Robust Stability Analysis of Large Power Systems Using the Structured Singular value Theory

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Abstract

This paper examines the application of structured singular value (SSV) theory to analyse robust stability of complex power systems with respect to a set of structured uncertainties. Based on SSV theory and the frequency sweep method, techniques for robust analysis of large-scale power systems are developed. The main interest is focused on determining robust stability for varying operating conditions and uncertainties in the structure of the power system. The applicability of the proposed techniques is verified through simulation studies on a large-scale power system. In particular, results for the system are considered for a wide range of uncertainties of operating conditions. Specifically, the developed technique is used to estimate the effect of variations in the parameters of a major system intertie on the nominal stability of a critical inter-area mode.

1. Introduction

The evaluation of uncertainty effects on system dynamic performance is emerging as an area of increasing importance in the analysis of stability and control of power systems. Uncertainties in the power system model can be unstructured (i.e. non-bounded parameter uncertainties) \cite{1,2} or structured associated with loading and other varying operating conditions \cite{3,4} and can result in conservative assessment of system stability. In such cases, robustness is an important issue in control design and is also of interest to assess performance analysis.

With the increasing use of large control devices along with more variable and unpredictable operating conditions, the problem of plant uncertainty in the analysis of MIMO systems is becoming more important. This has led to the development of new tools such as the structured singular value theory, which allows for the precise measurement of the effects of system uncertainties on stability robustness and performance robustness.

In recent years, the issue of plant uncertainty in power system models has been considered by several researchers. Various approximations have been utilized in the modelling of uncertain systems including polynomial approaches, Lyapunov state-space based procedures and \( \mu \) synthesis. The most general and accurate means of analysing and characterising the effect of system uncertainty on robust performance and stability is the structured singular value framework developed by Doyle and other researchers \cite{5–7}.

Early applications of these methods were at first concerned with interpreting system uncertainty and were restricted to systems with small dimensions. More recent applications of these methodologies include the analysis of robust stability and control performance in power systems with variations in the operating conditions \cite{8–10}.

The structured singular value (SSV) theory has been applied successfully by Djukanovic et al. \cite{3,4} to determine robust stability of a power system for a wide range of power system operating conditions. The method is especially well suited for analysing the stability robustness of power systems with parameter variations whose exact values are unknown but which are known to lie between some
minimum and maximum values, and can be extended to include the representation of different types of uncertainties. However, there are still several major problems associated with this algorithm, such as the computational resources required for the analysis of realistic power systems. Further, the procedures are not general and may not be easily incorporated into existing commercial software.

This research work examines the application of SSV theory to the problem of evaluating the robust stability of large power systems with structured parametric uncertainties. In this approach, variations in system operating conditions and system topology are modelled as structured uncertainties and included in the nominal power system representation.

The stability of the resulting closed-loop power-system model is then evaluated in terms of its robustness in the presence of these uncertainties. Specifically, attention is focused on the study of uncertainties in the nominal system representation arising from two different sources, namely, variations in the level of power transfer across a critical system interconnector, and variations in the interconnecting tie-line reactance. SSV analysis is used to estimate both, the limiting loading conditions and the uncertain elements that dominate the robust performance. Such an accurate prediction capability would significantly reduce the amount of computational resources required for assessing the maximum change in expected operating conditions to make the system unstable. Moreover, the proposed procedures are general and can be easily incorporated into existing small signal stability software.

The performance of the proposed method is verified through simulation studies on a large-scale power system. In particular, results for the system are considered for an extensive range of uncertainties of operating conditions. The validation is based on comparison with results from repeated detailed eigenvalue analysis using a small signal stability program and detailed time-domain simulations. Simulation results of this system for several changes in operating conditions show that the proposed method is both accurate and flexible and can be used for robust stability and control analysis in large power systems.

