Winter February 1, 2016

Learning to Apply Algebra in the Community for Adults with Intellectual and Developmental Disabilities

Anthony Rodriguez

INTELLECTUAL AND DEVELOPMENTAL DISABILITIES
2016, Vol. 54, No. 1, 19–31

DOI: 10.1352/1934-9556-54.1.19

Learning to Apply Algebra in the Community for Adults With Intellectual Developmental Disabilities

Anthony M. Rodriguez

Abstract

Students with intellectual and developmental disabilities (IDD) are routinely excluded from algebra and other high-level mathematics courses. High school students with IDD take courses in arithmetic and life skills rather than having an opportunity to learn algebra. Yet algebra skills can support the learning of money and budgeting skills. This study explores the feasibility of algebra instruction for adults with IDD through an experimental curriculum. Ten individuals with IDD participated in a 6-week course framing mathematics concepts within the context of everyday challenges in handling money. The article explores classroom techniques, discusses student strategies, and proposes possible avenues for future research analyzing mathematics instructional design strategies for individuals with IDD.

Key Words: *adults with intellectual and developmental disabilities; algebra; experimental curriculum; money; mathematics instruction*

Typically, students with intellectual and developmental disabilities (IDD) learn basic, remedial mathematics skills that consist of adding, subtracting, multiplying, and dividing numbers (Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008). They then progress to classes that focus on life skills (Browder, Ahlgrim-Delzell, Courtade-Little, & Snell, 2006; Browder et al., 2008; Xin, Grasso, DiPipi-Hoy, & Jitendra, 2005). More often than not, students with IDD are not exposed to higher-order math concepts such as algebra. Martinez's (1998) adaptation of an algebra curriculum for two students with Down syndrome was the first known example of individuals with IDD learning algebra in an inclusive setting. A more recent study used systematic instruction for basic pre-algebra skill acquisition, a common goal of many remedial programs (Jimenez, Browder, & Courtade, 2008). A unique feature of this study was that adults with IDD explored algebraic ways of thinking and skill acquisition, within an applied, context and feedback-rich environment with real world applications (Chan, Konrad, Gonzalez, Peters, & Ressa, 2014). The mathematics lessons were used to understand the individual's experiences, and to link academic mathematical skills with life skills in a specific community of practice (Lave & Wenger, 1991; Gutstein, 2003). Though the math education literature on students with IDD is growing, little is known about how these individuals respond to learning algebra and what techniques are useful for teaching algebra to students with IDD.

Research has established the development of informal number sense as a strong predictor of future success in mathematics (Duncan et al., 2007). A few have uncovered the need for structured-discovery learning for developing meaning in mathematics for individuals who struggle (Bottge, Rueda, Serlin, Hung, & Kwan, 2007). Further, adult learners will engage in learning when it is relevant to their experience (Gregson & Sturko, 2007). Combining the need for number sense, the support of structured discovery in what the adult perceives as "necessary information," and the utility of algebra in applied real-life settings is a relatively new avenue for structuring learning activities for adults with IDD. The purpose of this study is to explore the feasibility of algebra instruction for adults with IDD and to propose priority avenues for future research on classroom analysis and instructional design for these individuals. Policy documents that have guided needed reforms in mathematics education in the United States recommend that all students should solve real-world problems every day in novel ways (see, for example, NCTM, 1989; 1991; 2000; NSF,

DOI: 10.1352/1934-9556-54.1.19

1996). All students include those with disabilities. Moses and Cobb (2001) extend this notion of access by declaring the learning of algebra as a civil right. Why not students with IDD?

Algebra is generalized numeracy (Usiskin, 1988) and can be taught to most people at any level. Algebraic strategies are an ideal way of finding optimal paths, comparing different rates, and revealing patterns. For example, when securing a loan, interest rates, time, and payments are calculated by algebraic variables. Understanding the math provides a better understanding of fees and interest. One can work with algebra at most levels of numerical understanding.

A common argument is that given pressure for academic coverage and functional outcomes, time is better spent on a curriculum of practical applications and skill development (Ayers, Lowry, Douglas, & Sievers, 2011). Algebra is simply a way of understanding and applying numbers through relevant practice. Educators can deepen content, shrink coverage, and develop meaningful mathematics in real contexts using standards-based instruction (Courtade, Spooner, Browder, & Jimenez, 2012; NCTM, 1989, 1991, 2000; NGA & CCSSO, 2010) through the mindful implementation of algebra in daily practice for people with IDD.

Published studies of algebra instruction for those with IDD (Jimenez, Browder, & Courtade, 2008; Rodriguez, 2013) or explorations of math learning among students with significant cognitive disabilities (Browder et al., 2008; Jimenez & Kemmery, 2013) focus on children. Yet, what happens for the many adults who missed this opportunity and are in need of foundational number-sense skills? Is it feasible to teach algebra through unscripted, yet methodically developed lessons grounded in the learner's daily experiences?

In this study, adults with IDD were taught applied problem solving (Ernest, 1988; Hiebert et al., 1996) in algebra, using real situations pertinent to their lives. The general approach was to encourage conversation on meaningful topics in class and mine the responses for mathematical content to be used to develop problems that affected their lives.

