Design of multiple FACTS controllers for damping inter-area oscillations: A decentralized control approach

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Design of multiple FACTS controllers for damping inter-area oscillations: a decentralised control approach

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Abstract

In this paper, a frequency-domain methodology based on decentralised control theory is proposed to co-ordinate multiple FACTS controllers as well as to minimise the potential for adverse interaction between control loops. First, the concept of dynamic loop interaction in multiple-input multiple-output (MIMO) control systems is introduced. Using the notion of a generalised dynamic relative gain in decentralised control systems, a method is then proposed to determine the best pairing of inputs and outputs for the design of MIMO systems. Finally, a residue-based method is adopted to tune system controllers. The design methodology is tested on a 6-area, 130-machine and 351-bus practical power system in which several large static VAR compensators are embedded in a complex AC network.

Keywords: FACTS devices; Inter-area oscillations; Decentralised control theory

1. Introduction

The application of multiple flexible AC system (FACTS) controllers has received considerable recent interest as a means of enhancing system dynamic performance [1–4]. Most available techniques for their design focus on the use of simple eigenvalue-based study techniques derived from sensitivity analyses of the state representation. While these approaches are very simple, leading to standard eigenvalue problems, FACTS controllers may interact with other controllers leading to performance deterioration [5].

In recent years there has been renewed interest into the application of decentralised control techniques to design control devices in power systems. Previous work by Zhang and Messina [6] and Milanovic and Serrano-Duque [7] has addressed control structuring and its application to power systems control design. The above-mentioned analytical methods are based on static measures of interaction and are, therefore, not suitable for the accurate study of dynamic interaction within the frequency range of inter-area oscillations. In the context of dynamic interaction, Gibbard et al. [8] investigated the problem of dynamic interaction involving multiple controllers using the concepts of synchronising and damping torques in synchronous generators. Further, Castro and Silva de Araujo developed a transfer function approach to the analysis of interaction and the study of modal controllability and observability [9]. These approaches, however, require knowledge of control structure and are therefore suitable for the final stage of control design. More recently, several authors have investigated the application of decentralised control methods for damping inter-area oscillations [10,11] but the analysis of dynamic interaction between existent system controllers has not been considered explicitly. The study of this subject is considered in this research.

This paper examines the application of decentralised power system controllers for damping low-frequency inter-area oscillations in large interconnected power systems. A two-stage frequency-domain design methodology based on decentralised control theory and residue analysis is proposed to co-ordinate controllers as well as to minimise the potential for adverse interaction between control loops. First, a comprehensive method based on the concept of a generalised dynamic relative gain in MIMO systems is proposed to determine the best pairing of inputs and outputs for the design of multi-loop control systems. The method takes into account both, the dynamic of the process and the dynamics of the controllers. A residue-based method is then used to tune system controllers.

The design methodology is tested on a 6-area, 130-machine and 351-bus practical power system in which several large static VAR compensators (SVCs) are embedded in a complex AC network. Special efforts...
are devoted to quantify the influence of existing SVC dynamic support to enhance the damping of three critical inter-area modes. Time-domain analyses are finally conducted to check the validity of the analysis as well as to assess the impact of dynamic voltage support on transient system behaviour.

The paper is organised as follows. In Section 2, a brief introduction to the problem of control system design is presented. Section 3 presents the development of a dynamic interaction measure for multivariable control processes based on the concept of a dynamic relative gain. Previous work on this area is reviewed and a computational procedure to measure loop-interaction in MIMO systems is proposed. Section 4 describes the controller design approach. Section 5 presents a large case study illustrating the proposed method. In Section 6, the general conclusions of this work and suggestions for future work are given.

2. Overview of the study approach

2.1. Basic concepts

Consider a linear time-invariant dynamical MIMO system described by the linear state representation

\[ \dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) \]  

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^k \) is the vector of control inputs and \( y \in \mathbb{R}^k \) is the vector of system outputs.

