When Good Bankers Go Bad: Is Moral Hazard Evolutionarily Stable?

Atin Basuchoudhary, Virginia Military Institute
Troy Siemers, Virginia Military Institute
Sam Allen, Virginia Military Institute
When Good Bankers Go Bad: Is Moral Hazard Evolutionarily Stable?

A Cultural Approach.

Atin Basuchoudhary  
Department of Economics and Business  
Virginia Military Institute  
Email: basuchoudharya@vmi.edu

Sam Allen  
Department of Economics and Business  
Virginia Military Institute  
Email: allensk@vmi.edu

Troy Siemers  
Department of Mathematics and Computer Science  
Virginia Military Institute  
Email: siemerstj@vmi.edu

Very preliminary draft. Please do not quote without permission.

12/1/2013

We apply existing theory as a preliminary analysis of whether efficient contracts can evolve naturally. Any banker could belong to one of two cultures – patient and impatient. We suggest that the interaction of patient bankers with other patient bankers is a critical element in the success of efficient contracts while the interaction of impatient bankers with other impatient bankers leads to the spread of moral hazard in the banking system. We show that the success (or failure) of efficient contracts depends on the initial proportion of bankers who are part of the patient culture. We further show that regulatory uncertainty could lead to the systematic spread of inefficient contracts. We find the same effect if corporate governance weakens. Thus, our paper places agency theory where patience is a cultural paradigm. This uses well-known theory to understand the system wide impact of regulatory change. As a result we suggest that well intentioned regulation may have counter-productive effects. Moreover, we suggest that a culture that rewards inefficient contracts is more likely to take root when crony capitalism weakens governance.
Culture changes over time to facilitate or to hinder aggressively competitive and predatory activities. Because these cultural changes are difficult to quantify, and fall outside the field of economics, they are rarely connected by economists to economic fluctuations. They should. -- (Akerlof and Shiller 2009)

Introduction

Agents (bankers in this paper) can use private information to their advantage because the principals (stockholders) act within the confines of bounded rationality. Ideally, contracts can reduce the effect of moral hazard. In fact, repetition can lead to efficient trades via the Folk theorem (see e.g. Fudenberg and Tirole 1991 or Myerson 1991 for a standard treatment of Folk Theorems). We, however, model our agents with certain pre-existing cultural traits. Mere repetition is insufficient to ensure efficient outcomes once interaction among agents with different cultural traits is repeated in an evolutionary framework. We base these results on the assumption that patience is a cultural trait. We further suggest that poor corporate governance makes it systematically harder for a culture that incentivizes efficient contracts to take hold. Last, we find that the probability of institutional change can incentivize a culture that allows moral hazard to persist throughout the banking system. Thus, in this paper we use well known theoretical results to shed light on recent concerns about systemic problems in the financial sector (Basuchoudhary & Razzolini, 2014; Basuchoudhary, Mazumder, & Simoyan, 2013; and Basuchoudhary, Siemers, & Allen, 2009). We use an existing theory of patience as a cultural trait and apply it to explain the persistence of inefficient outcomes in the banking sector.

A banker may be compensated in one of two ways (Akerlof & Romer, 1993). The standard, economist approved, approach would be to lend prudently. This sort of lending would generate a long stream of future profits. These profits would then be reflected in the stock price
and maximize shareholder value. Alternatively, bankers could take advantage of informational asymmetries and focus on short term gains that would increase their own returns at the expense of long term shareholder value.\(^1\) We suggest that patience is the source of the first approach while impatience lies at the heart of the second approach. Specifically, in our model a banker or agent can be part of one of two strategies or cultures – to be patient or to be impatient. Impatient bankers make imprudent loans – NINJA (no income no job or assets) loans for example -- that are highly profitable in the short term. Patient bankers cannot show the same profits in the short term, however over the long term their loans are much more profitable. In other words, patient bankers maximize the firm’s profits but impatient bankers maximize their own profits by succumbing to moral hazard.

