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Normal form analysis of stressed power systems: incorporation of SVC models

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Abstract

In this paper a comprehensive analytical technique based on normal form theory and symbolic computer algebra is proposed for the analysis of non-linear inter-area oscillations in stressed power systems that incorporate the operation of Static VAR Compensators (SVCs). A second-order representation of the power system is derived that considers the explicit representation of synchronous machines and multiple SVCs in the state representation. On the basis of this model, normal form theory is used to study non-linear power system behaviour following large disturbances. The proposed approach is general and may be extended to include other FACTS devices. A simplified 4-machine, 11-bus test system is used to study the effect of SVC voltage support on system damping. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: System oscillations; Normal form theory; Static VAR compensation

1. Introduction

Normal form theory has received considerable recent interest as a means of dynamic analysis of power system non-linear behaviour. With this technique, it is possible to obtain the simplest form of a set of non-linear differential equations and hence, to identify and study the nature of system oscillations using conventional linear analysis methodologies [1,2].

In the past few years, the normal form method (NFM) has been applied to determine the dominant modes of oscillation following large perturbations as well as to identify structural properties responsible for several aspects of non-linear modal interactions. In Refs. [3,4] the NFM was used to assess the onset of the inter-area phenomenon using a simplified representation of the power system. This basic procedure was later extended to include the transient representation of synchronous machines and their controllers [5,6]. More recently, Ni et al. [7] used normal form theory to investigate the presence of non-linear modal behaviour in a simplified ac/dc system. Further investigations suggest that non-linear modal interaction may significantly influence control system performance, namely the excitation system of generators [5,8]. The study of non-linear interactions arising from the application of control devices on the transmission network, however, has not been properly investigated.

In this paper, a comprehensive analytical technique based on normal form theory and symbolic computer algebra is proposed for the study and control of non-linear inter-area oscillations in power systems incorporating FACTS controllers. Emphasis is being placed on the analysis and simulation of static VAR compensation at critical system buses. A second-order model of the power system is developed which considers the representation of synchronous machines and static VAR compensation. On the basis of this model, normal form theory is used to study the non-linear power system behaviour following large disturbances. The proposed method is general and may be extended to include other FACTS devices. A simplified 4-machine, 11-bus test system is used to illustrate the proposed methodology. Detailed non-linear time-domain simulations are finally conducted to check the validity of the analysis as well as to assess the effect of SVC voltage support on system damping. For completeness, an overview of the NFM is also provided, together with a brief description of the adopted approach.
2. The adopted power system model

The power system is seen as constituted of the dynamic models of synchronous machines and static VAR compensators interacting through the quasi-stationary network representation. In this model, each synchronous machine is represented by an internal voltage source $E_0 = E_0^d + jE_0^q$ behind a transient reactance $x_0 = x_0^d = x_0^q$. Static VAR compensators are modelled as variable shunt impedances. Fig. 1 shows a schematic representation of the adopted system model.

2.1. Generator dynamic equations

Each synchronous generator is represented using the two-axis model and a static excitation system. A block diagram of the excitation system model is shown in Fig. 2. The differential equations describing the dynamic behaviour of the $k$th generator and the excitation system are given by Eqs. (1)–(4).

Rotor swing

$$\dot{\delta}_k = \omega_k - \omega_{\text{ref}} = f_{\delta_k},$$

(1)

$$\dot{\omega}_k = \frac{1}{M_k} \left( P_m - (E_{dq}^d I_d + E_{dq}^q I_q) - D_k \left( \frac{\omega_k}{\omega_0} \right) \right) = f_{\omega_k}$$

Internal voltage equations

$$E_{d_k} = \frac{1}{T_{d_k}} (E_{d_k}^d I_d - (x_d - x_d^q) I_d - E_{q_k}^d) = f_{E_{d_k}}$$

(2)

$$E_{q_k} = \frac{1}{T_{d_k}} (-E_{q_k}^d + (x_d - x_d^q) I_d) = f_{E_{q_k}}$$

Static excitation system

$$E_{f_k} = \frac{1}{T_{\text{exc}}} \left( -E_{f_k}^d + K_{\text{exc}} (V_{\text!rms}} + V_{\text{ref}} - V_{\text{exc}} \right) = f_{E_{f_k}}$$

(3)