Let the eigenvalues \( \lambda_i, i = 1, 2, ..., n \) of matrix \( A \) be distinct. The output of the system can then be expressed in vector form as [12]

\[ y(s) = G(s)u(s) = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i} u(s) \]  

where

\[ G(s) = CU(sI - A)^{-1}VB \]  

is the open-loop transfer function matrix (TFM) of the system with elements \( g_{ij}(s) \) for inputs \( u_i \) and outputs \( y_j \), \( i, j = 1, ..., k \). \( A = \text{diag}\{\lambda_1, \lambda_2, ..., \lambda_n\} \) and \( U \) and \( V \) are, respectively, the right and left eigenvector matrices of \( A \); the \( R_i = (s - \lambda_i)C(sI - A)^{-1}B \mid_{s=\lambda_i}, i = 1, ..., n \) are the residue matrices.

Let now \( r \) be the vector of reference signals, \( r_i, i = 1, ..., k \), for the closed-loop system and \( G_C(s) \) be the TFM of the feedback controllers. A control process is said to be decentralised if the off-diagonal elements of the controller TFM, \( G_C(s) \), are zero as shown in Fig. 1.

With reference to Fig. 1, the closed-loop TFM of the system, \( H(s) \), is

\[ H(s) = (I - G(s)G_C(s))^{-1}G(s) \]  

where \( G_C(s) = \text{diag}\{g_{ci}(s), i = 1, ..., k\} \)

From Eq. (4) it follows that if \( g_{ij}(s) = 0, (i \neq j) \), then each controller \( g_{ci}(s), i = 1, ..., k \) can be designed for the isolated subsystem \( g_{ii}(s) \) without any loss of performance or interaction with other control loops. The system, in fact, consists of a set of single-input single-output (SISO) loops.

In the more general case, if \( g_{ij}(s) \neq 0 (i \neq j) \), it is necessary to structurally decompose the decentralised control system into individual SISO loops to reduce the problem of interaction between control loops. Note, however, that these techniques aim at minimising the interaction that might occur between control loops by selecting appropriate feedback signals for the controllers; no attempts are made to eliminate the interaction.

It should be emphasised that the use of decentralised control structures may lead to performance deterioration when compared to a system with full feedback. Decentralised control, however, leads to hardware and tuning simplicity and reduces the number of parameters to be optimised and is widely used by the industry.

2.2. Design of decentralised controllers

In dealing with interacting power system controllers using decentralised control theory, two major design aspects must be addressed: the first aspect is the study of the extent of interaction between the control loops and the second aspect involves assessment of the nature of this interaction, i.e. whether or not the interaction is favourable.

The overall design of decentralised controllers can be broadly divided into two main stages [13]:

- The selection of feedback signals for the controllers, or pairing of inputs and outputs.
- The design of supplemental power system damping controllers (PSDC), \( g_{ci}(s), i = 1, ..., k \) for each SISO control loop to provide an acceptable performance.

In the first design stage, the controlled variables and the manipulated variables are paired such that the interaction
between the resulting control loops is reduced; this facilitates the independent design of controllers and leads to tuning simplicity. The design of decentralised controllers, on the other hand, requires the tuning of a $k \times k$ diagonal TFM of controllers, $G_C(s)$, using sequential loop-closing techniques or, alternatively, the independent design of each control loop. The basis of the selection of feedback signals and the design of controllers proposed in this research is dealt within the following sections.

### 3. Dynamic interaction analysis

From a control design perspective, it is desirable to choose the feedback signals for the controllers such that the resulting control strategy exhibits the overall minimum interaction between control loops. This is referred to, as the problem of pairing of inputs and outputs or the selection of variable pairing [12]. Once the input–output pairing providing the least interaction is chosen, the individual controllers can be designed using sequential or independent loop techniques.

To gain additional insight into the nature of loop interaction, consider now a MIMO control process in which the manipulated variable $u_i \in \{u_1, \ldots, u_l\}$ is chosen to pair with the controlled variable $y_i \in \{y_1, \ldots, y_k\}$ with a feedback controller $g_{ci}(s)$. Further, it is assumed that other control loops are closed as shown in Fig. 2.

It follows that control action in the $u_i - y_i$ loop in response to changes and/or disturbances in this control loop initiates control action in one or more control loops to keep the variables, $y_i$, $l \neq i$ at their set points. This problem is known as interaction and may result in a degraded overall performance for the MIMO system [14,15].