We couch this interaction in the context of an evolutionary “Stag Hunt” or assurance game where the impatient banker has an incentive to pursue a personally beneficial though morally hazardous policy. Patient bankers can cooperate to maximize the profits of the firm while impatient bankers work separately to maximize their personal profit. In biology, a strategy would be an inherited trait. However, in a social setting Gintis (2009, p. 229) points out that “society has the strategy set (the set of alternative cultural forms) and individuals inherit or choose among them.” Thus, the strategy set for this society of bankers includes patience and impatience. A strategy that performs better than others is, according to evolutionary game theory, more fit than the others and is more likely to pervade society through a process of natural selection. In evolutionary games, payoffs determine this fitness (Harrington, 2009). The theory

\(^1\) These short term gains may be legal or outright fraudulent. For example they could simply kite the value of the stock or take out the money invested with them through inflated salaries, sweetheart deals, or bonuses. We do not make a distinction between legal and illegal expressions of the moral hazard problem.
of the basic game structure we use is quite well known. For example, it is well known that inefficient outcomes often prevail or persist in evolutionary game theory even when efficient outcomes are predicted in standard game theory.\textsuperscript{2} However, our use of patience as a cultural trait, i.e. as components of a social strategy set, to explain systemic failure in banking is novel.\textsuperscript{3} Thus, an evolutionary approach can determine if a culture of impatience can invade a society where patient bankers diligently pursue the interests of stockholders. We show that it can as a function of certain parameters. These parameters include the divergence between the level of patience that exists in the patient culture relative to the impatient culture, the probability that institutions (i.e. the game itself) persist, and good corporate governance that limits the divergence between the interests of the principals (stockholders) and bankers (agents). This link between institutional/governance parameters and a culture that leads to inefficient outcomes may have interesting policy consequences. In other words, we suggest that an evolutionary game theoretic approach can be an interesting approach to mechanism design that accounts for prevailing social and cultural trends.

\textsuperscript{2} Even some recent undergraduate textbooks outline this process. For example both the Stag Hunt and the Prisoner’s Dilemma game – set in an evolutionary framework – are outlined in Harrington (Harrington 2009, p. 515-529)

\textsuperscript{3} We use this approach in two other papers (Basuchoudhary, Siemers and Allen, 2009) and Basuchoudhary, Mazumder and Simoyan, 2013) to explain systematic failure in entire societies and why biologists do not cooperate. The current paper merely uses the same analysis to an extremely another subset of society – the banking system. That the same analysis can provide insights at the general societal level as well as to specific sub sectors of society is, in our opinion, testament to the methodological advantage of an evolutionary approach. In fact a more general version of the model presented here is developed in Basuchoudhary & Razzolini, (2014).
We present our model in Section I. In Section II we present equilibrium strategies and present some comparative static results. Section III concludes.

I. The Model

We model bankers (agents) in a banking system as members of two possible cultures – patient and impatient. A banker can be a member of either culture – but not both at the same time. These bankers interact in a society – an assurance game – that has these two cultures as the strategy set. In the game this binary cultural trait is represented by the discount rate \( r \) – bankers in a patient culture have a lower discount rate \( r_L \) than bankers in the impatient high discount rate \( r_H \) culture. Players from both cultures interact in repeated random pairings. In these pairings the agents play strategies based on their culture (“genome” in Maynard Smith’s parlance) but not on previous history of play.\(^4\) In other words players, as is commonly accepted in evolutionary game theory, are boundedly rational.

We note that patient bankers find it incentive compatible to maximize shareholder value while impatient bankers maximize their own payoffs at the expense of the shareholder. Players receive payoffs that are interpreted in terms of “fitness” of their respective strategies or cultures. Fitness tracks the likelihood that a particular strategy is more likely to prevail in a population. These payoffs depend on certain parameters. The total payoff from a contract that maximizes shareholder value \( e \) defines the efficient outcome. Presumably, some fraction \( \rho \) of this efficient outcome is payment to the agent. To reduce complexity we arbitrarily normalize \( \rho \)

\(^4\) Our model follows the standard procedure established by John Maynard Smith in his seminal work *Evolution and the Theory of Games* (Smith, 1982).
to 1. That is, the interests of the owners and the bankers are exactly aligned to the point they are indistinguishable from each other. In addition, \( \alpha \in (0,1/2) \) tracks the proportion of the efficient outcome \( e \) appropriated by an agent who succumbs to moral hazard. Thus, \( \alpha \) tracks the gains to the agent at the expense of the principal. Thus a higher \( \alpha \), to the extent that it benefits the banker at the expense of the shareholder, represents a failure of corporate governance. In addition to these parameters we add \( p \) – the probability the game continues. We suggest that \( p \) tracks institutional stability. For example, the introduction of a new regulatory regime implies the end of the current game. Thus, a falling \( p \) would indicate an increased likelihood of an end to the current regulatory regime; i.e. an increased likelihood of institutional change. We assume that \( p \) is independent of history.

