Acoustically Driven Oscillations of Freely Suspended Liquid Crystal Filaments

J. Petzold
A. Nemes
A. Eremin
C. Bailey, Kent State University - Kent Campus
N. Diorio, Kent State University - Kent Campus, et al.
Acoustically driven oscillations of freely suspended liquid crystal filaments

J. Petzold, a A. Nemes, a A. Eremin, a C. Bailey, b N. Diorio, b A. Jáklí b and R. Stannarius* a

Received 18th March 2009, Accepted 2nd June 2009
First published as an Advance Article on the web 2nd July 2009
DOI: 10.1039/b905459f

Some liquid crystalline phases of bent-core mesogens are known to form stable freely-suspended filaments with length to diameter ratios of 1000 and larger. These structures can behave like thin liquid chords. We study filament oscillations excited with harmonic sound waves. From amplitudes of the filament motion and phase shifts respective to the harmonic excitation signal we develop a model for the filament dynamics. Like in solid chords, the resonance frequency \( f_0 \) is inversely proportional to their length. The dependence of \( f_0 \) upon the filament radius allows one to draw conclusions on the nature of the filament tension. For thin filaments, this tension can be largely attributed to surface tension, while for thick filaments there must be other, bulk contributions in addition. The decay time of the filament oscillations is proportional to the filament length. This can be explained by the assumption that dissipation is restricted to the two filament ends. An important observation is that thick filaments often deviate significantly from cylindrical shape.

I. Introduction

Thin fibers are of significant importance within a widespread range of materials and soft matter structures. They are found in biological systems, in many technological applications, and in everyday life. Particularly interesting objects are thin filaments formed by complex liquids. Newtonian fluids cannot form stable filaments, since the Rayleigh–Plateau instability destroys liquid cylinders with length to diameter ratio (slenderness ratio) larger than \( \pi \). Surface energy causes such structures to become unstable with respect to long wavelength radius fluctuations. They decompose into droplets. The situation is qualitatively similar in a large variety of liquid crystalline phases. In nematics, the influence of orientational anisotropy on fiber stability is insignificant, nematic bridges break at the same aspect ratio of \( \pi \) like ordinary isotropic liquids.\(^1\) Even simple smectic layer structures like smectic A do not stabilize free-standing cylinders in air substantially; a critical slenderness ratio of 4.2 was reported for them.\(^2\) Stable smectic A filaments were found so far exclusively in isotropic solvents.\(^2,4\) It seems that calamitic liquid crystals must be discarded as candidates for freely suspended filament formation.

Bent-core mesogens, discovered about one decade ago,\(^4,\text{5–10} \) have a tendency to produce more complex layered phases than calamitics. Some bent-core mesophases exhibit a rich variety of filaments with helical superstructures in the isotropic melt. A few layered mesophases from bent-core mesogens form free-standing fibers in air with slenderness ratios larger than 1000.\(^5,7\) Those structures have remarkable dynamic properties. They behave in a liquid-like manner when their length is changed by slowly varying the separation between the supports. On the other hand, they can be excited to vibrations like solid chords. Investigations of structure and dynamics of such filaments were performed over the past decade in order to understand the mechanisms that stabilize the filaments against the Rayleigh–Plateau instability, and to describe their mechanical properties.\(^5,7\)

So far, the dynamic response has been studied under excitation with electric fields, including constant fields,\(^\text{11} \) switched dc electric fields\(^9\) and ac electric excitation.\(^13,\text{14} \) In this type of experiments, electrical and mechanical interactions are in general superimposed, and the separation of individual dynamic parameters is very difficult. The electric interactions are in general non-linear in the field strength. At the very least, linear and quadratic terms have to be taken into account. The equation of motion that describes the oscillations in a sinusoidal electric field normal to the filament can probably be expressed by a periodically driven Mathieu equation.\(^19\) Solutions are available only numerically, and complex dependencies on excitation amplitudes and frequencies are found. Therefore, useful results were obtained only from the analysis of free oscillations after the electric field is switched off.\(^9\) Even then, electric forces cannot be excluded completely. The filaments charge during exposure to the deflection field,\(^9\) and Coulomb interactions of the filaments with the electrodes are still present after the electric excitation is removed.

A more reliable alternative is a purely mechanical excitation, in order to dispense with the electric field completely. This is demonstrated in the present study. One could, in principle, pluck the filaments with a short mechanical pulse and achieve similar free oscillations like at electric excitation. A more time-consuming but more accurate method is the harmonic analysis of forced oscillations under a periodic excitation with sound. We demonstrate the equivalence of results obtained with the latter technique to data from electrically plucked filaments. The frequency dependence of the dynamic response for filaments of different geometrical sizes is analyzed. Two materials in different mesophases are compared.