2. General framework for robust stability analysis

2.1. Linear fractional transformation (LFT)

Fig. 1 shows the general framework for robust stability analysis. Here, \( M(s) \) represents the generalized plant model and \( \Delta \) represents the structured uncertainties in the system due to parameter variations or variations in the operating conditions [11]. The inputs and outputs corresponding to the uncertainties in the system are \( w \) and \( z \) respectively. Further, it is assumed that both \( \Delta \) and \( M(s) \) are stable.

The resulting closed-loop system is defined by the linear fractional transformation (LFT) of \( M \) closed with \( \Delta \) such that

\[
F_u = (M, \Delta) = M_{22} + M_{21} \Delta (I - M_{11} \Delta)^{-1} M_{12},
\]

\[
F_l = (M, \Delta) = M_{11} + M_{12} \Delta (I - M_{22} \Delta)^{-1} M_{21}
\]

where \( l \) and \( u \) indicate that the lower and upper loops are closed, respectively. In robustness analysis, the size of the smallest destabilizing perturbation in \( \Delta \) is characterized by calculating the structured singular value of \( M \) as explained below.

2.2. Small-gain theorem

The system in Fig. 1 is internally stable for all \( \Delta(s) \in \Delta \) with \( ||\Delta(j\omega)||_\infty \leq 1 \) if and only if \( M(s) \) is stable, and \( ||M(j\omega)||_\infty < 1 \). This statement is referred to as the small gain theorem for structured perturbations [6]. The small gain theorem may be conservative because it does not account for structure in the uncertainty operator. The structured singular value is defined as an alternative measure of robustness.

2.3. Structured singular value

The SSV, \( \mu_\Delta(M) \), of a \( nxn \) complex transfer function \( M(s) \) with respect to the set \( \Delta \) of allowable perturbations is defined as [5,6]

\[
\mu_\Delta(M) = \begin{cases} 
0 & \text{if } \det(I - M\Delta) \neq 0 \\
1 & \text{else} \\
\min_{\Delta \in \Delta} \sigma_{\max}(\Delta) & \text{such that } |I - M\Delta| = 0
\end{cases}
\]

where \( \Delta \) is a subset of the allowable perturbations describing the uncertainty structure, i.e. the set of all
block-diagonal matrices with some specific structure, and 
\(\sigma_{\text{max}}(\Delta)\) denotes the largest singular value of \(\Delta\); the set of all 
allowable distinct real uncertainties, \(\Delta(s)\), used in this work 
is defined as 
\[
\Delta = \text{diag}(\delta_1^l, \ldots, \delta_m^l, \delta_{n-1}^l) 
\]
where \(\delta_i^l \in \mathbb{R}\) and \(\sum_{i=1}^n \delta_i = n\). This definition can easily be 
extended to system with complex or repeated uncertainties.

### 2.4. Robust stability in the presence of multiple structured uncertainties

The analysis of robust stability is concerned with finding necessary 
and sufficient conditions on \(M(s)\) that guarantee the stability of the closed-loop system for all perturbations 
\(\Delta(s)\). Referring to Fig. 1, the nominal performance is 
obtained when \(\Delta = 0\); the system is said to be robustly stable if 
the system is stable for all permissible \(\Delta(s)\). Perturbations 
in this model can be classified as structured if they can be 
attributed to parameter uncertainties in the nominal model. 

Let the uncertainty, \(\Delta(s)\) represent the set of all allowable 
variations of these parameters and \(M(s)\) be a stable matrix. It can be 
shown [5,6] that the system in Fig. 1 is stable for all \(\Delta(s)\) with 
\[\|\Delta(j\omega)\|_{s}\leq 1\] if and only if \(M(s)\) is stable and 
\[\max_{\Delta, \mu} \|\Delta(M)\| < 1\] if this theorem is satisfied, the system is 
said to be robustly stable, i.e. if the system is stable for all 
permissible \(\Delta(s)\).

It should be noted that \(\mu\) is dependent on the block 
structure of \(\Delta\); the robust stability properties computed by 
SSV will only be accurate if a realistic uncertainty operator 
is chosen. The determination of the maximum \(\mu\) that will 
drive the system unstable is an important practical problem 
and will be used in this paper for the determination of 
extreme loading conditions.

