Brain mappings of the arithmetic processing in children and adults.

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Abstract

Despite the increasing number of experimental mapping showing that human arithmetic cognition is supported by widely spread neural circuits; the theoretical reasoning about these data remains mostly metaphorical and guided by a connectionist approach. Although neurons at distinct areas in the brain are assumed to take charge of different duties in the solution of the experimental task, the results are always discussed by hypothesizing some association between the different areas without questioning any difference of behavior at the level of the neurons at each of these areas. Here, the brain is assumed as Distributed Intelligent Processing System (DIPS) formed by collections of loosely interacting specialized agents (neurons), each agent specializing, for example, in data collection (sensors), problem solving (associative neurons), data communication (interneuronal systems) and in acting upon the surrounding environment (motorneurons). A new technique for EEG brain mapping is proposed and used to study arithmetic cognition in elementary school aged children and adults. Factor analysis showed three distinct patterns of neuronal recruitment for arithmetic calculations in all experimental groups which varied according to the type of calculation, age and sex.

Keywords: Arithmetic; Learning; Brain mapping; Cognition; EEG

1. Introduction

How and why the human arithmetic cognitive abilities evolved? It is tempting to answer this question by the obvious assumption that the larger the brain the more its owner capacity to adapt and survive in aggressive and/or rapidly changing environments and that mutation and natural selection provides the mechanism for such brain enlargement. But one could then argue that, in addition to the size of the brain, neuronal specialization and its configuration have a role to play too. But, are there different cerebral regions responsible for mathematical thinking?

Both experiments carried out in the early 1980s and recent experiments with animals and EEG and fMRI in humans, demonstrated a higher cerebral activity during numerical performance, in particular the inferior parietal cortex as well as multiple regions of the prefrontal cortex (e.g., Refs. [2,3,6–8,15,20–23,26,27,29,44,46,50]). It is now accepted that the inferior parietal region is important for the transformation of numerical symbols into quantities, and the representation of relative number magnitudes. The prefrontal cortex is proposed to be responsible for sequential ordering of successive operations, control over their execution, error correction, inhibition of verbal responses, etc. Moreover, other central and temporal cortex areas are also involved in many arithmetic calculations. It seems, therefore, that human arithmetic capabilities are the result of a complex cerebral processing involving different types of neurons widely distributed over the brain, each of them in charge of solving a particular subtask of the whole problem.

Despite the increasing number of authors (e.g., Refs. [1,5,8,21,49]) experimentally mapping a widely spread neural circuit subserving the human arithmetic cognition,
the theoretical reasoning about these data remains mostly metaphorical and guided by a connectionist approach. Although neurons at distinct areas in the brain are assumed to take charge of different duties in the solution of the experimental task, the results are always discussed by hypothesizing some association between the different areas without questioning any distinct behavior at the level of the neurons at each of these areas. Additionally, the role played by each brain area on the different arithmetic tasks is in general metaphorical instead of formally discussed.

Since the classical Galvani’s experiments, electrical membrane gradients and their induced variations play an important role in the understanding of the cerebral physiology. The Nobel-prized work of Hodgkin and Huxley [18] in the first half of the 20th century clearly identified the main membrane components governing its electrical behavior and formalized this behavior as equivalent to that of a dynamic system having two stable and one unstable equilibrium states. McCulloch and Pitts [25] proposed a logical calculus immanent from these ideas, which has been used since then to formalize the neuron in the artificial neural nets, the classical modeling in the connectionist approach. However, molecular neurobiology has been accumulating evidences for complex biochemical transactions among neurons, which supports a more complex neuronal modeling by Fuzzy Formal Languages and the Theory of Distributed Intelligent Processing Systems [38–40,42].

A Distributed Intelligent Processing System (DIPS) is (e.g. Refs. [32,33,39,41]) formed by collections of loosely interacting specialized agents (neurons), each agent specializing in data collection (sensors), problem solving (associative neurons), data communication (interneuronal systems) and in acting upon the surrounding environment (motor-neurons). DIPS reasoning specialized agents (cells) that may provide the solution of a problem. Decentralized means that both control and data are logically and often spatially distributed; there is neither global control nor global data storage. The programming intends to build models in which the control structure emerges as a pattern of passing messages among the agents being modeled. Task distribution is an interactive process between an agent with a task to be executed and a group of agents that may be able to execute the task.

In this kind of system, cognition becomes a function of both the types of agents the system is composed of, and of how and with what purpose these agents are used for. Cognition becomes dependent on both the behavior of the specialized agents that are in charge of solving specific tasks and on how versatile are the relations shared by these specialized agents; of how plastic may be the commitments for actions among these agents. Of course, the complexity of the tasks solvable by a DIPS determines the number of agents to be enrolled in their solution [39,40,42].