2.2. Representation of static VAR compensators

Fig. 3 shows a block representation of the transfer function of the controller used for the static VAR compensator. For SVC $i$, the dynamic equation describing the dynamic behaviour of the output susceptance $b_{\text{svc}}$ is

$$\dot{b}_{\text{svc}} = \frac{1}{T_{\text{svc}}} \left( -b_{\text{svc}} + K_{\text{svc}} (V_{\text{svc}} + V_{\text{ref}} - V_{\text{exc}}) \right) = f_{b_{\text{svc}}}$$

(5)

where $V_{\text{ref}}$ is the SVC reference voltage and $V_{\text{exc}}$ is the supplementary control signal derived from the power system damping controller (PSDC); the magnitude of the terminal voltage at bus $p$, $V_t$, is defined as:

$$V_t = \sqrt{V^2_d + V^2_q}$$

(6)

2.3. Network representation

The network is represented by a quasi-stationary model: loads are treated as constant impedances and the transient
generator impedances \( r_e + jx_e \) are included in the augmented network admittance matrix. To illustrate the nature of the proposed model, consider a general ng-generator system with nc SVCs. Assume, for the sake of simplicity, that the internal generator nodes are ordered in the form \([1, \ldots, ng]\), followed by load nodes where SVCs are applied \([ng + 1, \ldots, ng + nc]\) and finally, the load buses without voltage support \([ng + nc + 1, \ldots, N]\). Eliminating the terminal generator buses and load buses without SVCs, the nodal balance equation is expressed in network \(D–Q\) coordinates as

\[
\begin{bmatrix}
I_g \\
\vdots \\
0
\end{bmatrix} = \begin{bmatrix}
\dot{\tilde{Y}}_{gg} & \cdots & \dot{\tilde{Y}}_{gg} \\
\vdots & \ddots & \vdots \\
0 & \cdots & \dot{\tilde{Y}}_{gg}
\end{bmatrix}
\begin{bmatrix}
V_g \\
\vdots \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix}
\]

(7)

where \(\dot{\tilde{Y}}_{gg}, \dot{\tilde{Y}}_{gl}, \dot{\tilde{Y}}_{lg}\text{ and }\dot{\tilde{Y}}_{ll}\) are submatrices of the reduced admittance matrix, and

\[
I_g = \begin{bmatrix}
I_{D_1} + jI_{Q_1} & I_{D_2} + jI_{Q_2} & \cdots & I_{D_{ng}} + jI_{Q_{ng}}
\end{bmatrix}
\]

\[
V_g = \begin{bmatrix}
E'_{D_1} + jE'_{Q_1} & E'_{D_2} + jE'_{Q_2} & \cdots & E'_{D_{ng}} + jE'_{Q_{ng}}
\end{bmatrix}
\]

\[
V_l = \begin{bmatrix}
V_{D_{ng+1}} + jV_{Q_{ng+1}} & V_{D_{ng+2}} + jV_{Q_{ng+2}} & \cdots & V_{D_{ng+nc}} + jV_{Q_{ng+nc}}
\end{bmatrix}
\]

\[
\beta_{svc} = \text{diag}[\beta_{svc_1}, \beta_{svc_2}, \ldots, \beta_{svc_{nc}}]
\]

2.4. Coordinate transformation

In order to express network currents in machine \((d–q)\) coordinates, let the complex transformation be introduced

\[
I_g = I_{dq} = TI_{dq}
\]

\[
V_g = V_{dq} = TV_{dq}
\]

(8a)

(8b)

where

\[
T = \text{diag}[e^{j\delta_1}, e^{j\delta_2}, \ldots, e^{j\delta_{ng}}]
\]

and

\[
E_{dq} = \begin{bmatrix}
E'_{d_1} + jE'_{q_1} & \cdots & E'_{d_{ng}} + jE'_{q_{ng}}
\end{bmatrix}^T
\]

Substituting Eqs. (8a) and (8b) into Eq. (7) leads to

\[
I_{dq} = T^{-1}\dot{\tilde{Y}}_{gg}TE_{dq} + T^{-1}\dot{\tilde{Y}}_{gl}V_l
\]

(9)

and

\[
0 = \dot{\tilde{Y}}_{lg}TE_{dq} + \dot{\tilde{Y}}_lV_l + j\beta_{svc}V_l
\]

(10)

Solving for the load terminal voltages \(V_l\) results in

\[
V_l = -(\dot{\tilde{Y}}_{ll} + j\beta_{svc})^{-1}\dot{\tilde{Y}}_{lg}TE_{dq}
\]

