left( \frac{\partial \phi_{\text{in}}(x)}{\partial x} \right) q_{i,\text{in}} \left( \frac{\partial \phi_{\text{out}}(x)}{\partial x} \right) q_{i,\text{out}}
\]

(13)

The kinetic energy term, \(T_{\text{coupled}}\), calculated for the \(i\)th blade and then summed over three blades is shown in the following equation:

\[
T_{\text{coupled}} = \frac{1}{2} \sum_{i=1}^{3} \int_0^L \mu(x) v_i^2 dx + \frac{1}{2} M_n q_{i,\text{in}}^2 + \frac{1}{2} M_n q_{i,\text{out}}^2
\]  

(14)

where \(v_i\) is the absolute velocity of the \(i\)th blade, \(q_{i,\text{in}}\) is the absolute in-plane velocity of the tower/nacelle, \(q_{i,\text{out}}\) is the absolute out-of-plane velocity of the tower/nacelle and \(M_n\) is the mass of the tower/nacelle.

The wind turbine model described in this section considers both in-plane (with primarily edgewise contribution) and out-of-plane blade vibrations (with primarily flapwise contribution) which are coupled. Due to the rotation of the blades the model was formulated using a position vector from a stationary reference point. The position vector, \(r(t)\), of a given point on
the $i$th blade at a distance of $x$ from the hub has the following representation in the rotating coordinate system with the base unit vectors $(i(t), j(t), k(t))$ (Fig. 1).

$$
\begin{align*}
    r_i(t) &= (x + q_{n,in} \sin \Psi_i) i(t) \\
          &+ (\dot{q}_{in}(x) q_{i,in} + q_{n,in} \cos \Psi_i) j(t) \\
          &+ (\dot{q}_{out}(x) q_{i,out} + q_{n,out}) k(t)
\end{align*}
$$

(15)

The velocity vector of a point $x$ along the blade, $v_i$, is obtained by differentiating $r_i$ with respect to time

$$
\begin{align*}
    v_i(t) &= \dot{r}_i(t) = \left(\dot{q}_{n,in} \sin \Psi_i - \Omega \dot{q}_{in}(x) q_{i,in}\right) i(t) \\
          &+ \left(\Omega x + \dot{q}_{in}(x) q_{i,in} + q_{n,in} \cos \Psi_i\right) j(t) \\
          &+ \left(\dot{q}_{out}(x) q_{i,out} + q_{n,out}\right) k(t)
\end{align*}
$$

(16)

The expressions for potential and kinetic energy are substituted back into Eq. (1) to derive the equations of motion. The equations of motion are of the form

$$
[M_u(t)]\ddot{\bar{q}} + [C_u(t)]\dot{\bar{q}} + [K_u(t)]\bar{q} = [Q_L] + [Q_g]
$$

(17)

where $[M_u(t)]$, $[C_u(t)]$ and $[K_u(t)]$ are the time dependent uncontrolled mass, damping and stiffness matrices respectively. The details of these matrices are provided in the appendix. The term $[Q_L]$ is the aerodynamic loading vector and $[Q_g]$ is the gravity loading vector, both of which are discussed later.

2.1. Structural damping

The Euler–Lagrangian formulation used to develop the models in this paper does not account for the structural damping of the wind turbine blades or of the nacelle/tower. Structural damping is a nonconservative force depending on the inherent properties of the structural materials and as such cannot be taken into account in the energy formulation. Blade structural damping terms, $C_b$, and nacelle/tower structural damping terms, $C_n$, are included in the equations of motion separately. This structural damping is assumed to be in the form of stiffness proportional damping. The structural damping is specific to the particular blades and nacelle/tower being modeled. Each blade or nacelle/tower will have its own associated damping ratio. The structural damping for the blades is given as

$$
C_b = \frac{2\xi_b}{\omega_b} K_b
$$

(18)

where $\xi_b$ is the damping ratio of the blade, $\omega_b$ is the natural frequency of the blade and $K_b$ is the stiffness of the blade.

Similarly, the structural damping for the nacelle/tower is given as

$$
C_n = \frac{2\xi_n}{\omega_n} K_n
$$

(19)

where $\xi_n$ is the damping ratio of the nacelle/tower, $\omega_n$ is the natural frequency of the nacelle/tower and $K_n$ is the stiffness of the nacelle/tower.

3. TMDs for control of in-plane blade vibration

TMDs are now placed inside the blades and the nacelle to control the in-plane vibrations of the blades. The model is reformulated to incorporate TMDs within the blades and in the nacelle. The equations for the potential and kinetic energies are outlined. A schematic demonstrating the new model is shown in Fig. 3.

