Co-ordinated application of FACTS devices to enhance steady-state voltage stability

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to enhance steady-state voltage stability

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Abstract

This paper examines the co-ordinated application of flexible ac transmission system (FACTS) technologies to extend system voltage stability margins. A systematic analysis and design method, based on the singular value/eigenvalue decomposition analysis of the load-flow Jacobian and the study of the controllability characteristics of an equivalent state model is used to study the voltage instability phenomenon as well as to assess the potential for small-signal voltage stability improvement by means of FACTS compensation. The method is of particular interest for the preliminary design of new system devices and the study of interactions among existing controllers. Results obtained using a practical system representative of the Central American interconnected network are presented, illustrating the application of FACTS technologies to extend voltage stability limit.

Keywords: Flexible ac transmission system devices; Voltage collapse; Singular value decomposition analysis

1. Introduction

Voltage stability is becoming a major cause for concern in the design and operation of many power systems [1–4]. In weakly interconnected systems in developing countries, voltage instability is often associated with system structure and the lack of adequate reactive power support at critical system locations. Flexible ac transmission system (FACTS) devices may provide significant benefits in terms of both, greater operating flexibility and extended voltage stability margins.

FACTS devices are being increasingly utilised in many electric power systems to enhance voltage control and system dynamic performance. Among the existing devices, static VAR compensators (SVCs) have been found to improve voltage regulation as well as to increase voltage stability margins in several practical applications [5–7]. Other emerging FACTS technologies have also the potential for enhancing system performance as suggested by several researchers [5,6]. The co-ordinated application of FACTS devices, however, requires a careful appraisal of system characteristics and expected operating conditions.

Footnotes:

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well as transfer function residues are then used to identify the best locations for system reinforcement.

A complex system representative of the Central American interconnected network is used to illustrate the potential for improved voltage stability by means of FACTS technologies. The study focuses on the analysis of three critical voltage stability modes associated with insufficient reactive power support at critical system locations. Singular value decomposition (SVD) analysis of the extended power flow Jacobian matrix is used to identify critical areas of system, the system prone to voltage instability as well as to develop remedial actions including the co-ordinated application of multiple FACTS controllers. Study results include the identification of critical system locations. Singular value decomposition (SVD) analysis of three critical voltage stability modes associated with insufficient reactive power support at critical system locations.

2. Basic concepts

2.1. Small-signal voltage stability

Small-signal voltage stability is usually studied assuming that the system is described by a steady-state operating point and that the system parameters vary slowly and continuously. For a power system with n nodes and ng voltage controlled buses, the power system is represented by the steady-state power balance equations linearised about a steady-state operating point [10]

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = 
\begin{bmatrix}
J_{pp} & J_{pv} \\
J_{qv} & J_{qq}
\end{bmatrix} \begin{bmatrix}
\Delta V \\
\Delta \theta
\end{bmatrix} = J(V, \theta) \begin{bmatrix}
\Delta V \\
\Delta \theta
\end{bmatrix}
\]

(1)

where \(\Delta P\) is the incremental change in bus real power injection of order \(n - 1\), \(\Delta Q\) is the incremental change in bus reactive power injection of order \(n - ng\), \(\Delta \theta\) is the incremental change in bus voltage angle of order \(n - 1\), and \(\Delta V\) is the incremental change in bus voltage magnitude of order \(n - ng\).

The matrix \(J(V, \theta)\) in Eq. (1) is the load-flow Jacobian obtained from a conventional load-flow program, modified to include the voltage dependent static load characteristics while vectors \(\Delta P\), \(\Delta Q\) include the representation of constant power static load characteristics. Solving for the bus voltage magnitudes and phase angles in Eq. (1) yields the sensitivity relation

\[
\begin{bmatrix}
\Delta V \\
\Delta \theta
\end{bmatrix} = J(V, \theta)^{-1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

(2)

Eq. (2) provides an explicit relationship that allows focusing on the study of the effect of power injection perturbations on the change in bus voltage magnitudes and phase angles. Several variants of this static approach have been proposed for the study of voltage stability and the design of remedial measures [5,6,11].

