Adaptive Hypersonic Flight Control via Backstepping and Kriging Estimation

Bin Xu, Nanyang Technological University, Singapore
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Bin Xu, Fuchun Sun, Shixing Wang
Department of Computer Science and Technology
Tsinghua University
Beijing, China
smileface.binxu@gmail.com

Hao Wu
Department of Mathematics and Computer Science
Free University of Berlin
Berlin, Germany
hwu@zedat.fu-berlin.de

Abstract—This paper investigates the adaptive Kriging controller for the longitudinal dynamics of a generic hypersonic flight vehicle (HFV). For the altitude subsystem, the dynamics are transformed into the strict-feedback form where the back-stepping scheme is employed. Considering the nonlinearity of the dynamics, the nominal feedback is included in the controller while Kriging system is designed to estimate the uncertainty. With the proposed controller, the almost surely bounded stability is guaranteed. The simulation study is presented to show the effectiveness of the proposed control approach.

Index Terms—Hypersonic flight control, back-stepping, recursive Kriging estimation, discrete-time

I. INTRODUCTION

The recent success of NASA’s X-43A experimental airplane in flight testing has affirmed the feasibility of Hypersonic Flight vehicles (HFVs). HFVs are intended to present a reliable and more cost efficient way to access space. The longitudinal model of HFV is known to be unstable, non-minimum phase with respect to the regulated output, and affected by significant model uncertainty. Therefore hypersonic flight vehicles are extremely sensitive to changes in atmospheric conditions as well as physical and aerodynamic parameters. For the controller design for hypersonic aircraft, it must guarantee stability and provide a satisfied control performance.

Recently many methods have been proposed for HFV control, such as the sliding mode control [1], the adaptive control [2] and neural robust control [3]. The sequential loop closure controller design [4] is based on the decomposition of the equations into functional subsystems. As described in [5][6] the altitude subsystem can be transformed into the strict-feedback form and the back-stepping design [7] is adopted. Based on system transformation, one high gain observer based controller [8] is proposed. Also different considerations are included such as the aerothermoelastic effects [9], the control constrain [10] and the improvement of control performance [11].

However, most of the research of HFV is focusing the control design in continuous-time domain. In this paper, the discrete controller via back-stepping is proposed. For the system uncertainty, we took Kriging estimation [12]. Compared with other estimators, it can obtain the best unbiased estimates and evaluate the reliability of the estimations based on the hypothesis of rationalized variables. To deal with the online handling of the data, we adopt the recursive form of the Kriging estimator [13] with only a finite number of data to be recorded.

This paper is organized as follows. Section II describes the longitudinal dynamics of a generic HFV. The modeling and transformation is shown in Section III. The brief description of Kriging estimator is explained in Section IV. Section V presents the adaptive Kriging controller design. The simulation result is included in Section VI. Section VII presents several comments and final remarks.

II. HYPERSONIC VEHICLE MODELING

The control-oriented model of the longitudinal dynamics of HFV considered in this study is given by [1]. This model is comprised of five state variables \(X = [V, h, \alpha, \gamma, q]^T\) and two control inputs \(U_c = [\delta_c, \beta]^T\) where \(V\) is the velocity, \(\gamma\) is the flight path angle, \(h\) is the altitude, \(\alpha\) is the attack angle, \(q\) is the pitch rate, \(\delta_c\) is elevator deflection and \(\beta\) is the throttle setting.

\[
\dot{V} = \frac{T \cos \alpha - D}{m} - \frac{\mu \sin \gamma}{r^2} \tag{1}
\]

\[
\dot{h} = V \sin \gamma \tag{2}
\]

\[
\dot{\gamma} = \frac{L + T \sin \alpha}{mV} - \frac{(\mu - V^2r) \cos \gamma}{V r^2} \tag{3}
\]

\[
\dot{\alpha} = q - \dot{\gamma} \tag{4}
\]

\[
\dot{\beta} = \frac{M_{yy}}{T_{yy}} \tag{5}
\]

where \(m, I_{yy}, \mu\) are the mass of aircraft, moment of inertia about pitch axis and gravity constant. \(r\) is the radial distance from center of the earth. \(T, D, L\) and \(M_{yy}\) represent thrust, drag, lift-force and pitching moment respectively.