Methods

Participants

To recruit participants for the study, I solicited the help of a nonprofit organization that trains self-advocates in a wide range of areas, such as job skills, life skills, and relationship skills, throughout the state of New Mexico. Persons served by the organization exhibit a range of developmental disabilities, including intellectual disability, autism, cerebral palsy, spastic triplegia, traumatic brain injury, and multiple disabilities. I had volunteered at the center in the past and was impressed with the way in which individuals with IDD worked on self-advocacy and community-based activism. These individuals looked like the optimal group to work with on an experimental math program. Through my experience teaching math to children with disabilities, I understood the value of linear algebra and its natural application for understanding financial concerns by modeling of raw numbers and projecting financial models across time.

Using the offices of the league, we posted flyers inviting individuals to participate in an opportunity to learn financial literacy through the Money Club. The Money Club was used as a means of teaching algebra through real-life examples taken from students' capacities, interests, and habits (Dewey, 1929). Many members of the group had money problems, histories of being cheated by scams, and gambling concerns. The title of the course was "The Money Club" to encourage the participants to attend, combine finance with algebra, and lower their anxiety related to mathematics.

Criteria for inclusion were that participants must be at least 18 years of age, that they were able to provide consent from a guardian or assent from the individual, and have IDD. Participants of the study varied in terms of identity markers and geographic location. There were a total of 10 participants (see Table 1)—six men and four women. There were five Caucasians, two Mexican Americans, one Hispanic, and two of mixed ancestry of Hispanic and Caucasian. (Due to cultural distinctions specific to this region of the country, Mexican American and Hispanic categories were disaggregated to honor these differences.) Participant ages ranged from 22 to 45 years, with a mean age of 31. Eight participants lived in the city, and two were from rural areas. We met twice a week for 2 hr per class over a span of 6 weeks, for a total of 12 classes. All participants had a 75% attendance rate or higher in the 6 weeks of classes.

The study and all specifics for participation were reviewed with the individual and guardian, if applicable, at the organization's location. A number of individuals took advantage of the

Table 1
Participants With IDD at a More Severe Level: Background and Participation

Participant	Gender/Ethnicity	Age	Diagnosis	Preinterviews	Postinterviews	Attendance
Lynne	F/Mixed	45	Bipolar Disorder/IDD	Yes	Yes	75% (9/12)
Helen	F/Caucasian	42	Autism/TBI	Yes	Yes	100% (12/12)
Amy	F/ Caucasian	26	IDD	Yes	Yes	100% (12/12)
Mindy	F/Caucasian	27	IDD/SLD	Yes	Yes	92% (11/12)
Wes	M/Mixed	44	Multiple Disabilities	Yes	Yes	92% (11/12)
Robert	M/Caucasian	30	Multiple Disabilities	Yes	Yes	83% (10/12)
Antonio	M/Mexican American	22	Autism	Yes	No	83% (10/12)
Gary	M/Hispanic	27	Autism	Yes	No	92% (11/12)
Patrick	M/Caucasian	22	IDD	Yes	No	75% (9/12)
Alejandro	M/Mexican American	24	IDD	Yes	No	75% (9/12)

Note. F = female; M = male; IDD = intellectual and developmental disabilities; TBI = Traumatic Brain Injury; SLD = Specific Learning Disability.

opportunity to participate in the Club without participating in the study (their audiotape data was excluded from analysis). Due to the open, "drop-in club" environment of the league, this type of participation was common. The University of New Mexico conducted a full Institutional Review Board (IRB) and permitted the study.

Instructional Approach

The instruction approach was guided by the *design experiment* framework, a qualitative form of inquiry rooted in research that emerged from teachers' daily practices in the Netherlands (Gravemeijer, 1994). The design experiment approach is used to generate theory through practice with detailed documentation of the adjustments the teacher makes to affect instruction (Brown, 1992; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; diSessa & Cobb, 2004; Edelson, 2002; Gravemeijer, 1994; Stephan & Cobb, 2003; Stephan, Cobb, & Gravemeijer, 2003). The approach was selected for use in this study because of the method's focus on capturing data during classroom instruction, using it to formatively assess instruction and revise teaching in real time. Through these "minicycles" of instruction, assessment, and revision, the approach attempts to optimize curricular innovations through structured analysis and reflection components, which lead to new lessons and supports.

The *design experiment* approach is comprised of two interwoven components: instructional design and *classroom analysis of mathematics practices* (Cobb et al., 2003; Stephan & Cobb, 2003). Data consisted of classwork, preinterview and postinterview transcripts, taped class sessions, and instructor notes and reflections. Data were reduced into themes in a poststudy analysis (Cobb & Whitenack, 1996).

Instructional Design

Within the 6-week unit on the financial applications of algebra, there were 12 classes (see Table 2). Each class was composed of minicycles that had four components: (a) the preteaching instructor notes that framed the lesson plan; (b) the lesson plan; (c) class-based instruction; and (d) the writing of instructor notes for the next class, which completed the cycle. The intent of the "minicycle" is to analyze and improve on instruction during the flow of instruction.

Classroom Analysis

Data were analyzed concurrently throughout the study to reach a grounded understanding of the classroom experience; a primary feature of a *design experiment* approach is to develop theory in support of learning by generating and refuting conjectures about the process of teaching and learning.