The game is represented in Figure 1. Each randomly chosen player interacts with another for a very long time.\(^5\) The culture is then transmitted to other bankers in the population through a replicator dynamic process.\(^6\) We deliberately avoid parameter ranges that generate either a prisoner’s dilemma or chicken games, i.e. we additionally restrict the value of \( \alpha < 1/2 \).

Note that for any \( \alpha \in (0,1/2) \) the highest possible fitness arises when people from a patient culture interact with other people from the patient culture. However, a proportion \( x \) of the population has a culture of patience. Thus, there is a chance \((1-x)\) that patient bankers may

\(^5\) This is a fairly standard formulation in the way repetition is treated in the evolutionary game theory literature. The literature on the evolution of cooperation for example represents repetition in the stage game (See Bendor and Swistak 1997 or Milinski 1987). The stage game is then used to motivate the replicator dynamic. We use a similar approach as well. This allows us to talk about discounting in the stage game itself.

\(^6\) The exact process through which this cultural information is transmitted is unclear. However, it is well established that evolutionary models mimic models of learning (Mailath, 1998).
interact with impatient bankers. In this interaction a culture of impatience is fitter than; i.e. more likely to displace; a culture of patience. There is then, an \( x \) chance that a culture of patience will be relatively fitter than a culture of impatience while there is a \( (1-x) \) chance that a culture of impatience will be relatively fitter than a culture of impatience will prevail. Note further that the parameter restrictions on \( \alpha \) suggest that the overall fitness of a world where moral hazard prevails is less than the overall fitness of a world where efficient contracts are the norm. This, perhaps, should not be surprising to the reader. However, overall, if a particular culture has an expected payoff above the average fitness of the entire population then the percentage of bankers belonging to that culture will increase. And of course, overall, if a particular culture has an expected fitness below the average fitness of the entire population then the percentage of bankers belonging to that culture will decrease.

Thus, there is an incentive to be impatient, as a reflection of moral hazard, in our model.

However, whether a culture that incentivizes moral hazard will prevail or not depends on the relative fitness of that strategy. In section II below we adress the conditions under which this may happen.

### II. Equilibrium and Comparative Statics
The equilibrium outcomes of evolutionary assurance games are quite well known. In this section, though, we reinterpret this equilibrium to gain some insight into whether a culture of impatience can dislodge efficient outcomes in the banking sector. We find that there is a “tipping point” for the number of impatient people it takes to eliminate a culture of patience. More specifically, a culture that incentivizes moral hazard may prevail even though efficient contracts are generally a fitter strategy. We then derive how the different parameters of our model, \( a, r_H, r_L, \) and \( p \) affect this tipping point.

a. Equilibrium

Here we show that the success of a culture of patience depends on whether the actual proportion of the banker population exceeds some critical level \( x^* \). There are, however, no a priori reasons to believe that a specific proportion of the population will be patient. Thus, a culture of impatience may prevail simply because random chance dictates that the proportion of patient bankers is too small. We note here that the fundamental theory behind this result is not new – any number of textbooks deal with the evolutionary outcomes of assurance games. Indeed, our interpretation of this theory as a motivator for the evolution of patience is also well laid out in Basuchoudhary, Siemers and Allen (2009) and in Basuchoudhary, Mazumder and Simoyan (2013). In fact all the formal proofs of the propositions provided here are also available in Basuchoudhary, Siemers and Allen (2009) and in Basuchoudhary, Mazumder and Simoyan (2013). The novelty in this paper lies in the interpretation of the evolution of patience as an outcome that makes the banker’s incentives compatible with that of the owners of the firm as an application of existing theory. In this section, therefore, we argue that the development of contracts that make the agents (bankers) incentives compatible with that of the principal (shareholders) is a matter of pure chance. In fact, whether these contracts prevail or not is
fundamentally determined by the proportion of people who subscribe to a culture of patience. Thus, ultimately, “good” contracts are determined by culture. And there are no guarantees that a culture that incentivizes good contracts, i.e. the patient culture, will prevail.