### 3. Power system representation in the SSV framework

Let the state system representation be given by

\[
\dot{x} = Ax 
\]
where matrix \(A \in \mathbb{R}^{n \times n}\) is the constant state system matrix. 
The uncertainties in the state model are incorporated into the robustness problem using the method of Djukanovic et al. 
[3,4], modified to consider numerical rather than analytical formulations. This approach allows handling systems of practical dimensions and general structure without modelling constraints.

Following the nomenclature used in [3], let \(p_1, p_2, \ldots, p_m\) be the set of uncertain parameters which are assumed to vary within practical limits. It follows that the simultaneous variations of these parameters will result in a change of the coefficients \(a_{ij} \in A\) that can be expressed in the form [4]

\[
a_{ij}^{\text{var}} = f_{ij}(p_1, p_2, \ldots, p_m) 
\]
where the nonlinear functionals, \(f_{ij}\), are analytical mathematical expressions to be determined. Various analytical approaches to estimate the change in these parameters have been proposed in the literature [3,4]. Common to all these techniques is the reliance on analytical models.

Due to the high order of the plant, however, the general calculation of analytical models may become prohibitive 
and depends on the nature and modelling characteristics of the power system model under consideration. An alternative 
to introducing uncertainty in the system model is to perform 
several successive load flow studies for the considered 
variation in one or more system parameters; the resulting change in several system parameters is then approximated 
by a second (or higher) order polynomial approximation 
using a least-square minimization technique.

It is important to notice that in interpreting parameter variations in the uncertainty framework two fundamental operations are involved: identifying the varying elements of the state-space matrix \(A\) affected by a change in the set of uncertain parameters, \(p_1, p_2, \ldots, p_m\) and introducing an uncertain representation of such a change in the nominal plant model.

To illustrate the details of the proposed procedure consider the case of a single varying parameter, \(p\). Making use of (5), the resulting change in a given coefficient, \(a_{ij}^{\text{var}}\), is then approximated by the polynomial function

\[
a_{ij}^{\text{var}} = f(a_{ij}, p) = a_{ij0} + a_{ij1}p_a + a_{ij2}p_b \]
where the polynomial coefficients, \(a_{ij}\), are identified using a least-squares fit procedure, and each value of the operating parameter is within a given range \(p_1^{\text{min}} \leq p_1 \leq p_1^{\text{max}}\). A general algorithm for evaluating these coefficients is given in Section 4.

From the theory of robust systems, parametric uncertainty 
can be expressed as a parameter set of the form [11]

\[
p_1 = \bar{p}_1(1 + r_1 \delta_1) \]
where \(p_1 = \bar{p}_1 + p_1^{\text{min}}\) and \(r_1 = \frac{p_1^{\text{max}} - p_1^{\text{min}}}{p_1^{\text{max}} + p_1^{\text{min}}}\)

Following the nomenclature used in [3], let \(A_0, A_1, \ldots, A_m\) be the set of uncertain parameters which are assumed to vary within practical limits. It follows that the simultaneous variations of these parameters will result in a change of the coefficients \(a_{ij} \in A\) that can be expressed in the form [4]

\[
a_{ij}^{\text{var}} = f_{ij}(p_1, p_2, \ldots, p_m) 
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\[
p_1 = \bar{p}_1(1 + r_1 \delta_1) \]
in which

\[
\bar{p}_1 = \frac{p_1^{\text{max}} + p_1^{\text{min}}}{2}, \quad r_1 = \frac{p_1^{\text{max}} - p_1^{\text{min}}}{p_1^{\text{max}} + p_1^{\text{min}}} \]

where \(\bar{p}_1\) is the mean parametric value and \(r_1\) is the relative uncertainty in the parameter; \(\delta_1 \in \mathbb{R}\) is a real scalar such that \(-1 \leq \delta_1 \leq 1\).