(11)

and

\[
I_{dq} = T^{-1}\dot{\tilde{Y}}_{gg}TE_{dq} - T^{-1}\dot{\tilde{Y}}_{gl}(\dot{\tilde{Y}}_{ll} + j\beta_{svc})^{-1}\dot{\tilde{Y}}_{lg}TE_{dq}
\]

(12)

Eqs. (11) and (12) allow us to obtain the non-linear power system model as described below. Substitution of Eqs. (11) and (12) into Eqs. (1)–(5) provides the analytical expression needed to represent multiple SVCs in the system representation. Observe that the above model is general and may be used to represent other controllers. It should be emphasised that Eq. (12) involves the inverse of an analytical expression. This computation is done using the symbolic calculation capability within MAPLE [10].

To gain further insight into the nature of the system model, assume, for the sake of simplicity, that a single SVC is applied at load bus \(p\). From Eq. (12) the current of the \(k\)th generator can then be expressed on \(d–q\) axes as

\[
I_{d_k} + jI_{q_k} = \sum_{m=1}^{ng} \tilde{Y}_{km} e^{-j\delta_k - \delta_m}(E'_{d_m} + jE'_{q_m})
\]

\[
- (\dot{\tilde{Y}}_{lp} + j\beta_{svc}) \sum_{m=1}^{ng} \tilde{Y}_{pm} e^{-j\delta_k - \delta_m}(E'_{d_m} + jE'_{q_m})
\]

(13)

Hence, solving for the real and imaginary components yields

\[
I_{d_k} = \sum_{m=1}^{ng} F_{G+p}(\delta_{km})E'_{d_m} - F_{B-C}(\delta_{km})E'_{q_m}
\]

\[
- K_1 \sum_{m=1}^{ng} \tilde{F}_{G+p}(\delta_{km})E'_{d_m} - \tilde{F}_{B-C}(\delta_{km})E'_{q_m}
\]

\[
+ K_2 \sum_{m=1}^{ng} \tilde{F}_{G+p}(\delta_{km})E'_{q_m} + \tilde{F}_{B-C}(\delta_{km})E'_{d_m}
\]

(14)

and

\[
I_{q_k} = \sum_{m=1}^{ng} F_{G+p}(\delta_{km})E'_{q_m} + F_{B-C}(\delta_{km})E'_{d_m}
\]

\[
- K_1 \sum_{m=1}^{ng} \tilde{F}_{G+p}(\delta_{km})E'_{q_m} + \tilde{F}_{B-C}(\delta_{km})E'_{d_m}
\]

\[
- K_2 \sum_{m=1}^{ng} \tilde{F}_{G+p}(\delta_{km})E'_{d_m} - \tilde{F}_{B-C}(\delta_{km})E'_{q_m}
\]

(15)

for \(k = 1, \ldots, ng\), where the \(G_{km} + jB_{km}\) is the transfer admittance between buses \(k\) and \(m\), and:

\[
K_1 = \begin{bmatrix}
\frac{G_{pp}G_{kp}}{G_{pp}^2 + (B_{pp} + \beta_{svc})^2} & \frac{(B_{pp} + \beta_{svc})B_{kp}}{G_{pp}^2 + (B_{pp} + \beta_{svc})^2}
\end{bmatrix}
\]

(16)

\[
K_2 = \begin{bmatrix}
\frac{G_{pp}B_{kp}}{G_{pp}^2 + (B_{pp} + \beta_{svc})^2} & -\frac{(B_{pp} + \beta_{svc})G_{kp}}{G_{pp}^2 + (B_{pp} + \beta_{svc})^2}
\end{bmatrix}
\]

\[
F_{G+p}(\delta_{km}) = G_{km} \cos(\delta_k - \delta_m) + B_{km} \sin(\delta_k - \delta_m)
\]

\[
F_{B-C}(\delta_{km}) = B_{km} \cos(\delta_k - \delta_m) - G_{km} \sin(\delta_k - \delta_m)
\]

\[
\tilde{F}_{G+p}(\delta_{km}) = G_{pm} \cos(\delta_k - \delta_m) + B_{pm} \sin(\delta_k - \delta_m)
\]