\[
L = \frac{1}{2} \rho V^2 SC_L
\]

\[
D = \frac{1}{2} \rho V^2 SC_D
\]

\[
T = \frac{1}{2} \rho V^2 SC_T
\]

\[
M_{yy} = \frac{1}{2} \rho V^2 SC [C_M(\alpha) + C_M(\delta_c) + C_M(q)]
\]

\[
r = h + R_E
\]

The engine dynamics are modeled by a second order system:

\[
\ddot{\beta} = -2\xi\omega_n\beta - \omega_n^2\beta + \omega_n^2\beta_c \tag{6}
\]
III. SYSTEM TRANSFORMATION

A. Strict-Feedback Formulation

From [8], the dynamics are decoupled into two functional subsystems. Given the tracking reference $V_i$ and $h_d$, we design the velocity and altitude controller separately.

(A) Velocity subsystem

The velocity subsystem (1) can be rewritten as

$$\dot{V} = f_V + g_V u_V$$
$$u_V = \beta_e$$
$$\hat{V} = V$$

where $f_V = - (D/m + \mu \sin \gamma/r^2)$, $g_V = \dot{q}S \times 0.02576 \cos \gamma/m$ if $\beta_e < 1$. Otherwise $f_V = - (D/m + \mu \sin \gamma/r^2) + \dot{q}S \times 0.0224 \cos \alpha/m$, $g_V = \dot{q}S \times 0.00336 \cos \gamma/m$.

(B) Altitude subsystem

Since $\gamma$ is quite small, we take $\sin \gamma \approx \gamma$ in (2). The thrust term $T \sin \alpha$ in (3) can be neglected because it is generally much smaller than $L$. Since the sampling period is short, the velocity can be approximately considered to be constant during the controller design.

Define $\theta = \alpha + \gamma$, $X_A = [x_1, x_2, x_3, x_4]^T$, $x_1 = h$, $x_2 = \gamma$, $x_3 = \theta$, $x_4 = q$, $x_5 = \delta_e$. The dynamics of (2)-(5) can be written as the strict-feedback form:

$$\dot{x}_i = f_i + g_i x_{i+1}$$
$$y_A = x_1$$

where $f_1 = 0$, $g_1 = V$, $f_2 = -(\mu - V^2 r) \cos \gamma/(V r^2) - 0.6203 q S \gamma/(m V)$, $g_2 = 0.6203 q S/(m V)$, $f_3 = 0$, $g_3 = 1$, $f_4 = \dot{q} S \tilde{c} [M(\alpha) + C_M(q) - 0.0292 \alpha]/I_{yy}$, $g_4 = 0.0292 \dot{q} S \tilde{c}/I_{yy}$, $i = 1, 2, 3, 4$.

Assumption 1: $f_i$ and $g_i$ are unknown smooth functions and can be decomposed into the nominal part $f_{iN}, g_{iN}$ and the unknown part $\Delta f_i, \Delta g_i$. There exist constants $g_i$ and $g_{iN}$ such that $g_i \geq g_{iN} > 0$ and $g_i \geq g_{iN} > 0$, $i = 2, 4, V$.

The nominal part will be considered as the feedback item for the controller design in Section V. Though some items are constants such as $f_1$ and $f_3$, the expression is remained to make Eq.(8) and the future controller design more explicit.

B. Discrete-time Model

By first-order Taylor expansion with sample time $T_s$, we have

$$V(k + 1) = V(k) + T_s [f_V(k) + g_V(k) u_V(k)]$$
$$x_i(k + 1) = x_i(k) + T_s [f_i(k) + g_i(k) x_{i+1}(k)]$$

where $i = 1, 2, 3, 4$.

It is noted that systems (9) and (10) are in strict-feedback form. The first-order Taylor expansion is just for analysis and controller design. The control inputs will be applied on the real system (1)-(5) with (6).