Substituting Eqs. (7) and (8) into (6), the uncertain plant 
with structured uncertainty becomes

\[
A = A_0 + L^T[A_1(\delta_1 I) + A_{11}(\delta_1^2 I)]R \]
in which \(A_1 = A_{11} = 0\) yields the nominal linearised model of the power system, and

\[
A_0 = A_0^{\text{var}} + [A_0^{\text{var}} + A_{11}^{\text{var}} p_a] p_b 
\]

\[
A_1 = [A_1^{\text{var}} + 2A_{11}^{\text{var}} p_a] p_b 
\]
\[ A_{11} = A_{11}^{\text{var}}p_b \]

\[ p_a = \tilde{p}_1 \]

\[ p_b = \tilde{p}_1 - p_{\min} \]

where \( A_o \) represents the nominal plant, and \( A_{11}^{\text{var}} \) represent varying matrices combining all uncertainties; matrices \( A_{11}^{\text{var}} \) contain the effects of the polynomial function coefficients. In addition, the \( L_{rAc} \times nve \) and \( R_{cAc} \times nve \) matrices are composed by 0's and 1's values, as described in reference [4]. In this formulation, \( nve \) is the number of states, and \( rAc \) and \( cAc \) are the number of rows and columns of the state matrix \( A \) that change due to the varying operating conditions.

The framework for robust stability and performance analysis requires that the uncertain system in (9) be written as a LFT of the uncertain parameter \( d \). From (9), the uncertain plant with structured uncertainty can be described by

\[ x = A_0x + Ww \quad (10) \]

where matrix \( W \) accounts for structured uncertainties in the model. Noting that (refer to Figs. 1 and 2)

\[ L^T A_1(\delta_1 I)R = w_1, \quad L^T A_{11}(\delta_1 I)R = w_2 \quad (11) \]

enables the LFT of the uncertain system to be written as

\[ x = \Gamma_{11}x + \Gamma_{12}w, \quad z = \Gamma_{21}x + \Gamma_{22}w \quad (12) \]

\[ w = \Delta z \quad (13) \]

where \( \Delta(\delta) \) has the diagonal structure

\[ \Delta(\delta) = \text{blockdiag}(\delta_1 I, \delta_2 I) \]

and

\[ z = [z_1, z_2]^T, \quad w = [w_1, w_2]^T \]

\[ \Gamma_{11} = A_o; \quad \Gamma = [L^T A_1 L^T A_{11}] \]

\[ \Gamma_{21} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \Gamma_{22} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \]

It follows that the upper LFT of \( \Delta(\delta) \) with the coefficient matrix \( \Gamma \) can be expressed as

\[ F_u(\Gamma, \Delta(\delta)) = \Gamma_{22} + \Gamma_{21} \Delta(\delta)(I - \Gamma_{11}\Delta(\delta))^{-1}\Gamma_{12} \quad (14) \]

Eq. (14) can be interpreted as the transfer matrix from \( w \) to \( z \), such that \( w = F_u(\Gamma, \Delta)z \). Robust stability of the closed-loop system is then evaluated using \( \mu \)-analysis techniques. This approach is applicable to a variety of uncertainties of operating conditions and can be easily incorporated into existing small signal stability software. Details of the implementation are discussed below.

4. Computational algorithm

The procedure aforementioned was implemented in Matlab using the \( \mu \)-analysis framework toolbox, Musyn [12]. An overview of the algorithm utilized for assessing robust stability is shown in Fig. 3, illustrating the combined application of conventional small signal analysis and \( \mu \)-synthesis techniques for robust stability assessment of power systems. Notice that step 1 in Fig. 3 is performed using a commercial small signal stability software.

The robust stability technique is implemented as follows.

1. Define a set of parameter variations \( p_1, p_2, \ldots, p_m \). For each operating condition, perform a load flow
simulation and obtain the associated set of state matrices that correspond to a discrete combination of uncertain parameters.