\[
\tilde{F}_{B-C}(\delta_{km}) = B_{pm} \cos(\delta_k - \delta_m) - G_{pm} \sin(\delta_k - \delta_m)
\]
Further, the $D-Q$ axes voltages at the SVC location are obtained from Eq. (11) as
\[ V_{D_p} = -K_3 \sum_{m=1}^{ng} \tilde{F}_{G-B} G_{pm} E_{dp}' + \tilde{F}_{B-G} \delta_m E_{dp}' \]
\[ - K_4 \sum_{m=1}^{ng} \tilde{F}_{G-B} G_{pm} E_{dp}' + \tilde{F}_{B-G} \delta_m E_{dp}' \tag{17} \]
\[ V_{Q_p} = -K_3 \sum_{m=1}^{ng} \tilde{F}_{G-B} G_{pm} E_{dq}' + \tilde{F}_{B-G} \delta_m E_{dq}' \]
\[ - K_4 \sum_{m=1}^{ng} \tilde{F}_{G-B} G_{pm} E_{dq}' + \tilde{F}_{B-G} \delta_m E_{dq}' \tag{18} \]
where
\[ \tilde{F}_{G-B} G_{pm} \cos(\delta_m) - B_{pm} \sin(\delta_m) \tag{19a} \]
\[ \tilde{F}_{B-G} \delta_m G_{pm} \sin(\delta_m) + B_{pm} \cos(\delta_m) \tag{19b} \]
\[ K_3 = \left( \frac{G_{pp}}{G_{pp}^2 + (B_{pp} + \beta_{svc})^2} + \frac{(B_{pp} + \beta_{svc})}{G_{pp}^2 + (B_{pp} + \beta_{svc})^2} \right) \tag{20a} \]
\[ K_4 = \left( \frac{G_{pp}}{G_{pp}^2 + (B_{pp} + \beta_{svc})^2} - \frac{(B_{pp} + \beta_{svc})}{G_{pp}^2 + (B_{pp} + \beta_{svc})^2} \right) \tag{20b} \]
In the following sections, perturbation theory is used to obtain a second order representation of the power system that considers the explicit representation of synchronous machines and SVCs.

3. Non-linear power system representation

Let $x = [\delta, \omega, E_{d}', E_{q}', E_{d}, E_{q}, \beta_{svc}] \in \mathbb{R}^n$ be the overall state vector. Substituting Eqs. (14), (15), (17) and (18) into Eqs. (1)–(6) yields the non-linear power system representation
\[ x = f(x) = [f_1, f_2, f_3, f_4, f_5, f_6]' \tag{21} \]
where $f(x)$ represents a smooth $n$-dimensional vector field in the theory of dynamical systems [2].

3.1. Second order power system representation

Assuming that $f$ is continuous and can be expanded, the Taylor or power series expansion up to order 2 of Eq. (21) about a stable equilibrium point results in [3]
\[ \dot{x} = Ax + \frac{1}{2} x^T H x \]
\[ \dot{x} = Ax + \frac{1}{2} H x \]
where matrix $A$ represents the linear part of the non-linear vector field at the origin, and $F_2(x)$ is a vector of power series in the Taylor expansion in $x$ of degree 2; the $H^k$ ($k = 1, \ldots, n$) are the Hessian matrices of second order derivatives. In the adopted model, the linear state matrix is of the form
\[ A = \begin{bmatrix} 0 & \frac{\partial f_1}{\partial \delta} & 0 & 0 & 0 & 0 \\
\frac{\partial f_2}{\partial \delta} & 0 & \frac{\partial f_2}{\partial \omega} & \frac{\partial f_2}{\partial E_d} & 0 & \frac{\partial f_2}{\partial E_q} \\
\frac{\partial f_3}{\partial \delta} & 0 & \frac{\partial f_3}{\partial \omega} & \frac{\partial f_3}{\partial E_d} & 0 & \frac{\partial f_3}{\partial E_q} \\
\frac{\partial f_4}{\partial \delta} & 0 & \frac{\partial f_4}{\partial \omega} & \frac{\partial f_4}{\partial E_d} & 0 & \frac{\partial f_4}{\partial E_q} \\
\frac{\partial f_5}{\partial \delta} & 0 & \frac{\partial f_5}{\partial \omega} & \frac{\partial f_5}{\partial E_d} & 0 & \frac{\partial f_5}{\partial E_q} \\
\frac{\partial f_6}{\partial \delta} & 0 & \frac{\partial f_6}{\partial \omega} & \frac{\partial f_6}{\partial E_d} & 0 & \frac{\partial f_6}{\partial E_q} \end{bmatrix} \]
where the first-order derivatives are given by
\[ \frac{\partial f_1}{\partial \omega_k} = 1.0 \]
\[ \frac{\partial f_2}{\partial \omega_k} = -M_k \left[ \frac{\partial I_{d_k}}{\partial E_{dq}} E_{dq} + \frac{\partial I_{q_k}}{\partial E_{dq}} E_{dq} + \frac{\partial E_{dq}}{\partial I_{q_k}} I_{q_k} \right] \]
\[ \frac{\partial f_3}{\partial \omega_k} = -M_k \left[ \frac{\partial I_{d_k}}{\partial E_{dq}} E_{dq} + \frac{\partial I_{q_k}}{\partial E_{dq}} E_{dq} + \frac{\partial E_{dq}}{\partial I_{q_k}} I_{q_k} \right] \]
\[ \frac{\partial f_4}{\partial \omega_k} = -M_k \left[ \frac{\partial I_{d_k}}{\partial E_{dq}} E_{dq} + \frac{\partial I_{q_k}}{\partial E_{dq}} E_{dq} + \frac{\partial E_{dq}}{\partial I_{q_k}} I_{q_k} \right] \]
\[ \frac{\partial f_5}{\partial \omega_k} = \frac{1}{T_D} \left[ \frac{\partial I_{d_k}}{\partial \delta_m} (x_{d_k} - x_k) \right] \]
\[ \frac{\partial f_6}{\partial \omega_k} = \frac{1}{T_D} \left[ \frac{\partial I_{d_k}}{\partial \delta_m} (x_{d_k} - x_k) \right] \]
\[
\frac{\partial f_k}{\partial \delta_m} = \frac{1}{T_{q_k}} \left[ \frac{\partial I_{q_k}}{\partial \delta_m} (x_{q_k} - x_k') \right];
\]

