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Metric modifications for a massive spin 1 particle

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Previous evaluations of long range one-photon and one-graviton-loop corrections to the energy-momentum tensor and the metric tensor describing spacetime in the vicinity of massive spinless and spin 1/2 systems have been extended to particles with unit spin and speculations are confirmed concerning universal properties of such forms.

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I. INTRODUCTION

In earlier papers, we described calculations of one-loop corrections to the energy-momentum tensor of a charged or neutral spinless or spin 1/2 particle of mass m and focused exclusively on the *nonanalytic* components of such results [1,2]. This is because such nonanalytic pieces involve terms with singularities at small momentum transfer q which, when Fourier-transformed, yield long distance corrections to the energy-momentum tensor, as well as—via the Einstein equations—large distance corrections to the metric tensor. This procedure is valid *both* in the charged and neutral cases. In the former the corrections are due to the one-photon-loop diagrams shown in Fig. 1 and are straightforward. In the latter the corrections arise from the one-graviton exchange diagrams shown in Fig. 2, but while such graviton-loop corrections are in general nonrenormalizable, this feature is not an problem for the *nonanalytic* component which we study. Indeed such diagrams do contain divergences, but these are ultraviolet infinities and are confined to the short distance—large momentum transfer—sector. The nonanalytic pieces are in the infrared region and are associated with long distance—small momentum transfer—pieces of the amplitude. They are completely well defined and unique and must be reproduced in any future successful theory of quantum gravity. In fact the classical component of these corrections to $T_{\mu\nu}$ is generated by the classical electromagnetic or gravitational field outside the massive system and *must* be the result of any such calculation. This procedure of isolating robust features of quantum gravity by excising the ill-defined short distance sector while retaining the well-defined long distance component is the basic idea of the effective theory of quantum gravity and has been developed in a series of papers by various authors over the past decade [3].

A. Charged particle

In the case of a charged system having mass m and charge e, for both the spinless and spin 1/2 field cases, the longest range component of the $T_{\mu\nu}$ modifications given by one-photon exchange is associated with the simple classical electric and magnetic fields outside an elementary charged system, but they are accompanied by shorter range quantum corrections. Likewise the metric was demonstrated to be modified from its simple Schwarzschild form via terms proportional to $G\alpha$, where G is the gravitational constant and $\alpha = e^2/4\pi$ is the fine structure constant. Specifically, in harmonic gauge, the metric for a spinless particle has the form

$$
\begin{aligned}
{}^{\rm ch}g_{00}^{S=0}(\vec{r}) &= 1 - \frac{2Gm}{r} + \frac{G\alpha}{r^2} - \frac{8G\hbar}{3\pi m r^3} + \ldots, \\
{}^{\rm ch}g_{0i}^{S=0}(\vec{r}) &= 0, \\
{}^{\rm ch}g_{ij}^{S=0}(\vec{r}) &= -\delta_{ij} - \delta_{ij}\frac{2Gm}{r} + G\alpha\frac{r_i r_j}{r^4} \\
&\quad + \frac{4}{3\pi}\frac{G\alpha\hbar}{mr^3}\left(\frac{r_i r_j}{r^2} - \delta_{ij}\right) - \frac{4}{3\pi}\frac{G\alpha\hbar(1-\log\mu r)}{mr^3} \\
&\quad \times \left(\delta_{ij} - 3\frac{r_i r_j}{r^2}\right) + \ldots. \qquad (1)
\end{aligned}
$$

(Note: the dependence on the arbitrary scale factor μ can be removed by a coordinate transformation.) The classical—$\hbar$-independent—pieces of these α-dependent modifications are well known and can be found by expanding the familiar Reissner-Nordström metric, describing spacetime around a charged, massive object [4]. On the other hand, the loop calculation also yields quantum mechanical—$\hbar$-dependent—pieces which are new and whose origin can be understood qualitatively as arising from zitterbewegung [1].

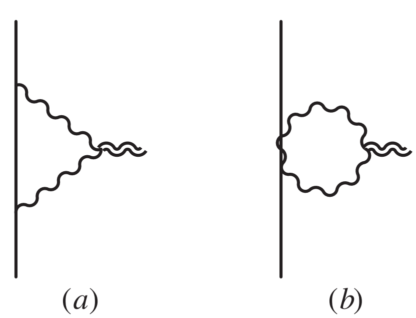
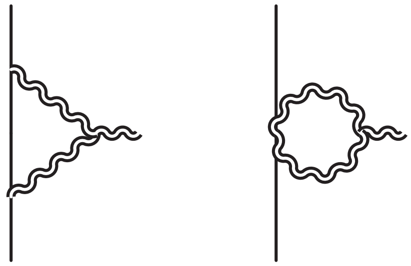


FIG. 1. Feynman diagrams having nonanalytic components. Here the single wiggly lines represent photons while the double wiggly line indicates coupling to a graviton.



(a) (b)

FIG. 2. Feynman diagrams having nonanalytic components. Here the doubly wiggly lines represent gravitons.

In the case of a spin $1/2$ system the 00 and ij components of the metric are identical to Eq. (1) but include a factor $\chi_f^\dagger \chi_i$ which vanishes if initial and final spins are orthogonal. There exists, in addition a nonvanishing $0i$ component of the metric, whose origin is the spin

$$\begin{aligned}
{}^{\mathrm{ch}}g_{00}^{S=1/2}(\vec{r}) &= {}^{\mathrm{ch}}g_{00}^{S=0}(\vec{r}) \cdot \chi_f^\dagger \chi_i, \\
{}^{\mathrm{ch}}g_{0i}^{S=1/2}(\vec{r}) &= (\vec{S} \times \vec{r})_i \left(\frac{2G}{r^3} - \frac{G\alpha}{mr^4} + \frac{2G\alpha\hbar}{\pi m^2 r^5} + \ldots \right), \\
{}^{\mathrm{ch}}g_{ij}^{S=1/2}(\vec{r}) &= {}^{\mathrm{ch}}g_{ij}^{S=0}(\vec{r}) \cdot \chi_f^\dagger \chi_i,
\end{aligned} \tag{2}$$

where $\vec{S} \equiv \chi_F^\dagger \vec{\sigma} \chi_i/2$. Here the classical component of this modification can be found by expanding the Kerr-Newman metric [5], which describes spacetime in the vicinity of a charged, massive, and spinning particle, and again there exist quantum corrections due to zitterbewegung [1].

B. Neutral particle

In the case of neutral particles, for a spinless field the metric arising from one-graviton exchange was shown to be modified from its lowest order form by long distance forms proportional to G^2—in harmonic gauge

$$\begin{aligned}
{}^{\mathrm{neu}}g_{00}^{S=0}(\vec{r}) &= 1 - \frac{2Gm}{r} + \frac{2G^2m^2}{r^2} + \frac{7G^2m\hbar}{\pi r^3} + \ldots, \\
{}^{\mathrm{neu}}g_{0i}^{S=0}(\vec{r}) &= 0, \\
{}^{\mathrm{neu}}g_{ij}^{S=0}(\vec{r}) &= -\delta_{ij}\bigg[1 + \frac{2Gm}{r} + \frac{G^2m^2}{r^2} + \frac{14G^2m\hbar}{15\pi r^3} \\
&\quad - \frac{76}{15}\frac{G^2m\hbar}{\pi r^3}(1 - \log\mu r) \bigg] - \frac{r_i r_j}{r^2}\bigg[\frac{G^2m^2}{r^2} \\
&\quad + \frac{76G^2m\hbar}{15\pi r^3} + \frac{76}{5}\frac{G^2m\hbar}{\pi r^3}(1 - \log\mu r) \bigg] + \ldots.
\end{aligned} \tag{3}$$

Again the classical—$\hbar$-independent—pieces of these modifications are well known and can be found by expanding the familiar Schwarzschild (Kerr) metric, which describes spacetime around a massive (spinning) object [4]. On the other hand, the calculation also yields new quantum mechanical—$\hbar$-dependent—pieces.

In the case of a spin $1/2$ system the 00 and ij components of the metric are the same as those for spin 0 times the factor $\chi_f^\dagger \chi_i$, which vanishes for orthogonal spins, and there exists, in addition, a nonvanishing $0i$ piece of the metric

$$\begin{aligned}
{}^{\mathrm{neu}}g_{00}^{S=1/2}(\vec{r}) &= {}^{\mathrm{neu}}g_{00}^{S=0}(\vec{r}) \cdot \chi_f^\dagger \chi_i, \\
{}^{\mathrm{neu}}g_{0j}^{S=1/2}(\vec{r}) &= (\vec{S} \times \vec{r})_i \left(\frac{2G}{r^3} - \frac{2G^2m}{r^4} + \frac{3G^2\hbar}{\pi r^5} + \ldots \right), \\
{}^{\mathrm{neu}}g_{ij}^{S=1/2}(\vec{r}) &= {}^{\mathrm{neu}}g_{ij}^{S=0}(\vec{r}) \cdot \chi_f^\dagger \chi_i,
\end{aligned} \tag{4}$$

where the spin vector is $\vec{S} = \chi_f^\dagger \vec{\sigma} \chi_i/2$ as above. Here the classical component of this modification can be found by expanding the Kerr-Newman metric [5], describing spacetime around a neutral spinning mass and once again there exist quantum corrections due to zitterbewegung.