2) Compute coefficients of the second-order approximating polynomials in (6) for each varying element of matrix $A$ using the least-squares minimization technique. To assess the extent to which a parameter $a_{ij}$ is modified by the change in operating conditions compute the metric

$$d_{ij}^{\text{diff}} = |a_{ij}^{\text{nom}} - a_{ij}^{\text{num}}| > \epsilon$$

in which:

- $a_{ij}^{\text{nom}}$ represents the nominal value of the coefficient $a_{ij}$,
- $a_{ij}^{\text{num}}$ is the numerical value of the nominal coefficient obtained from the load flow simulation; $\epsilon$ is an appropriate accuracy criterion. For each varying element of the state matrix, obtain the polynomial coefficients using a least-squares minimization technique.

3) Evaluate matrices $\Gamma_{11}, \Gamma_{12}, \Gamma_{21}$ and $\Gamma_{22}$ using the procedure given in Section 3.

4) Generate the $M$-$\Delta$ structured by using Eq. (12). Express the effect of varying parameters and changing operating conditions as structured uncertainties, $\Delta$.

5) Evaluate stability robustness using $\mu$ analysis. Compute the upper bound $\mu_{\text{plot}}$ on the structured singular value of matrix $M(\mu)$ using the optimal multiplier method [12]. Determine the size of the smallest $\Delta$ (limiting loading condition) which destabilizes the system by computing $\delta_1 = 1/\mu_{\text{upper}}$.

Using the algorithms described above it is possible to determine worst-case operating conditions as well as to identity which elements dominate the robustness measure.

5. Results on a complex system

5.1. Small signal characteristics

The robustness evaluation technique is demonstrated on a 6-area model of the Mexican interconnected system (MIS). A simple schematic representation of the MIS used in the studies is shown in Fig. 4. System studies are based on a dynamic model of the system that includes the detailed representation of 377 generators, 3759 buses, 2936 branches and 1986 transformers.

The base case condition is the summer peak-load 2002; the overall state space model of the system has 2256 dynamic states.

In this representation, 25 machines are equipped with PSSs. Further, loads are represented as 70% constant current and 30% constant impedance characteristics for both active and reactive power. The 6-area MIS model has a complex dynamic behaviour associated with the strong presence of several critical inter-area modes that involve the participation of machines from different geographical areas.

Table 1 summarizes the main characteristics of some
selected modes showing their oscillation pattern and swing frequency.

The study focuses on the analysis of the effects of varying parameters on the stability of the slowest mode in the MIS model. This mode is referred to as the 0.3 Hz north–south inter-area mode 1 and has very complex dynamic characteristics involving the interaction of most machines in the north and south systems.

In this analysis, two critical system parameters were used to assess the influence of uncertainty of operating conditions on the nominal stability of this mode, namely the north–south power transfer HUI-PRD 400 kV and the north–south intertie reactance and (refer to Fig. 4). These operating conditions are known from previous studies to excite the slowest inter-area modes.

Fig. 5 shows, schematically, the nature of the varying operating conditions considered in the studies. Under normal operating conditions, power is transmitted southward from steam and combined-cycle power stations in the north systems through two 400 kV transmission lines. The weak nature of this intertie poses a limit to power transfers between the north and south systems and is a major cause of the onset of low-frequency oscillations.

The validation is based on comparisons with results from conventional eigen-analysis using a commercial small-signal stability software.

### 5.2. Robust stability assessment of limiting tie-line reactance

In this study, the interconnecting tie-line reactance, $X_{\text{tie-line}}$, was varied from the nominal operating condition with the two circuits in service, to a stressed system condition associated with the loss of one the circuits of the 400 kV HUI-LAJ transmission line (refer to Fig. 4). The base-case (nominal) condition corresponds to a 875 MW north–south intertie transference with all PSSs considered.

Eigenvalue studies were performed to assess limiting conditions. Table 2 depicts the damping of the critical 0.3 Hz north–south inter-area mode 1 for different values of the interconnecting reactance. As expected from physical considerations, the analysis reveals that as the interconnecting reactance is increased, the damping ratio of the critical inter-area mode drops off rapidly. Note, however, that the conventional determination of the critical parameter comes at a high computational expense due to the large number of simulations required to estimate the stability limit.