2.2. The singular value decomposition (SVD) approach

Let the system Jacobian \(J\) in Eq. (1) be factored in the form \(J = UV\Sigma^T\), where \(U = \text{col}(u_1, u_2, ..., u_N)\) is an \(N \times N\) orthogonal matrix, \(V = \text{col}(v_1, v_2, ..., v_N)\) is an \(N \times N\) orthogonal matrix, and \(\Sigma(J)\) is an \(N \times N\) matrix whose off-diagonal entries are all 0s and whose diagonal elements satisfy that \(\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_N \geq 0\). It can be proved that the rank of matrix \(J\) equals the number of nonzero singular values and that the magnitudes of the nonzero singular values provide a measure of how close \(J\) is to a matrix of lower rank [12].

Using the orthogonality conditions \(J^{-1} = V\Sigma^{-1}U^T\) and solving for the terminal voltages yields

\[
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix} = V\Sigma^{-1}U^T \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = \sum_{j=1}^{\min(s, r)} \frac{v_j u_j^T}{\sigma_j} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

(3)

where

\[
S_j = u_j^T \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

(4)

and \(r\) represents the number of dominant singular values. In the vicinity of the singular point (\(\sigma_N = 0\)), a small change in real and reactive power injections will result in a large change in bus voltage magnitudes and angles. In practice, however, the point of collapse is not usually obtained from a conventional load-flow program, thus two or three terms are required to approximate Eq. (3).

As noted by several researchers, the magnitude of the minimum singular value (MSV) of the Jacobian matrix, \(\sigma_N(J)\) provides a useful measure of how close the system is to the voltage collapse or singular point [8,9]. Moreover, the maximum entries in \(\Sigma\) indicate the most sensitive bus voltage magnitudes while the maximum entries in \(u_N\) correspond to the most sensitive direction for changes of power injections. From this representation, bus as well as branch and generator participations can be readily determined.

3. Modified power system representation

The standard steady-state system power voltage equations in Eq. (1) do not provide explicit relationships that allow predicting and comprehending the influence of system controllers on the voltage stability problem. In this
section, a comprehensive incremental model of the power system is proposed of interest for the study of voltage stability and the development of remedial measures based on FACTS compensation.

3.1. Modelling of FACTS devices

FACTS devices are introduced in the basic model in Eq. (1) as linearised relationships between the small changes in real and reactive power injections and the corresponding changes in terminal voltage at the connecting nodes. The generic model used to represent shunt-connected devices, namely SVCs and STATCOM is of the form

$$
\begin{align*}
\Delta P_d &= J_{Pb} \Delta \theta_d + J_{PB} \Delta V_d + J_{Qb} \Delta \theta_d \\
\Delta Q_d &= J_{Pq} \Delta \theta_d + J_{QV} \Delta V_d + J_{QV} \Delta \theta_d
\end{align*}
$$

where $$\Delta P_d$$ is the incremental change in the device real power injection, $$\Delta Q_d$$ incremental change in the device reactive power injection, $$\Delta V_d$$ incremental change in the device voltage magnitude, $$\Delta \theta_d$$ incremental change in the device voltage angle, and $$\Delta \beta_d$$ incremental change in the device susceptance.

Here the coefficients $$J_{Pb}, J_{Pq}, J_{Qb},$$ and $$J_{QV}$$ represent appropriate sensitivity relations which are used to modify the power flow Jacobian matrix; the terms $$J_{PB}, J_{QV}$$, on the other hand, represent the sensitivity of the active and reactive power injections drawn by the device to changes in the controllable susceptances. Fig. 1(a) and (b) depict the general nature of the adopted model for representing SVCs and TCSCs. With appropriate considerations, these models can be used to represent other system controllers and the characteristics of static voltage-dependent load characteristics.

Further, Fig. 1(c) shows the adopted UPFC model used in the studies. The model is used to maintain both, a specified real and reactive power transfer from bus $$m$$ to bus $$k$$ as well as to maintain constant the voltage magnitude at bus $$k$$ at $$V_{nom}$$. In the proposed formulation, bus $$m$$ is represented as a PQ bus, while bus $$k$$ is represented as a PV bus. Details can be found in Ref. [13]. Operating limits for synchronous machines and FACTS controllers are approximated in the power load-flow equations as linear relationships between changes in power injections and the change in voltage magnitudes following the approach in Ref. [14].
3.2. Overall system representation