\[
\frac{\partial f_k}{\partial E_{q_k}} = \frac{1}{T_{q_k}} \left[ \frac{\partial I_{q_k}}{\partial E_{q_k}} (x_{q_k} - x_k') \right];
\]

\[
\frac{\partial f_k}{\partial E_{d_k}} = \frac{1}{T_{d_k}} \left[ \frac{\partial I_{d_k}}{\partial E_{d_k}} (x_{d_k} - x_k') \right] - \frac{\partial E_{d_k}}{\partial E_{d_k}}.;
\]

\[
\frac{\partial f_k}{\partial \beta_{nvc}} = \frac{1}{T_{q_k}} \left[ \frac{\partial I_{q_k}}{\partial \beta_{nvc}} (x_{q_k} - x_k') \right]
\]

where matrix \( A \) contains the real part of the original vector field and each \( F_k \) is a real vector-valued homogeneous polynomial of degree \( k \) in \( x \) that represents non-linear effects of order 2 and higher.

The physical notion behind the NFM is to use a sequence of non-linear co-ordinate transformations defined in the neighbourhood of the origin to eliminate or simplify system non-linearities in Eq. (23). This process usually includes three main steps: (i) the system is expanded in power series about an initial condition, (ii) the resulting system is represented in the Jordan canonical form by a linear transformation of the original variables, and (iii) the system is represented in the normal form by a sequence of non-linear coordinate transformations. The details of these computations are described below.

4.2. Reduction of the system to a normal form

To transfer Eq. (23) into the normal form, assume that it is desired to study the behaviour of the system in the neighbourhood of a SEP. Let the state matrix \( A \) has a set of distinct eigenvalues \((\lambda_1, \lambda_2, \ldots, \lambda_n)\) with a corresponding set of right eigenvectors \( U = \text{col}(u_1, u_2, \ldots, u_n) \). Then, the linear change of co-ordinates \( x = Uy \) transforms the second-order system model into its Jordan form

\[
y = Jy + U^{-1}F_2(Uy) = Jy + \hat{F}_2(y) + \sum_{k=2}^{\infty} \hat{F}_k(y)
\]

where \( y \in \mathbb{R}^p \) is the vector of Jordan-form co-ordinates, and \( J = U^{-1}AU = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) \); the polynomial vector \( \hat{F}_2(y) \) contains second-order effects in the state representation. For the \( j \)th state, Eq. (24) can be further expressed in the uncoupled form

\[
y_j = \lambda_j y_j + \sum_{m=1}^{n} \sum_{p=1}^{n} C_{mnp}y_my_p + \cdots \quad (j = 1, \ldots, n)
\]

where the \( C_{mnp} \) are the second-order coefficients of the Jordan form variables. To remove second-order terms in Eq. (25), let the following near-identity non-linear co-ordinate transformation be introduced next

\[
y = z + h_2(z)
\]
Eqs. (26) into Eq. (25) results in the normal form system

\[ z = (I + Dh_2(z))^{-1}[J(z + h_2(z)) - Dh_2(z)z] + \tilde{F}_2(z) + O(3) \]  