C. A speculation

Based on the feature that the 00, ij components of both the energy-momentum tensor and the metric were found to have identical forms for both spin 0 and $1/2$ up to a factor which requires that the components of initial and final spins be identical, it is tempting to speculate that the leading 00, ij pieces of the metric about a particle of *arbitrary* spin has a universal form. That universality is true for the leading off-diagonal—spin-dependent—component cannot be determined from a single spin $1/2$ calculation, but it seems reasonable to speculate that this is also the case. However, whether these assertions are generally valid can be found only by additional calculation, which is the purpose of the present note, wherein we evaluate the nonanalytic piece of the energy-momentum tensor for a spin 1 particle having mass m—e.g., the $W^\pm$ or Z^0 boson. We also make the connection with and examine the universality issue for the metric tensor describing spacetime around such a spin 1 particle.

In the next section then we briefly review the results of the previous papers, and follow with a discussion wherein these calculations are extended to the case of a massive spin 1 particle. Results are summarized in a brief concluding section.

II. LIGHTNING REVIEW

Since it important to the remainder of this note, we first present a concise review of the results obtained in our previous papers [1]. In the case of a spin 0 system having mass m, the general form of the energy-momentum tensor is

$$\begin{aligned}
\langle p_2 | T_{\mu\nu}(x) | p_1 \rangle_{S=0} &= \frac{e^{i(p_2 - p_1)\cdot x}}{\sqrt{4E_2E_1}} [2P_\mu P_\nu F_1^{S=0}(q^2)) \\
&\quad + (q_\mu q_\nu - q^2 \eta_{\mu\nu}) F_2^{S=0}(q^2)],
\end{aligned} \tag{5}$$

where $P = \frac{1}{2}(p_1 + p_2)$ is the average momentum, while $q = p_1 - p_2$ is the momentum transfer. The tree level values for these form factors are

$$F^{S=0}_{1,\text{tree}} = 1, \qquad F^{S=0}_{2,\text{tree}} = -\tfrac{1}{2}. \tag{6}$$

In the case of a spin 1/2 system, again of mass m, there exists an additional form factor—$F_3^{S=1/2}(q^2)$—associated with the existence of spin—

$$\begin{aligned}\langle p_2|T_{\mu\nu}(x)|p_1\rangle_{S=1/2} = \frac{e^{i(p_2-p_1)\cdot x}}{\sqrt{E_1E_2}}\bar{u}(p_2)\bigg[&P_\mu P_\nu F_1^{S=1/2}(q^2)\\ &+\frac{1}{2}(q_\mu q_\nu - q^2\eta_{\mu\nu})F_2^{S=1/2}(q^2)\\ &-\left(\frac{i}{4}\sigma_{\mu\lambda}q^\lambda P_\nu + \frac{i}{4}\sigma_{\nu\lambda}q^\lambda P_\mu\right)\\ &\times F_3^{S=1/2}(q^2)\bigg]u(p_1)\end{aligned} \tag{7}$$

and the tree level values for these form factors are

$$F^{S=1/2}_{1,\text{tree}} = F^{S=1/2}_{3,\text{tree}} = 1, \qquad F^{S=1/2}_{2,\text{tree}} = 0. \tag{8}$$

A. Charged particles

Now consider the modification due to radiative (photon loop) corrections in the case that the spin 0 or spin 1/2 particle is charged. As discussed in the introduction we will retain only the leading nonanalytic pieces of such corrections, since these are model-independent and determine the long distance corrections to the energy-momentum tensor and to the metric. Such nonanalytic pieces arise only from Figs. 1(a) and 1(b) so we consider solely these diagrams. Here we have defined

$$L = \log\left(\frac{-q^2}{m^2}\right) \quad \text{and} \quad S = \pi^2\sqrt{\frac{m^2}{-q^2}}.$$

we find, for spin 0 and 1/2, the results

$$\begin{aligned}{}^{\text{ch}}F^{S=0}_{1,\text{loop}}(q^2) &= {}^{\text{ch}}F^{S=1/2}_{1,\text{loop}}(q^2) = \frac{\alpha}{16\pi}\frac{q^2}{m^2}(8L+3S),\\ {}^{\text{ch}}F^{S=0}_{2,\text{loop}}(q^2) &= {}^{\text{ch}}F^{S=1/2}_{2,\text{loop}}(q^2) = \frac{\alpha}{24\pi}(8L+3S),\\ {}^{\text{ch}}F^{(S=1)/2}_{3,\text{loop}}(q^2) &= \frac{\alpha}{24\pi}\frac{q^2}{m^2}(4L+3S).\end{aligned} \tag{9}$$

Such nonanalytic forms, which are singular in the small-q limit, are present due to the presence of *two* massless propagators in the Feynman diagrams [6] and can arise even in electromagnetic diagrams when this situation exists [7]. Upon Fourier-transforming, the piece proportional to S is found to give classical ($\hbar$-independent) behavior while the term involving L is found to yield quantum mechanical ($\hbar$-dependent) corrections.[1] The feature that the form factors $F_1(q^2=0)$ and $F_3(q^2=0)$ retain their values of unity even when electromagnetic corrections are included arises from the stricture of energy-momentum conservation in the case of $F_1(q^2)$ and angular momentum conservation in the case of $F_3(q^2)$. An intriguing implication of the latter result is the absence of any anomalous gravitomagnetic moment. (Note that there exists no constraint on $F_2^{S=0}(q^2=0)$.)

The universality of these radiatively corrected forms is suggested by the identity of ${}^{\text{ch}}F^{S=0}_{1,2}(q^2)$ and ${}^{\text{ch}}F^{S=1/2}_{1,2}(q^2)$. Of course, the spin-dependent gravitomagnetic form factor ${}^{\text{ch}}F_3^{S=1/2}(q^2)$ has no analog in the spin 0 sector.

The connection with the metric tensor described in the introduction arises when these results for the energy-momentum tensor are combined with the (linearized) Einstein equation [8]

$$\Box h_{\mu\nu} = -16\pi G\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right), \tag{10}$$

where we have defined

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{11}$$

and

$$T \equiv \text{Tr}T_{\mu\nu}. \tag{12}$$

Taking Fourier transforms, we find—for spin 0

$$\begin{aligned}{}^{\text{ch}}h^{S=0}_{00}(\vec{r}) &= -16\pi G\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot\vec{r}}\frac{1}{\vec{k}^2}\left(\frac{m}{2} - \frac{\alpha\pi|\vec{k}|}{8} - \frac{\alpha\vec{k}^2}{3\pi m}\log\frac{\vec{k}^2}{m^2}\right) + \ldots,\\ {}^{\text{ch}}h^{S=0}_{0i}(\vec{r}) &= 0,\\ {}^{\text{ch}}h^{S=0}_{ij}(\vec{r}) &= -16\pi G\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot\vec{r}}\frac{1}{\vec{k}^2}\left(\frac{m}{2}\delta_{ij} + \left(\frac{\alpha\pi}{16|\vec{k}|} + \frac{\alpha}{6\pi m}\log\frac{\vec{k}^2}{m^2}\right)(k_ik_j - \delta_{ij}\vec{k}^2)\right) + \ldots,\end{aligned} \tag{13}$$

while for spin 1/2 we find

$$\begin{aligned}{}^{\text{ch}}h^{S=1/2}_{00}(\vec{r}) &= {}^{\text{ch}}h^{S=0}_{00}(\vec{r})\cdot\chi_f^\dagger\chi_i,\\ {}^{\text{ch}}h^{S=1/2}_{0i}(\vec{r}) &= -16\pi Gi\int\frac{d^3k}{(2\pi)^3}\frac{1}{\vec{k}^2}\left(\frac{e^{i\vec{k}\cdot\vec{r}}}{2} - \frac{\alpha\pi|\vec{k}|}{16m} - \frac{\alpha\vec{k}^2}{12\pi m^2}\log\frac{\vec{k}^2}{m^2}\right)(\vec{S}\times\vec{k})_i\\ {}^{\text{ch}}h^{S=1/2}_{ij}(\vec{r}) &= {}^{\text{ch}}h^{S=0}_{ij}(\vec{r})\cdot\chi_f^\dagger\chi_i,\end{aligned} \tag{14}$$

where the spin vector $\vec{S}$ is as defined above. Evaluating the various Fourier transforms, we find the results quoted in the introduction [9].

B. Neutral particles

If the system being considered is neutral the radiative corrections found above vanish and the leading modifications to the energy-momentum tensor arise from graviton-loop corrections. Again we emphasize that although such quantum gravity corrections are themselves nonrenorma-

[1] The at first surprising feature that a loop calculation can yield classical physics is explained in Ref. [6]

lizable, meaning that the short distance effects are infinite and ill-defined, the nonanalytic components, which give rise to long range corrections, are entirely well defined and robust—they must arise in any future theory of quantum gravity. In this case the corrections arise from the diagrams shown in Fig. 2(a) and 2(b) and have the form, for spin 0 and 1/2

$$\begin{aligned} {}^{\rm neu}F^{S=0}_{1,\rm loop}(q^2) &= {}^{\rm neu}F^{S=1/2}_{1,\rm loop}(q^2) = \frac{Gq^2}{\pi}\left(-\frac{3}{4}L+\frac{1}{16}S\right), \\ {}^{\rm neu}F^{S=0}_{2,\rm loop}(q^2) &= {}^{\rm neu}F^{S=1/2}_{2,\rm loop}(q^2) = \frac{Gm^2}{\pi}\left(-2L+\frac{7}{8}S\right). \end{aligned} \tag{15}$$

As found in the case of charged systems, and required by energy-momentum and angular momentum conservation, the formfactors ${}^{\rm neu}F_1(q^2=0)$ and ${}^{\rm neu}F_3(q^2=0)$ are unaffected by loop corrections while there exists no such restriction on ${}^{\rm neu}F_2(q^2)$. Also, we confirm the universality of the neutral system results via the identity of loop corrections to the form factors neu$F^{S=0}_{1,2}$ and neu$F^{S=1/2}_{1,2}$.