IV. KRIGING ESTIMATORS

Given a domain $D \in \mathbb{R}^d$, a random function $Y$ is a collection of random variables $\{Y(x) | x \in D\}$ that can be characterized by the set of all its $k$-variate cumulative distribution functions for any number $k$: $F(x_1, ..., x_k; y_1, ..., y_k) = \Pr \{Y(x_1) \leq y_1, ..., Y(x_k) \leq y_k\}$.

We denote by $m(x)$ and $\sigma(x, x')$ the mean function and covariance function of $Y$. If it is a joint multivariate Gaussian distribution, then $Y$ is a Gaussian random function. It is easy to see that a Gaussian random function can be specified by its mean function and covariance function as

$$(Y(x_1) \cdots Y(x_k)) \sim N((m(x_1) \cdots m(x_k))^T, \Sigma)$$

where $N(., .)$ denotes the Gaussian distribution, and $\Sigma \in \mathbb{R}^{k \times k}$ is the covariance matrix, with $\Sigma_{ij} = \sigma(x_i, x_j)$.

Suppose that the function $y(x)$ is a realization of $Y$, and the values of the variable $y(x)$ are known at $k$ points $x_1, ..., x_k$. For a new point $x^0$, the value of $y(x^0)$ can be estimated by Simple Kriging Estimator [12] as

$$y_{SK}(x^0) = m(x^0) + \Sigma_0 \Sigma^{-1} (y - m)$$

where $\Sigma_0 = \sigma(x_1, x_1) \cdots \sigma(x_k, x_k)$, $y = (y(x_1) \cdots y(x_k))^T$, $m = (m(x_1) \cdots m(x_k))^T$.

To avoid the infinite data size, we take the recursive Kriging estimation with the following assumption.

Assumption 2: Each unknown function $d_i$ is the realization of a random function $D_i$ with covariance $\sigma_i$. The time correlation of $D_i$, is local, that is

$$\sigma_i(\theta_i(k_1), \theta_i(k_2)) = 0, |k_1 - k_2| > M$$

where $M$ is the positive integer.

The related covariance function is defined as

$$\sigma_i(\theta_i(k_1), \theta_i(k_2)) = C_i^0 \exp \left( - \frac{||\theta_i(k_1) - \theta_i(k_2)||^2}{h_0^2} \right) C_i^1 |k_1 - k_2|$$

where $h_0^2$ and $C_i^0$ are design constants.

V. DISCRETE CONTROL DESIGN

A. Adaptive Kriging Control via Back-stepping for the Altitude Subsystem

As the elevation deflection occurs in the final equation, we perform the back-stepping design procedure. During each step, one virtual control input is designed and the error dynamics are obtained. For the uncertainty, the Kriging system is taken for estimation.

$$z_1(k) = x_1(k) - x_{1d}(k)$$
$$z_2(k) = x_2(k) - x_{2d}(k)$$
$$z_3(k) = x_3(k) - x_{3d}(k)$$
$$z_4(k) = x_4(k) - x_{4d}(k)$$

where $x_{2d}(k)$, $x_{3d}(k)$, $x_{4d}(k)$ are the virtual control inputs, $x_{1d}(k) = h_d(k)$.
To make the expression for Kriging estimation explicit, we have the following definitions: $X_i(k) = [x_1(k), \cdots, x_i(k)]^T$, 
$\Delta X_i(k) = [\Delta x_1(k), \cdots, \Delta x_i(k)]^T$ with $\Delta x_i(k) = x_i(k) - x_i(k-1)$ and $X_{1d}(k+i) = [x_{1d}(k), \cdots, x_{1d}(k+i)]^T$.

**Step 1.** From (13)
\[
\begin{align*}
  z_1(k+1) &= x_1(k) + T_s \left[ f_1(k) + g_1(k)x_2(k) \right] - x_{1d}(k+1) \\
  &= h_d(k+1) \quad (17)
\end{align*}
\]
where $x_{1d}(k+1) = h_d(k+1)$ is the reference command.