In order to examine these effects analytically, assume that only the $i$th loop undergoes set point changes ($\Delta r_i = 0$, $l \neq i$). The closed-loop TFM of the system with the $u_i - y_i$ loop open, $H'(s)$, may then be expressed as (Fig. 2)

$$H'(s) = [I - G(s)G_C(s)]^{-1}G(s) = [H'_0]$$

where $G_C(s)$ is the diagonal matrix of system controllers with $g_{cr} = 0$. Therefore, the $i$th output of the system, $y_i(s)$, can be expressed in the form

$$y_i = h'_{il}(s)r_i = (g_{ir}(s) + \delta_i(s))r_i$$

where

$$\delta_i(s) = \sum_{j=1 \atop j \neq i}^k g_{ij}g_{cj}(s)h'_{jl}$$

In the equations above, $g_{ir}(s)$ represents the interaction-free transfer function of the system, and $\delta_i(s)$ represents the additional dynamics in the $i$th loop resulting from other control loops. It then follows that is the special case of a diagonal system, $g_{ir}(s) = 0$, $i \neq j$, $\delta_i(s) = 0$, and Eq. (6) reduces to $y_i = h'_{il}(s)r_i = g_{ir}r_i$. The resulting non-interacting control of the MIMO system implies one-to-one correspondence between a reference input and a controlled output.

Successive application of the above criteria to all control loops enables to derive a measure of the relative effect $u_i$ has on $y_i$ compared with other pairing alternatives as explained below.

Several approaches to the determination of interaction between controllers have been proposed in the literature. Foremost, among the existing techniques, the relative gain array (RGA) has been extensively used as a measure of static interaction in invertible square systems. In this approach, the relative gain between the input $u_i$ and the output $y_i$ is defined as [16,17]

$$\text{RGA}_{ij}\{G(0)\} = \left[\frac{\partial y_i}{\partial u_j}\bigg|_{u=0} - \delta_{ij}\right]^{-1} = \frac{g_{ij}(0)}{\tilde{g}_{ij}(0)}$$

where $g_{ij}(0) = \frac{\partial y_i}{\partial u_j}\bigg|_{u=0}$ is the steady-state gain between $u_j$ and $y_i$ when only the control $u_j$ is applied to the system, and $\tilde{g}_{ij}(0) = \frac{\partial y_i}{\partial u_j}\bigg|_{y=0}$ is the steady-state gain between $u_j$ and $y_i$ when feedback control is applied, such that in the steady state all $y_i$ ($i = 1, \ldots, k; l \neq i$) are held at their nominal value, i.e., assuming perfect control. The term $\tilde{g}_{ij}$ is the inverse of the $(i,j)$th element of matrix $G^{-1}(s)$ [15].

From the previous definition, it follows that the matrix of relative gains can be expressed as

$$\text{RGA}\{G(0)\} = [G(0)]\otimes[G^{-1}(0)]^T$$

where $\otimes$ denotes the element-by-element product of the two matrices.

It should be emphasised that the $\text{RGA}_{ii}$ coefficients provide a measure of how the stationary open-loop gain $g_{ii}$ is changed due to the closing of others loops, which, in turn, is reflected in $\tilde{g}_{ii}$. Ideally, $\text{RGA}_{ii} = 1$ and $\text{RGA}_{ij} = 0$ representing a fully decoupled control system. In this case, the RGA matrix is the identity matrix.
While this approach has been extended to include dynamic transfer functions rather than steady-state gains, the method assumes perfect closed loop control. This has limited its usefulness to accurately measure interactions specially in practical applications where there exists bias in the outputs, such as in many cases of proportional control and \( H_{\infty} \) control. This fact has motivated the development of several alternate analytical techniques.

A more useful measure of dynamic interaction that takes into account both, the dynamics of the controller and the dynamics of the process is obtained from the generalised dynamic relative gain (GDRG) [18]

\[
\text{GDRG}_{ij}(s) = \frac{g_{ij}(s)}{h_{ij}(s)}
\]

where

- \( g_{ij} \) is the \((i, j)\)th element of matrix \( G(s) \), and
- \( h_{ij} \) is the \((i, j)\)th element of matrix \( H(s) \) defined in Eq. (5).