The connection with the metric tensor can now be found by combining these corrections with the (linearized) Einstein equation [8]. Taking Fourier transforms, we find—for spin 0—the results

$$\begin{aligned} {}^{\rm neu}h^{S=0}_{00}(\vec r) &= -16\pi G\int\frac{d^3k}{(2\pi)^3}e^{i\vec k\cdot\vec r}\frac{1}{\vec k^2}\left(\frac{m}{2}-\frac{Gm^2\pi|\vec k|}{4}+\frac{7Gm\vec k^2}{8\pi}\log\frac{\vec k^2}{m^2}\right)-\frac{43G^2m\hbar}{15\pi r^3} \qquad {}^{\rm neu}h^{S=0}_{0i}(\vec r)=0 \\ {}^{\rm neu}h^{S=0}_{ij}(\vec r) &= -16\pi G\int\frac{d^3k}{(2\pi)^3}e^{i\vec k\cdot\vec r}\frac{1}{\vec k^2}\left[\frac{m}{2}\delta_{ij}-\delta_{ij}\left(\frac{Gm^2\pi|\vec k|}{32}-\frac{3Gm\vec k^2}{8\pi}\log\frac{\vec k^2}{m^2}\right)+\left(k_ik_j+\frac{1}{2}\vec k^2\delta_{ij}\right)\left(\frac{7Gm^2\pi}{16|\vec k|}-\frac{Gm}{\pi}\right.\right. \\ &\qquad \left.\left.\times\log\vec k^2\right)\right]+4G^2m^2\left(\frac{\delta_{ij}}{r^2}-2\frac{r_ir_j}{r^4}\right)+\frac{G^2m\hbar}{15\pi r^3}\left(\delta_{ij}+44\frac{r_ir_j}{r^2}\right)-\frac{44G^2m\hbar}{15\pi r^3}\left(\delta_{ij}-3\frac{r_ir_j}{r^2}\right)(1-\log\mu r), \end{aligned} \tag{16}$$

while for spin 1/2

$$\begin{aligned} {}^{\rm neu}h^{S=1/2}_{00}(\vec r) &= {}^{\rm neu}h^{S=0}_{00}(\vec r)\cdot\chi_f^\dagger\chi_i, \\ {}^{\rm neu}h^{S=1/2}_{0i}(\vec r) &= -16\pi G\frac{i}{2}\int\frac{d^3k}{(2\pi)^3}\frac{e^{i\vec k\cdot\vec r}}{\vec k^2}\left(1-\frac{Gm\pi|\vec k|}{4}\right. \\ &\qquad \left.-\frac{G\vec k^2}{4\pi}\log\frac{\vec k^2}{m^2}\right)(\vec S\times\vec k)_i+\frac{21G^2\hbar}{5\pi r^5}(\vec S\times\vec r)_i, \\ {}^{\rm neu}h^{S=1/2}_{ij}(\vec r) &= {}^{\rm neu}h^{S=0}_{ij}(\vec r)\cdot\chi_f^\dagger\chi_i. \end{aligned} \tag{17}$$

Evaluating the various Fourier transforms, we find the results quoted in the introduction [9].[2]

The purpose of the present note is to study how these results generalize to the case of higher spin and thereby to check our universality assumption. Specifically, we shall examine the radiative corrections to the energy-momentum tensor of a spin 1 system having mass m for both the charged and neutral cases.

III. SPIN 1

A neutral spin 1 field $\phi_\mu(x)$ having mass m is described by the Proca Lagrangian, which is of the form [10]

$$\mathcal{L}(x) = -\tfrac{1}{4}U_{\mu\nu}(x)U^{\mu\nu}(x)+\tfrac{1}{2}m^2\phi_\mu(x)\phi^\mu(x), \tag{18}$$

where

$$U_{\mu\nu}(x) = i\partial_\mu\phi_\nu(x)-i\partial_\nu\phi_\mu(x) \tag{19}$$

is the spin 1 field tensor.

A. Charged spin 1 interactions

If the particle has charge e, we can generate a gauge-invariant form of Eq. (18) by use of the well-known minimal substitution [11]—defining

$$\pi_\mu = i\nabla_\mu - eA_\mu(x) \tag{20}$$

and

$$U_{\mu\nu}(x) = \pi_\mu\phi_\nu(x)-\pi_\nu\phi_\mu(x) \tag{21}$$

the charged Proca Lagrangian density becomes

$$\mathcal{L}(x) = -\tfrac{1}{2}U^\dagger_{\mu\nu}(x)U^{\mu\nu}(x)+m^2\phi^\dagger_\mu(x)\phi^\mu(x). \tag{22}$$

Introducing the left-right derivative

$$D(x)\overleftrightarrow{\nabla}F(x)\equiv D(x)\nabla F(x)-(\nabla D(x))F(x) \tag{23}$$

the single-photon component of the interaction can be written as

$$\begin{aligned} \mathcal{L}_{\rm int}(x) &= ieA^\mu(x)\phi^{\alpha\dagger}(x)[\eta_{\alpha\beta}\overleftrightarrow{\nabla}_\mu-\eta_{\beta\mu}\nabla_\alpha]\phi^\beta(x) \\ &\quad +\eta_{\alpha\mu}(\nabla_\beta\phi^{\alpha\dagger}(x))\phi^\beta(x) \end{aligned} \tag{24}$$

so that the on-shell matrix element of the electromagnetic current becomes

[2] Here the r-dependent corrections proportional to $\hbar$ arise from the graviton vacuum polarization correction, while those independent of $\hbar$ arise from corrections to the linear Einstein equation [1].

$$\frac{1}{\sqrt{4E_fE_i}}\langle p_f,\epsilon_B|j_\mu|p_i,\epsilon_A\rangle = -\frac{e}{\sqrt{4E_fE_i}}[2P_\mu\epsilon_B^*\cdot\epsilon_A - \epsilon_{A\mu}\epsilon_B^*\cdot q + \epsilon_{B\mu}^*\epsilon_A\cdot q], \tag{25}$$

where we have used the property $p_f\cdot\epsilon_B^* = p_i\cdot\epsilon_A = 0$ for the Proca polarization vectors. If we now look at the spatial piece of this term we find

$$\frac{1}{\sqrt{4E_fE_i}}\langle p_f,\epsilon_B|\vec{\epsilon}_\gamma\cdot\vec{j}|p_i,\epsilon_A\rangle \simeq \frac{e}{2m}\vec{\epsilon}_\gamma\times\vec{q}\cdot\hat{\epsilon}_B^*\times\hat{\epsilon}_A = \frac{e}{2m}\langle 1,m_f|\vec{S}|1,m_i\rangle\cdot\vec{B}, \tag{26}$$

where we have used the result that in the Breit frame for a nonrelativistically moving particle

$$i\hat{\epsilon}_B^*\times\hat{\epsilon}_A = \langle 1,m_f|\vec{S}|1,m_i\rangle \tag{27}$$

which we recognize as representing a magnetic moment interaction with $g=1$. On the other hand if we take the time component of Eq. (25), we find, again for the Breit frame and a nonrelativistically moving system

$$\frac{1}{\sqrt{4E_fE_i}}\langle p_f,\epsilon_B|\epsilon_{0\gamma}j_0|p_i,\epsilon_A\rangle \simeq -e\epsilon_{0\gamma}\left[\epsilon_B^*\cdot\epsilon_A + \frac{1}{2m}(\epsilon_{A0}\hat{\epsilon}_B^*\cdot\vec{q} - \epsilon_{B0}^*\hat{\epsilon}_A\cdot\vec{q})\right]. \tag{28}$$

Using

$$\epsilon_A^0 \simeq \frac{1}{2m}\hat{\epsilon}_A\cdot\vec{q}, \qquad \epsilon_B^0 \simeq -\frac{1}{2m}\hat{\epsilon}_B^*\cdot\vec{q}, \qquad \epsilon_B^*\cdot\epsilon_A \simeq -\hat{\epsilon}_B^*\cdot\hat{\epsilon}_A - \frac{1}{2m^2}\hat{\epsilon}_B^*\cdot\vec{q}\hat{\epsilon}_A\cdot\vec{q}, \tag{29}$$

where the hat indicates the rest frame value, we observe that

$$\frac{1}{\sqrt{4E_fE_i}}\langle p_f,\epsilon_B|\epsilon_{0\gamma}j_0|p_i,\epsilon_A\rangle \simeq e\epsilon_{0\gamma}\hat{\epsilon}_B^*\cdot\hat{\epsilon}_A, \tag{30}$$

which is the expected electric monopole term—any electric quadrupole contributions have cancelled [12]. Overall then, Eq. (25) corresponds to a simple E0 interaction with the charge accompanied by an M1 interaction with g-factor unity, which is consistent with the speculation by Belinfante that for a particle of spin S, $g=1/S$ [13].