The virtual control input $x_{2d}(k)$ is designed as
\[
\begin{align*}
  x_{2d}(k) &= \frac{-x_1(k) + c_1 z_1(k) + x_{1d}(k+1) - T_s f_1(k)}{T_s g_1(k)} \\
  \text{where } 0 < c_1 < 1 \text{ and } g_1(k) &= V(k).
\end{align*}
\]
Combining (14), (17) and (18), the following equation can be obtained:
\[
\begin{align*}
  z_1(k+1) &= c_1 z_1(k) + T_s g_1(k) z_2(k) \quad (19)
\end{align*}
\]

**Step 2.** From (14)
\[
\begin{align*}
  z_2(k+1) &= x_2(k) + T_s \left[ f_2(k) + g_2(k)x_3(k) \right] - x_{2d}(k+1) \\
  &= x_2(k) + T_s \left[ f_{2N}(k) + g_{2N}(k)x_{3d}(k) + g_{2N}(k) z_3(k) \right] \\
  &\quad + [T_s \Delta f_2(k) + T_s \Delta g_2(k)x_3(k) - x_{2d}(k+1)] \\
  \text{Define } z_{u2}(k) &= T_s \Delta f_2(k) + T_s \Delta g_2(k)x_3(k) + x_{2d}(k) - x_{2d}(k+1) \\
  z_{u2}(k) &= z_{u2}(k-1) + d_2(\theta_2(k)) \quad (21)
\end{align*}
\]

where $f_{2N}(k)$ and $g_{2N}(k)$ are the nominal parts of $f_2(k)$ and $g_2(k)$, $x_{2d}(k+1)$ is the future desired control input value as designed in (18), $\theta_2(k) = [X_3^T(k-1), \Delta X_3^T(k), X_{1d}(k+2)]^T$ and $d_2(\theta_2(k))$ is the unknown function.

The virtual control input $x_{3d}(k)$ is designed as
\[
\begin{align*}
  x_{3d}(k) &= \frac{(c_2 - 1) z_2(k) - T_s f_{2N}(k)}{T_s g_{2N}(k)} - z_{u2}(k-1) - \tilde{d}_2(\theta_2(k)) \\
  \text{where } 0 < c_2 < 1 \text{ and } \tilde{d}_2(\theta_2(k)) &= \text{the Kriging estimation of } d_2(\theta_2(k)).
\end{align*}
\]
Combining (15), (20) and (22), the following equation can be obtained:
\[
\begin{align*}
  z_2(k+1) &= (c_2 - 1) z_2(k) + T_s g_{2N}(k) z_3(k) + \tilde{d}_2(\theta_2(k)) \\
  \text{where } \tilde{d}_2(\theta_2(k)) &= d_2(\theta_2(k)) - \tilde{d}_2(\theta_2(k)).
\end{align*}
\]

**Step 3.** From (15)
\[
\begin{align*}
  z_3(k+1) &= x_3(k) + T_s \left[ f_3(k) + g_3(k)x_4(k) \right] - x_{3d}(k+1) \\
  \text{Define } z_{u3}(k) &= \frac{x_{3d}(k) - x_{3d}(k+1)}{T_s} \\
  z_{u3}(k) &= z_{u3}(k-1) + d_3(\theta_3(k)) \\
  \text{where } \theta_3(k) &= [X_4^T(k-1), \Delta X_4^T(k), X_{1d}(k+3)]^T \text{ and } d_3(\theta_3(k)) \text{ is the unknown function.}
\end{align*}
\]
The virtual control input $x_{4d}(k)$ is designed as
\[
\begin{align*}
  x_{4d}(k) &= \frac{(c_3 - 1) z_3(k)}{T_s} - z_{u3}(k-1) - \tilde{d}_3(\theta_3(k)) \quad (26)
\end{align*}
\]
where $0 < c_3 < 1$ and $\tilde{d}_3(\theta_3(k))$ is the Kriging estimation of $d_3(\theta_3(k))$.

Combining (16), (24) and (26), the following equation can be obtained:
\[
\begin{align*}
  z_3(k+1) &= (c_3 - 1) z_3(k) + T_s z_4(k) + T_s \tilde{d}_3(\theta_3(k)) \quad (27)
\end{align*}
\]
where $\tilde{d}_3(\theta_3(k)) = d_3(\theta_3(k)) - \tilde{d}_3(\theta_3(k))$.