In the DIPS approach, the task of counting [40,42] (Rocha et al., 2004), for instance, may be assumed to require a set of sensory agents in order to visually inspect the environment and to identify the elements to be counted (Fig. 1). It may also require motor control agents in charging of positioning the eyes over the elements to be counted and controlling the fingers to point or to mark the identified elements. The modeling may also demand that the recognized quantities to be accumulated in a set of accumulator neurons (A in the Fig. 1) whose outputs are classified by two different other sets of neurons composing two different circuits. On one hand, each quantifier neuron Q of the K Fuzzy Numbers (KFN) circuit (Fig. 1) represents a number, and its output measures in the closed interval [0,1], how much the actual value in the accumulator matches (1) or not (0) the quantity it represents. Such an encoding is of the type described for primate neurons’ quantity encoding neurons [29,30,44]. On the other hand, each neuron I of the Crisp Base Number (CBN) circuit (Fig. 1) also represents a number, but its output now is proportional to the quantity it represents. Counting is performed by these circuits as a serial operation coordinated by the set of control neurons (C in Fig. 1) in charge of serially controlling the focus of the eyes over the objects to be counted and fingers movements marking the same objects. These neurons are of the type described by Carpenter et al. [3] in the monkey motor cortex. Each time objects are focused and recognized (R neurons in the figure), the corresponding quantity is loaded in gate neurons (G in Fig. 1) controlled by the C neurons. The gate neurons direct these quantities to be accumulated by the A neurons and to be classified (recognized) by the Q and I neurons. On one hand, Q neurons control motor neurons in charge of phonating or writing the corresponding number name. On the other hand, I neurons are in charge of identifying the quantity associated to a heard or read number. The basic arithmetic calculus (addition, subtraction, multiplication and division) is assumed to be processed by this type of circuit as both formal or simulated operations [40,42].

Counting is performed as a serial operation coordinated by frontal neurons in charge of controlling the focus of the eyes over the objects to be counted and fingers movements marking the same objects. Each time the eyes are focused by the neuron C over an object of a set of objects to be counted, the event is accumulated by the A neurons. The gate neurons direct these quantities to be accumulated by the A neurons and to be classified by the Q and I neurons. Q neurons control the hand and phonation to speak about the counting. I neurons are used to decode numerals or number names. See text for further details.

To learn to count implies, according to this model, the control of DNA reading both to specialize different agents to accumulate and classify data, as well as to control the communication resources among these agents at the synaptic level [34,39,40]. The arithmetic properties of the circuits in Fig. 1 are strongly dependent on different electrical encodings of the same type of information by distinct types of neurons, and this neural specialization is
proposed to be governed by controlling different DNA readings of genes specifying ionic gates [40,42]. According to this model, arithmetic learning during the school days should involve the increase of gene expression defining I neurons, besides improving the control of counting motor pathway, as two of the most important learning issues.

The importance of the gender differences in cognitive abilities has been stressed by many authors, and especially sex differences in mathematics achievement are well documented…”, as pointed by Halpern [17] in her 1992 book: Sex differences in cognitive abilities. The importance of this subject is clearly acknowledge in the special issue (1999/24) of the journal Contemporary Educational Psychology devoted to the Math-Fact Retrieval Hypothesis, proposed by Royer and Tronsky to explain that males outperformed females in the SAT-Math exam because they are quicker in math-fact retrieval. In addition, Gallager et al. [11] proposed that “… strategy flexibility is a source of gender differences in mathematical ability assessed by SAT-M and GRE-Q problem solving”. Rocha et al. [40,42] showed that gender differences starts to appear at the very beginning of the arithmetic academic training and persisted until adult ages. It must be remarked that the adult group, in this study, was composed by exact sciences graduate students, such that the authors claimed that their results could not be explained by any strong claim that math gender differences is the result of a biased education, but seemed to point to a phylogenetic explanation.

Another interesting DIPS property is that many different solutions are tried to the very same arithmetic problem. Indeed, this seems to be the strategy used by the brain in the case of arithmetic processing. Many studies on how children in different countries solve standard arithmetic problems revealed that they use multiple different strategies such as [2,6,13,14,19,24,28,45]:

(a) total manipulation: the child counts separately each set to be processed by pointing each element, and then counts by the same process each element of the union, or the complement, etc., of these sets to get the final result;

Fig. 1. Modeling neural arithmetic circuits. Frontal cortical circuits C are in charge of controlling the motor trajectory focusing the eyes over the elements to be counted. Whenever one or more of such elements are recognized by sensory neurons R, the corresponding amount of pulses is sent to gate neurons G and released to be accumulated by the neurons A. The output of these accumulators is classified by a set of quantifier neurons Q or I. Quantifier neurons Q activate neurons at Broca area to name the counting result, and/or neurons in the hand’s area to write the same name. Any spoken or written numeral is decoded by connecting the visual neurons responsible for the numeral recognition, to the corresponding I quantifier that loads the accumulator with the adequate charge.
(b) simplified manipulation: the result is obtained by counting each element of the union, or the complement, etc., of the sets to be processed;

(c) optimized manipulation: the result is obtained by performing the minimum counting, which varies according to the type of calculation to be performed, and

(d) mental calculation: the result quickly processed (retrieved from memory) by specialized circuits.