Despite this suggestively simple result, however, Eq. (18) does *not* correctly describe the interaction of the charged W-boson field, due to the feature that the $W^\pm$ are components of an SU(2) vector field [14]. The proper Proca Lagrangian has the form

$$\mathcal{L}(x) = -\tfrac{1}{4}\vec{U}_{\mu\nu}^\dagger(x)\cdot\vec{U}^{\mu\nu}(x) + \tfrac{1}{2}m_W^2\vec{\phi}_\mu(x)\cdot\vec{\phi}^\mu(x), \tag{31}$$

where the field tensor $\vec{U}_{\mu\nu}(x)$ contains an additional piece on account of gauge invariance

$$\vec{U}_{\mu\nu}(x) = \pi_\mu\vec{U}_\nu(x) - \pi_\nu\vec{U}_\mu(x) - ig\vec{U}_\mu(x)\times\vec{U}_\nu(x) \tag{32}$$

with g being the SU(2) electroweak coupling constant. The Lagrange density Eq. (31) then contains the term

$$\mathcal{L}_{\rm int}(x) = -gW^{0\mu\nu}(x)(W_\mu^{+\dagger}(x)W_\nu^+(x) - W_\mu^{-\dagger}(x)W_\mu^-(x)) \tag{33}$$

among (many) others. However, in the standard model the neutral member of the W-triplet is a linear combination of Z^0 and photon fields [15]—

$$W_\mu^0 = \cos\theta_W Z_\mu^0 + \sin\theta_W A_\mu \tag{34}$$

and, since $g\sin\theta_W = e$, we have a piece in the interaction Lagrangian

$$\mathcal{L}_{\rm int}^{(1)}(x) = -eF_{\mu\nu}(x)(W_\mu^{+\dagger}(x)W_\nu^+(x) - W_\mu^{-\dagger}(x)W_\mu^-(x)), \tag{35}$$

which represents an additional interaction that must be appended to the convention Proca result. In the Breit frame and for a nonrelativistically moving system we have

$$\frac{1}{\sqrt{4E_fE_i}}\langle p_f,\epsilon_B|\vec{\epsilon}_\gamma\cdot\vec{j}^{(1)}|p_i,\epsilon_A\rangle \simeq \frac{e}{2m_W}\vec{\epsilon}_\gamma\times\vec{q}\cdot\hat{\epsilon}_B^*\times\hat{\epsilon}_A = \frac{e}{2m_W}\langle 1,m_f|\vec{S}|1,m_i\rangle\cdot\vec{B} \tag{36}$$

and

$$\frac{1}{\sqrt{4E_fE_i}}\langle p_f,\epsilon_B|j_0^{(1)}|p_i,\epsilon_A\rangle \simeq -e\frac{1}{2m_W}(\epsilon_A^0\hat{\epsilon}_B^*\cdot\vec{q} - \epsilon_{B0}^*\hat{\epsilon}_A\cdot\vec{q}) = -\frac{e}{2m_W^2}\hat{\epsilon}_B^*\cdot\vec{q}\hat{\epsilon}_A\cdot\vec{q}. \tag{37}$$

The first piece—Eq. (36)—constitutes an additional magnetic moment and modifies the W-boson g-factor from its Belinfante value of unity to its standard model value of 2. Using

$$\frac{1}{2}(\epsilon_{Bi}^*\epsilon_{Aj} + \epsilon_{Ai}\epsilon_{Bj}^*) - \frac{1}{3}\delta_{ij}\hat{\epsilon}_B^*\cdot\hat{\epsilon}_A = \left\langle 1,m_f\left|\frac{1}{2}(S_iS_j + S_jS_i) - \frac{2}{3}\delta_{ij}\right|1,m_i\right\rangle \tag{38}$$

we observe that the second component—Eq. (37)—implies the existence of a quadrupole moment of size $Q = -e/M_W^2$. Both of these results are established predictions of the standard model for the charged vector bosons [16].

In fact, it has recently been argued, from a number of viewpoints, that the "natural" value of the gyromagnetic ratio for a particle of *arbitrary* spin should be $g=2$ [17], as opposed to the value $1/S$ from the Belinfante conjecture, and we shall consequently employ $g=2$ in our spin 1 calculations below.

B. Photon-loop corrections: Spin 1

Having obtained the appropriate Langrangian for the interactions of a charged spin-1 system,

$$\mathcal{L}(x) = -\frac{1}{2}U^{\dagger}_{\mu\nu}(x)U^{\mu\nu}(x) + m^2\phi^{\dagger}_{\mu}(x)\phi^{\mu}(x) - eF^{\mu\nu}(x) \times (\phi^{+\dagger}_{\mu}(x)\phi^{+}_{\nu}(x) - \phi^{-\dagger}_{\mu}(x)\phi^{-}_{\mu}(x)) \quad (39)$$

we can now calculate the matrix elements which will be needed in our calculation. Specifically, the general single-photon vertex for a transition involving an outgoing photon with polarization index μ and four-momentum $q = p1 - p2$, an incoming spin one particle with polarization index α and four-momentum p_1 together with an outgoing spin one particle with polarization index β and four-momentum p_2 is [18]

$$V^{(1)}_{\beta,\alpha,\mu}(p_1,p_2) = -ie[(p_1+p_2)_\mu\eta_{\alpha\beta} - (gp_{1\beta} - (g-1)p_{2\beta})\eta_{\alpha\mu} - (gp_{2\alpha} - (g-1)p_{1\alpha})\eta_{\beta\mu})], \quad (40)$$

while the two-photon vertex with polarization indices μ, ν, an incoming spin one particle with polarization index α and four-momentum p_1 together with an outgoing spin one particle with polarization index β and four-momentum p_2 has the form [19]

$$V^{(2)}_{\beta,\alpha,\mu\nu}(p_1,p_2) = -ie^2(2\eta_{\mu\nu}\eta_{\alpha\beta} - \eta_{\alpha\mu}\eta_{\beta\nu} - \eta_{\alpha\nu}\eta_{\beta\mu}). \quad (41)$$

The energy-momentum tensor connecting an incoming vector meson with polarization index α and four-momentum k_1 with and outgoing vector with polarization index β and four-momentum k_2 is found to be [20]

$$\langle k_2,\beta|T^{(0)}_{\mu\nu}|k_1,\alpha\rangle = (k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu})\eta_{\alpha\beta} - k_{1\beta}(k_{2\mu}\eta_{\alpha\nu} + k_{2\nu}\eta_{\alpha\mu}) - k_{2\alpha}(k_{1\nu}\eta_{\beta\mu} + k_{1\mu}\eta_{\beta\nu}) + (k_1\cdot k_2 - m^2)(\eta_{\beta\mu}\eta_{\alpha\nu} + \eta_{\beta\nu}\eta_{\alpha\mu}) - \eta_{\mu\nu}[(k_1\cdot k_2 - m^2)\eta_{\alpha\beta} - k_{1\beta}k_{2\alpha}] \quad (42)$$

and that between photon states is identical, except that the terms in m^2 are absent. The leading component of the on-shell energy-momentum tensor between charged vector meson states is then

$$\langle k_2,\epsilon_B|T^{(0)}_{\mu\nu}|k_1,\epsilon_A\rangle = (k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu})\epsilon^*_B\cdot\epsilon_A - k_1\cdot\epsilon^*_B(k_{2\mu}\epsilon_{A\nu} + k_{2\nu}\epsilon_{A\mu} - k_2\cdot\epsilon_A(k_{1\nu}\eta^*_{B\mu} + k_{1\mu}\epsilon^*_{B\nu}) + (k_1\cdot k_2 - m^2)\times(\epsilon^*_{B\mu}\epsilon_{A\nu} + \epsilon^*_{B\nu}\epsilon_{A\mu}) - \eta_{\mu\nu}[(k1\cdot k_2 - m^2)\epsilon^*_B\cdot\epsilon_A - k_1\cdot\epsilon^*_B k_2\cdot\epsilon_A] \quad (43)$$

and the focus of our calculation is to evaluate the one-loop electromagnetic corrections to Eq. (43), via the diagrams shown in Fig. 1, keeping only the leading nonanalytic terms. Details of the calculation are outlined in the appendix, and the results are

(a) Seagull loop diagram [Fig. 1(a)]

$$Amp[a]_{\mu\nu} = \frac{L\alpha}{48\pi m}\Big[\epsilon_A\cdot\epsilon^*_B\Big(2q_\mu q_\nu + \frac{1}{2}q^2\eta_{\mu\nu}\Big) - \epsilon_A\cdot q\epsilon^*_B\cdot q\eta_{\mu\nu} + \epsilon_A\cdot q(\epsilon^*_{B\mu}q_\nu + \epsilon^*_{B\nu}q_\mu) + \epsilon^*_B\cdot q(\epsilon_{A\mu}q_\nu + \epsilon_{A\nu}q_\mu) - 2(\epsilon_{A\mu}\epsilon^*_{B\nu} + \epsilon_{A\nu}\epsilon^*_{B\mu})q^2\Big]. \quad (44)$$