The formulation of Step 3 is actually the same to Step 2. The difference is due to the fact that $f_3 = 0$ and $g_3 = 1$.

**Step 4.** From (16)
\[
\begin{align*}
  z_4(k+1) &= x_4(k) + T_s \left[ f_4(k) + g_4(k)u_A(k) \right] - x_{4d}(k+1) \\
  &= x_4(k) + T_s f_{4N}(k) + T_s g_{4N}(k)x_5(k) \\
  &\quad + [T_s \Delta f_4(k) + T_s \Delta g_4(k)x_5(k) - x_{4d}(k+1)] \\
  \text{Define } z_{u4}(k) &= T_s \Delta f_4(k) + T_s \Delta g_4(k)u_A(k) + x_{4d}(k) - x_{4d}(k+1) \\
  z_{u4}(k) &= z_{u4}(k-1) + d_4(\theta_4(k)) \quad (29)
\end{align*}
\]
where $f_{4N}(k)$ and $g_{4N}(k)$ are the nominal parts of $f_4(k)$ and $g_4(k)$, $x_{4d}(k+1)$ is the future desired control input value as designed in (26), $\theta_4(k) = [X_4^T(k-1), \Delta X_4^T(k), X_{1d}(k+4)]^T$.

The actual control input of elevator deflection is designed as follows.
\[
\begin{align*}
  x_5(k) &= \frac{(c_4 - 1) z_4(k) - T_s f_{4N}(k) - z_{u4}(k-1) - \tilde{d}_4(\theta_4(k))}{T_s g_{4N}(k)} \\
  \text{where } 0 < c_4 < 1 \text{ and } \tilde{d}_4(\theta_4(k)) &= \text{the Kriging estimation of } d_4(\theta_4(k)).
\end{align*}
\]
Combining (28) and (30), the following equation can be obtained:
\[
\begin{align*}
  z_4(k+1) &= (c_4 - 1) z_4(k) + T_s g_{4N}(k) \tilde{d}_4(\theta_4(k)) \\
  \text{where } \tilde{d}_4(\theta_4(k)) &= d_4(\theta_4(k)) - \tilde{d}_4(\theta_4(k)).
\end{align*}
\]
The value of the uncertainty at time $k$ in (21), (25) and (29) can be calculated from the dynamics (10) with (18), (22) and (26) at time $k+1$. Especially we have the following calculation:
\[
\begin{align*}
  \Delta f_i(k) + \Delta g_i(k)x_{i+1}(k) &= x_i(k+1) - x_i(k) \\
  - T_s f_{iN}(k) - T_s g_{iN}(k)x_{i+1}(k) \\
  \text{where } \theta_i(k) &= [X_i^T(k-1), \Delta X_i^T(k), X_{1d}(k+3)]^T \text{ and } d_i(\theta_i(k)) \text{ is the unknown function.}
\end{align*}
\]

**Theorem 1:** Considering system (10) under Assumptions 1-3 with the controller (18), (22), (26), (30) where $\tilde{d}_i(\theta_i(k))$, $i = 2, 3, 4$ is the Kriging estimation with (12). Provided the following conditions hold

(i) The Gaussian functions of $d_i$, $i = 2, 3, 4$ are independent.
(ii) $E \left[ d^2_j(\theta_j(k)) \right] \leq a_j ||\theta_j(k) - \theta_j(k-1)|| + c_j, j = 2, 3, 4$

then all the signals involved are almost surely bounded and the tracking errors $z_i(k), k \geq 2, i = 1, 2, 3, 4$ are exponentially bounded in mean square.

The proof is similar to [13] and thus omitted in this paper.