For a power system with \( m \) FACTS devices, the relation between the change in the bus power injections and the corresponding change in bus voltages and the controllable susceptances can be obtained by substituting Eq. (5) in Eq. (6) as

\[
\begin{align*}
\Delta P & = \begin{bmatrix} J_{p_0} & J_{p_v} \end{bmatrix} \Delta \theta + \begin{bmatrix} J_{q_0} & J_{q_v} \end{bmatrix} \Delta V \\
\Delta Q & = \begin{bmatrix} J_{p_0} & J_{p_v} \end{bmatrix} \Delta \theta + \begin{bmatrix} J_{q_0} & J_{q_v} \end{bmatrix} \Delta V + \begin{bmatrix} J_{p_p} & \Delta \beta_{\text{FACTS}} \end{bmatrix} \Delta V
\end{align*}
\]

where the submatrices \( J_{p_0}, J_{p_v}, J_{q_0}, J_{q_v} \) include the effect of FACTS devices, and the submatrices \( J_{p_p}, J_{q_p} \) represent the sensitivity of the active and reactive power injections to changes in the susceptance of FACTS devices, respectively. The vector \( \Delta \beta_{\text{FACTS}} = [\Delta \beta_{d}\Delta \beta_{q}\cdots\Delta \beta_{r}]^T \) contains the incremental changes in the individual susceptances. It should be emphasized that for assessing the placement of new devices, namely SVCs and TCSCs, submatrices \( J \) in Eq. (6) are zero valued, since \( \beta_{d} = 0 \).

From Eq. (6) it follows that

\[
\begin{align*}
\Delta \theta & = (J^\nu)^{-1} \Delta P - (J^\nu)^{-1} \begin{bmatrix} J_{p_p} & \Delta \beta_{\text{FACTS}} \end{bmatrix} \\
\Delta V & = (J^\nu)^{-1} \Delta Q - (J^\nu)^{-1} \begin{bmatrix} J_{q_p} & \Delta \beta_{\text{FACTS}} \end{bmatrix}
\end{align*}
\]

where

\[
J^\nu = \begin{bmatrix} J_{p_0} + J_{p_v} & J_{p_v} \\ J_{q_0} + J_{q_v} & J_{q_v} \end{bmatrix}
\]

Hence, assuming that the real load power change is independent of voltage changes (\( \Delta P = 0 \)), and solving for the terminal voltages in Eq. (7) yields the reduced order model

\[
\Delta V = (J^\nu_K)^{-1} \Delta Q - (J^\nu_K)^{-1} J^\nu_p \Delta \beta_{\text{FACTS}}
\]

where

\[
J^\nu_K = J_{q_v}^\nu - J_{q_p}(J_{p_p})^{-1} J_{p_v}^\nu = \frac{\partial Q}{\partial V}
\]

is the reduced Jacobian matrix of the system, and the relation

\[
J^\nu_R = J_{q_0}^\nu - J_{q_p}(J_{p_p})^{-1} J_{p_0} = \frac{\partial Q}{\partial \beta_{\text{FACTS}}}
\]

represents the sensitivity of the bus voltage magnitudes to changes in the output of FACTS controllers. Eq. (8) describes the combined effect of reactive load changes and control action on the bus voltage magnitudes.

4. Placement of FACTS controllers

4.1. Transformation to an equivalent representation

Important insight into the mechanism of voltage stability may be obtained from the application of linear system theory. Following Viera et al. [15], a useful analogy to the study of system eigenvalues and singular values in the developed model can be derived from the analysis of the characteristic equation

\[
|\lambda + (J^\nu_\theta)^{-1} \lambda| = 0
\]

It follows from Eq. (11) that voltage stability can be studied using the equivalent state-space representation

\[
\Delta V = -(J^\nu_\theta)^{-1} \Delta V + \Delta Q - J^\nu_\theta \Delta \beta_{\text{FACTS}}
\]

where \( y \) is the output vector that includes selected bus voltage magnitudes. Fig. 2 shows a schematic representation of the equivalent state model illustrating the connection between system variables. In this model, \( \Delta Q \) represents the input vector of specified changes in reactive power injections and \( \Delta V \) represents the change in power injection resulting from the change in terminal voltages; the state vector \( \Delta V(s) \) accounts for the bus voltage magnitude deviations in Eq. (7).