(27)

where \( Dh_2(z) \) is the Jacobian of \( h_2(z) \) and \( O(3) \) denotes an expression containing non-linear terms in \( z \) of order 3 and higher which have been modified by the transformation. A conventional approach for solving Eq. (27) is to use the truncated Taylor expansion \([1, 2]\)

\[ (I + Dh_2(z))^{-1} = I - Dh_2(z) + (Dh_2(z))^2 + \cdots \]  

(28)

For sufficiently small \( z \), the inverse of \( (I + Dh_2(z))^{-1} \) exists and can be approximated by the first terms of the Taylor expansion as \( (I + Dh_2(z))^{-1} = I - Dh_2(z) + O(2) \). Hence, the second-order normal form can be written as

\[ z = Jz + Dh_2(z) - Dh_2(z)z + \tilde{F}_2(z) + O(3) \]

\[ = Jz + F^*_2(z) + O(3) \]

(29)

where the \( F^*_2 \) are resonance terms that cannot be eliminated by the non-linear transformation and the vector \( h_2(z) \) is then chosen, ideally, to eliminate or simplify the terms \( F^*_2(z) \). It can be proved that if no resonance conditions are found, the system can be expressed in the uncoupled form

\[ z = \Lambda z \]  

(30)

where

\[ \Lambda = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_n]. \]

4.3. Approximate time-domain solutions

Following Thapar et al. \([3] \), approximate time-domain solutions in the original co-ordinate system can be obtained by using the normal form solution in Eq. (30). Let the state
solution for the $k$th state be written in the form

$$z_k(t) = z_k^0 e^{\lambda_k t}, \quad k = 1, \ldots, n$$  \hspace{1cm} (31)

where $z_k^0$ is the initial condition of the normal-form variable $z_k$. Closed loop second-order time-domain solutions can then be obtained by introducing the second-order inverse transformation $y = z + h_2(z)$ and $x = U y$. Once the eigenvalues and associated right and left eigenvectors have been found, the approximate closed-form solution for the $k$th state can be expressed as

$$x_k(t) = \sum_{j=1}^{n} u_{kj} z_j^0 e^{\lambda_j t} + \sum_{j=1}^{n} u_{kj} \left( \sum_{l=1}^{n} h_{kl} z_l^0 e^{\lambda_l t} \right) e^{(\lambda_k + \lambda_l) t},$$

where the initial conditions in $z$ co-ordinates are obtained solving the non-linear optimisation problem

$$\min f(z^0) = z^0 + h_2(z^0) - y(z^0)$$  \hspace{1cm} (33)


4.4. Procedure for computation of normal form coefficients

The proposed methods were implemented in a computer package for investigation of power system non-linear behaviour under stressed operating conditions using MATLAB. Computation of normal form coefficients and identification of non-linear modal interaction is carried out as follows:

(a) For a given disturbance, determine the post-disturbance stable equilibrium point $x_{SEP}$. Move the origin to the post-disturbance SEP.

(b) Obtain the second-order power system representation of the system about $x_{SEP}$. Determine the initial conditions in the Jordan and normal form variables from the relations $y^0 = U^{-1} x^0$. Compute the initial conditions of normal form variables from Eq. (33).

(c) Compute normal form coefficients following the procedure in Section 4. Obtain explicit time-domain closed-form solutions using Eq. (32).

5. Application

A single-line diagram of the test system is shown in Fig. 5. The test system comprises four generators representing equivalent electric areas, 11 buses and three loads. The base case condition is essentially that provided in Ref. [12] modified to stress the system and to include the representation of an SVC. The generator and SVC parameters are given in Appendix A. Loads are modelled as constant admittances.

5.1. Damping characteristics of the study system

Table 1 summarises the main characteristics of the system modes showing their swing frequency and source. The analysis focuses on the study of the potential for non-linear interaction between electromechanical modes. No specific studies to assess the influence of the control modes of excitors on system damping were undertaken.