DOI: 10.1352/1934-9556-54.1.19

Table 2
Themes of the 12 Classes

Themes	Description
Patterns in nature and the community	Finding numerical patterns and symmetry throughout the day (trees, equal seats on each side of the bus, tiles)
Chopping up wholes: Fractions	Folding and tearing paper, breaking paint stirrers
The art of being cheap: Living within our means	Looking at our earnings and our spending
Why you never buy anything except fountain soda at a gas station mart (smart shopping)	Comparing prices of discount, standard, and corner stores
Daily compounding savings: Brew your own coffee	Comparing daily coffee purchases vs. brewing coffee
Composition and decomposition: How to make a 21 (Blackjack joining and separating)	Using playing card values to make a 21
Pawn shop mathematics	Finding percentages of numbers and values of pawned items vs. purchase price
Credit cards are evil	Internet search of variables in credit cards and application of variables in a monthly bill
Pay day loans are predatory animals (applying for a real loan)	Calculating interest accrual with a pay day loan vs. a loan from a bank
Don't trust an investor in a bow tie (investing and Ponzi schemes)	The math and history behind a Ponzi Scheme, finding rates of return at each level
Saving and investing using 2 step algebra: Tying and untying of knots	Taking money earned and expenditures to calculate how much money they can save each week
Fraud and you	Tips for identifying fraud and what to do

Instructor notes documented the daily choices, understandings, and actions within the classroom and how I adapted curriculum to meet the needs of the Money Club participants. The notes were also used as a formative assessment of participants' learning and my teaching and were the central lynchpin that connected the other data streams together.

I wrote instructor notes each day, Monday through Friday. On Monday, I wrote down ideas that I had over the weekend, organized notes from the week before, and made the outline of a lesson plan. The rest of the week I wrote about my own perceptions of the learning and the contexts from which the learning took place grounded in the actions of the class from warm-up problem solving to group interactions. The content and structure of the lessons were derived from these forms, creating the diverse and multileveled foundations for student learning, as well as the math problems and groupings within the classroom.

Learning was documented through discussion, group and individual work, student participation, student questions, and problem solving. Open coding was used (Emerson, Fretz, & Shaw, 2011) to generate as many codes and ideas as possible. Later, a more focused coding structure was used to identify embedded patterns and relationships.

Procedures

Before the unit began, each participant was interviewed for approximately 30 min to get a sense of the participant's background in mathematics, education, interests, life concerns, and ways in which the participant learns best. There were a total of 24 hr of taped classroom observations, 50 pages of transcribed preinterviews and postinterviews, and 96 transcribed pages of instructor notes, as well as extensive lesson plans.

There were 20 original open codes related to the classroom analysis and instructional design.

DOI: 10.1352/1934-9556-54.1.19

These were reduced into four focused codes or main themes of the study found at the bottom of Table 3.

Results: Classroom Learning and Instructional Design

The results suggest how teachers can instruct individuals with IDD to learn algebra in the context of personal finances. The results are organized under the two core components of a *design experiment*: *Classroom Analysis of Mathematics Practices* and *Instructional Design* (Stephan & Cobb, 2003). Both components provide an understanding of student learning in the classroom and a means to improve daily instruction.

Classroom Analysis of Mathematics Practice

Leaving markers. Two of the participants (Helen and Wes) used a problem-solving approach that left out key steps normally used in solving the problem. Both reported solving the problem mentally and would simply write down numbers as "markers." According to them, recording the numbers as markers to each line in each step of a single problem, gave them enough support to proceed to the next task of problem solving and to finish the problem (see Table 4). This tool reduced the cognitive demand of the problem by not having to hold many numbers in their heads while combining it with the next variable, reminiscent of Baroody (1984, 1986, 1988). By reducing cognitive demand and lowering the amount of working memory needed to accomplish complex tasks, the learner can complete higher-level mathematics problems. Helen talked about solving the problems in "her head," but had difficulty stating why she took such an approach. Helen's ability to problem solve without being able to articulate the rationale is not unlike how people perform skilled behaviors without thinking about how they do it.

The variable x in this problem stands for *hours* worked, -5 (constant) is the *cost* for a round trip bus ticket ($2.50 each way) to work, the 5

Table 3
Open Coding Framework for Development of Themes

Classroom Analysis of Mathematics Practice	Instructional Design
Open Codes	
• Invented strategy	• Opening sequence (none, 10–30 minute)
• *Standard* algorithm	• Participation in (large group, small group, independent)
• Show pathway for problem solving (full/partial/none)	• Hands raised
• Math in their heads	• Talking about math
• Shelving numbers	• Equity in participation (high, medium, low)
• Counting (fingers, manipulatives, etc.)	• Small-group sharing load of work (high, medium, low)
• Using (+, –, x, ÷) to solve problem	• Feedback from teacher (specific, general, none)
• Using a reference when talking about math	• Role models in groups
• (Concrete, representational, abstract, none)	• Direct instruction in small groups from teacher as (short tune-ups, continuous, none)
• Finishing work (early, with group, lagging behind)	• Participation (math relevant, theme/object relevant, off topic)
• Disruptions, complaints, walkouts, transitions	
Focused Codes/Themes	
• Leaving markers: Freeing up cognitive space for problem solving (invented strategies vs. standard algorithm)	• Front loading developing concepts first to support problem solving
• Concrete supports drive mathematics investigations	• Importance of full participation in the Money Club

DOI: 10.1352/1934-9556-54.1.19

Table 4
Standard Algorithm Versus Helen's Way

Standard Process for Solving 2-Step Equations	Helen's Way Stashing Numbers Aside to Open Cognitive Space and Further Process Problems
5X − 5 = 20	5X − 5 = 20
+ 5 +5	25
5X = 25	
5 5	5
X = 5	

(coefficient in front of the x) is for the *hourly rate* estimated after taxes ($7.50 per hour at the standard $\frac{2}{3}$ take-home rate adjusted for taxes and fees), and the number 20 is for 1 day's take-home pay for work when she works 5 hours. Normally, the participants show how they increase by 5 on both sides to get a 25 on one side of the equal sign and a $5x$ on the other, and then divides by 5 on both sides to get $x = 5$. Helen bypasses these procedures and just "unties the knot" or decomposes the number in her head, leaving markers to guide her for each particular step (25 for one step, and 5 for the second one). One interpretation of Helen's strategy is that it takes fewer steps, freeing her up to complete the problem (recalling two distinct steps instead of the multiple number placements and steps in standard algorithm).