![Fig. 3. TMD inside blade.](image-url)
The position vector of the damper in the $i$th blade, positioned at a location $x_0$ along the blade is given by

$$r_{d,i}(t) = (x_0 + q_{n,in} \sin \Psi_i) i(t) + (\phi_{in}(x)q_{i,in} + q_{n,in} \cos \Psi_i + d_i + y_0) j(t) + (\phi_{out}(x)q_{i,out} + q_{n,out}) k(t)$$

(20)

where $d_i$ is the in-plane motion of the damper relative to the blade and $y_0$ is the reference position of the damper mass from the centreline of the blade along the longitudinal axis. The potential energy, $V_{\text{coupled damping}}$, and the kinetic energy, $T_{\text{coupled damping}}$, of the system are respectively given by

$$V_{\text{coupled damping}} = \frac{1}{2} \sum_{i=1}^{3} (k_{r} + k_{g} + k_{gr}) q_i^2 + \frac{1}{2} \sum_{i=1}^{3} (k_{r} + k_{g} + k_{gr}) q_i^2$$

$$T_{\text{coupled damping}} = \frac{1}{2} \sum_{i=1}^{3} \int_0^L \mu(x)v_i^2 \, dx + \frac{1}{2} M_n q_{n,in}^2 + \frac{1}{2} M_n q_{n,out}^2 + \frac{1}{2} m_{bd} \sum_{i=1}^{3} \dot{v}_d^2 + \frac{1}{2} m_{nd} \dot{d}_{n,in}^2$$

(21)

where $m_{bd}$ is the mass of the blade dampers, $m_{nd}$ is the mass of the nacelle dampers, $k_{bd}$ is the stiffness of the blade dampers and $k_{nd}$ is the stiffness of the nacelle dampers. The motion of the in-plane nacelle damper is $q_{n,in}$ and the motion of the out-of-plane nacelle damper is $d_{n,out}$.

Substituting the terms for the kinetic and potential energies into Eq. (1) yields the equations of motion for the passively controlled 12 degree of freedom system. There are two degrees of freedom for each blade (in-plane and out-of-plane tip displacement), two degrees of freedom for the nacelle/tower (in-plane and out-of-plane displacement), one degree of freedom for each blade damper (displacement relative to the blade) and one degree of freedom for the nacelle damper (in-plane displacement).

The equations of motion for the passively controlled system also lead to a system with time varying mass, stiffness and damping matrices.

4. CCATMDs for control of in-plane blade vibration

4.1. Concept of a CCATMD

Active control of the in-plane blade vibration is now investigated. The ATMD is mounted at a certain distance from the tip of the blade. A cable is attached to the ATMD on one end and is connected to the blade tip at the other end. A tensile force is introduced in the cable. When the ATMD moves the cable tension at the blade tip acts at an inclined angle to the radial axis of the blade a component of which opposes the in-plane loading on the blade (see Fig. 4).
The cable has a pretension or prestress. When the ATMD vibrates due to the blade vibration two actions on the blade are generated. First, a restraining force acts on the blade due to the ATMD to reduce blade vibration. In addition, when the ATMD vibrates a component of the tension in the connected prestressed cable is also activated. This component also has the action of opposing the blade vibration due to the in-plane loading.

The control scheme is illustrated in Fig. 5. The introduction of active control and the presence of the pre-tensioned cable change the equations of motion of the system. The equations of motion for the new system with CCATMDs are outlined as follows:

\[
\frac{1}{2} \mathbf{M}_C(t) \ddot{\mathbf{q}} + [\mathbf{C}_c(t)] \dot{\mathbf{q}} + [\mathbf{K}_c(t)] \mathbf{q} = \{\mathbf{Q}_l\} + \{\mathbf{Q}_g\} + \{\mathbf{U}_m\} + \{\mathbf{U}_{cab}\}
\]

where \(\mathbf{M}_C(t)\), \(\mathbf{C}_c(t)\), and \(\mathbf{K}_c(t)\) are the time dependent controlled mass, damping and stiffness matrices respectively for the actively controlled system. The details of these matrices are provided in the appendix. The term \(\{\mathbf{Q}_l\}\) is the aerodynamic loading vector and \(\{\mathbf{Q}_g\}\) is the gravity loading vector, both of which are discussed later. \(\{\mathbf{U}_m\}\), the generalized ATMD control force vector, and \(\{\mathbf{U}_{cab}\}\), the vector of the actuator control forces, are defined later.

The equations of motion are represented in state-space form

\[
\dot{\mathbf{q}} = [\mathbf{A}]\mathbf{q} + \{\mathbf{Q}_l\} + \{\mathbf{Q}_g\} + \{\mathbf{B}\} \mathbf{U}_a + \{\mathbf{U}_{cab}\}
\]

where

\[
[A] = \begin{bmatrix}
0_{12 \times 12} & I_{12 \times 12} \\
-M_c^{-1}K_c & -M_c^{-1}C_c
\end{bmatrix}, \quad \{\mathbf{Q}_l\} = \begin{bmatrix}
0_{12 \times 1}
M_c^{-1}Q_l
\end{bmatrix},
\]

\[
\{\mathbf{Q}_g\} = \begin{bmatrix}
0_{12 \times 1} \\
M_c^{-1}Q_g
\end{bmatrix}, \quad [B] = \begin{bmatrix}
0_{12 \times 4}
M_c^{-1}[\mathbf{B}]
\end{bmatrix}
\]

and

\[
\{\mathbf{U}_{cab}\} = \begin{bmatrix}
0_{12 \times 1} \\
M_c^{-1}U_{cab}
\end{bmatrix}
\]

4.2. ATMD control force vector

\(\{\mathbf{U}_m\}\) is the generalized ATMD control force vector. The principle of virtual work is used to obtain \(\{\mathbf{U}_m\}\). The virtual work is expressed as

\[
\delta w = \sum_{i=1}^{3} (\delta d_i + \delta q_{in} \phi_{in}(\mathbf{x}_0)) u_{ai} - \sum_{i=1}^{3} \delta q_{in} \phi_{in}(\mathbf{x}_0) u_{ai} + (\delta d_n + \delta q_{n,in}) u_{an} - \delta q_{n,in} u_{an}
\]