In addition, the type of function explaining the number size effect reported in the literature correlates with the kind of manipulation used (e.g., Ref. [24]). Finally, training changes the frequency of use of the different types of strategy, such that adults tend to rely more on mental calculation than children [35,45]. Rocha et al. [40,42] showed that children and adults used different strategies for solving any kind of arithmetic calculations, because they found different types of correlation between the calculation time and the size of the different operands. Additionally, they showed that the size effect dependence was more complex for adults in comparison with the children, and they concluded that learning enriches arithmetic knowledge by increasing the number of available strategies for the same calculations.

The purpose of this paper is to study the brain activity associated with arithmetic operations (addition, subtraction, multiplication and division) in the same groups of children and adults used by Rocha et al. [40,42] to study the chronometry of arithmetic calculus.

2. Methods

2.1. The experimental groups

Data were obtained from the same three experimental groups used by Rocha et al. [40,42], equally gender balanced and differing in the mean age and cognitive development (see Table 1):

1) Group CHI2: 20 children—mean age 7 years, 7 months; enrolled at 2nd and 3rd semesters of the elementary school, mastering addition and subtraction, besides being able to read simple phrases, and

2) Group CHI4: 24 children—mean age 8 years, 3 months; enrolled at 4th and 5th semesters of the elementary school, mastering addition, subtraction, multiplication and in their initial training for division, besides being able to read simple texts and write simple phrases,

3) Group AD: adults—mean age 28 years; enrolled in graduate courses in the field of exact sciences of one university near São Paulo and attending a special training program in Biotechnology.

Programmable arithmetic games were used to randomly generate addition, subtraction, multiplication or division, involving numbers up to 20. Two different problem displays were available (Fig. 2). In the upper examples, the calculus was presented in a formal way and in blocks of three problems. In the lower examples, the calculus was presented formally and illustrated with distinct elements of the same semantic group. The volunteer had to choose the correct answer by selecting one of the numbers displayed as answer numbers.

The children were selected from those whose parents agreed with explained experimental protocol and signed a special permission. Each group was formed by children from two equivalent classrooms concerning the cognitive profile, as evaluated by the principal of a middle class school in the city of Guarulhos. Adults volunteered after having the experimental protocol explained and also signed the permission, whose terms were approved by the university’s Ethics Committee.

2.2. The arithmetic questions

The experimental protocol consisted in solving 30 different questions for each arithmetic calculation while the volunteer’s EEG was recorded (Fig. 2). Each question (Fig. 2) was visually presented in a computer screen, and the volunteer had to choose the correct response among a set of displayed numbers. The questions were presented into two different visual formats. In one of them (VA format) the quantities were represented by numbers and elements of a given class of objects (toys, fruits, etc.), whereas in the other format (VB format) quantities were only represented by numbers. Two different series of questions were prepared, each concerning a given display format, and containing 15 problems each. The questions in each series were randomly selected when they were initially programmed. So the sequence of quantities calculations was the same for all volunteers. Each question in the series was numbered to allow the study of any dependence of performance concerning the order of question presentation. Whereas adults solved both series, the groups CHI2 and CHI4 manipulated only the questions in the VA format. This was mainly because initial tests revealed that children could become tired if submitted to long series of calculations. CHI4 children were tested in two different epochs: at the end of the 3rd and 4th semesters in an attempt to quantify a possible learning by reduction in errors and response time. Adults solved the VA series for all four types of arithmetic calculations before the VB series were presented. This was

<table>
<thead>
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done in order to allow comparisons between children and adults to be studied independently from the type of visual presentations, and to assure that the VA format could not be interpreted as infantile by the adults. All mistakes and correct responses were clearly signalized to the volunteers after they selected the number they assumed as the answer to the proposed question. The set of numbers available for response selection were displayed in lines of increasing but not consecutive quantities. These sequences were randomly selected by the time of programming the experimental protocol, such that all volunteers were presented with the same responses set for each proposed question. All sessions were videotaped for further inspection of the volunteers’ performance whenever necessary. Adults were encouraged to comment on the experiment at the end of the session.

2.3. The game event related activity

Each subject solved the tests while his/her EEG was registered with 20 electrodes placed according to the 10/20 system; impedance smaller than 10 kΩ; low band passing filter 50 Hz; sampling rate of 256 Hz and 10 bits resolution (Fig. 2). Two networked personal computer were used: one for the EEG recording and the other for game playing. Timing of test events (e_1, e_2, ...) like the beginning of the visual information display and the decision making, were written as corresponding marks (m_1, m_2, ...) in the file of the recorded EEG. The EEG was visually inspected for artifacts before its processing, and the events associated to a bad EEG were discarded. The mean time for solving the test was calculated to each experimental group. The recorded EEG was averaged for epochs of duration equal to this mean time, referred to the beginning of the visual display, in order to generate the Game Event Related Activity (GERA) file for each volunteer (Fig. 2). These GERAs were used to calculate different brain mappings according to the following rationale [10] (Rocha et al., 2004). The linear correlation coefficients r_{i,j} for the averaged activity at each recording site d_i referred to the averaged activity for each other 19 derivations r_j were calculated for each GERA (Fig. 2). The entropy h(r_{i,j}) of the regression coefficients for each recording site d_i was calculated as follows [10,42]:

Fig. 2. The game event-related activity. The EEG is recorded while the volunteer is solving the proposed test. The first step in the EEG analysis is to calculate the game event-related activity. This is obtained by averaging the EEG epochs associated to a given game event (e.g., response selection, or information display, etc.). The next step is to calculate all 400 linear regressions among each EEG electrode d_i and the other electrodes d_j. In the sequence, the entropy h(r_{i,j}) is calculated for each d_i (see table in the figure) according to Eqs. (1)–(7) (see text) in order generate the mapping (CCM) of the cerebral activity supporting the cognitive function being studied.
The calculated normalized values $N(-h(r_i))$, values of each $h(r_i)$ were

$$N(-h(r_i)) = ((-h(r_i) - \min(-h(r_i))) / \max(-h(r_i)) - \min(-h(r_i)))$$

These values were plotted to produce the CCM encoding the min($-h(r_i)$) in black and max($-h(r_i)$) in red, and (max($-h(r_i)$)−min($-h(r_i)$))/2 in yellow.

(b) FM: three factors were extracted by using Principal Components Analysis and the varimax normalized method. If these extracted factors explained more than 50% of the total $h(r_i)$ variability, then the analysis was considered acceptable, and the color plots of the calculated correlation coefficient for each electrode and each factor were produced. The values loading factors $l(f(r_i))$ for each recording site $d$, and calculated for each factor $F_{1-3}$ were plotted to produce each FM $1-3$. $l(f(r_i))=0$ is encoded in black and $l(f(r_i))=1$ is encoded in red, $l(f(r_i))=0.5$ is encoded in yellow.

3. Results

3.1. Entropy statistical differences

The mean, maximum and minimum values of the actual $h(r_i)$ calculated for each EEG site $d$, concerning each group and each type of calculation are shown in Table 2. These mean and max/min $h(r_i)$ values differed at the level of $p<0.05$ among groups and type of calculations as calculated by means of Yates Corrected Chi-squared. The differences between the mean $h(r_i)$ values obtained for the adults and children were statistically different for all type of calculations. No $h(r_i)$ statistical difference was observed between CHI2 and CHI4 groups and those $h(r_i)$ mean differences for male and females are marked by * in Table 2.

3.2. Adults CCMs

The adult CCMs were very similar for male and females (Fig. 4). High values of $h(r_i)$ for the frontal and central electrodes where obtained bilaterally in the case of addition and division and mostly over the left hemisphere in the case of multiplication. Different from the CCMs, the factor mappings FM1, FM2 and FM3 greatly differed among sexes.

3.3. Adults factors

On the one hand, male FM1 shows that $h(r_i)$ for left frontal and central electrodes (mainly F1, F3, FZ, C3 and CZ) covaried for all calculations with the exception of division. In this latter case, covariance involved right frontal (FP1, F4) and central areas (FP1, F4, FZ and CZ). On the other hand, female FM1 shows a $h(r_i)$ covariance: (a) for FP1, FP2 and FZ that it is strong in the case of addition and subtraction and less prominent for division; (b) that is important for FZ, FP2, F4, F8 and T4 in the case of multiplication, and (c) that is very strong over the posterior areas (P3, PZ, P5, O1, O2 and OZ) in the case of division.

Male FM2 greatly varied according to the type of calculation. Covariance is (a) left-lateralized (F7, T3, T5 and P3) in the case of addition; (b) posterior (T5, P3, O1, OZ and O2) for subtraction; (c) central and parietal (C3, CZ, P3 and PZ) in the case of multiplication, and (d) right posterior (PZ, P4, T6, O2 and OZ) and left-medial (F3, C3 and P3) for division. Female FM2 disclose (a) an important central-parietal covariance (mainly C3, CZ, P3 and PZ) for both addition and multiplication; (b) a strong association between central and medial frontal areas (C3, CZ, C4, F3 and F4) for division and (c) a posterior association among parietal and occipital areas in the case of the subtraction.

On one hand, male FM3 was predominantly right lateralized (mostly F8, T4 and T6; and occasionally F4, F8, C4 and O2) for all types of calculations, although $h(r_i)$ covaried also for F7, T3 and T5 in the case of division. On the other hand, female FM3 shows a covariance similar to that encountered in the case of men for both subtraction and division. Female $h(r_i)$ covariance is bilateral and posterior in the case of addition, and restricted only to O2 and OZ for multiplication.

3.4. CHI4 CCMs

The CCMs of the CHI4 children were very dissimilar for males and females (Fig. 4). Male CCMs exhibit high $h(r_i)$ values mostly over frontal and central areas, bilaterally. The number of electrodes associated to these high values varied according to the type of calculation, and are the electrodes: (a) FP1, FP2, F3, F4, FZ, and C3 in the case of addition,
although CZ, C4, PZ, P4, OZ, O2 and T6 were moderately involved too; (b) FP1, FP2, F3, F4 and FZ for subtraction; (c) FP2, F4, FZ and C4 in the case of multiplication, and (d) FZ, FP1, FP2 and P4 in the case of subtraction. Female CCMs show widely spread covariance for all types of arithmetic calculus.