Invented strategies versus standard algorithm. The members of the Money Club used both invented strategies and standard algorithms. Invented strategies are creative ways of problem solving designed by the participants that are not the traditional path to the answer. After the first few classes, participants began to take risks and use invented strategies to solve problems. For example, Lynne developed a "half of a half" invented strategy similar to the work of Baroody, Ginsberg, and Waxman (1983) for solving problems involving fractions with even denominators. For example, when finding $\frac{1}{4}$ of the number 8, you break the initial number in half (4), then break the second number in half to get a fourth (2). She claimed to have thought it up in her head and was not sure she was doing the mathematics correctly. She tested out her method, finding that it worked in other fraction problems, and then employed it as a tool for all fractions problems. After solving one problem well, she was confident that she could continue to solve others. Her initial successes created momentum in problem solving, deepening her understanding and skill in using this procedure in mathematics.

Concrete supports drive math investigations. Another effective system for the Money Club was first to teach with concrete, three-dimensional mathematics "models" (e.g., broken rulers, string, blocks) as supports and graphical illustrations. The instruction then moved toward more abstract models, a technique found to be successful in both science (Goldstone & Son, 2005; Moreno, Ozugul, & Reisslein, 2011) and algebra instruction (Witzel, 2005) as individuals became more secure in their understanding and abilities. When the class discussed mathematical concepts with a model or drew pictures on the whiteboard, the lesson began to gain traction. The models stimulated discussion and connecting prior knowledge and understanding of mathematics to the concepts of the day. For example, during an exercise on breaking and rejoining, Robert modeled breaking a paint stirrer; and the others joined in the excitement and broke their stirrers. The pieces were mixed up in a pile in the middle of the classroom and students were asked, "If we broke these rulers and you had to take one piece and toss the other in a pile with all of ours, how would you know if you found your missing piece?" They were told to piece their paint stirrer back together. After they found the missing

DOI: 10.1352/1934-9556-54.1.19

piece and rejoined the paint stirrer, they were prompted with, "What does algebra mean?" Mindy said, "fractions," while Robert stated, "You split them up and then you put them back together," which is an exact definition of rejoining in algebra. They had to find the missing piece and rejoin the whole part. Afterward, the class discussed other instances, such as bone fragments in crime scene investigations (CSIs), for which mathematics is useful for thinking of how to piece the broken pieces back together.

As part of the discussion, the group was asked for ideas on how to engage math in investigations to find the completed whole. The examples varied from concrete to abstract. To facilitate the discussion, I demonstrated one problem-solving technique. Then, the class was divided into four groups and asked to formulate group examples for problem solving. Different operations for finding the missing piece emerged from the discussions. For example, Amy's approach involved the process of elimination and then summing the two pieces ($23 + X = 53$, $X = 30$) to find the whole. That is, Amy added up from 23 to find 53, then would search through the pile and add the measurement of each piece over and over again until she found a chard that was exactly 30 cm long. In contrast, Robert took the whole measurement of a joined piece (53 cm) and subtracted his piece (23 cm) to know that the missing piece must be 30 cm. They had to find alternate routes to solve the problem, which created many connections to the math problem and paths to the answer. This approach (see Table 5) served many purposes, one of which was to reinforce learning from prior classes while developing the disposition for mathematics found in Common Core Math Practice 1(CCSS. MATHPRACTICE.MP1, NGA &CCSSO, 2010).

Table 5
How Participants Found the Missing Pieces After Breaking Paint Stirrer in Two

How They Found the Missing Pieces	Participant
"I fit them together."	Amy
"I added them together."	Lynne
"I used subtraction, actually."	Wes
"I used my mathematics skills."	Robert
"I narrowed it down."	Mindy
"I checked them."	Antonio
"I just figured out the piece by looking."	Patrick

Another example of exercises in the application of algebra concepts emerged from the participants reporting drinking coffee purchased daily at the local convenience store or coffee shop. Many of the participants lived below the poverty line. The question was posed, "Can brewing your own coffee get you to Hawaii?" The students engaged in a comparative analysis where they determined the cost of buying coffee versus self-brewing. Using unit cost of bulk coffee, milk, and sugar, the students used calculators to solve for per-cup costs of purchased versus self-brewed coffee. The group graphed the amounts of money saved per week to determine the number of weeks it would take to purchase a plane ticket.

Instructional Design

Front-loading and developing concepts first to support problem solving. "The study indicated that "front-loading" concepts, or slowing down instruction in the beginning to help with understanding, improved or sped up subsequent learning. Use of front-loading was influenced by Claxton's (1997) concept of "slow knowing," similar to Hiebert and Carpenter's (2005) recommendation to emphasize conceptual development first, with skill work later in each learning cycle. Primary support came from the group leaders, who helped struggling classmates in small groups by giving their peers a clear understanding of the task and its meaning. Initial conceptual development sometimes ran 30 min or more and included eliciting class participation to bridge participants' prior knowledge and experience to the mathematic concept being taught. Investing time at the beginning of class to develop concepts allowed participants to ease into problem solving while building communication, building trust, and ultimately increasing participation.