We use a replicator dynamic approach to solve for the Nash equilibrium. The formal proof of proposition 1 is in Appendix A. We, however, note here that a Nash equilibrium strategy profile in an evolutionary framework is also an evolutionary stable strategy (ESS) profile. An ESS is stable in the sense that small mutations, i.e. small proportions of a population playing the non ESS strategy, cannot invade a population successfully. This is a static concept. A replicator dynamic approach suggests that an ESS is stable precisely because it arises out of the dynamics of a process that generates the fittest strategy.

**Proposition 1.** The impatient culture is an evolutionary stable strategy (ESS) iff \( x < \frac{1}{a(1-\delta_L)^{-1}} \).

First of all, we note that the replicator dynamic approach to solving this game allows us to find evolutionary stable strategies. Since this is a simple 2 x 2 game the attractor in the replicator dynamic is equivalent to a Nash Equilibrium, i.e an Evolutionarily Stable Strategy (ESS). This implies that small mutations around \( x^* = \frac{1}{a(1-\delta_L)^{-1}} \) will shift the equilibrium towards a culture where either efficient contracts prevail or where inefficient contracts prevail. Thus even if, for example, a small \( \epsilon \) proportion of the patient population makes a mistake (mutates) and becomes impatient the culture of the entire banking system cascades towards the point where only a culture of impatience prevails. The opposite happens if a small \( \epsilon \) proportion of the population

---

7 For notational simplicity we set \( \delta_H = \frac{p}{1+r_H} \) and \( \delta_L = \frac{p}{1+r_L} \).
mutates and becomes patient. Thus, second, we note that this proposition suggests that the patient culture can prevail only if the proportion $x$ of patient people in a population satisfies $x > x^*$. Since there is no a priori reason to believe that $x$ will always exceed $x^*$ we cannot predict that the culture of patience will succeed. For example if $x^*$ is 8% while the actual proportion of patient bankers in a population is 7.9% then this the banking system will be doomed to be plagued by moral hazard. But all will be well if the actual proportion of patient bankers happens to be 8.1%. In the context of this paper, this result suggests that a culture that incentivizes moral hazard may well become the dominant culture in the banking system even though it is clearly not optimal for the system. This may explain the systemic failure of the financial system in 2008.

\textit{a. Comparative Statics}

In this section we will look at how $x^*$ responds to changes in some of the parameters in our model. Recall that $x^*$ represents a sort of tipping point for whether one culture will prevail over the other or not. Thus if $x^*$ rises then for the patient culture to succeed a larger proportion of the population needs to be patient. Mutations that change the actual proportion of a population would be rare and small. Thus, we argue that a rising $x^*$ makes it harder for the patient culture to succeed. With this outcome in mind we introduce three propositions that show how $x^*$ responds to changes in $\alpha$, $p$, and $r_H$. We normalize $r_L$ to zero.

\textit{Proposition 2.} A culture that incentivizes moral hazard spreads through society as corporate governance weakens since $\frac{dx^*}{d\alpha} > 0$.

The proof that $\frac{dx^*}{d\alpha} > 0$ is available in Appendix B. As $\alpha$ rises so does $x^*$. This happens because as the incentive to be impatient grows, so does the need for more patient people in
society to achieve the efficient outcome. In other words, a rising $x^*$ implies that a higher proportion of the population needs to be part of a patient culture for a patient culture to prevail. This makes it harder for the patient culture to prevail. For example, say 10% of a society is patient. Further, assume that $x^*$ is 8%. According to Proposition 1 a patient culture will prevail in this society. Now say that a worsening of corporate governance through, e.g., an easing of board oversight makes $\alpha$ rise. As a result say $x^*$ rises to 12%. Now suddenly our society, where the proportion of patient people is at 10%, finds itself on the wrong side of $x^*$ and begins to move towards the impatient equilibrium. Thus fewer contracts that make a random banker’s incentives compatible with that of shareholders are actually written. In this scenario more and more bankers succumb to moral hazard at the expense of those who do not – effectively crowding out patient bankers whose incentives match those of shareholders. We note here that asymmetric information is not the source of our results. Thus, an exogenous change in corporate governance may well work through cultural forces and exacerbate the moral hazard problem. In other words, poor corporate governance makes it systematically harder for a culture that incentivizes contracts that promote efficient trades from taking root. We therefore suggest that a weakening of corporate governance specifically may help explain why, in spite of huge injections of cash, the US financial system continues to be plagued by inefficiencies.