4.2. Ranking of remedial measures

Two complementary analytical approaches to identify the best locations for system reinforcement as well as to analyse the potential for adverse interactions among controllers have been developed in this research. The first method is based on the analysis of transfer function residues from each FACTS device location to a given bus voltage magnitude. In this approach, two alternative transfer function matrices are of interest, namely:

1. The transfer function matrix relating the output vector to the changes in the system demand

\[
G_L(s) = \frac{\Delta y(s)}{\Delta Q(s)} = C[sI + (J^\nu_\theta)^{-1} \lambda]^{-1} \Delta Q
\]

2. The transfer function matrix between the susceptance of FACTS controllers and the controlled bus voltage...
deviations

\[ G_{\text{FACTS}}(s) = \frac{\Delta y(s)}{\Delta \rho_{\text{FACTS}}(s)} = C[sI + (J_k^0 Q)]^{-1}J_k^0 \]

\[ = \left[ \sum_{i=1}^{n} \frac{R_k}{s - \lambda_k} \right] \]

(14)

where \( R_k = C_p q_k^T J_{y0} \) is the residue matrix, and \( p_k \) and \( q_k^T \) are the right and left eigenvectors of the state matrix associated with mode \( \lambda_k \) [16], computed at the point of maximum loadibility. At \( s = 0 \), Eqs. (13) and (14) provide convenient forms to assess the effect of the voltage dependent load characteristics and FACTS compensation on voltage stability. In the latter case, existing or new devices can be incorporated in the developed technique.

The second analysis method is based on the analysis of controllability characteristics of the equivalent system model. Let the input matrix of FACTS controllers be expressed as \( J_k^0 = \frac{1}{2} j_{y0}^2 \cdot J_{y0}^2 \). Given the system of equations (12) it can be proved that the distance \( d \) between the pair \( (J_k^0 Q, J_k^0) \) and the nearest set of uncontrollable systems is given by the minimum of the smallest singular values [12]

\[ d = \min_{\lambda_i} \sigma_{\text{min}}[sI + (J_k^0 Q), J_k^0] \]

with respect to \( \lambda \), where \( \sigma_{\text{min}} \) is the MSV of the augmented matrix. Therefore, a practical measure of the modal controllability of the \( i \)th the voltage stability mode from the \( k \)th FACTS device is obtained from the analysis of the augmented matrix:

\[ W_k^i = \sigma_{\text{min}}[\lambda_i I + (J_k^0 Q), J_k^0] \]

(15)

It can be deduced from Eq. (16), that a voltage stability mode associated with \( \lambda_i \) is more controllable from the \( k \)th input than from the \( j \)th input if and only if \( \sigma_{\text{min}}(W_k^i) > \sigma_{\text{min}}(W_j^j) \). Eq. (16) can be used to identify the best location for adding FACTS devices as well as to determine adverse interactions among controllers.

4.3. Algorithm for the calculation of voltage stability margins and the development of remedial measures

The following procedure is used for finding system areas prone to voltage instability as well as to identify remedial measures based on FACTS technologies:

Step 1
Starting with a base case condition, increase load in steps and determine the point of maximum loadibility. Then, compute the set of critical singular values \( \sigma_i \) (\( i = 1, \ldots, r \)) and the associated right and left singular vectors.

Fig. 3. Detail of the study system illustrating the location of major transmission facilities and areas prone to voltage instability.
Step 2
For each critical singular value, compute bus, generator and branch participations.

Step 3
Compute transfer function residues and controllability measures of selected modes at selected FACTS devices.

Step 4
Rank potential locations according to the magnitude of the controllability measures and residues.

The aforementioned approach allows determining the most effective locations for FACTS devices and can be used to identify adverse interactions between controllers.

5. Case study

5.1. Description of the system studied

Fig. 3 shows a simplified one-line diagram of the study system illustrating major transmission and generating facilities. The study system represents parts of the 230, 138 and 69 kV transmission network of the Guatemalan interconnected system (GIS) and its interconnection with other regional systems. In this diagram, legends SLU-69, SMM-69 and SAL-69 represent interconnections with neighbouring systems.

The GIS model used in these studies contains 116 buses, 98 transmission lines and 33 generators serving a load of nearly 705.0 MW for the base case operating condition. Voltage support in this system is essentially provided by conventional fixed shunt compensation. Presently, power transfer is essentially established by first contingency stability and voltage considerations.