(b) Born loop diagram [Fig. 1(b)]

$$Amp[b]_{\mu\nu} = \frac{\alpha}{48\pi m}\Big\{-3P_\mu P_\nu q^2\epsilon^*_B\cdot\epsilon_A(8L+3S) + [(P_\mu\epsilon_{A\nu} + P_\nu\epsilon_{A\mu})\epsilon^*_B\cdot q - (P_\mu\epsilon^*_{B\nu} + P_\nu\epsilon^*_{B\mu}\epsilon_A\cdot q)] \times q^2(4L+3S) - [\epsilon_A\cdot q(\epsilon^*_{B\mu}q_\nu + \epsilon^*_{B\nu}q_\mu) + \epsilon^*_B\cdot q(\epsilon_{A\mu}q_\nu + \epsilon_{A\nu}q_\mu)]L - \Big[q_\mu q_\nu(10L+3S) - q^2\eta_{\mu\nu}\Big(\frac{15}{2}L + 3S\Big)\Big]\epsilon^*_B\cdot\epsilon_A + 2(\epsilon^*_{B\mu}\epsilon_{A\nu} + \epsilon^*_{B\nu}\epsilon_{A\mu})q^2L + \ldots\Big\}. \quad (45)$$

The full loop contribution is then

$$Amp[a+b]_{\mu\nu} = \frac{\alpha}{48\pi m}\{\epsilon^*_B\cdot\epsilon_A[(q_\mu q_\nu - q^2\eta_{\mu\nu}) - 3P_\mu P_\nu q^2](8L+3S) + ((P_\mu\epsilon_{A\nu} + P_\nu\epsilon_{A\mu})\epsilon^*_B\cdot q - (P_\mu\epsilon^*_{B\nu} + P_\nu\epsilon^*_{B\mu})\epsilon_A\cdot q)q^2(4L+3S) + \ldots\}. \quad (46)$$

Because of covariance and gauge invariance the form of the matrix element of $T_{\mu\nu}$ between on-shell spin 1 states must be expressible in terms of *six* independent form factors

$$\begin{aligned}\langle p_2,\epsilon_B|T_{\mu\nu}(x)|p_1,\epsilon_A\rangle_{S=1} = -\frac{e^{i(p_2-p_1)\cdot x}}{\sqrt{4E_1E_2}}\Big[&2P_\mu P_\nu \epsilon_B^*\cdot\epsilon_A {}^{\text{ch}}F_1^{S=1}(q^2) + (q_\mu q_\nu - \eta_{\mu\nu}q^2)\epsilon_B^*\cdot\epsilon_A {}^{\text{ch}}F_2^{S=1}(q^2)\\ &+ [P_\mu(\epsilon_{B\nu}^*\epsilon_A\cdot q - \epsilon_{A\nu}\epsilon_B^*\cdot q) + P_\nu(\epsilon_{B\mu}^*\epsilon_A\cdot q - \epsilon_{A\mu}\epsilon_B^*\cdot q)]^{\text{ch}}F_3^{S=1}(q^2)\\ &+ [(\epsilon_{A\mu}\epsilon_{B\nu}^* + \epsilon_{B\mu}^*\epsilon_{A\nu})q^2 - (\epsilon_{B\mu}^*q_\nu + \epsilon_{B\nu}^*q_\mu)\epsilon_A\cdot q - (\epsilon_{A\mu}q_\nu + \epsilon_{A\nu}q_\mu)\epsilon_B^*\cdot q\\ &+ 2\eta_{\mu\nu}\epsilon_A\cdot q\epsilon_B^*\cdot q]^{\text{ch}}F_4^{S=1}(q^2) + \frac{2}{m^2}P_\mu P_\nu\epsilon_A\cdot q\epsilon_B^*\cdot q^{\text{ch}}F_5^{S=1}(q^2)\\ &+ \frac{1}{m^2}(q_\mu q_\nu - \eta_{\mu\nu}q^2)\epsilon_A\cdot q\epsilon_B\cdot q^{\text{ch}}F_6^{S=1}(q^2)\Big]. \end{aligned}\tag{47}$$

Here ${}^{\text{ch}}F_1^{S=1}(q^2)$, ${}^{\text{ch}}F_2^{S=1}(q^2)$, ${}^{\text{ch}}F_3^{S=1}(q^2)$ correspond to their spin 1/2 counterparts ${}^{\text{ch}}F_1^{S=1/2}(q^2)$, ${}^{\text{ch}}F_2^{S=1/2}(q^2)$, ${}^{\text{ch}}F_3^{S=1/2}(q^2)$, while ${}^{\text{ch}}F_4^{S=1}(q^2)$, ${}^{\text{ch}}F_5^{S=1}(q^2)$, ${}^{\text{ch}}F_6^{S=1}(q^2)$ represent new forms unique to spin 1. (Note that each kinematic form in Eq. (47) is separately gauge invariant.)

The results of the calculation described above can most concisely be described in terms of these form factors. Thus the tree level predictions can be described via

$$F_{1,\text{tree}}^{S=1} = F_{3,\text{tree}}^{S=1} = 1, \qquad F_{2,\text{tree}}^{S=1} = F_{4,\text{tree}}^{S=1} = -\tfrac{1}{2}, \qquad F_{5,\text{tree}}^{S=1} = F_{6,\text{tree}}^{S=1} = 0, \tag{48}$$

while the photon-loop corrected values are given by

$$\begin{gathered} {}^{\text{ch}}F_{1,\text{loop}}^{S=1}(q^2) = {}^{\text{ch}}F_{1,\text{loop}}^{S=1/2}(q^2) = {}^{\text{ch}}F_{1,\text{loop}}^{S=0}(q^2) = \frac{\alpha}{16\pi}\frac{q^2}{m^2}(8L+3S)+\ldots,\\ {}^{\text{ch}}F_{2,\text{loop}}^{S=1}(q^2) = {}^{\text{ch}}F_{2,\text{loop}}^{S=1/2}(q^2) = {}^{\text{ch}}F_{2,\text{loop}}^{S=0}(q^2) = \frac{\alpha}{24\pi}(8L+3S)+\ldots,\\ {}^{\text{ch}}F_{3,\text{loop}}^{S=1}(q^2) = {}^{\text{ch}}F_{3,\text{loop}}^{S=1/2}(q^2) = \frac{\alpha}{24\pi}\frac{q^2}{m^2}(4L+3S)+\ldots, \qquad {}^{\text{ch}}F_{4,\text{loop}}^{S=1}(q^2) = \frac{\alpha}{192\pi}\frac{q^2}{m^2}(16L-3S)+\ldots,\\ {}^{\text{ch}}F_{5,\text{loop}}^{S=1}(q^2) = \frac{\alpha}{384\pi}\frac{q^2}{m^2}(64L+9S)+\ldots, \qquad {}^{\text{ch}}F_{6,\text{loop}}^{S=1}(q^2) = \frac{\alpha}{192\pi}(64L+15S)+\ldots, \end{gathered}\tag{49}$$

and we note that these results confirm universality—${}^{\text{ch}}F_{1,2,3,\text{loop}}^{S=1}(q^2)$ as found for unit spin agree exactly with the forms ${}^{\text{ch}}F_{1,2,3,\text{loop}}^{S=1/2}(q^2)$ found previously for spin 1/2 and with ${}^{\text{ch}}F_{1,2,\text{loop}}^{S=0}(q^2)$ in the spinless case. The three "new" form factors ${}^{\text{ch}}F_{4,5,6}^{S=1}(q^2)$, which have no analog for lower spin systems are seen to be higher order and to have a quadrupole structure, which presumably itself has a universality generalization to higher spin systems.

We verify that both ${}^{\text{ch}}F_1^{S=1}(q^2=0)=1$ and ${}^{\text{ch}}F_3^{S=1}(q^2=0)=1$ as required by energy-momentum and angular momentum conservation. Interestingly, the form factors ${}^{\text{ch}}F_4^{S=1}(q^2)$ and ${}^{\text{ch}}F_5^{S=1}(q^2)$ are also unrenormalized from their tree level values and this fact has an interesting consequence. Since, in the Breit frame and using nonrelativistic kinematics we have

$$\begin{aligned}\langle p_2,\epsilon_B|T_{00}(0)|p_1,\epsilon_A\rangle \simeq m\Big\{&\hat{\epsilon}_B^*\cdot\hat{\epsilon}_A\Big({}^{\text{ch}}F_1^{S=1}(q^2) - \frac{q^2}{2m^2}F_2^{S=1}(q^2)\Big) + \frac{1}{2m^2}\hat{\epsilon}_B^*\cdot\vec{q}\hat{\epsilon}_A\cdot\vec{q}\Big[{}^{\text{ch}}F_1^{S=1}(q^2) - {}^{\text{ch}}F_2^{S=1}(q^2)\\ &- 2\Big({}^{\text{ch}}F_4^{S=1}(q^2) + {}^{\text{ch}}F_5^{S=1}(q^2) - \frac{q^2}{2m^2}F_6^{S=1}(q^2)\Big)\Big]\Big\}\\ &+\ldots\langle p_2,\epsilon_B|T_{0i}(0)|p_1,\epsilon_A\rangle \simeq -\frac{1}{2}[(\hat{\epsilon}_B^*\times\hat{\epsilon}_A)\times\vec{q}]_iF_3^{S-1}(q^2)+\ldots\end{aligned}\tag{50}$$

we can identify values for the gravitoelectric monopole, gravitomagnetic dipole, and gravitoelectric quadrupole coupling constants

$$\begin{gathered} K_{E0} = m^{\text{ch}}F_1^{S=1}(q^2=0), \qquad K_{M1} = \frac{1}{2}{}^{\text{ch}}F_3^{S=1}(q^2=0),\\ K_{E2} = \frac{1}{2m}[{}^{\text{ch}}F_1^{S=1}(q^2=0) - {}^{\text{ch}}F_3^{S=1}(q^2=0) - 2^{\text{ch}}F_4^{S=1}(q^2=0) - 2^{\text{ch}}F_5^{S=1}(q^2=0)]. \end{gathered}\tag{51}$$

Taking $Q_g \equiv m$ as the gravitational "charge," we observe that the tree level values—

$$K_{E0} = Q_g, \qquad K_{M1} = \frac{Q_g}{2m}, \qquad K_{E2} = \frac{Q_g}{m^2} \tag{52}$$

are *unrenormalized* by loop corrections—not only is there no any anomalous gravitomagnetic moment, as mentioned above, but also there exists no anomalous gravitoelectric quadrupole moment.