Observe, from Eq. (6), that \( h_{ij}^* \) in Eq. (10) represents a measure of how the other loops affect the \( i \)th control loop. For a given frequency of interest \( \omega_n \), the minimum interaction for the \( i \)th control loop with other controllers is obtained if

\[
|\text{GDRG}_{ij}(j\omega_n)| = 1
\]

(11)

Therefore, a practical measure of dynamic loop interaction of a multi-loop control system can be obtained from the GDRG number,

\[
N_{\text{GDRG}}(j\omega) = \sum_{i=1}^{k} |\text{GDRG}_{ij}(j\omega)| - 1
\]

(12)

From Eq. (12), it follows that a zero GDRG number indicates the absence of interaction between control loops at a frequency \( \omega \), and then it is desirable to have \( N_{\text{GDRG}}(j\omega) \) close to zero for the frequency range of interest. It should be emphasised that unlike the RGA, the GDRG can be applied to systems including any kind of controllers and takes into account the dynamic interaction between control loops. Clearly, the RGA is a particular case of the GDRG.

Practical computation of the GDRG number is obtained by noting that the transfer function \( h_{ij}(s) \) can be written as

\[
h_{ij}(s) = \frac{h_{ij}^*}{1 - h_{ij}^*g_{ci}}
\]

(13)

\[\begin{array}{c}
\text{r} \\
h_{ii} \\
\text{y} \\
\Rightarrow \quad \text{r} \\
h_{ij}^* \\
g_{ci} \\
\Rightarrow \quad \text{y} \\
h_{ij}^* \\
g_{cj} \\
\end{array}\]

Fig. 3. Schematic diagram illustrating the computation of \( h_{ij}^*(s) \) from the closed-loop representation.

Hence, solving for \( h_{ij}^* \) yields

\[
h_{ij}^*(s) = \frac{h_{ij}}{1 + h_{ij}g_{ci}}
\]

(14)

Using the above expression, an efficient procedure to compute the GDRG number for each control loop in a general system of \( k \times k \) dimensions can be formulated as follows:

1. Rearrange the system in Eq. (3) such that the \( i \)th output is controlled through the \( i \)th control input;
2. Construct the diagonal matrix of system controllers \( \mathbf{G}_c(s) = \text{diag}[g_{ci}(s), i = 1, \ldots, k] \);
3. Compute the closed-loop TFMs \( \mathbf{H}(s) \) from Eq. (14);
4. Then, for each control loop \( i \):
   - Compute \( h_{ij}^* \) from Eq. (14);
   - Evaluate the GDRG number, \( N_{\text{GDRG}}(j\omega) \), for the frequency range of interest using Eqs. (10) and (12);
5. Plot the \( N_{\text{GDRG}}(j\omega) \) for the frequency range of interest. For each frequency of concern \( \omega \), the GDRG number provides an estimate of the potential for dynamic interaction with the associated mode.

4. Controller design approach

A sequential loop-closure strategy was adopted for tuning PSDC. In this methodology, each supplemental control loop is designed independently of the other so as to enhance the damping of a specific swing mode.

The design of each controller involves the following steps:

1. For each pair \( u_j \) and \( y_j \), use transfer function residues for preliminary screening of alternatives for input–output pairings resulting in the best damping of critical inter-area modes.
2. Based on residue analyses in step 1, generate a feasible set of candidate controllers for damping inter-area oscillations and evaluate the GDRG number for each alternative.
3. Select the set with minimum interaction by applying the procedure described in Section 3.
4. Once the least interacting pairing of feedback loops is selected, design the individual SISO controllers for each individual control loop. The nature of this approach is described below.