Based on this analysis, the tie-line reactance, $X_{\text{tie-line}}$, was modelled as a structured uncertainty using the procedures developed in Section 3. In the study, a grid of operating conditions is determined from a parameter space of the form $p = [p_1, p_2, p_3]$ corresponding to various sections of the north–south interface out of service. The first value, $p_1$ corresponds to the normal operating condition with all circuits in service, whilst the second and third values, $p_2, p_3$ correspond to the case with one and two circuits out of service. This basic approximation is found to yield accurate enough results. From the $\mu$ plot in Fig. 6, we can see that $\mu$ has a dominant peak at about 0.30 Hz, which is the 0.30 Hz north–south inter-area mode 1. The magnitude of the peak is greater than 1 suggesting that the system is unstable.

Table 3 compares the exact tie-line reactance at which instability is detected, with the estimated reactance using $\mu$-analysis. For reference and comparison, the eigenvalue calculated at the exact reactance using a conventional small signal stability program is also included. It can be seen that uncertainty analysis gives a good prediction of the highest limiting reactance. The error in the maximum reactance estimate is about 6%. We remark

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**Table 1**

Critical inter-area modes of the 6-area MIS model

<table>
<thead>
<tr>
<th>Mode description</th>
<th>Eigenvalue*</th>
<th>Freq. (Hz)</th>
<th>Swing pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter-area mode 1</td>
<td>$-0.222 \pm j 2.19$</td>
<td>0.34</td>
<td>North systems vs. south systems</td>
</tr>
<tr>
<td>Inter-area mode 2</td>
<td>$-0.028 \pm j 3.28$</td>
<td>0.52</td>
<td>Peninsular system vs. western and southeastern systems</td>
</tr>
<tr>
<td>Inter-area mode 3</td>
<td>$-0.151 \pm j 4.92$</td>
<td>0.78</td>
<td>Western system vs. southeastern system</td>
</tr>
<tr>
<td>Mode 4</td>
<td>$-0.292 \pm j 7.62$</td>
<td>1.213</td>
<td>Local mode to the Peninsular system</td>
</tr>
<tr>
<td>Mode 5</td>
<td>$-0.601 \pm j 9.38$</td>
<td>1.49</td>
<td>Local mode to the southeastern system</td>
</tr>
</tbody>
</table>

* Real part in (1/s); imaginary part in (rad/s).

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**Table 2**

Damping of the north–south inter-area mode 1 as a function of the interconnecting tie-line reactance

<table>
<thead>
<tr>
<th>Tie-line reactance (pu)</th>
<th>Eigenvalue</th>
<th>Damping ratio (%)</th>
<th>Freq.(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03532</td>
<td>$-0.3646 \pm j 1.839$</td>
<td>19.44</td>
<td>0.2928</td>
</tr>
<tr>
<td>0.05532</td>
<td>$-0.1716 \pm j 1.6313$</td>
<td>10.46</td>
<td>0.2529</td>
</tr>
<tr>
<td>0.07532</td>
<td>$-0.0343 \pm j 1.3801$</td>
<td>2.48</td>
<td>0.2197</td>
</tr>
</tbody>
</table>

---

Fig. 5. Schematic representation of the north–south interface showing varying parameters used in the analysis.
that this case corresponds to nearly a 100% increase in the interconnecting tie-line reactance.

5.3. Robust stability assessment of maximum power transfer

High southwards power transfers across the north–south interface are known from previous studies to strongly stimulate the 0.3 Hz north–south mode 1. To assess the limiting loading condition, the north–south power transfer level was varied from the nominal operating condition to a stressed system condition with a high power flow representing the critical load transference. The base case (nominal) condition corresponds to a double circuit north–south inter-tie with all PSSs on. Several operating scenarios to stress the system are possible that result in decreased stability margins for the slowest inter-area mode. Among these alternatives, we favour the use of controllability-based scenarios since they enable to consider generation patterns that affect the mode of concern.