3.5. CHI4 factors

Male FM1 differed for each type of arithmetic calculus. It is mostly left lateralized (FP2, FP1, F3, CZ, P3, PZ and O2) in the case of addition. In the case of subtraction and multiplication, it involves bilaterally the anterior areas, although the pattern is a little different for each of these two calculus, involving mainly: T3, F7, F1, F2, F4, F8, T4 and CZ for subtraction, and F3, FP1, FP2, F4, C3 and CZ for multiplication. In the case of division, male FM1 is mostly right lateralized and involves T3, F7, F4, F8, C4 and T4. Female FM1 is more well behaved than the male one. It involves mainly the bilateral anterior electrodes. In the case of addition, it is mostly defined by T3, F7, FP1, FP2, F4, F8, and O2. In the case of multiplication, it is composed of T3, F7, C3, CZ, FP2, F4 and C4.

FM2 mappings greatly differed between boys and girls. On one hand, boys exhibited a bilateral pattern for addition

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**Table 1: Entropy Table**

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**Fig. 3. Calculating the brain mappings.** The entropy $h(r_i)$ values were used to produce two types of mappings: (1) the cerebral cognitive mapping (CCM) plots the normalized mean $h(r_i)$ values of $r_i$ for each recording site $d_i$ in each experimental group, and (2) the factorial mappings (FM) plot the loading factors $l_f(d_i)$ for each factor $F_i$ calculated for each recording site $d_i$. For all mappings: black color encodes $h(r_i)$ or $l_f(d_i)$ equal to zero, and: black color encodes $h(r_i)$ or $l_f(d_i)$ equal to one.
366

Table 2
Commitment \( (h(r_{ij})) \) vs. calculation

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
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</tbody>
</table>

AD—addition, SU—subtraction, MU—multiplication, DI—division. ADU—adults, CHI4—children enrolled at 2nd and 3rd semesters of the elementary school, CHI2—children enrolled at 4th and 5th semesters of the elementary school. Mean, Max and Min: Mean, maximum and minimum \( h(r_{ij}) \) values, respectively. Statistical differences at the level of \( \rho < 0.05 \) between male and female means are marked with *.

almost absent for addition and involved FZ, F4 and O2 in the case of female subtraction or FZ, FP2, F8, T4, C4 and PZ for male diminution.

4. Discussion

4.1. The entropy

Here, the brain is assumed as a Distributed Intelligent Processing System (DIPS) and the commitment \( h(r_{ij}) \) of any agent \( r_i \) in solving a given task—e.g., arithmetic calculation, was calculated according to Eqs. (1)–(3), taking into consideration that the correlation coefficient \( r_{ij} \) between the EEG activity recorded at the sites \( d_i, d_j \) measures the possibility \( \rho(n_i, n_j) \) that the neurons \( n_i, n_j \) in these sites enroll themselves in the solution of the proposed task. The present clearly shows (see the \( h(r_{ij}) \) max/min values in Table 1) the adequacy of this formal approach since the actual values \( h(r_{ij}) \) calculated for each arithmetic calculus and each experimental group obeyed the theoretical conditions required to guarantee that agent commitment has always a positive value. Also, the mean \( h(r_{ij}) \) was statistically smaller for male than females in all groups, which may be an explanation of the fact that the males were quicker than females in solving arithmetic questions [43,45]. Finally, the mean \( h(r_{ij}) \) was smaller for adults in comparison to children, but adults were faster than children in doing the same calculations.

The \( h(r_{ij}) \) values were used to construct different brain mappings—CCM and FMs—associated to each arithmetic task. These mappings corroborated the male/female and adult/children differences in performance described by Rocha et al. [40,42], and disclosed some interesting characteristics of the brain activity associated with the arithmetic calculations (Fig. 4). These results will be discussed here taking into account the model proposed by Rocha et al. [40,42] for the arithmetic neural circuits, discussed before and illustrated in Fig. 1.

4.2. Males×females

The adult CCMs were very similar for both male and females and for all types of calculation (Fig. 4). They show that anterior and bilateral areas were strongly committed to solving any type of arithmetic calculation. The three FMs show, however, that these anterior areas enrolled other neurons distributed over the entire brain in different ways when male and females are considered.