To illustrate the nature of the proposed method, let the transfer function of the \( j \)th PSDC be given by (Fig. 2)

\[
g_{cj}(s) = \frac{y_j(s)}{u_j(s)} = K_{pdc}M_j(s), \quad j = 1, \ldots, k
\]

(15)
\[ M_j(s) = \left( \frac{sT_{w}}{1 + sT_{w}} \right) \left( \frac{1 + sT_1}{1 + sT_2} \right)^{m_j} \]

where \( K_{psdc} \) is the PSDC gain, \( T_w \) is the washout time constant designed to remove the steady-state offset of the controller and \( T_1, T_2 \) are the lead-lag block time constants; \( m \) is the number of compensation stages. The design objective can be formulated as follows. Given a critical system mode \( \lambda_h \), the stabiliser constants \( K_{psdc}, T_1, T_2 \) are tuned such that the critical system eigenvalue is left-shifted. A first-order estimate of the sensitivity of a closed-loop eigenvalue \( \lambda_h \) to the change in the stabiliser gain \( K_{psdc} \) is given by [19]

\[
\Delta \lambda_h = \frac{R^h}{1 - R^h \frac{\partial g_c(\lambda_h)}{\partial \lambda_h}} M_j(\lambda_h) \Delta K_{psdc_j} \tag{16}
\]

where \( R^h \) is the open-loop residue from the \( j \)th input to the \( i \)th output of the state model, evaluated at the complex frequency \( \lambda_h \). It follows that when the stabiliser gain adjustment \( \Delta K_{psdc} \) is small enough such that \( R^h \frac{\partial g_c(\lambda_h)}{\partial \lambda_h} \ll 1 \), the resulting change in the eigenvalue \( \lambda_h \) may be approximated by

\[
\Delta \lambda_h \approx R^h M_j(\lambda_h) \Delta K_{psdc_j} \tag{17}
\]

For a PSDC with \( m_j \) compensation stages, the time constants and the gain of the stabilising controller are adjusted to attain maximum damping following the next procedure:

1. Compute \( R^h \) for each design alternative, and frequency of interest \( \lambda_h \) for \( h = 1, \ldots, k \). Adjust the time constants \( T_1, T_2 \) to ensure a negative real shift in the mode of interest \( \arg\{R^h M_j(\lambda_h)\} = \pm 180^\circ \) \tag{18}

2. Determine the feedback gain \( K_{psdc} \) by using Eq. (17) subjected to

\[
\left| R^h \frac{\partial g_c(\lambda_h)}{\partial \lambda_h} \right| \ll 1
\]

3. Recalculate system eigenvalues. If necessary go back to step 1.

5. Case study

A 6-area, 110-machine practical power system is used to illustrate the control design approach. Fig. 4 shows a single-line diagram of the system under investigation illustrating the location of major transmission and generation facilities. The transmission network consists of 351 buses, 470 branches, 130 generators and eight major SVCs.
encompassing parts of the bulk Mexican interconnected system (MIS).

5.1. Dynamic voltage support within the system

SVCs are installed at several key locations on the 400/230 kV network of the MIS to provide voltage support as well as to enhance system dynamic performance. Table 1 synthesises the main characteristics of SVCs considered in this study. Major control facilities include the ±300 MVar SVC at the Temascal substation on the 400 kV transmission network of the south-eastern system, the −90/+300 MVar SVC at the Guemez substation on the north–south interface, and two −90/+300 MVar SVCs at the Topilejo and Texcoco substations in the neighbourhood of the Mexico city metropolitan area. These devices provide voltage stabilisation during steady-state and transient conditions. The normal operating mode for the SVCs at Guemez and Texcoco is voltage control. The SVC at the Topilejo substation, on the other hand, incorporates a supplemental modulation control loop.

Other existing devices were found to have a rather marginal impact on overall system damping and are therefore not included in the following analyses.

5.2. Damping characteristics of the MIS

The six-area model of the MIS exhibits three critical lightly damped inter-area modes at 0.40, 0.71 and 1.12 Hz with damping ratios below 3%:

- Two critical inter-area modes at 0.40, and 0.71 Hz. These modes are referred to as the 0.4 Hz North–south inter-area mode 1 and the 0.7 Hz East–west inter-area mode 3 in Table 2.
- A higher frequency mode of essentially local nature at 1.12 Hz.

A root-loci of the dominant system eigenvalues for the base case initial conditions is shown in Fig. 5. Further, Table 2 synthesises the main characteristics of these modes showing their modal damping ratios and swing frequency. Of particular importance, the 0.4 Hz inter-area mode 1 involves the oscillation of machines in the north systems swinging against machines in the south system. Also of concern, the 0.72 Hz inter-area mode 3 is of relevance as it involves machines in the south-eastern swinging against machines in the central and western systems. Finally, mode 9 represents a localised oscillation involving machines in the central and western systems.