Finally, we note that the metric forms are also universal up to small (shorter distance) quadrupole corrections—

$$\begin{aligned} {}^{\mathrm{ch}}g_{00}^{S=1}(r) &= {}^{\mathrm{ch}}g_{00}^{S=0}(r) \times \hat{\epsilon}_B^* \cdot \hat{\epsilon}_A, \\ {}^{\mathrm{ch}}g_{0i}^{S=1}(\vec{r}) &= {}^{\mathrm{ch}}g_{0i}^{S=1/2}(\vec{r}), \\ {}^{\mathrm{ch}}g_{ij}^{S=1}(\vec{r}) &= {}^{\mathrm{ch}}g_{ij}^{S=0}(\vec{r}) \times \hat{\epsilon}_B^* \cdot \hat{\epsilon}_A, \end{aligned} \tag{53}$$

where in the case of spin 1 we use the spin vector $\vec{S} = i\hat{\epsilon}_B^* \times \hat{\epsilon}_A$—cf. Eq. (27).

C. Spin 1 neutral particle

Using the simple Proca Langrangian Eq. (18), we can calculate the matrix elements which will be required for our graviton-loop calculation. Specifically, the general single graviton vertex for a transition involving an outgoing graviton with polarization indices $\mu\nu$ and four-momentum $q = p_1 - p_2$, an incoming spin one particle with polarization index α and four-momentum p_1 together with an outgoing spin one particle with polarization index β and four-momentum p_2 is

$$\begin{aligned} V^{(1)}_{\beta,\alpha,\mu\nu}(p_1,p_2) = i\frac{\kappa}{2}\{&(p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu})\eta_{\alpha\beta} \\ &+ \eta_{\mu\nu}p_{1\beta}p_{2\alpha} - p_{1\beta}(p_{2\mu}\eta_{\nu\alpha} + p_{2\nu}\eta_{\alpha\mu}) \\ &- p_{2\alpha}(p_{1\mu}\eta_{\nu\beta} + p_{1\nu}\eta_{\beta\mu}) \\ &+ (p_1\cdot p_2 - m^2) \\ &\times(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta})\}, \end{aligned} \tag{54}$$

where $\kappa = \sqrt{32\pi G}$ is the gravitational coupling, while the two-graviton vertex with polarization indices $\mu\nu$ and $\rho\sigma$, an incoming spin one particle with polarization index α and four-momentum p_1 together with an outgoing spin one particle with polarization index β and four-momentum p_2 has the form

$$\begin{aligned} V^{(2)}_{\beta,\alpha,\mu\nu,\rho\sigma}(p_1,p_2) = -i\frac{\kappa^2}{4}\{&[p_{1\beta}p_{2\alpha} - \eta_{\alpha\beta}(p_1\cdot p_2 - m^2)](\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}) + \eta_{\mu\rho}[\eta_{\alpha\beta}(p_{1\nu}p_{2\sigma} \\ &+ p_{1\sigma}p_{2\nu}) - \eta_{\alpha\nu}p_{1\beta}p_{2\sigma} - \eta_{\beta\nu}p_{1\sigma}p_{2\alpha} - \eta_{\beta\sigma}p_{1\nu}p_{2\alpha} - \eta_{\alpha\sigma}p_{1\beta}p_{2\nu} + (p_1\cdot p_2 - m^2) \\ &\times(\eta_{\alpha\nu}\eta_{\beta\sigma} + \eta_{\alpha\sigma}\eta_{\beta\nu})] + \eta_{\mu\sigma}[\eta_{\alpha\beta}(p_{1\nu}p_{2\rho} + p_{1\rho}p_{2\nu}) - \eta_{\alpha\nu}p_{1\beta}p_{2\rho} - \eta_{\beta\nu}p_{1\rho}p_{2\alpha} - \eta_{\beta\rho}p_{1\nu}p_{2\alpha} \\ &- \eta_{\alpha\rho}p_{1\beta}p_{2\nu} + (p_1\cdot p_2 - m^2)\eta_{\alpha\nu}\eta_{\beta\rho} + \eta_{\alpha\rho}\eta_{\beta\nu})] + \eta_{\nu\rho}[\eta_{\alpha\beta}(p_{1\mu}p_{2\sigma} + p_{1\sigma}p_{2\mu}) - \eta_{\alpha\mu}p_{1\beta}p_{2\sigma} \\ &- \eta_{\beta\mu}p_{1\sigma}p_{2\alpha} - \eta_{\beta\sigma}p_{1\mu}p_{2\alpha} - \eta_{\alpha\sigma}p_{1\beta}p_{2\mu} + (p_1\cdot p_2 - m^2)(\eta_{\alpha\mu}\eta_{\beta\sigma} + \eta_{\alpha\sigma}\eta_{\beta\mu})] \\ &+ \eta_{\nu\sigma}[\eta_{\alpha\beta}(p_{1\mu}p_{2\rho} + p_{1\rho}p_{2\mu}) - \eta_{\alpha\mu}p_{1\beta}p_{2\rho} - \eta_{\beta\mu}p_{1\rho}p_{2\alpha} - \eta_{\beta\rho}p_{1\mu}p_{2\alpha} - \eta_{\alpha\rho}p_{1\beta}p_{2\mu} \\ &+ (p_1\cdot p_2 - m^2)(\eta_{\alpha\mu}\eta_{\beta\rho} + \eta_{\alpha\rho}\eta_{\beta\mu})] - \eta_{\mu\nu}[\eta_{\alpha\beta}(p_{1\rho}p_{2\sigma} + p_{1\sigma}p_{2\rho}) - \eta_{\alpha\rho}p_{1\beta}p_{2\sigma} - \eta_{\beta\rho}p_{1\sigma}p_{2\alpha} \\ &- \eta_{\beta\sigma}p_{1\rho}p_{2\alpha} - \eta_{\alpha\sigma}p_{1\beta}p_{2\rho} + (p_1\cdot p_2 - m^2)(\eta_{\alpha\rho}\eta_{\beta\sigma} + \eta_{\beta\rho}\eta_{\alpha\sigma})] - \eta_{\rho\sigma}[\eta_{\alpha\beta}(p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu}) \\ &- \eta_{\alpha\mu}p_{1\beta}p_{2\nu} - \eta_{\beta\mu}p_{1\nu}p_{2\alpha} - \eta_{\beta\nu}p_{1\mu}p_{2\alpha} - \eta_{\alpha\nu}p_{1\beta}p_{2\mu} + (p_1\cdot p_2 - m^2)(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\beta\mu}\eta_{\alpha\nu})] \\ &+ (\eta_{\alpha\rho}p_{1\mu} - \eta_{\alpha\mu}p_{1\rho})(\eta_{\beta\sigma}p_{2\nu} - \eta_{\beta\mu}p_{2\sigma}) + (\eta_{\alpha\sigma}p_{1\nu} - \eta_{\alpha\nu}p_{1\sigma})\eta_{\beta\rho}p_{2\mu} - \eta_{\beta\mu}p_{2\rho}) \\ &+ (\eta_{\alpha\sigma}p_{1\mu} - \eta_{\alpha\mu}p_{1\sigma})(\eta_{\beta\rho}p_{2\nu} - \eta_{\beta\nu}p_{2\rho}) + (\eta_{\alpha\rho}p_{1\nu} - \eta_{\alpha\nu}p_{1\rho})(\eta_{\beta\sigma}p_{2\mu} - \eta_{\beta\mu}p_{2\sigma})\}. \end{aligned} \tag{55}$$