If addition and subtraction are considered, the male FM1 shows that the activity of neurons recorded by the electrodes over FP1, F3, FZ, C3, CZ and PZ are strongly correlated, and are also moderately associated with P3. The female MF1 and MF2 show a high correlation between the activity over FP1, FZ and FP2 (MF1) and over C3, CZ, P3 and PZ. Taking into account these data, it is possible to propose that...
Fig. 4. Brain mappings of the arithmetic calculations. CCM—$h(r)$ mean mappings. FM1, FM2 and FM3—factor mappings generated by principal components analysis. AD—addition. SU—subtraction. MU—multiplication. DI—division. CHI4—children enrolled at 4th and 5th semesters of the elementary school. CHI2—children enrolled at 2nd and 3rd semesters of the elementary school. All mappings are normalized into the closed interval [0,1], and color encoded according to the rule: 1 is red and 0 is black.
a set of left frontal neurons \( (N_f) \) and another of bilateral central-parietal \( (N_{cp}) \) cells are involved in solving both adult addition and subtraction. On one hand, the female \( N_f \) is more anterior than male \( N_f \) and this latter is more lateral than the former. On the other hand, \( N_{cp} \) may be assumed more similar for both genders, although sharing an association with \( N_f \) that is strong in the case of male and it is almost absent in the case of female. Male MF3 display a strong correlation among areas in the right hemisphere, whereas MF2 points to associations between sites in the left hemisphere and the posterior cortex. Female MF3 display patterns similar to male MF3 and includes also some of the areas appearing in man’s MF2. Thus, it may also be proposed the existence of a third set of neurons \( (N_v) \) distributed more laterally in both hemispheres. Again, the neurons of this third set are more correlated in male than in the female. Finally, it may be assumed that \( N_f \) and \( N_{cp} \) are sets of neurons involved with the task solving whereas \( N_v \) is in charge of the task visual computations.

### 4.3. Some proposals from the literature

Frontal and central-parietal or temporal and parietal components have been described in many fMRI studies about addition and subtraction \([1,4,5,8,21,27,37,48,49]\) and ERPs \([20,23,46]\). All these authors speak of widespread areas involved in arithmetic calculations, and stressed both left frontal and parietal areas as common and important components of the arithmetic brain. Some of these authors have suggested that the frontal component of such circuitry (our \( N_f \)) is much involved with the complexity of calculation, besides other duties \([21,23,27,36,48,49]\). The \( N_{cp} \) component is generally assumed having a bilateral distribution and a specific role for arithmetic computations, and its activity is described as mostly dependent on the type of calculation and the number size \([4,5,8,13,16,27,31,37,48]\). Some authors have also referred to other visual and verbal components associated to arithmetic calculations that involve other neural circuits (e.g., Refs. \([5,8,49]\)). Among all the papers listed above, only Skandries et al. \([46]\) have reported that female consistently have larger global field power in EEG than males, and they also displayed different scalp field topography of various components. These authors also stressed that early visual processing ERP components were gender sensitive. The gender differences on calculation time for both addition and subtraction \([40,42]\) may be explained by the use of different strategies by man and woman, that are reflected in a better coordination between \( N_f \) and \( N_{cp} \) (as shown by FM1) in the case of male than the association disclosed by MF1 and MF2 in the case of females. In addition, these differences seem to be supported by a different \( N_v \) enrollment in both male and female task solving.

Rocha et al. \([40,42]\) have shown that bilateral but not left or right parietal congenitally brain lesions considerably impairs the capability of this disabled children to learn crisp numbers and arithmetic calculations. Additionally, they reported that left frontal lesion delays but does not prevent arithmetic learning, although it may results in clear difficulty in handling ordinal in respect to cardinal numbers. These results may be assumed to support the above proposition of a left \( N_f \) and a bilateral \( N_{cp} \) circuit.

### 4.4. A proposed model

Now, let us propose the \( N_{cp} \) as the set of neurons in charge of implementing the set of accumulators A and the KFN and CBN circuits (Fig. 1), whereas \( N_f \) is the set of neurons implementing both the ordinal numbering control of the counting pathway \([40,42]\) and the gate control of the KFN and CBN circuits exercised by the C neurons, one of the components of \( N_f \). If this is such, then addition and subtraction solution may be solved by using the KFN and CBN circuits by means of both simulation and formal calculation, as proposed by Rocha et al. \([40,42]\). In the case of formal calculation, the visual displayed operands are recognized by visual neurons (R neurons) that project directly to the corresponding I neurons. In this way, e.g., \( 4+3 \) (or \( 7-3 \)) involves the decoding of the numerals \( 4 \) (7) and \( 3 \) by some specific neurons in the visual associative cortex and their semantic evaluation by means of the I neurons. The type of calculation \( + \) or \( - \) is visually recognized and semantically decoded by other neurons of the \( N_f \) set, since the C neurons are in charge of the different calculus simulations proposed by different authors in the literature \([2,6,9,24,45,47]\). In the case of simulated operations, numbers representing quantities may first be decoded as a set of elements that are then used to mentally simulate the process of counting (up or down) sets representing the operands. Hybrid calculations may be processed by loading one of the numbers directly on the accumulator a?A using the corresponding identifier i?A and decoding the other as a set of elements, whose elements are sequentially accumulated on the same a?A through the adequate gate g?G. If the operands are first ordered, then this hybrid calculation may be optimized by loading directly the highest operand and decoding the other as a set of elements. Thus, both the KFN and CBN may allow male and female to use distinct adding/subtracting strategies (e.g.,\([12]\) and Ref. \([45]\), what in turn shapes their \( N_f \) and \( N_{cp} \) and results in their distinct MFs. It is easily verified that simulated calculations may render addition time dependent on the size of operands, whereas the formal calculation makes it constant. Since different sets of agents may enroll to solve the same task proposed to a DIPS, then it may be proposed here that man/woman may use different strategies of recruiting the KFN and CBN for their calculations that could explain the gender differences observed for the calculation response time \([40,42]\) and for the MFs.