Two main control alternatives to enhance damping of these modes are discussed in the following subsections; the co-ordinated application of existing SVCs and the use of PSSs at critical machines.

5.3. Application of multiple SVC dynamic support

Transfer function residues between the reference voltage of SVCs, \( V_{ref} \) and locally measurable input signals at the SVC bus, \( y_i \), were computed. Signals chosen as inputs to the PSDC included tie-line power (\( P_{line} \)) and tie-line current (\( I_{line} \)). Table 3 synthesises the largest residues of critical system modes and various controller inputs for the SVCs at Temascal, Topilejo and Guemez.

The analysis of transfer function residues in Table 3 indicates that the SVC at the Guemez substation is expected to provide damping to both, the 0.4 Hz North–south inter-area mode 1 and the 0.6 Hz mode. The SVCs at the Temascal and Topilejo substations, on the other hand, are expected to provide stabilization to modes 3 and 9, respectively. Other SVCs were found to have a negligible effect on the damping of critical modes.

From Table 3, a three SVCs alternative was chosen to aid damping of critical inter-area modes, namely:

- The SVC at Guemez substation tuned to damp the 0.40 Hz North–south inter-area mode 1 and the 0.59 Hz inter-area mode 2.
- The SVC at Temascal tuned to damp the 0.72 Hz East–west inter-area mode 3.
- The SVC at Topilejo tuned to damp mode 9.

Table 4 summarises the main characteristics of these sets together with the control objective and the input signals chosen for the PSDC.

To assess the extent of interaction of loop interaction between SVCs, the GDRG approach set out in Section 3 was used. Fig. 6 depicts the magnitude of the GDRG number for each set as a function of frequency. Examination of these results shows that sets 3 and 4 exhibit the smallest degree of
interaction for the frequency range of interest, particularly for the East–west inter-area mode 3. Notice, however that, whilst the adopted procedure ensures the minimum interaction between SVC controllers, it does not directly reveal whether or not the interaction is favourable.

Based on this analysis, set 4 was chosen as the most desirable alternative. The time constants and gains of the PSDC were tuned one by one for each specific system mode following the procedure described in Section 4. To simplify the analysis, a washout time constant of $T_w = 10.0$ s was selected. Further, a minimum damping ratio $\zeta = 4\%$ was chosen as the design objective. Table 5 summarises the parameters of the synthesised controllers for the case with SVCs at Guemez, Temascal and Topilejo. The closed-loop eigenvalues are given in Table 6. It is seen that the improvement in the modal damping ratio of inter-area modes 1 through 3 is noticeable. As expected from residue analysis, however, the SVC at Topilejo is not able to provide the desired damping enhancement for mode 9, suggesting the application of other control alternatives or the use of remote input signals [20].

5.4. Application of PSSs

This alternative considers the application of PSSs at critical system generators. The system inputs and

<table>
<thead>
<tr>
<th>Mode</th>
<th>SVC location</th>
<th>Input signal to transfer function</th>
<th>Residue magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Guemez</td>
<td>Guemez-HUI ($P_{line}$)</td>
<td>5.257</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Guemez-HUI ($I_{line}$)</td>
<td>4.990</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Guemez-ALT ($P_{line}$)</td>
<td>4.858</td>
</tr>
<tr>
<td>2</td>
<td>Guemez</td>
<td>Guemez-HUI ($P_{line}$)</td>
<td>2.019</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Guemez-HUI ($I_{line}$)</td>
<td>1.931</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Guemez-ALT ($P_{line}$)</td>
<td>1.862</td>
</tr>
<tr>
<td>3</td>
<td>Temascal</td>
<td>JUI-Temascal ($I_{line}$)</td>
<td>5.674</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Temascal-TCL ($I_{line}$)</td>
<td>4.506</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Temascal-MID ($I_{line}$)</td>
<td>4.007</td>
</tr>
<tr>
<td>9</td>
<td>Topilejo</td>
<td>Topilejo-SNB ($I_{line}$)</td>
<td>0.271</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Topilejo–Texcoco ($I_{line}$)</td>
<td>0.134</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Topilejo-TCL ($P_{line}$)</td>
<td>0.132</td>
</tr>
</tbody>
</table>
outputs used are the exciter voltage references and local rotor speeds, respectively. Residues for the transfer function \( D_v = D_{V_{ref}} \) for the critical modes are given in Table 7.