---

8 There may be any number of reasons, ranging from lack of independence to insufficient incentives, to insufficient attention, that may reduce the level of corporate governance provided by the board of directors. Indeed corporate governance may decline for reasons not connected to the board (Tirole, 2006).

9 Our results are not amenable to a “market for lemons” (Akerlof, 1970) sort of explanation since our agents are not differentiated by the nature of information they have.
Proposition 3. A culture where the incentives of bankers coincide with that of shareholders is more likely to prevail as the likelihood that the current institutional structure continues, \( p \), rises because \( \frac{dx^*}{dp} < 0 \).

The formal proof for this proposition is in Appendix C. Mathematically, as \( p \) falls the present value of future payoffs to both cultures diminish. However, the payoff to the impatient culture falls relatively less than that for the patient culture since the impatient culture places less value on the future anyway. Thus as \( p \) falls the impatient culture becomes fitter (has a relatively higher payoff) than the patient culture. This leads to an increase in the proportion of impatient bankers in the population. Therefore, ensuring that the patient culture prevails requires that there be more patient bankers.

Recall that we interpret \( p \) as the probability that a particular institution continues.\(^{10}\) This proposition then essentially means that if any institutional arrangement becomes more likely to change then it becomes harder for the patient culture to prevail. Say the government is contemplating a change in the regulatory structure that impacts how bankers interact with each other – for example if all banking functions are subject to the same regulatory oversight\(^{11}\) -- then even if the purpose of the regulation is to reduce moral hazard and improve efficiency its effect will be to make it harder for banker’s incentives to be compatible with that of shareholders. Thus such reforms may do little to stem the tide of the financial crisis and may indeed exacerbate it.

\(^{10}\) An institution, of course, is an “An established custom, practice, or relationship in a society” (The American Heritage Dictionary, 1994). That is, the rules of a game are an institution. Thus any change in the rules of the game -- signifying an end to the game itself -- indicates the end of an institution.

\(^{11}\) Sen. Dodd’s Finance Reform Bill (Brady, 2009) for example fits this criterion.
Proposition 4. A culture where the incentives of bankers coincide with that of shareholders is more likely to prevail if impatient people care even less about the future $\frac{dx^*}{dr_H} < 0$.

The formal proof for this proposition is in Appendix C. As impatient bankers receive less benefit from future payoffs their expected fitness diminishes relative to that of patient bankers. Thus, a higher discount rate reduces the proportion of impatient bankers in the population. Therefore, ensuring that the patient banking culture prevails requires fewer patient bankers.

III. Conclusion.

In this paper we suggest that the success, or failure, of the banking system is tied to cultural factors. While these cultural factors are not deterministic they are certainly slow to change. In this environment inefficient contracts rife with moral hazard may prevail even if the benefits of efficient contracts are well known. We further demonstrate the ease, or difficulty, with which inefficiency may permeate a financial system as a function of certain parameters that include the the level of corporate governance and the likelihood of exogenous institutional change.
Bibliography


Appendix

Proposition 1. The impatient culture is an evolutionary stable strategy (ESS) if and only if
\[ x < \frac{1}{a\delta_L - 1} \]

Proof of Proposition 1

The expected payoff from the Patient \( r_L \) strategy is
\[ E(r_L) = \frac{x e}{2} \left( \frac{1 + r_L}{1 + r_L - p} \right) \] (1)

and from Impatient \( r_H \) strategy is
\[ E(r_H) = \alpha e x \left( \frac{1 + r_H}{1 + r_H - p} \right) + \frac{ae}{2} \left( 1 - x \right) \left( \frac{1 + r_H}{1 + r_H - p} \right) \] (2)

The patient and impatient strategy provide the same expected payoff when \( E(r_L) = E(r_H) \), i.e.,

\[ \frac{x e}{2} \left( \frac{1 + r_L}{1 + r_L - p} \right) = \alpha e x \left( \frac{1 + r_H}{1 + r_H - p} \right) + \frac{ae}{2} \left( 1 - x \right) \left( \frac{1 + r_H}{1 + r_H - p} \right) \]

Or

\[ \frac{x e}{2} \left( \frac{1 + r_L}{1 + r_L - p} \right) = \left( \frac{1 + r_H}{1 + r_H - p} \right) \left( \alpha e x + \frac{ae}{2} \left( 1 - x \right) \right) \] (3)