Voltage stability studies in this research are focused on the study of three voltage stability modes associated with the lack of voltage support. The geographical location for the critical areas determined from the mode shape of the right singular vector is shown in Fig. 3.

5.2. Determination of the limiting loading condition

The critical operating point was obtained by increasing the system load in order to steer the power system to its maximum loadability condition. At each step, the new equilibrium point was determined from the corresponding load-flow solution. The procedure was continued until the point where the power flow solution failed to converge and the singular values were then computed for the last converged load-flow condition.

Table 1 lists the five smallest singular values for the base case and the critical operating condition without FACTS compensation. The areas associated with the critical voltage stability modes are shown in Fig. 3 as determined from the study of the right singular vector. It should be noted that at the condition of maximum loadibility, the system is voltage unstable ($\lambda_1 \approx -0.0031$).

5.3. Application of FACTS devices

The approach described in Section 4 was applied to investigate the benefits to be derived from the application of FACTS compensation. Cases of special interest investigated included: (1) the addition of an SVC/STATCOM at critical system buses, (2) the application of a UPFC/TCSC on weak transmission lines, and (3) the co-ordinated application of both technologies.

Transfer function residues and controllability indices of the most critical voltage stability modes were used to determine the best locations for FACTS controllers. Table 2 synthesises the largest transfer function residues associated with critical voltage stability modes for the transfer functions $\Delta V_j(s)/\Delta Q_j(s)$ and $\Delta V_j(s)/\Delta \beta_{SVC}(s)$. For the critical singular value $\sigma_1$, the analysis of residues singles out the 69 kV bus HUE-69 at the end of a radial network as the most effective location for adding voltage support. The neighbouring 69 kV buses, POL-69 and ESP-69 are also

### Table 1
The five smallest singular values for the base case and the critical operating condition

<table>
<thead>
<tr>
<th>Singular value ($\sigma$)</th>
<th>Base case</th>
<th>Critical case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1$</td>
<td>0.0443</td>
<td>0.0031</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.0993</td>
<td>0.0897</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>0.1848</td>
<td>0.1671</td>
</tr>
<tr>
<td>$\sigma_4$</td>
<td>0.2896</td>
<td>0.2762</td>
</tr>
<tr>
<td>$\sigma_5$</td>
<td>0.3519</td>
<td>0.3142</td>
</tr>
</tbody>
</table>

### Table 2
Normalised transfer function residues

<table>
<thead>
<tr>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>Residue of $\Delta V(s)/\Delta \beta_{SVC}(s)$</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>HUE-69</td>
<td>1.000</td>
</tr>
<tr>
<td>POL-69</td>
<td>0.9227</td>
</tr>
<tr>
<td>ESP-69</td>
<td>0.8636</td>
</tr>
<tr>
<td>TOT-69</td>
<td>0.7954</td>
</tr>
<tr>
<td>SOL-69</td>
<td>0.7363</td>
</tr>
</tbody>
</table>
identified as having a large participation in this mode. Controllability analyses, participation factors and the mode shape of right singular vectors, on the other hand, led to similar conclusions and are therefore omitted from the following analyses.

Modal analysis was further conducted to determine the best locations for adding series FACTS compensation. Special attention was given to the analysis of the critical voltage stability mode. Table 3 lists the ranking of compensation alternatives provided by the analysis of residues and controllability for this mode. For comparison purposes, branches with high participations in the critical mode are also listed.

While these approaches lead to a different ranking of compensation alternatives, the results show that the best locations for placement of series and shunt compensation are the transmission lines on the 230 kV network along with transmission lines on the 69 kV network.

To verify the correctness of the results, FACTS controllers were applied at selected transmission lines and buses and the MSV was computed for each compensation alternative. In these studies, the 69 kV buses HUE-69 and POL-69 within the critical area (Fig. 3) for the critical mode were initially chosen for placement of SVCs. Conversely, several transmission lines on the 230, 138 and 69 kV networks were chosen for adding a UPFC/TCSC. In all cases, the UPFC was designed to maintain both, constant real power along the interconnector and constant voltage at the extreme end of the transmission line while the TCSC was chosen to be 30% of the line’s inductive reactance. Further, the UPFC rating considered for the series part and shunt parts is 30 MVAR. The SVC rating was chosen to be $-30$ MVAR/$30$ MVAR with a control slope of 3% for all studies.