For the purposes of this study, an SVC was applied at bus 9. No supplemental modulation controls and limits were used in the normal form analysis. Fig. 6 depicts the root-loci of electromechanical modes showing the effect of SVC voltage support on system damping. It is seen that SVC voltage support improves the damping of the local plant mode 9 and has a small positive contribution to the damping of inter-area modes 11 and 13. The analysis of the root-loci in Fig. 6b on the other hand, shows no adverse interaction between the SVC control and control modes associated with the excitation systems in generators.

To examine the effect of dynamic voltage support on system damping, a three-phase fault was applied at bus 7 cleared by opening the tie-line to bus 5. This fault is known to excite the inter-area modes leading to poorly damped inter-area oscillations. Examination of dynamic system performance in Fig. 7 confirms that the addition of an SVC provides positive damping to system oscillations.

5.2. Effect of stress on system damping

In order to assess the effect of increased system stress on system behaviour, several disturbances and loading scenarios were developed. Cases of special interest selected for analysis included the study of the combined effect of increased generation at machine 2 and a three-phase short circuit at bus 7 cleared by opening the tie-line to bus 5, with different clearing times. Table 2 summarises the operating conditions adopted in the study.

Normal form coefficients were computed following the procedure described in Section 4. In the normal form solution in Eq. (32), non-linear effects appear both, in the
Table 1
Eigenvalues of the test system for the base case condition without SVC voltage support

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Frequency (Hz)</th>
<th>Damping</th>
<th>Mode type</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>-24.50 ± j22.53</td>
<td>3.5865</td>
<td>-0.736</td>
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<tr>
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<td>-0.1063</td>
<td>Inter-area</td>
<td>Gen #2 vs. Gen #4</td>
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<td>Gen #1 vs. Gen #2,3,4</td>
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</table>

Fig. 6. Root-loci of system eigenvalues. (a) Electromechanical modes and (b) control modes.

Fig. 7. Comparison of relative rotor angle swings and speed deviation for generator 2 with and without SVC voltage support. (a) Rotor angle swing and (b) relative speed deviation.
Table 2
Selected operating conditions

<table>
<thead>
<tr>
<th>Case description</th>
<th>Clearing time $T_{cl} \text{ (s)}$</th>
<th>Generation loading $P_{ng}$, $P_{pm}$</th>
<th>$P_{ng}$, $P_{pm}$</th>
</tr>
</thead>
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<td>$P_{ng}$, $P_{pm}$</td>
<td>1.60, 1.60</td>
</tr>
<tr>
<td>Stressed condition</td>
<td>0.15</td>
<td>$P_{ng}$, $P_{pm}$</td>
<td>1.60, 2.32</td>
</tr>
</tbody>
</table>

Table 3
Second-order non-linear interaction coefficients at different operating conditions

| Electromechanical mode $\lambda_l$ | $|h_{2pl}|$ | $|h_{2mp}\phi_{pl}|$ |
|----------------------------------|------------|-----------------|
| **Base case condition**          |            |                 |
| 9                                | 0.235 (20,9) | 0.092 (17,17) |
| 9                                | 0.107 (13,13) | 0.049 (4,17) |
| 9                                | 0.020 (13,9) | 0.035 (5,17) |
| 9                                | 0.018 (19,9) | 0.029 (13,9) |
| 11                               | 0.133 (13,13) | 0.076 (17,17) |
| 11                               | 0.046 (11,20) | 0.070 (11,17) |
| 11                               | 0.028 (11,14) | 0.064 (17,3) |
| 11                               | 0.021 (11,15) | 0.056 (18,17) |
| 13                               | 0.044 (20,14) | 0.027 (17,17) |
| 13                               | 0.006 (15,19) | 0.010 (17,4) |
| 13                               | 0.004 (13,13) | 0.009 (17,5) |
| 13                               | 0.004 (14,15) | 0.005 (17,14) |
| **Stressed operating condition** |            |                 |
| 9                                | 0.217 (20,9) | 0.856 (17,17) |
| 9                                | 0.035 (13,13) | 0.603 (13,13) |
| 9                                | 0.017 (19,9) | 0.474 (13,19) |
| 9                                | 0.016 (14,9) | 0.462 (13,9) |
| 11                               | 0.057 (13,13) | 2.351 (17,17) |
| 11                               | 0.399 (11,20) | 1.216 (17,11) |
| 11                               | 0.017 (11,13) | 0.548 (4,17) |
| 11                               | 0.013 (11,15) | 0.363 (17,8) |
| 13                               | 0.112 (14,20) | 0.330 (17,17) |
| 13                               | 0.10 (14,14) | 0.108 (12,11) |
| 13                               | 0.007 (19,15) | 0.089 (18,18) |
| 13                               | 0.005 (11,15) | 0.072 (17,11) |

The analysis of the stressed operating condition in Table 3, on the other hand, reveals an increase in the product $|h_{2pl}\phi_{pl}|$ showing the presence of non-linear modal interaction between inter-area modes and control modes associated with generator 3 and to a less extent with the control modes of exciters. These results suggest the need to tune the control system at this generator to decrease modal interaction to aid in improving the damping of these modes.