Eq. (25) leads to the generalized control force vector \(\{\mathbf{U}_m\}\)

\[
\{\mathbf{U}_m\} = \left[\frac{\delta w}{\delta \mathbf{q}_i}\right] = [\mathbf{B}] \{\mathbf{U}_a\}
\]
where the control influence matrix, \([\mathbf{B}]\) is given by

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]  \(\text{(27)}\)

The vector of the actuator control forces \(\{U_a\}\) is given by

\[
\{U_a\} = \begin{bmatrix} u_{a1} \\ u_{a2} \\ u_{a3} \\ u_{am} \end{bmatrix}
\]  \(\text{(28)}\)

where \(u_{a1}\), \(u_{a2}\) and \(u_{a3}\) are the control forces on the actuators connected to the blade dampers for the first, second and third blade respectively and \(u_{am}\) is the control force on the actuator connected to the in-plane damper located at the top of the tower.

### 4.3. Cable force vector

The vector \(\{U_{cab}\}\) is a force vector which appears as an additional vector on the right hand side of Eq. (23) due to the tension in the cable.

The cable arrangement is shown in Fig. 4. The tensile force in the cables is dependent on the motion of the ATMD within the blade and is therefore time varying in nature. The tensile force in the cable, \(\{U_{cab}\}\) is derived using the principle of virtual work as follows. The virtual work due to the cable force on the \(i\)th blade is

\[
\delta w = T_{cab} \sin \theta_{cab} \cos \alpha \phi_{in}(L) \delta q_{in} + T_{cab} \cos \theta_{cab} \sin \alpha \phi_{out}(L) \delta q_{out}
\]  \(\text{(29)}\)

where \(\alpha\) is the blade pre-twist angle and \(T_{cab}\) is the pretension in the cable.

Because of the pre-twist the cable does not lie on the plane of the rotor. However, in this paper it is assumed that \(\alpha\) is small such that the virtual work due to the cable force on the \(i\)th blade is approximated by

\[
\delta w = T_{cab} \sin \theta_{cab} \phi_{in}(L) \delta q_{in}
\]  \(\text{(30)}\)

Eq. (30) leads to

\[
\frac{\delta w}{\delta q_{in}} = T_{cab} \sin \theta_{cab}
\]  \(\text{(31)}\)

for the normalized mode with \(\phi_{in}(L) = 1\). Therefore, \(\{U_{cab}\}\) is given by

\[
\{U_{cab}\} = \begin{bmatrix} T_c \\ \theta_{6\times1} \end{bmatrix}
\]  \(\text{(32)}\)

where \(\{T_c\}\) is given by

\[
\{T_c\} = \begin{bmatrix} T_{cab} \sin \theta_{cab} \\ -T_{cab} \sin \theta_{cab} \\ T_{cab} \sin \theta_{cab} \\ -T_{cab} \sin \theta_{cab} \\ T_{cab} \sin \theta_{cab} \\ -T_{cab} \sin \theta_{cab} \end{bmatrix}
\]  \(\text{(33)}\)

In Eqs. (29) and (31)–(33), \(\theta_{cab}\) is given as

\[
\theta_{cab} = \frac{d_i}{L-x_0}
\]  \(\text{(34)}\)

Since \(d_i\), the position of the ATMD within the \(i\)th blade varies with time, \(\theta_{cab}\) will vary with time. This leads to a nonlinear time varying tensile force vector \(\{U_{cab}\}\).

### 4.4. CCATMD control scheme

An appropriate control scheme is required to implement the active vibration control strategy. In this paper an optimal control scheme is chosen to obtain the control forces on the actuators, \(\{U_a\}\). The external disturbance \(U_{cab}\) is not considered in the design of the feedback controller. The effect of \(U_{cab}\) is considered indirectly by virtue of state feedback that may lead to a sub-optimal controller. The Linear Quadratic Regulator (LQR) is used in this paper as it is a well-known method for optimal control. The LQR is a feedback controller used to operate a dynamic system at minimum cost [20]. The linear state feedback
is given by

$$\{U_a\} = [G_{LQR}] \{\Phi\}$$  \hspace{1cm} (35)

where $[G_{LQR}]$ is the feedback gain and $\{\Phi\}$ is the state vector. The value for $[G_{LQR}]$ is optimized such that the following performance criterion is minimized

$$J = \min_{U_a} \left\{ \int_0^T \left[ \{\Phi\}^T \{Q\} \{\Phi\} + \{U_a\}^T \{R\} \{U_a\} \right] dt \right\}$$  \hspace{1cm} (36)

where $[Q]$ and $[R]$ are weighting matrices for the response and the control force respectively.

Appropriate choice of the weighting matrices is crucial when performing LQR control. In this study the LQR cost function consists of applying weights to individual state and input values separately (i.e. all non-diagonal $[Q_{LQR}]$ and $[R_{LQR}]$ matrix elements are set to zero). Importance is placed on maintaining small values for blade displacements thus their weighting factors are chosen to be large. Additionally, because each CCATMD has a limited range of motion (limited by the chord length of the blade at the location of the damper installation), similar importance is placed on maintaining small values for the displacements of the CCATMDs, and their weighting factors are chosen to be large as well.

After the gain matrix, $[G_{LQR}]$, is obtained by the LQR method the control input is then calculated and incorporated into the equations of motion.