Ideas on supported problem solving. As the study progressed, many participants gained confidence in their problem-solving abilities and cited examples from the Money Club to back up their statements, suggesting the malleability of disposition and drive (Pink, 2009). Analyses of interviews, daily notes, and recorded class sessions revealed improved confidence in problem solving as students were exposed to more material. The negative experiences from past mathematics classes must be addressed if risk taking is to occur in class. Robert's disposition toward problem solving

DOI: 10.1352/1934-9556-54.1.19

changed markedly. When challenged at the beginning of the program, he would give up and begin a sequence of negative self-talk. For example, he regularly said, "I am not actually good at math" and "I am afraid of getting the wrong answer." When asked during the postinterview what he would say to someone who doesn't like math, he stated:

> R: Well, that's an easy question. I would tell them, "Look, you've gotta know you're mathematics, if not, you're gonna end up with a problem." Let's say, hang on, let me say this, let's say we have 4 or 5 cheeseburgers and I have, eh, you have one and I have one, and the rest would be like, for Mindy, for Matilda, for Gary, for the rest of the guys. One thing is for sure, is that all of us have to divide how many cheeseburgers we're gonna eat.

In his unscripted problem idea, Robert and I would have a whole cheeseburger and the remaining burgers would then be divided among the rest of the group, leaving them with less than a full burger. His response was a word problem with fractions that he extemporaneously designed.

Ancillary instructional strategies. Several additional instructional strategies were found to be effective in the Money Club: (a) Positive, pointed, and success-oriented feedback regarding mathematics attached to clear redirections for improvement; (b) in whole groups, steady and deliberate speech underlying the concepts first followed up with skills; (c) small groups driven by well-prepared peer models; and (d) brief one-on-one teacher-led "tune-ups" to deal with problems and quick release to the other small groups. Similar observations have been made by Chan et al. (2014), Chappuis, Stiggins, Chappuis, and Arter (2012), Hiebert and Carpenter (2005) and Roscoe and Chi (2007) in their studies of the role of feedback in instruction, multiple ways of assessing knowledge, ways to develop conceptual understanding, and the ways in which peer tutors develop and deliver this knowledge.

Importance of motivation and full participation in the Money Club. Student participation was most consistent in whole-group instruction. Small-group instruction of four or fewer students had high but inconsistent participation depending on the peer model who led the group. The peer model would need to be both well prepared through the whole-group instruction and work consistently in delivering knowledge to the group successfully, as found in Roscoe and Chi (2007). One-on-one direct instruction by the teacher was subject to the law of diminishing returns. Initial burst of participation and independence following instruction deteriorated over time. Thus, I needed engaged and motivated peer models to encourage peers to work at high levels.

As the Money Club progressed, participation increased. Members noted that Money Club was fun and they were learning; they knew the flow of the class and could anticipate how a lesson was going to proceed. Individuals were excited, vying for center stage, each wanting to contribute an answer or tell a story that led to improved questioning of content, offering a "feedback-rich environment" to support every learner, found by Chan et al. (2014). This feeling reinforced was evidence that the club had built a community of practice with a culture of support, acceptance, and relevant learning. An illustration of this community of practice was an extension activity regarding fractions and percentages. The class shot basketball-style free throws into an empty wastepaper basket in a lesson on fractions and percentages (made shots divided by attempted shots). Lynne struggled converting fractions to percentages, similar to normal development of understanding decimals and fractions in Hiebert, Wearne, and Tabor (1991). She could figure out any percent based on 10 attempts but could not with other denominators. Having made two out of five shots, Lynne replied 10% when asked what percentage it was. I redirected Lynne to her "half of a half" strategy with the question, "Are you thinking that if you make the same amount with fewer attempts the percentage goes down?" and was then instructed to "Think . . . 'It doubles.'" The teacher replied, "What is $\frac{2}{5}$ as a percentage?" Lynne responded correctly this time with "40%." Many people would have had a hard time sticking with the line of questioning in front of a class, but Lynne kept at it. After many rounds of converting fractions to percentages, the class created a bar graph on the board, displaying everyone's shooting percentages. Based on the results, we decided that Gary, Wes, and Mindy would be our three best shooters in clutch situations.

DOI: 10.1352/1934-9556-54.1.19

Discussion

The two components—classroom analysis and instructional design—suggest that the adults in this study engaged in applied problem solving by employing markers and invented strategies. Participants used algebraic processes to solve problems, as well as using manipulated and visual models and graphs. The curriculum design components that emerged included "front-loading" concepts at the start of instruction, making connections between mathematics concepts and real-world contexts, and emphasizing the basic number sense to teach algebra. Finally, my observations suggest the need for all learners to develop a disposition for engaging in increasingly difficult mathematics work.

Algebra as Generalized Numeracy

The crucial supports necessary to learning algebra for this group were methodical explanation of concepts attached to real-life examples from their community (front-loading) with the employment of mathematics models, followed by repetitive skill practice in applied settings. Identification of peer models and extensive planning for daily changes in the skills of participants were essential elements of instructional design. Developing lessons relating finance to algebra that were of personal interest to the participants was critical in engaging and increasing participation. All of this was accomplished with basic numeracy skills using algebra as a vehicle for understanding and high repetition skill practice. The familiarity of using basic numeracy gave them the confidence to continue in the formative practice of algebra.