The present findings about multiplication may easily be understood according to the proposed model, too. Both male and female FM2 show that \( N_{cp} \) neurons are engaged in this
task, being left (male FM1) or right (female FM1) controlled by the Ne circuit (see Fig. 4). Many fMRI studies have also disclosed both frontal and parietal components associated to multiplication \([4,37,46,49]\). FM3 (associated to Ne) shows a strong correlation among right hemisphere areas in the case of man, and its almost absent in females. Again, the neuronal enrollment is different between sexes. Counting by block (or multiple counting) may explain product solving \([40,42]\). Either a formal block counting is left controlled in the case of man or a block counting is visually (right) simulated by woman. This could explain male being faster than female, although the difference being the smallest observed for all calculations. This small difference may be accounted by the heavy dependence of multiplication solving on a sex equally MF2 associated to the Ncp set of neurons. It must be remembered that Skrandies et al. \([46]\) also pointed that scalp field distributions were affected by sex, indicating the activation of different neuronal assemblies during visual information processing of males and females.

Division is the operation that most differed concerning the factor mappings. In the case of man, MF1 showed a strong correlation for FP2, F8, FZ and CZ, whereas MF2 displayed an association between F3, C3, P3, PZ, P4, O2 and OZ. It also resulted in a bilateral pattern of MF3. To the best of our knowledge, only Skrandies et al. \([46]\) have also included division among the calculations proposed to their volunteers, but these authors did not described their results taking into consideration the distinct types of calculus. Perhaps, adults make more use (as reflect by MF3) of verbally encoded rules of thumbs (e.g., all even number are divided by even numbers, etc.) to orient calculation. On one hand, perhaps male used this type of information to better control the Ncp neurons distributed bilaterally (see male division MF2 in Fig. 4), or to perform other visual formal operations (see male division MF1 in Fig. 4) such as line number sectioning \([50]\) as reported, at least by one volunteer. On the other hand, female may have used the same type of information to orient different counting up/down strategies to solve division since her cerebral pattern for this operation has some similarity to her subtraction MFs if one consider that most of subtraction MF2 is present in MF1 division and that there is some resemblance between subtraction MF3 and division MF2. This may be an indication that female made more use of the multiple counting up/down strategies discussed by Rocha et al. \([40,42]\), and may explain why division calculation time differences between man and woman was the second most important.

4.5. Adults and children

The CHI4 and ADU brain mappings for addition and subtraction were more similar than those for multiplication and division. But children were less trained in these latter calculations than on addition and summation. Their addition and subtraction calculation times are statistically smaller than the corresponding multiplication and division times, too \([40,42]\). It is also interesting to note that children used a lot of finger point and marking while solving any arithmetic question, whereas adults used mental simulations instead of these overt manipulations for the same purpose. This could be the main source of the adult/children brain mapping differences and explains the fact that children are slower and err more than adults in doing any kind of arithmetic calculations.

CHI4 and ADU male addition mappings share more similarities that the female ones. The boys’ CCM includes the adult cerebral areas plus a moderate activation over P4, T5 and OZ; their MF1 disclose a correlation pattern very similar to the adult one, and their MF2 may be assumed as a combination of the adult MF2 and MF3. On the contrary, female mappings differed between adults and children, if MF3 is not considered. High \(h(r)\) values were obtained for the CHI4 girls over C3, FP1, F4, FZ, C4, PZ and P4 and OZ and these areas seemed to be well correlated as disclosed by MF1. Additionally, F3, C3 and O1 are the only high correlated areas in children MF2. The similarities between children and adults are even greater in the case of subtraction, although the gender differences appear to be greater between boys and girls than between man and woman. These boy and girl differences and the similarities between children and adults may be understood as if children were still developing the Nf and Ncp set and that boys were more advanced than girls in building them. This could explain the boys being faster than girls in arithmetic calculation.

There are more similarities between girls’ multiplication and division mappings than between the corresponding woman or boy mappings. In addition, these arithmetic mapping differences were clear between men and boys. Taken together, the MFs disclose patterns of greater coordination among enrolled areas in the case of boys compared to girls and in the case of adults compared to children. Also, the CCMs seem to evidence a neuronal recruitment to solve multiplication and division that is greater for girls than for boys and adults. All of this may be understood as multiplication and division children Nf and Ncp buildup that is less developed than their corresponding modeling of addition and multiplication, a fact that parallels the difference in the amount of their training on these different arithmetic operations.

Finally, although there are some similarities among the CHI2 and CHI4 CCMs, the differences between their MFs are well defined. But the CHI2 calculation times were also very different from those obtained for the CHI4 group. Again MF patterns seem to correlate with the amount of training experienced by and the degree of proficiency attained by children.