To stress the system, the generation was increased in machines in the northern and northeastern systems determined from modal analysis. The generation at selected machines in the south systems was decreased accordingly to achieve a given level of power transfer across the north–south interface. The objective was to supply the demand in the central system which is the main load centre in the system.

Table 4 depicts the damping of the critical north–south inter-area mode as a function of the inter-tie power flow. The analysis reveals that the critical inter-area mode becomes unstable at a transfer level of about 1171 MW.

The analysis of the $\mu$ plot in Fig. 7, on the other hand, shows two dominant peaks suggesting a more complex behaviour; a peak at about 0.30 Hz associated with the critical inter-area mode 1 and a peak at about 0.81 Hz associated with the critical inter-area mode 3. The magnitude of the peaks for the 0.3 Hz inter-area mode 1 and the 0.78 Hz inter-area mode 3 is greater than 1 showing that the system is robustly unstable.

Table 5 shows the estimated and exact equivalent power flow along with the eigenvalue calculated at the exact power flow. By using $\mu$-analysis, the limiting loading condition can be estimated with reasonable accuracy. As shown in Table 5, uncertainty analysis gives a good prediction of limiting loading conditions even for the most stringent operating conditions.

5.4. Computational aspects

Of fundamental importance to the practical application of the proposed methodology are the computational requirements of the method. Table 6 shows the CPU time required
for robust stability analysis of the power transfer limit for the north–south corridor. The software is executed using an IBM-PC (Pentium IV/1.5 GHz).

In this study, computation of the state representation for the three operating conditions in Table 3 takes about 2010 s. In addition, the time spent on frequency response analysis, and \( \mu \)-analysis is 857 s whereas the time spent on computing other activities is about 930 s, namely the identification of varying coefficients, polynomial fitting and the computation of matrix \( M \). The total time to assess the limiting loading condition is 3797 s. Of particular relevance, the computational effort to assess the limiting condition of the Mexican system is equivalent to 5 small signal stability simulations using SSAT. It should be emphasized, however, that determining the exact instability condition using conventional small signal analysis may require many simulations depending on the knowledge of the system and the expertise of the analyst. This is particularly true in the case of simultaneous uncertainties occurring simultaneously.

The analysis shows that the computational burden and memory requirements for \( \mu \)-analysis are reasonable for the analysis of realistic power systems. CPU time requirements appear to compare favourably with those reported in the literature [2] but the nature of test systems used in these studies prevents direct comparison between these approaches.

### 5.5. Model validation

Detailed time domain simulations have been undertaken to validate system results Fig. 8 shows the time domain response of unit no. 1 of machine Carbon Dos located in the northeastern network of the system following outage of the unit no. 1 (650 MW) of the Laguna Verde nuclear power station, as a function of the north–south reactance. This contingency is known from previous studies to strongly excite the 0.34 Hz inter-area mode 1. Simulation results agree very well with the determination of small signal stability margins. As shown, the stability limit is close to \( X_{tie-line} = 0.0787 \text{p.u.} \) in close agreement with small signal stability results in Tables 2 and 3. Fourier analysis on the other hand (not shown) enables to confirm that the dominant component is the 0.34 Hz mode inter-area mode 1 as suggested from conventional eigen-analysis.

### 6. Conclusions

In this paper, a systematic analysis technique for robust stability analysis of power systems has been proposed. The approach can accommodate several types of uncertainties common in power system of realistic dimensions. Further, modelling capacities are practically unlimited since are those associated with the small signal stability software used in the analysis.

The developed robustness evaluation methodology has been incorporated into a production-grade small signal stability software and has successfully been tested on a practical test system. The practical implementation is validated through a wide range of load levels and different operating conditions.

Simulation results indicate that the proposed method can accurately be used to assess robust stability of large complex systems. Differences that do exist warrant additional investigation. The analysis shows that small signal analysis and interpretation are highly sensitive to the introduction of uncertainty.

Additional applications of robustness analysis hold particular promise for the simultaneous analysis of key varying parameters on nominal stability. Other aspects such as the extension of the above algorithms to design system
controllers and the improvement of $\mu$ computations are currently under development.

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**References**


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