Following the same approach as that used for the case with SVCs, two sets of input–output pairings were chosen.

<table>
<thead>
<tr>
<th>Set</th>
<th>Inter-area mode 1</th>
<th>Inter-area mode 3</th>
<th>Inter-area mode 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \Delta V_{ref} ) SVC at Guemez/( \Delta P_{line} ) Guemez-HUI</td>
<td>( \Delta V_{ref} ) SVC at Temascal/( \Delta P_{line} ) Temascal-MID</td>
<td>( \Delta V_{ref} ) SVC at Topileo/( \Delta P_{line} ) TCL-Topileo</td>
</tr>
<tr>
<td>2</td>
<td>( \Delta V_{ref} ) SVC at Guemez/( \Delta P_{line} ) Guemez-HUI</td>
<td>( \Delta V_{ref} ) SVC at Temascal/( \Delta P_{line} ) Temascal-PBD</td>
<td>( \Delta V_{ref} ) SVC at Topileo/( \Delta P_{line} ) Topileo–Texcoco</td>
</tr>
<tr>
<td>3</td>
<td>( \Delta V_{ref} ) SVC at Guemez/( \Delta P_{line} ) Guemez-ALT</td>
<td>( \Delta V_{ref} ) SVC at Temascal/( \Delta P_{line} ) Temascal-PBD</td>
<td>( \Delta V_{ref} ) SVC at Topileo/( \Delta P_{line} ) Topileo–Texcoco</td>
</tr>
<tr>
<td>4</td>
<td>( \Delta V_{ref} ) SVC at Guemez/( \Delta P_{line} ) LAJ-Guemez</td>
<td>( \Delta V_{ref} ) SVC at Temascal/( \Delta P_{line} ) Temascal-PBD</td>
<td>( \Delta V_{ref} ) SVC at Topileo/( \Delta P_{line} ) Topileo–Texcoco</td>
</tr>
</tbody>
</table>

![Graph](image-url)  
**Fig. 6.** Magnitude of GDRG as a function of frequency. SVCs at critical substations.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Generator</th>
<th>Location</th>
<th>Residue magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MTY</td>
<td>NE</td>
<td>0.7855</td>
</tr>
<tr>
<td></td>
<td>MPD</td>
<td>SE</td>
<td>0.0866</td>
</tr>
<tr>
<td></td>
<td>MZD</td>
<td>N</td>
<td>0.0740</td>
</tr>
<tr>
<td>2</td>
<td>SLM</td>
<td>W</td>
<td>0.1330</td>
</tr>
<tr>
<td></td>
<td>SLP</td>
<td>W</td>
<td>0.0923</td>
</tr>
<tr>
<td></td>
<td>MPD</td>
<td>SE</td>
<td>0.0829</td>
</tr>
<tr>
<td></td>
<td>SLM</td>
<td>W</td>
<td>0.1887</td>
</tr>
<tr>
<td>3</td>
<td>SLM</td>
<td>W</td>
<td>0.0283</td>
</tr>
<tr>
<td></td>
<td>SLP</td>
<td>W</td>
<td>0.0158</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Set</th>
<th>Inter-area mode 1</th>
<th>Inter-area mode 3</th>
<th>Inter-area mode 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \Delta \omega /\Delta V_{ref} ) MTY</td>
<td>( \Delta \omega /\Delta V_{ref} ) SLP</td>
<td>( \Delta \omega /\Delta V_{ref} ) SLM</td>
</tr>
<tr>
<td>2</td>
<td>( \Delta \omega /\Delta V_{ref} ) MTY</td>
<td>( \Delta \omega /\Delta V_{ref} ) MPD</td>
<td>( \Delta \omega /\Delta V_{ref} ) SLM</td>
</tr>
</tbody>
</table>
Table 8 lists the selected sets, while Fig. 7 provides the GDRG number for the frequency range of interest. From these results, set 1 was chosen as a desired control alternative. Table 9 shows the closed loop eigenvalues while the PSDC designed for the selected generators are given in Table 10.