Fig. 4 shows the effect of adding a UPFC at several selected transmission paths on the voltage stability margin for the critical mode. The results are in good agreement with the analysis of controllability and the study of transfer function residues in Table 3. Similar results were obtained for the analysis of the effect of SVC and TCSC on voltage stability and are therefore omitted.

To assess the effect of multiple FACTS compensation on voltage stability margins, a single controller was added at a time to improve the stability of the critical mode and the transfer function residues and controllability indices were then computed again to apply a second device. Comparisons were made with the best control strategy for each individual mode. The analysis of the proposed system reinforcement measures in Table 4 shows an improvement on overall voltage stability margins. Clearly the application of SVC

<table>
<thead>
<tr>
<th>Order</th>
<th>Residue of $\Delta V/\Delta \beta_{TCSC}$</th>
<th>$\sigma_{\text{min}} (W_i)$</th>
<th>Branch participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>ESC-69-3214</td>
<td>ESC-69 CL-69</td>
<td>HUE-69-POL-69</td>
</tr>
<tr>
<td>9</td>
<td>ESC-138 JUR-138</td>
<td>ESP-69 SMA-69</td>
<td>PAL-69 SAL-69</td>
</tr>
<tr>
<td>10</td>
<td>ESC-69 3110</td>
<td>GSU-69-CEN-69</td>
<td>GSU-69 3204</td>
</tr>
<tr>
<td></td>
<td>SID-230 3113</td>
<td>ESC-69 PNT-69</td>
<td>CEN-69 3202</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compensation alternative</th>
<th>Critical singular values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_1$</td>
</tr>
<tr>
<td>Base case without FACTS compensation</td>
<td>0.0031</td>
</tr>
<tr>
<td>Fixed bank of capacitors of 15 MVAR at bus HUE-69</td>
<td>0.0792</td>
</tr>
<tr>
<td>SVC at bus HUE-69</td>
<td>0.2210</td>
</tr>
<tr>
<td>UPFC in line ESP-69-SMA-69</td>
<td>0.2235</td>
</tr>
<tr>
<td>TCSC in line GNO-230-GES-230</td>
<td>0.1311</td>
</tr>
<tr>
<td>UPFC in line GNO-230-GES-230</td>
<td>0.2480</td>
</tr>
<tr>
<td>SVC at bus ABO-69 and UPFC in line ESP-69 SMA-69</td>
<td>0.3891</td>
</tr>
<tr>
<td>SVCs at buses HUE-69 and ABO-69</td>
<td>0.3849</td>
</tr>
</tbody>
</table>

Fig. 4. Effect of adding a UPFC on the voltage stability margin for the critical mode.

Table 3
Ranking of compensation alternatives for the critical singular value obtained from residues and controllability studies

Table 4
Effect of FACTS devices on voltage stability margins

and UPFC was found to be more desirable than the application of TCSC. It is also worth noting that the coordinated application of two devices allows for extended voltage stability improvement of the three most critical modes without undue interaction.

Voltage stability enhancement was finally determined by evaluating contingency cases with and without FACTS controllers. Table 5 summarises the effect of several compensation alternatives on voltage stability improvement for both, the base case and the post-contingency cases. As shown, FACTS devices can significantly increase the voltage stability margin for the most stringent operating conditions. Moreover, FACTS compensation allows extending voltage stability margins for the most severe contingencies.

Examination of system results shows also that a UPFC placed at a weak transmission line can have a similar performance on voltage stability improvement than an SVC placed at a critical bus. As shown, however, uncoordinated application of FACTS controllers can actually reduce the stability of the critical modes.

5.4. Comparison with other approaches

Several conventional studies based on well-established analysis techniques were conducted to verify the appropriateness of the developed models. Fig. 5 depicts the right singular vector for the critical singular value showing the effect of SVC and UPFC on voltage stability. As expected from previous studies, FACTS devices are seen to reduce the voltage sensitivity of critical nodes.

Studies were finally conducted to evaluate the improvement in voltage stability margins attained with the proposed system reinforcement measures. Fig. 6 shows the $Q-V$ relationship for various system configurations.