4.6. Conclusion

As a general conclusion, it would be said that the main difference between the adult and children brain activity patterns, as disclosed by MCs and MFs, is a larger children
neuronal enrollment in comparison to the adult due, perhaps, to more generalized use by children of simulated than formal counting and calculations in comparison to the adult, and also of a better adult Nf and N cp circuits development. The present results about the brain activity associated to arithmetic problem solving in children are, at the best of our knowledge, very original. Because of this, the present discussion about the young arithmetic brain is much more speculative than the present modeling of the adult arithmetic circuitry. The search in the literature for papers dealing with arithmetic, learning and brain activity selected only the work of Menon et al. [27], a fMRI study about optimization of arithmetic processing in perfect performers. These authors showed that activation of the left angular gyrus was training dependent.

In addition, the gender differences reported here, for both adults and children are very novel data. Although it is a well-established fact that boys outperformed females in the SAT-Math exam and it has been proposed that test solving speed and strategy flexibility may be explanations for such a finding [11,43], the present work seems to be one of the first attempts to provide an ample analysis of the possible brain activity differences between gender concerning arithmetic cognition. To our knowledge, only Skandries et al. [46] have reported that female consistently have larger global field power in EEG than males, and they also displayed different scalp field topography of various components. These authors also stressed that early visual processing ERP components were gender sensitive. The present results show marked differences between sexes on brain processing supporting arithmetic calculus and that these differences are present since the very beginning of the academic training. In addition, differences were found for male and female adults that experienced a more similar and important mathematical training because they are successful postgraduate students in the field of technology. All these facts seem to point toward an important phylogenetic component of these gender differences, that may in part be augmented by a gender influenced culture, which have also a phylogenetic root.

Appendix A. Studying the brain as a distributed intelligent processing system

The DIPS approach of the cerebral physiology results also into new technologies for analysis of the EEG activity related to cognitive tasks [10]. If ρ(ni,nk) measures, in the closed interval (0,1), the possibility of any two neurons ni,nk to jointly involve themselves in solving a task t, and

$$\rho_m(n_j) = 1/n \sum_{k=1}^n \rho(n_j, n_k)$$  (1)

$$\rho_m(n_j) - 0.5 = \xi$$  (2)

then

(a) N is said to be a strongly (un)connected system Nk if \(\xi = 0.5\) for most of \(a_i \in N\). Strongly (un)connected systems are of no interest if cognition is concerned, because either their agents have difficulties in enrolling together to solve any task, or the relations shared by their agents tend to be stereotyped rather than versatile; their agents are more likely to share strong than plastic commitments.

(b) N is said to be a loosely connected system Ni if \(\xi > 0\) for most of \(a_i \in N\) even if \(\rho(a_i,a_k) = 1\) or 0 for some, but not all \(a_k \in N_i\). A DIPS is a loosely connected system, because each of their agents retains the maximum capability to enroll with different groups of other agents to try to solve different tasks, since \(\rho_m(n_j) = 0.5\).

In this context: (a) the commitment \(h(n_j,k)\) of \(n_j, n_k\) in jointly solving a given task is calculated as:

$$h(n_j,k) = -\rho(n_j, n_k)\log_2\rho(n_j, n_k) - \sim \rho(n_j, n_k)\log_2 - \rho(n_j, n_k)$$

$$\sim \rho(a_i, a_k) = 1 - \rho(a_i, a_k)$$  (4)

(b) the enrollment capability \(h_m(n_j)\) of \(n_j\) is calculated as

$$h_m(n_j) = -\rho_m(n_j)\log_2\rho_m(n_j) - \sim \rho_m(n_j)\log_2 - \rho_m(n_j)$$  (5)

$$\sim \rho_m(n_j) = 1 - \rho_m(n_j)$$  (6)

that is, \(h_m(n_j)\) is a function of the mean possibility \(\rho_m(n_j)\) of \(n_j\) to communicate with the other agents \(n_k \in N_i\), and(c) the actual commitment \(h(n_j)\) of \(n_j\) to solve the task is

$$h(n_j) = \sum_{k=1}^n h_m(n_j) - h(n_j,k)$$  (7)

Now, the linear correlation coefficient \(r_{ik}\) for the EEG activities recorded at \(d_i,d_k\) is assumed to be an indirect measure of the possibility \(\rho(n_i,n_k)\) of the enrollment of the neurons \(n_i,n_k\) at these locations in solving a given task. In this way, the (mean) EEG activity recorded associated to a given event e of such the task t is used to calculate the corresponding \(h_e(n_j)\) for each recording site \(r_j\). The set \(H_e\) of all \(h_e(n_j)\) values for all similar events e are used to generate different brain mappings (Fig. 3) according to the statistics applied to analyze \(H_e\). Foz et al. [10] used this technique to study the plasticity of language neural circuits in brain-damaged children, having the classical Broca or Wernicke area’s destroyed during their fetal life. These children experienced severe language development delays, but by age of 11 years old, their brain mappings associated to oral charade and texts revealed an extensive use of the
right hemisphere areas to compensate the lesion of their homologue left sites.

References