5. Loading

The derived models will be subjected to realistic wind turbine loads and dynamic responses will be computed. The realistic loads are described and simulated in this section of the paper. The main sources of loading experienced by wind turbines are aerodynamic loading and gravity loading. Both of these load types are considered in this study.

5.1. Aerodynamic loading

In order to have a realistic estimate of the aerodynamic loading to which the rotor is subjected, models based on the Blade Element Momentum (BEM) theory have been adopted according to a method developed by Hansen. The BEM method computes the aerodynamic loads experienced by the rotor by taking into account the aerodynamic properties of the blade section airfoils, the structural and geometrical properties of the rotor, the wind speed and the rotational speed of the blades.

The BEM method couples momentum theory and blade element theory with the local events taking place at each blade element. BEM theory uses both axial and angular momentum balances to determine the flow and the resulting forces at the blade [21].

In BEM theory the rotor blade is analyzed in sections based on an assumption that the flow at a given annulus does not affect the flow at adjacent annuli (Fig. 6). The rotor blade is discretized into $N$ elements. Each element is located at a radial distance of $r$ from the hub and has a chord length of $c = c(r)$ and width $dr$. These elements are analyzed in sections along their length and the forces are summed over all sections to get the total force on the rotor Hansen. The rotor has a radius $L$ and angular velocity $\Omega$.

BEM accounts for a steady aerodynamic effect. Changes in the angle of attack are instantly felt in the aerodynamic loads and therefore the time scale for adjustment of the non-stationary flow is assumed to be small compared to the fundamental eigenfrequency of the blade. The effects of a discrete number of blades and far-field effects when the turbine is heavily loaded are not considered.

For more details about the specifics of implementation of Hansen’s modified BEM method see [21].

Fig. 6. Blade model according to the BEM theory.
5.2. Turbulence model

A rotationally sampled turbulence model has been assumed in this paper. In the 1980s the issue of rotationally sampled turbulence and the influence of a rotating blade in a turbulent wind field were first recognized by Madsen and Frandsen [22], Madsen et al. [23] and Connell [24]. Rotationally sampled turbulence transforms the energy of the longitudinal turbulence spectrum to multiples of the rotational frequency. The authors’ models were frequency domain models with longitudinal turbulence represented in terms of a coherence function and a wind power spectrum transformed to a rotational turbulence spectrum observed from the blade [25]. Veers [26] developed the Sandia method for generation of turbulence fields in the time domain. This model is the basis for most turbulence models to date. Rotationally sampled wind speed is synthesized by first applying an inverse Fourier transform to the power spectral density, via the Shinozuka technique (Shinozuka and Jan [27]) to generate a stochastic time series. Then a model of the cross-spectral density is used to estimate the wind that the blade would experience as it rotates through the turbulence [28].

The turbulent component of the wind velocity is quantified and included in the modified BEM method. This results in turbulent aerodynamic loading.

5.3. Virtual work to determine generalized aerodynamic loads

The aerodynamic loads normal to and tangential to the rotor plane (corresponding to the aerodynamic loads in the out-of-plane and in-plane direction, respectively) are calculated by the modified BEM method and are $p_N$ and $p_T$ respectively. Virtual work has been used to determine the generalized loads on the blades and the nacelle. The total virtual work done is given by

$$\delta w = \sum_{i=1}^{3} (\delta q_{i,in} P_{i,in} + \delta q_{i,out} P_{i,out} + \delta q_{n,in} P_{n,in} + \delta q_{n,out} P_{n,nout}) \quad (37)$$

where $P_{i,out}$ and $P_{i,in}$ are calculated for the $i$th blade by

$$P_{i,out} = \int_0^L p_{N,i}(x,t) \phi_{out}(x) \, dx$$
$$P_{i,in} = \int_0^L p_{T,i}(x,t) \phi_{in}(x) \, dx \quad (38)$$

and $P_{n,out}$ and $P_{n,in}$ are given by

$$P_{n,out} = \sum_{i=1}^{3} P_{n,out}(x,t)$$
$$P_{n,in} = \sum_{i=1}^{3} \int_0^L p_{T,i}(x,t) \cos \psi_i \quad (39)$$

The generalized loads for each generalized degree of freedom corresponding to the in-plane vibration of the blades are given by

$$Q_{i,in} = \frac{\delta w}{\delta q_{i,in}} = P_{i,in} \quad i = 1, 2, 3 \quad (40)$$

The generalized loads for each generalized degree of freedom corresponding to the out-of-plane vibration of the blades are given by

$$Q_{i,out} = \frac{\delta w}{\delta q_{i,out}} = P_{i, out} \quad i = 1, 2, 3 \quad (41)$$

The generalized load for the generalized degree of freedom corresponding to the in-plane vibration of the nacelle is given by

$$Q_{n,in} = \frac{\delta w}{\delta q_{n,in}} = P_{n,in} \quad (42)$$

The generalized load for the generalized degree of freedom corresponding to the out-of-plane vibration of the nacelle is given by

$$Q_{n,out} = \frac{\delta w}{\delta q_{n,out}} = P_{n,out} \quad (43)$$

It may be noted that the in-plane and out-of-plane loads are coupled for the model used in this paper.
5.4. Gravity loading

The loading effect of gravity has been considered. The gravity force causes a harmonic in-plane load on the blade. The gravity force acting on the blade is illustrated by Fig. 2. Resolving the gravity force leads to two forces, one normal to the blade:

\[ dG_N = \mu(x) \, dx \, g \sin \Psi_i \]  

(44)

and one along the blade:

\[ dG_A = \mu(x) \, dx \, g \cos \Psi_i \]  

(45)

where \( \mu(x) \) is the mass per unit length of the blade and \( g \) is the acceleration due to gravity, 9.81 m/s².