Conceptual Understanding in Mathematics

The design experiment process indicated the importance of conceptual understanding for adults with IDD as a major theme in designing instruction for these individuals. Conceptual understanding is rich in relationships and connections, which give background and purpose for solving new problems (Fuson et al., 1997; Hiebert & Carpenter, 2005). In this study, the "slow down to speed up" approach was employed to facilitate understanding of mathematics concepts. The student's grasp of concepts was understood through formative and summative assessment and pieced together by the teacher to reteach the student. Of course, teachers are faced with time limitations and must prioritize what must be taught. Conceptual understanding of mathematics is the foundational piece on which future learning depends; and concepts should ground each lesson to increase the chance of retention. For example, after an individual understands that Pi $\sim$ 3.14 is the diameter of a circle, they can use 3.14 to solve any problem involving circles. They need to know the diameter is added 3 times, end over end around the circumference, with the remaining 14/100 portion of the diameter added to complete the circle. They then have a deeper understanding of the concept and can use the efficiency of an algorithm ($A = \pi r^2$) as a tool to do their mathematical work.

During instruction, Wes and Helen were able to see the answers "in their minds" in a way that was similar to Fuson et al. (1997) in which the one- and two-step algebra equations and markers (see Table 4) enabled them to work on the next steps. They adapted to the demands of the problem and focused their working memory on one thing at a time by setting aside one answer to go after the next, then combining the two to get the complete answer for x. This method was not taught to them. By modeling the concept (e.g., breaking and joining), the teacher can explain how we find the missing pieces in algebra. Armed with the concept, the participants developed their own algorithm to solve the equations by setting aside partial answers so they could focus on each task individually to solve the whole problem.

Limitations

Caution must be taken when examining the results of the study. The individuals in this study were a select group primed for success by the support of their families and the self-advocacy organization that sponsored the Club. Not all people with IDD have a supportive network that enables such experimentation. Also, as is the standard concern with all teaching, both limited time and resources, we must prioritize what matters and teach the most generalizable and useful skills first. Long-term effects of algebra instruction need more attention in this population before making this a priority over all the required skills mandated by our districts and states.

Future Research

However, even with these concerns in mind, without access to algebra, people with IDD are

DOI: 10.1352/1934-9556-54.1.19

limited to learning purchasing, finance, and money-management skills without any context of how this works algebraically and conceptually nor why these skills should be used. Despite these challenges, evidence exists that general mathematical standards, paired with diverse and robust supports (Browder et al., 2007) and the use of clear targets (Chappuis et al., 2012) can support individuals in achieving at higher levels than previously thought.

This study was a feasibility effort and could be extended to evaluate the retention and use of skills and understanding in the real word. For example, does the learning of algebra affect the individual's purchases, use of credit cards, or use of other financial tools? Often, adults with IDD are a subset of "the working poor," on fixed incomes, living within walking distance of pawnshops, and payday loan businesses (Rivlin, 2010; She & Livermore, 2007). A concern is that these individuals are at risk for being victims of financial scams by predatory lenders (Rivlin, 2010) who often use algebra to calculate profits and gain monetary advantage over the consumer. If these individuals have a basic understanding of algebra, can they arm themselves with this knowledge to avoid being taken by such lenders?

Also, a more systematic evaluation of invented strategies would be required, including an effort to identify other novel approaches. The interaction of mathematical concepts with types of supports and methods of instruction should be explored. The degree of generalization of study findings to other participants with intellectual disability of varying levels of severity should be explored. The utility of study strategies to personal finance and scam avoidance will be needed.

Conclusion

The data in this study suggests that not only can adults with IDD learn the basics of algebra when applied to money and finance but also can think creatively and solve problems using invented strategies. Further, many of the participants, after taking this class, felt that if they could "do algebra," then they could do other things that were challenging and previously perceived as out of their reach. The Money Club became a safe place where adults both with and without disabilities could work side by side, socialize, and leave each night believing that they could do things they could not do previously.

This study addresses the importance of maintaining high expectations of all people, that hard work results in small improvements, and small improvements over a long period of time can equal tangible results. Throughout and during the post-interviews, many members specifically stated that they were shocked at how much they could do in mathematics. This feeling of empowerment may have happened because they knew they were the ones doing the "heavy lifting." Wes's postinterview reflects the change in self-perceptions:

> A: When I say the word "algebra," what do you think of?
>
> W: Um, it's very helpful, um, from what I've learned, I'm amazed, um, because, now I can do things, that I never thought I could…You know, I never thought I could do. It's a neat thing to have. You know, you know, since I was in special ed and stuff, they never taught anything like that. I figure, if you don't know something, it's always the best thing to do is try something that you don't think you can do, I think, it's, you would have, uh, a better life, if you'd at least give that a chance, you know.
>
> Wes (postinterview)

Students with learning disabilities are beginning to gain access to academic content in algebra (Foegen, 2008; Gagnon & Maccini, 2001; Kortering, deBettencourt, & Braziel, 2005; Strickland & Maccini, 2010). Is it feasible for students with IDD to enhance their mathematical performance and learning through algebraic concepts? If students with IDD can work at even incrementally higher levels of abstraction, then even partial understanding of how variables interact (fluctuation of prices from grocery store to corner mart) can help them with daily purchases and give concrete rationales for avoiding one place in favor of another. Basic conceptual understanding of compounded interest rates (credit cards or pawn shop loans) or purchases (coffee) can be the difference between sustainable financial freedom and dependency.