Or

\[ \frac{x e}{2} \left( \frac{1 + r_L}{1 + r_H} \right) \left( \frac{1 + r_H - p}{1 + r_L - p} \right) = \left( \alpha e x + \frac{ae}{2} \left( 1 - x \right) \right) \] (4)

Define \( \delta_L = p/(1 + r_L) \) and \( \delta_H = p/(1 + r_H) \). Thus,

\[ \frac{1 + r_L}{1 + r_H} = \frac{\delta_H}{\delta_L} \] (4)

And,

\[ \frac{1 + r_H - p}{1 + r_L - p} = \frac{\delta_H}{\delta_L}^{-1} \] (5)

---

12 All the proofs shown here are for the convenience of the reader. The proofs are not new. They are available in Basuchoudhary, Mazumder, & Simoyan (2013) and Basuchoudhary, Siemers, & Allen, (2009).
Substituting (4) and (5) into (3) gives us

\[ \frac{x e}{2} \left( \frac{1 - \delta_H}{1 - \delta_L} \right) = \left( a e x + \frac{a e}{2} (1 - x) \right) \]

Or,

\[ x^* = \frac{1}{\frac{1}{a(1 - \delta_L)^{\frac{1}{a}}} - 1} \quad (6) \]

Thus for the patient strategy to be preferred over the impatient strategy

\[ x > x^* = \frac{1}{\frac{1}{a(1 - \delta_L)^{\frac{1}{a}}} - 1} \quad (7) \]

**Proposition 2.** A culture that incentivizes moral hazard spreads through society as corporate governance weakens since \( \frac{dx^*}{da} > 0 \).

**Proof of Proposition 2**

Notice in equation (6) that as \( a \) rises the denominator of \( x^* \) becomes smaller. Thus, as \( a \) rises so does \( x^* \).

**Proposition 3.** A culture where the incentives of bankers coincide with that of shareholders is more likely to prevail as the likelihood that the current institutional structure continues, \( p \), rises because \( \frac{dx^*}{dp} < 0 \).

**Proposition 4.** A culture where the incentives of bankers coincide with that of shareholders is more likely to prevail if impatient people care even less about the future \( \frac{dx^*}{dr} < 0 \).
Proof of Propositions 3 and 4.

Proposition 3 states that \( x^* \) is decreasing in \( p \). Proposition 4 states that \( x^* \) is decreasing in \( r_H \).

To demonstrate these, we must show that the derivatives of \( x^* \) with respect to \( r_H \) and \( p \) are always negative. Recall from (7) that after substituting \( \delta_H \) and \( \delta_L \),

\[
x^* = \frac{1}{1 - \left( \frac{p}{1 + r_H} \right)} \left( \frac{1}{\alpha} - 1 \right)
\]

We take the term \( \left( \frac{1 - \frac{p}{1 + r_H}}{1 - \frac{p}{1 + r_L}} \right) \) from the denominator and take the first derivative with respect to \( p \) to find that

\[
d \left( \frac{1 - \frac{p}{1 + r_H}}{1 - \frac{p}{1 + r_L}} \right) = \left( \frac{r_H - r_L}{r_H + r_L} \right) \left( \frac{r_L + 1}{(r_L + p + 1)^2} \right)
\]

Notice, \( (r_H - r_L) > 0, r_H \geq 0, r_L \geq 0 \) by definition. Also, \( \left( \frac{r_L + 1}{(r_L + p + 1)^2} \right) > 0 \). Thus, \( \left( \frac{r_H - r_L}{r_H + r_L} \right) \left( \frac{r_L + 1}{(r_L + p + 1)^2} \right) > 0 \). In other words, the denominator of \( x^* \) rises with \( p \). Thus \( x^* \) itself falls as \( p \) rises.

Again, we take the term \( \left( \frac{1 - \frac{p}{1 + r_H}}{1 - \frac{p}{1 + r_L}} \right) \) from the denominator and take the first derivative with respect to \( r_H \). We assume without loss of generality, that \( r_L = 0 \). This gives us

\[
\frac{d}{dr_H} \left( \frac{1 - \frac{p}{1 + r_H}}{1 - \frac{p}{1 + r_L}} \right) = \frac{p}{(r_H + 1)^2(1 - p)}.
\]

Since \( 0 < p < 1 \) and \( r_H \geq 0 \) we can unambiguously say that the denominator of \( x^* \) rises with \( r_H \). Thus \( x^* \) itself falls as \( r_H \) rises.

This concludes the proofs for our propositions.