The analysis shows that with the speed-based stabilisers applied on critical machines, the damping ratio of inter-area modes 1, 2 and 9 is above the minimum damping chosen. However, the damping of inter-area mode 3, however, does not meet the specified design criteria. While several other control alternatives might be investigated, the analysis clearly suggests to use SVCs to aid damping of inter-area modes 1 and 3 and a PSS at machine SLM to enhance the damping of local mode 9.

5.5. Large performance studies

Selected transient stability studies were performed using a large-scale digital stability program to examine the effectiveness of SVC modulation for damping critical inter-area modes. The faults studied are:

Case A. Tie-line tripping without fault of the SLM-QRO 400 kV line in the Western system. This fault is known to excite the 0.7 Hz East–west inter-area mode 3.

Case B. Tie-line tripping without fault of one circuit of the north–south interface (ALT-PRD 400 kV line). This contingency is found to excite both, the 0.4 Hz North–south mode 1 and the inter-area mode 2.

Simulation results with and without SVC voltage support for the cases A and B and the compensation alternatives in Section 5.3 are shown in Figs. 8 and 9. Without voltage support, the system exhibits increased power and voltage oscillations along major interconnectors and machines. For case A, the critical contingency results in growing power and voltage oscillations having a frequency near 0.7 Hz on the 400 kV of the south-eastern system. Further, loss of one circuit of the ALT-PRD 400 kV line on the north–south interface leads to growing power oscillations characterised by a frequency of 0.4 Hz. The co-ordinated use of multiple SVC voltage support with supplementary modulation allows achieving satisfactory transient performance.

Table 9
Closed-loop system eigenvalues for the case with PSSs at MTY, SLM and MPD

<table>
<thead>
<tr>
<th>Inter-area mode</th>
<th>Eigenvalue</th>
<th>Damping ratio (%)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-0.1678 \pm j2.62$</td>
<td>6.30</td>
<td>0.418</td>
</tr>
<tr>
<td>2</td>
<td>$-0.3175 \pm j3.63$</td>
<td>8.71</td>
<td>0.570</td>
</tr>
<tr>
<td>3</td>
<td>$-0.1603 \pm j4.54$</td>
<td>3.52</td>
<td>0.724</td>
</tr>
<tr>
<td>9</td>
<td>$-0.3155 \pm j6.95$</td>
<td>4.53</td>
<td>1.107</td>
</tr>
</tbody>
</table>

Fig. 7. Magnitude of GDRG as a function of frequency. PSSs on dominant machines: (a) Active power flow and (b) rotor angle swings.

Table 10
Summary of PSS constants

<table>
<thead>
<tr>
<th>PSS location</th>
<th>$K_{psdc}$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$m_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTY</td>
<td>0.800</td>
<td>0.5995</td>
<td>0.0500</td>
<td>3</td>
</tr>
<tr>
<td>SLM</td>
<td>1.500</td>
<td>0.6397</td>
<td>0.0682</td>
<td>3</td>
</tr>
<tr>
<td>MPD</td>
<td>1.500</td>
<td>0.4268</td>
<td>0.0431</td>
<td>3</td>
</tr>
</tbody>
</table>
Fig. 8. Comparison of system performance with and without SVC voltage support for case A. (a) Active power flow and (b) Rotor angle swings.

Fig. 9. Comparison of system performance with and without voltage support for case B.
6. Conclusions

In this paper, a systematic frequency-domain design analytical methodology based on decentralised control theory and sensitivity analysis is proposed to co-ordinate multiple FACTS controllers as well as to minimise the potential for adverse interaction between control loops in MIMO systems. The proposed approach is of particular interest for studying the potential for dynamic interaction for power systems under decentralised control and can be used to assess overall system integrity. Further the developed algorithms can be easily implemented into existing computer software for power system analysis and design.

Study experience with a complex system shows that existing SVCs in the MIS can be effectively used to aid stabilization, especially under the most stringent operating conditions. The design strategy was shown to lead to satisfactory robust performance. Time-domain studies correlate well with small signal analyses showing the appropriateness of the developed models. Other aspects such as the assessment of the nature of modal interaction are to be further investigated.

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References


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