The total virtual work done by the gravity force normal to the blade, \( \delta w_g \), is obtained as

\[ \delta w_g = \sum_{i=1}^{3} \int_{0}^{L} (\mu(x) \, dx \, g \sin (\Psi_i)) (\delta q_{in} \phi(x) + \delta q_{in,in} \cos (\Psi_i)) \]  

(46)

Therefore the load on the \( i \)th blade due to gravity is given by

\[ Q_i = \frac{\delta w_g}{\delta q_{in}} = \int_{0}^{L} (\mu(x) \, dx \, g \sin (\Psi_i)) \]  

(47)

The load on the nacelle is given by

\[ Q_{n} = \frac{\delta w_g}{\delta q_{in,in}} = \sum_{i=1}^{3} \int_{0}^{L} (\mu(x) \, dx \, g \sin (\Psi_i) \cos (\Psi_i)) = 0 \]  

(48)

6. Numerical simulations

Numerical simulations have been performed to illustrate the effects of the passive and active control strategies developed. The NREL 5-MW baseline wind turbine [29] has been used to develop and test the model. The details of this turbine are provided in Table 1. The proposed model and control strategies were implemented in MATLAB.

The blade considered is the LM61.5 P2 (manufactured by LM Wind Power), which is 61.5 m long and has a total mass of 17,740 kg. Since the radius of the hub is 1.5 m, the total rotor radius is 63 m. The in-plane and out-of-plane modes shapes considered are shown in Fig. 7. These have been computed from blade structural data by using the MODES [30] finite element code. Modes performs eigenvalue analysis to compute mode shapes and frequencies for wind turbine blades.

The blades are modeled as cantilever beams fixed at the hub. The mode shapes must therefore have zero deflection and slope at the hub. The following sixth-order polynomials have been obtained from Modes [30], to be used as admissible

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Properties of NREL 5-MW baseline HAWT [29].</th>
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<tbody>
<tr>
<td><strong>NREL 5-MW baseline wind turbine properties</strong></td>
<td></td>
</tr>
<tr>
<td>Basic description</td>
<td>Max. rated power</td>
</tr>
<tr>
<td></td>
<td>Rotor orientation, configuration</td>
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<tr>
<td></td>
<td>Rotor diameter</td>
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<td></td>
<td>Hub height</td>
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<td></td>
<td>Cut-in, rated cut-out wind speed</td>
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<tr>
<td></td>
<td>Cut-in, rated rotor speed</td>
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<tr>
<td>Blade (LM 61.5 P2)</td>
<td>Length</td>
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<td></td>
<td>Overall (integrated) mass</td>
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<tr>
<td></td>
<td>Second mass moment of inertia</td>
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<td></td>
<td>1st in-plane mode natural frequency</td>
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<tr>
<td></td>
<td>1st out-of-plane mode natural frequency</td>
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<tr>
<td></td>
<td>Structural-damping ratio (all modes)</td>
</tr>
<tr>
<td>Hub + Nacelle</td>
<td>Hub diameter</td>
</tr>
<tr>
<td></td>
<td>Hub mass</td>
</tr>
<tr>
<td></td>
<td>Nacelle mass</td>
</tr>
<tr>
<td>Tower</td>
<td>Height above ground</td>
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<tr>
<td></td>
<td>Overall (integrated) mass</td>
</tr>
<tr>
<td></td>
<td>1st Fore-Aft mode natural frequency</td>
</tr>
<tr>
<td></td>
<td>1st Side-to-Side mode natural frequency</td>
</tr>
<tr>
<td></td>
<td>Structural-damping ratio (all modes)</td>
</tr>
</tbody>
</table>
shape functions:

\[
\phi_{\text{in}}(x) = -0.6893x^6 + 2.3738x^5 - 3.6043x^4 + 2.5737x^3 + 0.3461x^2
\]
\[
\phi_{\text{out}}(x) = -2.4766x^6 + 5.1976x^5 - 3.4820x^4 + 1.7085x^3 + 0.0525x^2
\]

where \( \bar{x} = x/L \), \( \phi_{\text{in}}(1) = 1 \) and \( \phi_{\text{out}}(1) = 1 \).

In all of the actively controlled simulations the following weighting matrices have been used for the LQR algorithm:

\[
\begin{bmatrix} Q \end{bmatrix} = \alpha I_{24 \times 24} \quad \begin{bmatrix} R \end{bmatrix} = \beta I_{4 \times 4}
\]

where \( \alpha = 1 \) and \( \beta = 10^{-10} \).