Through classroom analysis and instructional design, this study suggests that adults with IDD could invent strategies for solving fractions and

DOI: 10.1352/1934-9556-54.1.19

basic linear algebra equations using a context-rich, applied algebra program. When provided with robust and diverse supports such as mathematical manipulatives, concrete supports, and peer models with similar disabilities, the adults in this study demonstrated they could perform at higher levels than had been expected of them in the past.

References

Ayres, K. M., Lowrey, K. A., Douglas, K. H., & Sievers, C. (2011). I can identify Saturn but I can't brush my teeth: What happens when the curricular focus for students with severe disabilities shifts. *Education and Training in Autism and Developmental Disabilities, 46*(1), 11–21.

Baroody, A. J. (1984). The case of Felicia: A young child's strategies for reducing memory demands during mental addition. *Cognition and Instruction, 1*(1), 109–116. http://dx.doi.org/10.1207/s1532690xci0101_5

Baroody, A. J. (1986). Basic counting principles used by mentally retarded children. *Journal for Research in Mathematics Education, 17*(5), 382–389. http://dx.doi.org/10.2307/749330

Baroody, A. J. (1988). Number-comparison learning by children classified as mentally retarded. *American Journal of Mental Retardation, 92*(5), 461–471.

Baroody, A. J., Ginsberg, H. P., & Waxman, B. (1983). Children's use of mathematics structure. *Journal for Research in Mathematics Education, 14*(3), 156–168. http://dx.doi.org/10.2307/748379

Bottge, B. A., Rueda, E., Serlin, R. C., Hung, Y. H., & Kwon, J. M. (2007). Shrinking achievement differences with anchored math problems challenges and possibilities. *The Journal of Special Education*, *41*(1), 31–49. http://doi.org//10.1177/00224669070410010301

Browder, D. M., Ahlgrim-Delzell, L., Courtade-Little, G., & Snell, M. E. (2006). General curriculum access. In M. Snell & F. Brown (Eds.), *Instruction of students with severe disabilities* (6th ed., pp. 489–525). Upper Saddle River, NJ: Merrill/Prentice Hall.

Browder, D. M., Wakeman, S. Y., Flowers, C., Rickelman, R. J., Pugalee, D., Karvonen, M. (2007). Creating access to the general curriculum with links to grade-level content for students with significant cognitive disabilities: An explication of the concept. *The Journal of Special Education, 41*(1), 2–16. http://doi.org//10.1177/00224669070410010101

Browder, D. M., Spooner, F., Ahlgrim-Delzell, L., Harris, A., & Wakeman, S. (2008). A meta-analysis on teaching mathematics to students with significant cognitive disabilities. *Exceptional Children, 74,* 407–432.

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *The Journal of the Learning Sciences*, *2*(2), 141–178. http://dx.doi.org/10.1207/s15327809jls0202_2

Chan, P. E., Konrad, M., Gonzalez, V., Peters, M. T., & Ressa, V. A. (2014). The critical role of feedback in formative instructional practices. *Intervention in School and Clinic*, *50*(2), 96–104. http://doi.org/10.1177/1053451214536044

Chappuis, J., Stiggins, R., Chappuis, S., & Arter, J. (2012). *Classroom assessment for student learning: Doing it right, using it well*. Upper Saddle River, NJ: Pearson.

Claxton, G. (1997). *Hare brain, tortoise mind: How intelligence increases when you think less*. London, UK: Fourth Estate Limited.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13. http://doi.org/10.3102/0013189X032001009

Cobb, P., & Whitenack, J. (1996). A method for conducting longitudinal analyses of classroom video-recordings and transcripts. *Educational Studies in Mathematics, 30,* 213–228. http://doi.org/10.1007/BF00304566

Courtade, G., Spooner, F., Browder, D., & Jimenez, B. (2012). Seven reasons to promote standards-based instruction for students with severe disabilities: A reply to Ayres, Lowrey, Douglas, & Sievers (2011). *Education and Training in Autism and Developmental Disabilities, 47*(1), 3–13.

Dewey, J. (1929). My pedagogical creed. *Journal of the National Education Association*, *18*(9), 291–295.

diSessa, A. A., & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *The Journal of Learning Sciences,*

13(1), 77–103. http://dx.doi.org/10.1207/s15327809jls1301_4

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*(6), 1428–1446. http://dx.doi.org/10.1037/0012-1649.43.6.1428

Edelson, D. C. (2002). Design experiment: What we learn when we engage in design. *Journal of the Learning Sciences, 11,* 105–121. http://dx.doi.org/10.1207/S15327809JLS1101_4

Emerson, R. M., Fretz, R. I., & Shaw, L. L. (2011). Writing ethnographic fieldnotes. Chicago, IL: University of Chicago Press. http://dx.doi.org/10.7208/chicago/9780226206868.001.0001

Ernest, P. (1988). The problem solving approach to mathematics teaching. Teaching Mathematics and Its Applications 7(2), 82–92. http://dx.doi.org/10.1093/teamat/7.2.82

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Oliver, A. I., Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. Journal for Research in Mathematics Education, 28(2), 130–162. http://dx.doi.org/10.2307/749759

Gagnon, J. C., & Maccini, P. (2001). Preparing students with disabilities for algebra. Teaching Exceptional Children, 34(1), 8–15.