B. Adaptive Kriging Control for Velocity Subsystem

Define the velocity error

$$z_V(k) = V(k) - V_d(k) \quad (33)$$

From (33), we have

$$z_V(k+1) = V(k+1) - V_d(k+1)$$

$$= V(k) + T_s [f_{V_N}(k) + g_{V_N}(k)u_V(k)]$$

$$- V_d(k+1) + T_s [\Delta f_V(k) + \Delta g_V(k)u_V(k)]$$

The uncertainty is defined as

$$z_{UV}(k) = \frac{1}{T_s g_{V_N}(k)} T_s [\Delta f_V(k) + \Delta g_V(k)u_V(k)]$$

$$z_{UV}(k) = z_{UV}(k-1) + d_V(\theta_V(k))$$

where $f_{V_N}(k)$ and $g_{V_N}(k)$ are the nominal parts of $f_V(k)$ and $g_V(k)$, $V_d(k+1)$ is the future reference value, $\theta_V(k) = [X^T_d(k-1), \Delta X^T_d(k), V_k, \Delta V_k]^T$ and $d_V(\theta_V(k))$ is the unknown function.

The throttle setting is designed as

$$u_V(k) = \frac{c_V z_V(k) + V_d(k+1) - V(k) - T_s f_{V_N}(k)}{T_s g_{V_N}(k)}$$

$- z_{UV}(k-1) - \tilde{d}_V(\theta_V(k))$$

where $\tilde{d}_V(\theta_V(k))$ is the Kriging estimation of $d_V(\theta_V(k))$.

Eq.(34) can be derived as

$$z_V(k+1) = c_V z_V(k) + T_s g_{V_N}(k) \tilde{d}_V(\theta_V(k))$$

$$z_V(k+1) = c_V z_V(k) + T_s g_{V_N}(k) \hat{d}_V(\theta_V(k))$$

where $\hat{d}_V(\theta_V(k)) = d_V(\theta_V(k)) - \tilde{d}_V(\theta_V(k))$.

Theorem 2: Considering system (9) with the controller (36), where $\hat{d}_V(\theta_V(k))$ is the Kriging estimation with (12), the velocity is almost surely bounded. The proof is quite similar to Theorem 1 and omitted here.

VI. SIMULATIONS

In this section, we verify the effectiveness and performance of the proposed adaptive Kriging controller. Reference commands are generated by the filter:

$$\frac{h_d}{h_c} = \frac{\omega_n^2}{(s + \omega_n)(s^2 + 2 \epsilon \omega_n s + \omega_n^2)}$$

$$\frac{V_d}{V_c} = \frac{\omega_n^2}{(s + \omega_n)(s^2 + 2 \epsilon \omega_n s + \omega_n^2)}$$

where $\omega_n = 1, \omega_n = 0.5, \epsilon_c = 0.7, \omega_n = 1, \omega_n = 0.5, \epsilon_n = 0.7$. The perturbation is set to be 3% for the parameter set $(m, \mu, I_{yy}, S)$. 

![Fig. 1. Altitude and velocity response](image1)

![Fig. 2. Elevator Deflection and Throttle Setting](image2)

![Fig. 3. $Z_{U2}$ Tracking Error](image3)
The parameters for the controller are selected as $T_s = 0.05s$, $c_1 = 0.95$, $c_2 = 0.9$, $c_3 = 0.9$, $c_4 = 0.9$, $c_v = 0.9$. For the Kriging system, $h_0 = 0.1$, $C_0 = 1$, $\tau_0 = 1$, $M = 10$.

Fig.1 depicts the response performance that the altitude controller tracks the step change with magnitude 200ft while the velocity steps from 15060ft/s to 15160ft/s. The system converges to the reference command in about 20 seconds. The control inputs of the elevator deflection and the throttle setting are shown in Fig.2(A) and Fig.2(B). From Fig.(3)-Fig.(6), it shows that the Kriging system well tracks the defined uncertainty in (21),(25),(29) and (35).

VII. CONCLUSIONS

The adaptive Kriging controller via the back-stepping scheme is proposed in this paper. Considering the characteristics of HFV, the nominal nonlinearity during each step is eliminated. The system uncertainty is modeled as the realizations of Gaussian random functions and estimated by the recursive Kriging algorithm which eliminates the infinite increase of the data size. Simulation result shows the effectiveness of this method.
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