6.1. Simulation results

Simulations were performed to determine the effectiveness of the proposed control strategies. In all of the simulations the rotational speed of the turbine, \( \Omega \), is 12.1 rev/min. This is the rated rotor speed of the turbine. The first simulation performed considered a turbine operating at rated speed (12 m/s) with a turbulence intensity of 10 percent. The load applied to the rotor is shown in Fig. 8. The passive control strategy is investigated first. The TMDs are tuned to the blade’s fundamental in-plane frequency, 6.67 rad/s and have a mass ratio, \( \mu_{\text{damp}} \), of 3 percent. The blade dampers are located 75 percent along the blades i.e. \( x_0 = 0.75L \). At this location a damper stroke of approximately 3 m can be incorporated inside the blade. The TMD tuning ratio, \( \nu \), is taken as 1. The TMD damping is assumed to be optimal. There are many approximate and empirical expressions available in the literature for the evaluation of the optimal damping ratio of a TMD such as those formulated by Ghosh and Basu [31] and Luft [32]. In this paper the following expression given by Luft is used

\[
\zeta_{\text{damp}} = \frac{\sqrt{\mu_{\text{damp}}}}{2}
\]

Fig. 9 shows the modest performance of the passive control strategy. Fig. 10 shows that the active control strategy performs significantly better than the passive control strategy. The tension in the string is taken as 100 kN. A window of 100 s from each 10-min simulation is shown for illustration in Figs. 9(b) and 10(b). There are peak-to-peak reductions of up to 22.5 percent in the response when the passive TMDs are used. When the CCATMDs are used there is a reduction in the peak-to-peak response of up to 36 percent compared to the uncontrolled blade and a peak-to-peak reduction of 24 percent when compared to the passively controlled blade. The uncontrolled peak response is reduced by 15 percent when the CCATMDs are used. The standard deviation of the uncontrolled peak response is reduced by 23 percent when the CCATMDs are used. The passively controlled blade does not reduce the peak response or the standard deviation of the response.

The CCATMD results are now compared to the case of ATMDs without cables. Fig. 11 shows that the CCATMD scheme designed improves upon a traditional ATMD. When CCATMDs are used there is a reduction in the peak-to-peak response of up to 29 percent when compared to the ATMD response.

The control force required for the CCATMD controlled blade is given in Fig. 12.

For this particular load case a comparison can be made to the active tendon system designed by Staino et al. [13]. In Staino et al.’s work a similar load case was examined. The same 5 MW turbine operating at rated conditions with a turbulence intensity of 10 percent was studied. Staino et al.’s control scheme achieved peak-to-peak reductions of up to 32 percent compared to the uncontrolled system for a peak active control force requirement of 41.8 kN. The CCATMD controlled system proposed here can perform better for a reduced control effort. CCATMDs with a cable tension of 100 kN can reduce
peak-to-peak in-plane blade vibration by up to 36 percent compared to the uncontrolled system with a peak active control force requirement of 10 kN.

The focus of this paper is control of in-plane vibrations of wind turbine blades. However, since the in-plane and out-of-plane motions are coupled the out-of-plane response is shown for sake of completeness. Fig. 13 shows that there is a slight reduction in the out-of-plane response of the blade due to the CCATMD control scheme. Peak-to-peak reductions of up to 20 percent are shown.

For the next simulation the tension in the cable is increased to 200 kN, the loading is the same as the previous case. The passive control strategy employed is the same as the previous case. Comparison of Fig. 14 with Fig. 9 shows that the active control strategy again performs significantly better than the passive control strategy. A window of 100 s from each 10-min simulation is shown for illustration. The CCATMDs achieve peak-to-peak reductions of 41 percent compared to the uncontrolled blade and up to 38 percent when compared to the passively controlled blade. The CCATMDs also reduce the peak response of the uncontrolled blades by 10 percent and the standard deviation of the uncontrolled response by 34 percent.

Fig. 15 compares the CCATMD results to the case of ATMDs without cables. The CCATMD scheme designed improves upon a traditional ATMD. When CCATMDs are used there is a reduction in the peak-to-peak response of up to 43 percent when compared to the ATMD response. It is clear that increasing the tension in the cable has improved the response reduction.

The control force required for the actively controlled blade is given in Fig. 16.

A comparison can again be made to the active tendon system designed by Staino et al. [13]. As stated previously, for this load case Staino et al.’s control scheme achieved peak-to-peak reductions of up to 32 percent compared to the uncontrolled
system for a peak active control force requirement of 41.8 kN. The CCATMD controlled system again performs better for a reduced control effort. CCATMDs with a cable tension of 200 kN can reduce peak-to-peak in-plane blade vibration by up to 41 percent compared to the uncontrolled system with a peak active control force requirement of 5 kN.

Fig. 10. Comparison of uncontrolled system and CCATMD actively controlled system, $U = 12 \text{ m/s}$, $I = 10\%$, $T = 100 \text{ kN}$.

Fig. 11. Comparison of ATMD scheme and CCATMD scheme, $U = 12 \text{ m/s}$, $I = 10\%$, $T = 100 \text{ kN}$. 
The effect of the CCATMD control scheme on the out-of-plane blade response is again shown for sake of completeness. Fig. 17 shows that there is a slight reduction in the out-of-plane response of the blade due to the CCATMD control scheme. Peak-to-peak reductions of up to 23 percent are shown.

For the last simulation a highly turbulent case is investigated. It is important that the turbine can sustain the high loads from the turbulent wind without failure or damage and therefore vibration reduction is crucial when the wind speed and turbulence is high. In this case the wind speed is taken as 18 m/s and the turbulence intensity is 30 percent. Fig. 18 shows the loading applied to the blades. The passive control strategy employed is the same as the previous case. A window of 100 s from each 10-min simulation is shown in Figs. 19(b) and 20(b) for illustration. The actively controlled system again performs markedly better than the passive control strategy. The passively controlled blade achieves peak-to-peak reductions of 46 percent when compared to the uncontrolled blade and reduces the peak response of the blade by 8 percent. The CCATMDs achieve peak-to-peak reductions of 67 percent compared to the uncontrolled blade and 33 percent when compared to the...
Fig. 14. Comparison of uncontrolled system and CCATMD actively controlled system, $U=12 \text{ m/s}, I=10\%, T=200 \text{ kN}$.