Goldstone, R. L., & Son, J. Y. (2005). The transfer of scientific principles using concrete and idealized simulations. Journal of the Learning Sciences, 14, 69–110. http://dx.doi.org/10.1207/s15327809jls1401_4

Gravemeijer, K. (1994). Educational development and developmental research in mathematics education. Journal for Research in Mathematics Education, 25(5), 443–471. http://dx.doi.org/10.2307/749485

Gregson, J. A., & Sturko, P. A. (2007). Teachers as adult learners: Re-conceptualizing professional development. *Journal of Adult Education*, *36*(1), 1–18.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*(1), 37–73. http://dx.doi.org/10.2307/30034699

Hiebert, J., & Carpenter, T. P. (2005). Learning and teaching with understanding (pp. 65–97). In *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Uman, P. G., Murray, H. G., Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher, 25*(4), 12–21. http://dx.doi.org/10.3102/0013189X025004012

Hiebert, J., Wearne, D., & Taber, S. (1991). Fourth graders' gradual construction of decimal fractions during instruction using different physical representations. *The Elementary School Journal, 91*(4), 321–341. http://dx.doi.org/10.1086/461658

Jimenez, B. A., Browder, D. M., & Courtade, G. R. (2008). Teaching an algebraic equation to high school students with moderate developmental disabilities. *Education and Training in Developmental Disabilities, 43,* 266–274.

Jimenez, B., & Kemmery, M. (2013). Building the early numeracy skills of students with moderate intellectual disability. *Education and Training in Autism and Developmental Disabilities*, *48*(4), 479–490.

Kortering, L. J., deBettencourt, L. U., & Braziel, P. M. (2005). Improving performance in high school algebra: What students with learning disabilities are saying. *Learning Disability Quarterly, 28*(3), 191–203. http://dx.doi.org/10.2307/1593658

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511815355

Martinez, E. M. (1998). Teenagers with Down Syndrome study algebra in high school. *Down Syndrome Research and Practice, 5*(1), 34–38. http://dx.doi.org/10.3104/case-studies.73

Moreno, R., Ozogul, G., & Reisslein, M. (2011). Teaching with concrete and abstract visual representations: Effects on students' problem solving, problem representations, and learning perceptions. *Journal of Educational Psychology*, *103*(1), 32–47. http://dx.doi.org/10.1037/a0021995

Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Civil rights from Mississippi to the Algebra Project*. Boston, MA: Beacon Press.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Authors.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Authors.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Authors.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Authors.

Pink, D. H. (2009). *Drive: The surprising truth about what motivates us*. New York, NY: Penguin.

Rivlin, G. (2010). *Broke USA: From pawnshops to poverty inc. – How the working poor became big business*. New York, NY: HarperCollins.

Rodriguez, A. M. (2013). Teaching algebra to people with developmental disabilities: The practice, pitfalls, and promise of taking risks in the classroom. *Connections, 38*(4), 24–28.

Roscoe, R. D., & Chi, M. T. H. (2007). Understanding tutor learning: Knowledge-building and knowledge telling in peer tutors' explanation and questions. *Review of Educational Research, 77*(4), 534–574. http://dx.doi.org/10.3102/0034654307309920

She, P., & Livermore, G. A. (2007). Material hardship, poverty, and disability among workingage adults. *Social Science Quarterly*, 88(4), 970–989. http://dx.doi.org/10.1111/j. 1540-6237.2007.00513.x

Stephan, M., & Cobb, P. (2003). The methodological approach to classroom-based research. In *supporting students' development of measuring conceptions: Analyzing students' learning in social context* (pp. 36–50). Reston, VA: National Council of Teachers of Mathematics.

Stephan, M., Cobb, P., & Gravemeijer, K. (2003). Chapter 5: Coordinating social and individual analyses: Learning as participation in mathematical practices. *Journal for Research in Mathematics Education, 12*, 67–102.

Strickland, T. K., & Maccini, P. (2010). Strategies for teaching algebra to students with learning disabilities: Making research to practice connections. *Interventions in School and Clinic, 46*(1), 38–45. http://dx.doi.org/10.1177/1053451210369519

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. *The Ideas of Algebra, K–12, 8*, 7–13

Witzel, B. S. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities—A Contemporary Journal*, *3*(2), 49–60.

Xin, Y. P., Grasso, E., DiPipi-Hoy, C. M., & Jitendra, A. (2005). The effects of purchasing skill instruction for individuals with developmental disabilities: A meta analysis. *Exceptional Children, 71*(4), 379–400. http://dx.doi.org/10.1177/001440290507100401

Submitted 8/29/2014, accepted 10/9/2015.

I would like to thank Connie DeHerrera and Daniel Eckman for their generosity and support. Thanks to Ruth Luckasson, Susan Copeland, Kelley Peters, Jan Armstrong, Rick Kitchen, Liz Keefe, Frances Duff, Jeri Gillin, Julia Jordan-Zachery, Matthew Mercier, and David Roberts, as well as Glenn Fujiura and the entire editorial staff at AAIDD, for the many hours of analysis, feedback, generation of new ideas, and thoughtful assistance in this manuscript.

Author:
Anthony M. Rodriguez, Department of Education, Providence College, Providence, Rhode Island.

Correspondence concerning this article should be addressed to Anthony M. Rodriguez, Providence College, Department of Education, 1 Cunningham Square, Providence RI 02920 USA (e-mail: rodriguez@providence.edu).