II. Experimental setup and preparation techniques

The first material investigated (for chemical composition see Fig. 1) has a phase sequence isotropic 160 °C PM-SmCP 143 °C SmCP 90 °C crys.
We will refer to this material as I. Stable filaments can be prepared in the high temperature mesophase which is considered to be a polarization modulated Sm
\text{CP} phase (PM-SmCP).\textsuperscript{10,15} X-Ray data show that the filaments consist of cylindrically wrapped molecular layers.\textsuperscript{8} In general, filaments of more than \(~5\) \(\mu\)m diameter are composed of bundles of small fibers, which have diameters of the order of a few micrometres.\textsuperscript{10}

The second material (2) has the phase sequence isotropic \(180\) °C B7 (columnar) \(120\) °C crystalline (for chemical composition see Fig. 2). Stable filaments of this material can be drawn in the B7 phase, we assume that their internal structure is similar to filaments of the first material.

The experimental setup (Fig. 3(a) and (b)) is equivalent in its essential parts to the one used in previous investigations. Filaments are drawn in a nearly cubic copper heating box with a volume of about 100 cm\(^3\) that provides a constant temperature in the range from room temperature to about 200 °C. The box contains a support dish which holds a small drop of liquid crystalline material as the reservoir. A needle, attached to a stepper motor outside the box, is moved toward the drop until the tip is in contact with the mesogenic material. It is then slowly pulled away to draw a filament. We can produce and study filaments of about 4 cm maximum length. It is important that both needle and support dish can be synchronously rotated about the filament axis. This is necessary to check whether the filament is circular in the cross-section. We will discuss below that deviations of the filament from cylindrical geometry, which have earlier been ignored, represent a major error source in the interpretation of experimental data.

Lateral observation windows in the heating box allow one to study the films in transmitted light by means of a long-range microscope (Questar QM 100). A fast camera (Citius Imaging C10) is used to record videos of a small region near the central part of the filament. The viewing field is sketched schematically by the dotted boxes in Fig. 3(a) and (b). Frame rates are between 1000 fps (frames per second) and 5000 fps.

The setup in Fig. 3(a) is used for the electric measurements. The electric field is perpendicular to the observation axis, it deflects the filament in the field direction. The maximum deflection amplitude is of the order of 100 \(\mu\)m. This experiment is equivalent to that reported in ref. 9. When a DC electric field perpendicular to the filament axis is applied, the filament deflects, a well defined bend deformation is induced.\textsuperscript{11} The deflection shape is sinusoidal in good approximation, \(z(x) = z_0 \cos(\pi x / L)\) with the wave number \(k = \pi / L\) and filament length \(L\). The deflection amplitude \(z_0\) depends on the filament length \(L\) and diameter \(d\), and increases with the electric field strength. At the maximum applied field of 0.5 MV m\(^{-1}\), it is a few percent of \(L\). After the field is switched off, we observe damped oscillations of a spatially sinusoidal ground mode, and retrieve the dependence of both oscillation frequency and damping rate upon filament length, filament diameter and sample temperature. It has been shown that the analysis of the deflection amplitude \(z_0(t)\) is sufficient for the description of the essential filament dynamics, and that a linear oscillator model can be employed for \(z_0(t)\).\textsuperscript{9}

The acoustic setup is shown in Fig. 3(b). Sound-absorbing material covers the inner surface of the heating box. Before the acoustic experiments are performed, the two electrodes are removed without twisting or destroying the filament. A small glass plate with a diameter slightly larger than the filament length is positioned near the filament. This plate is connected to a speaker outside the heating box by a glass rod. The speaker is driven with a wave form generator in a frequency range between 100 Hz and 1 kHz. The relationship between electric voltage at the speaker and the amplitude of the plate oscillations is frequency dependent. The plate is adjusted so that the middle part of the excitation plate is recorded together with the filament (cf. dotted box in Fig. 3(b)). This allows a correction of slightly different excitation amplitudes during a frequency scan. The amplitude of the driving plate \(A_{plate}\) is chosen depending on the
driving frequency \( f_e \) so that the product \( (f_e \times A_{plate}) \) is constant during frequency sweeps (see section III B). \( A_{plate} \) values are typically in the range between 10 \( \mu \)m and 100 \( \mu \)m. The distance between plate and filament is approximately 0.5 mm.