Fig. 15. Comparison of ATMD scheme and CCATMD scheme, $U=12 \text{ m/s}, I=10\%, T=200 \text{ kN}$. 
passively controlled blade. The CCATMDs also reduce the peak response of the uncontrolled blades by 15 percent and the standard deviation of the uncontrolled response by 36 percent.

Fig. 21 compares the CCATMD results to the case of ATMDs without cables. The CCATMD scheme designed improves upon a traditional ATMD. When CCATMDs are used there is a reduction in the peak-to-peak response of up to 25 percent when compared to the ATMD response.

The control force required for the actively controlled blade is shown in Fig. 22.

The effect of the CCATMD control scheme on the out-of-plane blade response is again shown for sake of completeness. Fig. 23 shows that there is a slight reduction in the out-of-plane response of the blade due to the CCATMD control scheme. Peak-to-peak reductions of up to 29 percent are shown.

In each of the above cases the damper stroke required to achieve the reductions can be accommodated within the blade width. For the purpose of illustration the damper displacement required for the actively controlled blade in the last case (most critical) is shown in Fig. 24.
6.2. Choice of cable tension

Suitable cable tension values of 100 kN and 200 kN were arrived at by a parametric study. It is assumed the cables are made from mild structural steel with a yield stress of $\sigma = 250$ N/mm$^2$. It is desirable to keep the cross sectional area of the cables small due to the space available inside the blades and for implementation reasons. A cable tensile force of 100 kN
would require a cable diameter of 22 mm to ensure yielding does not occur. A cable tension of 200 kN requires a cable diameter of 32 mm to avoid yielding. These diameter values are acceptable. It was found through a parametric analysis that increasing the cable tension above 200 kN did not achieve significantly improved results.

Fig. 20. Comparison of uncontrolled system and CCATMD actively controlled system, $U = 18$ m/s, $I = 30\%$, $T = 200$ kN.

Fig. 21. Comparison of ATMD scheme and CCATMD scheme, $U = 18$ m/s, $I = 30\%$, $T = 200$ kN.
6.3. Analysis of simulation results

The simulations have shown that for a turbine operating at normal rated conditions the active control strategy can reduce in-plane vibrations by up to 41 percent compared to the uncontrolled blade responses. The control strategy also works for highly turbulent cases, reducing the blade vibration by up to 67 percent in high loading conditions compared to the uncontrolled blade responses. It has also been shown clearly that the CCATMD is a more effective control strategy than a simple ATMD. In each case the CCATMD produced larger response reductions than a traditional ATMD control scheme. As the turbulence intensity is increased it is noticed that both the passive and active control strategies become more effective. As the tensile force is increased in the cable the active control strategies also improved with a reduction in the actuator control force required for the TMD.

Fig. 22. Control force applied, $U = 18 \text{ m/s}$, $I = 30\%$, $T = 200 \text{ kN}$.

Fig. 23. Comparison of uncontrolled blade and CCATMD controlled blade (out-of-plane), $U = 18 \text{ m/s}$, $I = 30\%$, $T = 200 \text{ kN}$. 
7. Conclusions

In this study a new CCATMD for vibration control in wind turbine blades has been proposed and the performance investigated. The wind turbine was modeled as a time varying MDOF system under turbulent wind loading. The coupled MDOF model developed focused on the structural dynamics of the turbine including the interaction between the blades and the tower and the coupling between the in-plane and out-of-plane blade vibrations. Four mass dampers were added to the model, one attached to each blade and one at the top of the tower to control the response of each component. An active control strategy was developed using the LQR to control the in-plane displacement response of the blades and this was compared to the passively controlled system (passive mass dampers) and the uncontrolled system (no mass dampers). Numerical simulations were carried out to determine the effectiveness of the active control strategy for various turbulence levels as compared to passive and uncontrolled cases. It was found that the active control strategy employed achieved greater response reductions than the passive TMDs in general. This study has also shown that a cable connected ATMD control strategy is feasible for a wind turbine blade particularly for higher turbulent loadings with enough room for stroke within the blade.

Acknowledgments

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Appendix A

System matrices for uncontrolled and controlled cases.

Uncontrolled matrices:

\[
[M_u] = \begin{bmatrix}
    m_{2,\text{in}} & 0 & 0 & a_1 & 0 & 0 & 0 & 0 \\
    0 & m_{2,\text{in}} & 0 & a_2 & 0 & 0 & 0 & 0 \\
    0 & 0 & m_{2,\text{in}} & a_3 & 0 & 0 & 0 & 0 \\
    a_1 & a_2 & a_3 & N_m & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & m_{2,\text{out}} & 0 & 0 & b_1 & 0 \\
    0 & 0 & 0 & 0 & m_{2,\text{out}} & 0 & b_2 & 0 \\
    0 & 0 & 0 & 0 & 0 & m_{2,\text{out}} & b_3 & 0 \\
    0 & 0 & 0 & 0 & b_1 & b_2 & b_3 & N_m
\end{bmatrix}
\] (52)

where

\[
a_i = m_{1,\text{in}} \cos \Psi_i
\]