Table 5

<table>
<thead>
<tr>
<th>Line outage</th>
<th>Reinforcement measure</th>
<th>System status</th>
<th>Critical singular values</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESP-69-SMA-69</td>
<td>None</td>
<td>Unstable</td>
<td>$\sigma_1 = 0.1549$</td>
</tr>
<tr>
<td></td>
<td>SVC at bus HUE-69</td>
<td>Unstable</td>
<td>$\sigma_2 = 0.2448$</td>
</tr>
<tr>
<td></td>
<td>UPFC in line ESP-69-POL-69</td>
<td>Stable</td>
<td>$\sigma_3 = 0.4915$</td>
</tr>
<tr>
<td></td>
<td>SVC at bus ABO-69 and UPFC in line ESP-69-POL-69</td>
<td>Stable</td>
<td>$\sigma_3 = 0.1552$</td>
</tr>
<tr>
<td></td>
<td>SVC at ABO-69 and UPFC in line ESP-69-SMA-69</td>
<td>Stable</td>
<td>$\sigma_3 = 0.2602$</td>
</tr>
<tr>
<td></td>
<td>SVC at ABO-69 and UPFC at line ESP-69-POL-69</td>
<td>Stable</td>
<td>$\sigma_3 = 0.3309$</td>
</tr>
<tr>
<td>ESC-230-GSU-230</td>
<td>None</td>
<td>Unstable</td>
<td>$\sigma_1 = 0.0769$</td>
</tr>
<tr>
<td></td>
<td>SVC at bus HUE-69</td>
<td>Stable</td>
<td>$\sigma_2 = 0.2308$</td>
</tr>
<tr>
<td></td>
<td>UPFC in line ESP-69 SMA-69</td>
<td>Stable</td>
<td>$\sigma_3 = 0.3195$</td>
</tr>
<tr>
<td>GSU-69-CEN-69</td>
<td>None</td>
<td>Unstable</td>
<td>$\sigma_1 = 0.2312$</td>
</tr>
<tr>
<td></td>
<td>SVC at bus HUE-69</td>
<td>Stable</td>
<td>$\sigma_2 = 0.3317$</td>
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<td>UPFC in line ESP-69-POL-69</td>
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<td>$\sigma_3 = 0.4519$</td>
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<td>$\sigma_1 = 0.2370$</td>
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<td>SVC at ESP-69-SMA-69</td>
<td>Stable</td>
<td>$\sigma_2 = 0.3725$</td>
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<td>SVC at ABO-69 and UPFC in line ESP-69-POL-69</td>
<td>Stable</td>
<td>$\sigma_3 = 0.4542$</td>
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<td>SVC at ABO-69 and UPFC at line ESP-69-POL-69</td>
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<td>$\sigma_3 = 0.3745$</td>
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Fig. 5. Right singular vectors showing the impact of FACTS compensation.
curves computed at a critical bus in the electrical centre of the area identified for the critical mode, illustrating the impact of SVC and UPFC on voltage stability margins. As shown, the system operates very close to the stability margin for most critical areas as suggested from modal analysis in the previous sections. Of special significance, the bus HUE-69 shows the most reduced stability margin suggesting the need for voltage support. Again, these results correlate well with the modal analysis presented.

6. Conclusions

Voltage stability is becoming a limiting factor in the planning and operation of many power systems. In this paper, some major results of an analytical investigation aimed at identifying the potential for voltage stability improvement by means of the application of flexible ac transmission compensation are presented. Special emphasis is being placed on the analysis of poorly interconnected power systems characterised by several critical voltage stability modes.

Modal analysis of the power flow Jacobian matrix provides important information about the proximity of the system to voltage instability and can be used to identify system elements strongly involved in the phenomena as well as critical contingencies that could lead to instability. Present methodologies, however, may not identify the most effective locations for system reinforcement. Other quantities based on sensitivity relations or the computation of controllability and observability characteristics may help in the design process.

Study results show that several FACTS technologies have the potential for voltage stability improvement, especially under high stress conditions. Evaluation of flexible ac transmission technologies showed that a UPFC placed at the weakest transmission line could have a similar performance on voltage stability improvement that an SVC/STATCOM placed at the critical bus.

Other studies show that the uncoordinated application of multiple FACTS devices can actually decrease the stability margins of critical system modes. Controllability and observability characteristics are to be further investigated to identify potential adverse interaction between controllers.

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References