We also require the triple graviton vertex function, which is given by [21]

$$\begin{aligned} \tau^{\mu\nu}_{\alpha\beta,\gamma\delta}(k,q) = \frac{i\kappa}{2}\bigg\{&P_{\alpha\beta,\gamma\delta}\bigg[k^\mu k^\nu + (k-q)^\mu(k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\bigg] + 2q_\lambda q_\sigma[I^{\lambda\sigma,}{}_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta} + I^{\lambda\sigma,}{}_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta} \\ &- I^{\lambda\mu,}{}_{\alpha\beta}I^{\sigma\nu,}{}_{\gamma\delta} - I^{\sigma\nu,}{}_{\alpha\beta}I^{\lambda\mu,}{}_{\gamma\delta}] + [q_\lambda q^\mu(\eta_{\alpha\beta}I^{\lambda\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta}I^{\lambda\nu,}{}_{\alpha\beta}) + q_\lambda q^\nu(\eta_{\alpha\beta}I^{\lambda\mu,}{}_{\gamma\delta} + \eta_{\gamma\delta}I^{\lambda\mu,}{}_{\alpha\beta}) \\ &- q^2(\eta_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta}) - \eta^{\mu\nu}q^\lambda q^\sigma(\eta_{\alpha\beta}I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta}I_{\alpha\beta,\lambda\sigma})] + [2q^\lambda(I^{\sigma\nu,}{}_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}(k-q)^\mu \\ &+ I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}(k-q)^\nu - I^{\sigma\nu,}{}_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}k^\mu - I^{\sigma\mu,}{}_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}k^\nu) + q^2(I^{\sigma\mu,}{}_{\alpha\beta}I^\nu_{\gamma\delta,\sigma} + I^\nu_{\alpha\beta,\sigma}I^{\sigma\mu,}{}_{\gamma\delta}) \\ &+ \eta^{\mu\nu}q^\lambda q_\sigma(I_{\alpha\beta,\lambda\rho}I^{\rho\sigma,}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho}I^{\rho\sigma,}{}_{\alpha\beta})] + \bigg[(k^2 + (k-q)^2)\bigg(I^{\sigma\mu,}{}_{\alpha\beta}I^\nu_{\gamma\delta,\sigma} + I^{\sigma\nu,}{}_{\alpha\beta}I^\mu_{\gamma\delta,\sigma} - \frac{1}{2}\eta^{\mu\nu}P_{\alpha\beta,\gamma\delta}\bigg) \\ &- (k^2\eta_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta} + (k-q)^2\eta_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta})\bigg]\bigg\}, \end{aligned} \tag{56}$$

where we have defined

$$I_{\alpha\beta,\mu\nu} = \tfrac{1}{2}(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu}) \tag{57}$$

and

$$P_{\alpha\beta,\mu\nu} = I_{\alpha\beta,\mu\nu} - \tfrac{1}{2}\eta_{\alpha\beta}\eta_{\mu\nu}. \tag{58}$$

The final ingredient which we need is the harmonic gauge graviton propagator

$$D_{\alpha\beta,\mu\nu}(q) = \frac{i}{q^2 + i\epsilon} P_{\alpha\beta,\mu\nu}. \tag{59}$$

Again the leading piece of the spin 1 energy-momentum tensor is given in Eq. (47) but now the one graviton-loop corrections are calculated via Fig. 2 and are found to have the form

(a) Seagull loop diagram [Fig. 1(a)]

$$\begin{aligned}
&{}^{\rm neu}F^{S=1}_{1,{\rm loop}a}(q^2) = \frac{GLq^2}{\pi}\left(0+3-1-\frac{1}{2}\right) = \frac{3}{2}\frac{GLq^2}{\pi}, \quad {}^{\rm neu}F^{S=1}_{2,{\rm loop}a}(q^2) = \frac{GLm^2}{\pi}(-5+2-2+4) = -\frac{GLm^2}{\pi},\\
&{}^{\rm neu}F^{S=1}_{3,{\rm loop}a}(q^2) = \frac{GLq^2}{\pi}\left(0+\frac{3}{2}-1-\frac{1}{2}\right) = 0, \quad {}^{\rm neu}F^{S=1}_{4,{\rm loop}a}(q^2) = \frac{GLm^2}{\pi}\left(0+1-1+\frac{3}{2}\right) = \frac{3}{2}\frac{GLm^2}{\pi},\\
&{}^{\rm neu}F^{S=1}_{5,{\rm loop}a}(q^2) = \frac{GLm^2}{\pi}(0-3+0+0) = -3\frac{GLm^2}{\pi}, \quad {}^{\rm neu}F^{S=1}_{6,{\rm loop}a}(q^2) = \frac{GLm^2}{\pi}\left(-5-\frac{1}{2}+0+3\right) = -\frac{5}{2}\frac{GLm^2}{\pi}.
\end{aligned} \tag{60}$$

(b) Born loop diagram [Fig. 1(b)]

$$\begin{aligned}
{}^{\rm neu}F^{S=1}_{1,{\rm loop}b}(q^2) &= \frac{Gq^2}{\pi}\left[L\left(\frac{1}{4}-3+2-\frac{3}{2}\right)+S\left(\frac{1}{16}-1+1+0\right)\right] = \frac{Gq^2}{\pi}\left(\frac{1}{16}S-\frac{9}{4}L\right),\\
{}^{\rm neu}F^{S=1}_{2,{\rm loop}b}(q^2) &= \frac{Gm^2}{\pi}\left[S\left(\frac{7}{8}-1+2-1\right)+L(1-3+4-3)\right] = \frac{Gm^2}{\pi}\left(\frac{7}{8}S-L\right),\\
{}^{\rm neu}F^{S=1}_{3,{\rm loop}b}(q^2) &= \frac{Gq^2}{\pi}\left[S\left(0-\frac{1}{2}+\frac{1}{2}+\frac{1}{4}\right)+L\left(\frac{1}{6}-\frac{5}{4}+\frac{3}{4}+\frac{7}{12}\right)\right] = \frac{Gq^2}{\pi}\left(\frac{1}{4}S+\frac{1}{4}L\right),\\
{}^{\rm neu}F^{S=1}_{4,{\rm loop}b}(q^2) &= \frac{GLm^2}{\pi}\left(0-1+1-\frac{3}{2}\right)+\frac{Gq^2}{\pi}\left[L\left(-\frac{17}{8}+\frac{3}{8}-\frac{1}{2}+\frac{7}{8}\right)+S\left(-\frac{41}{128}+\frac{3}{16}-\frac{1}{4}+\frac{1}{16}\right)\right]\\
&= -\frac{3}{2}\frac{GLm^2}{\pi}-\frac{Gq^2}{\pi}\left(\frac{11}{8}L+\frac{41}{128}S\right),\\
{}^{\rm neu}F^{S=1}_{5,{\rm loop}b}(q^2) &= \frac{GLm^2}{\pi}(0+3+0+0)+\frac{Gq^2}{\pi}\left[S\left(\frac{5}{128}+\frac{3}{16}+0-\frac{3}{16}\right)+L\left(0+\frac{3}{4}+0-\frac{1}{2}\right)\right]\\
&= 3\frac{GLm^2}{\pi}+\frac{Gq^2}{\pi}\left(\frac{5}{128}S+\frac{1}{4}L\right),\\
{}^{\rm neu}F^{S=1}_{6,{\rm loop}b}(q^2) &= \frac{Gm^2}{\pi}\left[S\left(\frac{43}{64}-\frac{1}{8}+\frac{1}{4}-\frac{1}{8}\right)+L\left(\frac{13}{3}+\frac{1}{2}+\frac{1}{2}-\frac{7}{3}\right)\right] = \frac{Gm^2}{\pi}\left(3L+\frac{43}{64}S\right),
\end{aligned} \tag{61}$$

where we have divided each contribution into the piece which arises from the first four bracketed pieces of the triple graviton vertex above.[3]

The full results of this calculation can then be described via:

[3]There exists no contribution to the nonanalytic terms from the pieces in the fifth bracket since the intermediate gravitons are required to be on-shell.

$$\begin{gathered}
{}^{\rm neu}F^{S=1}_{1,\rm loop}(q^2) = {}^{\rm neu}F^{S=1/2}_{1,\rm loop}(q^2) = {}^{\rm neu}F^{S=0}_{1,\rm loop}(q^2) = \frac{Gq^2}{\pi}\left(-\frac{3}{4}L + \frac{1}{16}S\right) + \ldots \\
{}^{\rm neu}F^{S=1}_{2}(q^2) = {}^{\rm neu}F^{S=1/2}_{2,\rm loop}(q^2) = {}^{\rm neu}F^{S=0}_{2,\rm loop}(q^2) = \frac{Gm^2}{\pi}\left(-2L + \frac{7}{8}S\right) + \ldots \\
{}^{\rm neu}F^{S=1}_{3}(q^2) = {}^{\rm neu}F^{S=1/2}_{3,\rm loop}(q^2) = \frac{Gq^2}{\pi}\left(\frac{1}{4}L + \frac{1}{4}S\right) + \ldots \qquad {}^{\rm neu}F^{S=1}_{4}(q^2) = \frac{Gq^2}{\pi}\left(\frac{11}{8}L + \frac{41}{128}S\right) + \ldots \\
{}^{\rm neu}F^{S=1}_{5}(q^2) = \frac{Gq^2}{\pi}\left(\frac{1}{4}L + \frac{5}{128}S\right) + \ldots \qquad {}^{\rm neu}F^{S=1}_{6}(q^2) = \frac{Gm^2}{\pi}\left(\frac{1}{4}L + \frac{43}{128}S\right) + \ldots
\end{gathered} \tag{62}$$

and we confirm universality in that ${}^{\rm neu}F^{S=1}_{1,2,3,\rm loop}(q^2)$ as found for unit spin agree precisely with the forms ${}^{\rm neu}F^{S=1/2}_{1,2,3,\rm loop}(q^2)$ determined previously for spin 1/2 and with ${}^{\rm neu}F^{S=0}_{1,2,\rm loop}(q^2)$ in the spinless case. It is also interesting that the loop contributions to the new form factors ${}^{\rm neu}F^{S=1}_{4,\rm loop}(q^2)$, ${}^{\rm neu}F^{S=1}_{5,\rm loop}(q^2)$ which have no lower spin analog, vanish to order q^0 even though there exist nonzero contributions from both loop diagrams individually. Of course, the nonrenormalization of ${}^{\rm neu}F^{S=1}_{1}(q^2=0)$ and ${}^{\rm neu}F^{S=1}_{3}(q^2=0)$, required by energy-momentum and angular momentum conservation, is obtained, as required, meaning that, as noted above, there exists no anomalous gravitomagnetic moment. Likewise we observe that, as in the charged particle case there exists no anomalous gravitoelectric quadrupole moment, so this appears to be a general result.