\[
b_i = m_{1,\text{out}} \cos \Psi_i
\]

\[
N_m = 3m_0 + M_n
\]
\[
[C_\alpha] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
d_1 & d_2 & d_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(53)

where

\[
d_i = -2\Omega m_{1,\text{in}} \sin \Psi_i
\]

\[
[K_\alpha] = \begin{bmatrix}
k_{b,\text{in}} & 0 & 0 & 0 & k_c & 0 & 0 & 0 \\
0 & k_{b,\text{in}} & 0 & 0 & 0 & k_c & 0 & 0 \\
0 & 0 & k_{b,\text{in}} & 0 & 0 & 0 & k_c & 0 \\
c_1 & c_2 & c_3 & k_{n,\text{in}} & 0 & 0 & 0 & 0 \\
k_c & 0 & 0 & 0 & k_{b,\text{out}} & 0 & 0 & 0 \\
0 & k_c & 0 & 0 & 0 & k_{b,\text{out}} & 0 & 0 \\
0 & 0 & k_c & 0 & 0 & 0 & k_{b,\text{out}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{n,\text{out}} \\
\end{bmatrix}
\]

(54)

where

\[
k_{b,\text{in}} = k_{e,\text{in}} + k_{g,\text{in}} - k_{gr,\text{in}} - \Omega^2 m_{2,\text{in}}
\]

\[
k_{b,\text{out}} = k_{e,\text{out}} + k_{g,\text{out}} - k_{gr,\text{out}}
\]

\[
c_i = -\Omega^2 m_{1,\text{in}} \cos \Psi_i
\]

Controlled matrices:

\[
[M_\alpha] = \begin{bmatrix}
t_{1,\text{in}} & \nu & 0 & 0 & 0 & 0 & a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\nu & m_{bd} & 0 & 0 & 0 & 0 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & t_{2,\text{in}} & \nu & 0 & 0 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \nu & m_{bd} & 0 & 0 & b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & t_{2,\text{in}} & \nu & a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \nu & m_{bd} & b_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_1 & b_1 & a_2 & b_2 & a_3 & b_3 & N_m & m_{nd} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{nd} & m_{nd} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{1,\text{out}} & 0 & 0 & c & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{1,\text{out}} & 0 & c & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_{1,\text{out}} & 0 & c \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{nd} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{nd} & m_{nd} \\
\end{bmatrix}
\]

(55)

where

\[
a_i = m_{1,\text{in}} \cos \Psi_i + m_{bd}\phi_{in}(x_0) \cos \Psi_i
\]

\[
b_i = m_{bd} \cos \Psi_i
\]

\[
t_{i,\text{in}} = m_{2,\text{in}} + m_{bd}\phi_{in}(x_0)^2
\]

\[
t_{i,\text{out}} = m_{2,\text{out}} + m_{bd}\phi_{out}(x_0)^2
\]

\[
\nu = m_{bd}\phi_{in}(x_0)
\]

\[
c = m_{1,\text{out}} + m_{bd}\phi_{out}(x_0)
\]

\[
N_m = 3m_0 + 3m_{bd} + 2m_{nd} + M_n
\]
where

\[
d_i = -2\Omega m_{1,\text{in}} \sin \psi_i - 2\Omega m_{bd}\phi_{\text{in}}(x_0) \sin \psi_i
\]

\[
e_i = -2\Omega m_{bd} \sin \psi_i
\]

\[
[K_c] = \begin{bmatrix}
k_{b,\text{in}} & k_d & 0 & 0 & 0 & 0 & 0 & 0 & k_c & 0 & 0 & 0 & 0 \\
k_d & k_{d,t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{b,\text{in}} & k_d & 0 & 0 & 0 & 0 & k_c & 0 & 0 & 0 & 0 \\
0 & 0 & k_d & k_{d,t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_{b,\text{in}} & k_d & 0 & 0 & 0 & 0 & k_c & 0 & 0 \\
0 & 0 & 0 & 0 & k_d & k_{d,t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
f_1 & g_1 & f_2 & g_2 & f_3 & g_3 & k_{n,\text{in}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & k_{n,\text{out}} & 0 & 0 & 0 & 0 & 0 & 0 \\
k_c & 0 & 0 & 0 & 0 & 0 & 0 & k_{n,\text{out}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_c & 0 & 0 & 0 & 0 & 0 & k_{b,\text{out}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_c & 0 & 0 & 0 & 0 & k_{b,\text{out}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{b,\text{out}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{n,\text{out}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{n,\text{out}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
[K_e] = \begin{bmatrix}
\end{bmatrix}
\]

where

\[
k_{b,\text{in}} = k_{e,\text{in}} + k_{g,\text{in}} - \Omega^2 m_{2,\text{in}} - \Omega^2 m_{bd}\phi_{\text{in}}(x_0)^2
\]

\[
k_d = -\Omega^2 m_{bd}\phi_{\text{in}}(x_0)
\]

\[
k_{d,t} = -\Omega^2 m_{bd} + k_{bd}
\]

\[
k_{b,\text{out}} = k_{e,\text{out}} + k_{g,\text{out}} - k_{g,\text{out}}
\]

\[
f_i = -\Omega^2 m_{1,\text{in}} \cos \psi_i - \Omega^2 m_{bd}\phi_{\text{in}}(x_0) \cos \psi_i
\]

\[
g_i = -\Omega^2 m_{bd} \cos \psi_i
\]

References