Of particular interest, normal form analyses were further conducted to assess the potential for non-linear modal interaction between the inter-area modes and the SVC control mode. In these studies, the SVC gain was adjusted from a small value to the nominal control setting and the interaction coefficients were computed. Table 4 summarises these results. As shown, normal form analysis reveals a strong modal interaction between the SVC control mode (mode 19) and the inter-area mode 13. For the stressed condition, the analysis indicates that an increase in the SVC gain decreases the level of non-linear modal interaction between the inter-area mode 13 and the mode associated with the SVC control system. It is also shown that the potential for non-linear modal interaction between the inter-area mode 13 and the SVC control mode is highest when the SVC gain ($K_{svc}$) is increased to 100 p.u. The analysis suggests that the SVC control setting might be used to enhance system dynamic performance. Moreover, the analysis of the interaction coefficients associated with the SVC control mode allows confirming these findings. Adjusting the SVC gain $K_{svc}$ from 10 to 50 p.u. shows that the interaction between inter-area mode 13 and the SVC mode increases. Further adjustment of the gain constant (from 50 to 100 p.u.) is seen to result in a more dominant
participation of the SVC mode in this interaction. Clearly, non-linearity and non-linear modal interaction are not uniformly distributed and may exhibit complex characteristics. This phenomenon is rather complex and depends on several interacting factors such as the control system parameter setting and the system operating condition. Normal form analysis allows obtaining a clearer description of the nature of this interaction as well as to pinpoint the effect of specific system parameters on system non-linearity and non-linear modal interaction. The closed form solution provides significant information about these phenomena that is not available from linear analysis and time-domain simulations.

5.3. Model validation

Detailed time-domain simulations were finally conducted to check the accuracy of the developed algorithms as well as to gain insight into the effect of non-linear modal interaction on system damping. Figs. 8 and 9 show the dynamic response of system generators relative to generator 1 for the base case and the stressed operating conditions and considering the SVC at bus 9. For the sake of discussion, the results are compared with the detailed step-by-step solution (SBSS) obtained using a commercial transient stability program and the time-domain linear approximations. Study results show that second-order normal form solutions approximate the non-linear SBSS well for the highly stressed conditions while linear analysis is unable to provide a good approximation to system performance.

Also of interest, Fig. 10 shows the SVC output susceptance obtained from normal form analysis and SBSS for two different SVC gains; $K_{svc} = 50$ and $K_{svc} = 80$. The analysis shows that an increase in the SVC gain leads to a deterioration in the ability of the NFM to

![Fig. 8. Comparison of relative rotor angle swings for the base case condition. (a) Generator 3 and (b) generator 4.](image)

![Fig. 9. Relative rotor angle swings for the stressed operating condition. (a) Generator 2 and (b) generator 4.](image)
approximate the system response, as the degree of non-linearity increases. It should be stressed that the developed methodology based on normal form theory does not take into account the presence of limits in the excitation systems of generators and the SVC, which were fully represented in the detailed SBSS.

6. Conclusions

In this paper a systematic analytical technique based on normal form theory and symbolic computer algebra is proposed for the analysis of non-linear inter-area oscillations in stressed power systems that incorporate FACTS controllers. Emphasis is being placed on the representation of static VAR compensation. The developed algorithms are based on a symbolically assisted power system simulation software and can be extended to accommodate more complex power system representations.

The analysis of low-frequency inter-area oscillations in a test power system reveals that SVC voltage support at critical system locations can significantly influence non-linear system behaviour especially under stressed operating conditions. Preliminary investigations show that FACTS controllers may also affect non-linear modal interaction and system non-linearity.

The results obtained show a good agreement with detailed non-linear time-domain simulation. Other aspects such as tuning of system controllers and the analysis of the effect of load characteristics are to be addressed in future stages of this research.

Appendix A. Test system data

For the SVC it was assumed that $K_{\text{svc}} = 80$, $T_{\text{svc}} = 0.05$ (Table A1).

References


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