Finally, we note that the neutral particle metric forms are also universal up to small (shorter distance) quadrupole corrections—

$$\begin{aligned}
{}^{\rm neu}g^{S=1}_{00}(r) &= {}^{\rm neu}g^{S=0}_{00}(r) \times \hat{\epsilon}^*_B \cdot \hat{\epsilon}_A, \\
{}^{\rm neu}g^{S=1}_{0i}(\vec{r}) &= {}^{\rm neu}g^{S=1/2}_{0i}(\vec{r}), \\
{}^{\rm neu}g^{S=1}_{ij}(\vec{r}) &= {}^{\rm neu}g^{S=0}_{ij}(\vec{r}) \times \hat{\epsilon}^*_B \cdot \hat{\epsilon}_A,
\end{aligned} \tag{63}$$

where in the case of spin 1 we use the spin vector $\vec{S} = i\hat{\epsilon}^*_B \times \hat{\epsilon}_A$—cf. Eq. (27).

IV. CONCLUSIONS

Above we have calculated the long distance one-loop corrections to the energy-momentum tensor of a charged or neutral spin 1 system, which arise from the presence of nonanalytic pieces in such diagrams. We have confirmed the universality which was speculated in our previous work. In the case of the energy-momentum tensor we have confirmed for both photon-loop or graviton-loop contributions that

$$\begin{aligned}
F^{S=0}_{1,\rm loop}(q^2) &= F^{S=1/2}_{1,\rm loop}(q^2) = F^{S=1}_{1,\rm loop}(q^2), \\
F^{S=0}_{2,\rm loop}(q^2) &= F^{S=1/2}_{2,\rm loop}(q^2) = F^{S=1}_{2,\rm loop}(q^2), \\
F^{S=1/2}_{3,\rm loop}(q^2) &= F^{S=1}_{3,\rm loop}(q^2).
\end{aligned} \tag{64}$$

Likewise in the case of the metric tensor we found that, up to small quadrupole corrections,

$$\begin{aligned}
g^S_{00}(\vec{r}) &= g^{S=0}_{00}(\vec{r}) \cdot \langle S, m_f | S, m_i\rangle, \\
g^S_{0i}(\vec{r}, \vec{S}) &= g^{S=1/2}_{0i}(\vec{r}, \vec{S}), \\
g^S_{ij}(\vec{r}) &= g^{S=0}_{ij}(\vec{r}) \cdot \langle S, m_f | S, m_i\rangle,
\end{aligned} \tag{65}$$

where the spin vector $\vec{S}$ has the general form $\vec{S} \equiv \langle S, m_f|\vec{S}|S, m_i\rangle$. The universality in the case of the classical (square root) nonanalyticities is not surprising and in fact is *required* by the connection to the metric tensor and to the classical form of the energy-momentum tensorand of the metric tensor. In the case of the quantum (logarithmic) nonanalyticities, however, it is not clear why these terms must be universal. We also found additional higher order form factors for spin 1, which also receive loop corrections. It is tempting to conclude that this photon-loop and graviton-loop correction universality holds for arbitrary spin. However, it is probably not possible to show this by generalizing the calculations above. Indeed the spin 1 result involves *considerably* more computation than does its spin 1/2 counterpart, which was already much more tedious than that for spin 0. Perhaps a generalization such as that used in nuclear beta decay can be employed [22]. Work is underway on this problem and will be reported in an upcoming communication.

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APPENDIX

In this section we sketch how our results were obtained. The basic idea is to calculate the Feynman diagrams shown in Figs. 1 and 2. Thus for Fig. 1(a) we find [23]

$$Amp[a]_{\mu\nu} = \frac{1}{2!}\int\frac{d^4k}{(2\pi)^4}\frac{\epsilon_B^{*\beta}V^{(2)}_{\beta,\alpha,\mu\nu}(p1,p2)\epsilon_A^{\alpha}\langle k-q,\beta|T_{\mu\nu}|k,\alpha\rangle|_{m^2=0}}{k^2(k-q)^2}, \tag{A1}$$

while for Fig. 1(b) [23]

$$Amp[b]_{\mu\nu} = \int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k-q)^2((k-p)^2-m^2)}\times\epsilon_B^{\beta}V^{(1)}_{\beta,\lambda,\kappa}(p_2,p_1-k)\Big(-\eta^{\lambda\zeta}+\frac{(p_1-k)^\lambda(p_1-k)^\zeta}{m^2}\Big)\\ \times V^{(1)}_{\zeta,\rho,\delta}(p_1-k,p_1)\epsilon_A^{\rho}\langle k,\kappa|T_{\mu\nu}|k-q,\delta\rangle_{m^2=0} \tag{A2}$$

with obvious generalizations for the one graviton-loop case shown in Fig. 2. Here the various vertex functions are listed in Sec. III, while for the integrals, all that is needed is the leading nonanalytic behavior. Thus we use

$$I(q) = \int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k-q)^2} = \frac{-i}{32\pi^2}(2L+\ldots),\qquad I_\mu(q) = \int\frac{d^4k}{(2\pi)^4}\frac{k_\mu}{k^2(k-q)^2} = \frac{i}{32\pi^2}(q_\mu L+\ldots),$$
$$I_{\mu\nu}(q) = \int\frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{k^2(k-q)^2} = \frac{-i}{32\pi^2}\Big(q_\mu q_\nu\frac{2}{3}L - q^2\eta_{\mu\nu}\frac{1}{6}L+\ldots\Big), \tag{A3}$$
$$I_{\mu\nu\alpha}(q) = \int\frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu k_\alpha}{k^2(k-q)^2} = \frac{i}{32\pi^2}\Big(-q_\mu q_\nu q_\alpha\frac{L}{2}+(\eta_{\mu\nu}q_\alpha+\eta_{\mu\alpha}q_\nu+\eta_{\nu\alpha}q_\mu)\frac{1}{12}Lq^2+\ldots\Big)$$

for the "bubble" integrals and

$$J(p,q) = \int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k-q)^2((k-p)^2-m^2)} = \frac{-i}{32\pi^2m^2}(L+S)+\ldots,$$
$$J_\mu(p,q) = \int\frac{d^4k}{(2\pi)^4}\frac{k_\mu}{k^2(k-q)^2((k-p)^2-m^2)} = \frac{i}{32\pi^2m^2}\Big[p_\mu\Big(\Big(1+\frac{1}{2}\frac{q^2}{m^2}\Big)L-\frac{1}{4}\frac{q^2}{m^2}S\Big)-q_\mu\Big(L+\frac{1}{2}S\Big)+\ldots\Big],$$
$$J_{\mu\nu}(p,q) = \int\frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{k^2(k-q)^2((k-p)^2-m^2)}\\ = \frac{i}{32\pi^2m^2}\Big[-q_\mu q_\nu\Big(L+\frac{3}{8}S\Big)-p_\mu p_\nu\frac{q^2}{m^2}\Big(\frac{1}{2}L+\frac{1}{8}S\Big)+q^2\eta_{\mu\nu}\Big(\frac{1}{4}L+\frac{1}{8}S\Big)\\ +(q_\mu p_\nu+q_\nu p_\mu)\Big(\Big(\frac{1}{2}+\frac{1}{2}\frac{q^2}{m^2}\Big)L+\frac{3}{16}\frac{q^2}{m^2S}\Big),$$
$$J_{\mu\nu\alpha}(p,q) = \int\frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu k_\alpha}{k^2(k-q)^2((k-p)^2-m^2)}\\ = \frac{-i}{32\pi^2m^2}\Big[q_\mu q_\nu q_\alpha\Big(L+\frac{5}{16}S\Big)+p_\mu p_\nu p_\alpha\Big(-\frac{1}{6}\frac{q^2}{m^2}\Big)+(q_\mu p_\nu p_\alpha+q_\nu p_\mu p_\alpha+q_\alpha p_\mu p_\nu)\Big(\frac{1}{3}\frac{q^2}{m^2}L+\frac{1}{16}\frac{q^2}{m^2}S\Big)\\ +(q_\mu q_\nu p_\alpha+q_\mu q_\alpha p_\nu+q_\nu q_\alpha p_\mu)\Big(\Big(-\frac{1}{3}-\frac{1}{2}\frac{q^2}{m^2}\Big)L-\frac{5}{32}\frac{q^2}{m^2}S\Big)+(\eta_{\mu\nu}p_\alpha+\eta_{\mu\alpha}p_\nu+\eta_{\nu\alpha}p_\mu)\Big(\frac{1}{12}q^2L\Big)\\ +(\eta_{\mu\nu}q_\alpha+\eta_{\mu\alpha}q_\nu+\eta_{\nu\alpha}q_\mu)\Big(-\frac{1}{6}q^2L-\frac{1}{16}q^2S\Big)\Big]+\ldots \tag{A4}$$

for their "triangle" counterparts. Similarly higher order forms can be found, either by direct calculation or by requiring various identities which must be satisfied when the integrals are contracted with p^μ, q^μ or with $\eta^{\mu\nu}$. Using these integral forms and substituting into Eqs. (A1) and (A2), one derives the results given in Sec. III.

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