III. Experimental results and model predictions

A. Electric plucking

The electric experiment of ref. 9 is repeated in order to demonstrate the correspondence of the results with those obtained under acoustic excitation, for identical filaments. Experiments with electric plucking have been performed exclusively with material 1, in the PM-SmCP phase. Fig. 4 shows typical oscillations observed after an electric deflection of a filament. The damped oscillations after switching the field off can be satisfactorily fitted by an exponentially decaying cosine function

\[
z_0(t) = Z_0 \exp \left(-\frac{\gamma}{2} t\right) \cos(\omega_0 t - \varphi_0) \tag{1}\]

The solid line in Fig. 4 is a fit to eqn (1) with the parameters \( Z_0 = 0.161 \pm 0.005 \) mm, \( \gamma = 206 \pm 5 \) s\(^{-1}\), and \( f_0 = \omega_0/2\pi = 202.5 \pm 5 \) Hz. For different electric field strengths, we find different deflection amplitudes \( z_0 \) but the dynamic parameters, \( \omega_0 \) and \( \gamma \), are unaffected. This confirms that the filament can be described by a linear oscillator equation. The filament length \( L \) is the most important parameter in its dynamics. The oscillation frequency \( \omega_0 \) is inversely proportional to \( L \) when all other parameters (radius, temperature) are kept constant. This has been verified in a large number of experiments. Therefore we will employ the product \( f_0L = \omega_0L/2\pi \), when filaments of different lengths are compared. Another parameter that characterizes the filament is its thickness. In previous studies, we have assumed that the filaments are cylindrical to a good approximation. Then it would be sufficient to extract the diameter from a single optical image. However, as is demonstrated below, the filament cross-section can deviate considerably from circular shape. Such irregular shapes can be long-term persistent. For example, a twist of the fiber bundle does not relax within days.\(^7\) It was not tested whether the filament used in Fig. 4 had a cylindrical cross section. For a comparison of the dynamics at electric and acoustic excitation of the same filament, this detail plays no role.

B. Acoustic excitation

Under sine wave acoustic excitation, the filaments perform harmonic oscillations. We have verified that for small excitation amplitudes these oscillations represent a standing wave of the ground mode with wavelength \( \lambda = 2L \) and wave number \( k = \pi/L \), as in the electric case. The deflection of the filaments is sinusoidal at any moment. It is therefore sufficient to measure the motion of the central section of the filament \( z_0(t) \) as a representative measure of the filament dynamics.

The data acquisition in the acoustic experiments is shown exemplarily for the same filament as in Fig. 4 in the following images. Fig. 5 shows forced oscillation under acoustic driving with frequencies \( f_e = 150 \) Hz (far below resonance) and \( f_e = 200 \) Hz (near resonance). The amplitude of the filament oscillations and its phase shift with respect to the excitation source both change with \( f_e \). The Fourier spectrum obtained from a numerical transform shows that only the ground frequency is excited. Higher harmonics are below the detection threshold.

The filament response to the excitation \( F(t) \propto \cos(2\pi f_0 t) \) is described by the function

\[
z_0(t) = A \cos(2\pi f_e t + \phi) \]

with the excitation frequency dependent amplitude \( A(f_e) \) and phase shift \( \phi(f_e) \), with respect to the excitation plate. Amplitude

![Fig. 4](image-url) Free damped oscillation of a filament \((L = 4.3\ mm, \ apparent\ diameter\ d = 41\ \mu m)\) of substance 1 in the PM-SmCP phase at 150 °C after a voltage of 0.5 kV (electric field 83 kV m\(^{-1}\)) has been switched off. The solid line is a fit with eqn (1) and the parameters given in the text. The inset shows a larger time window of the same experiment.

![Fig. 5](image-url) Forced oscillation of a filament \((L = 4.3\ mm, \ apparent\ diameter\ d = 41\ \mu m)\) of substance 1 in the PM-SmCP phase at 150 °C under irradiation with (a) 150 Hz and (b) 200 Hz acoustic waves.
and phase are extracted from the Fourier transforms of the experimental data for different frequencies, scanned from 100 to 500 Hz in 4 Hz steps. Fig. 6 is an example of the frequency dependencies of amplitudes and phases obtained in the acoustic experiments. In order to interpret the phase $\phi$ correctly, one has to consider the relation between pressure and displacement in an acoustic wave, which is done in detail in the following paragraphs.

The amplitudes in Fig. 6 are fitted with:

$$A(\omega_k) = \frac{A_0}{\sqrt{(\omega_k^2 - \omega_0^2)^2 + \omega_k^2 \gamma^2}}$$  \hspace{1cm} (2)

where $\omega_k = 2\pi f_k$, and $\omega_0 = 2\pi f_0$. The fit parameters are $f_0 = 217.3 \pm 0.3$ Hz and $\gamma = 235 \pm 5$ s$^{-1}$. The difference to the eigenfrequency $f_0$ of the same filament in the electric experiment is about 7%. The phase in Fig. 6b has been fitted to

$$\phi = \arctan \left( \frac{\omega_k^2 - \omega_0^2}{\gamma \omega_k} \right) + \phi_{offs}$$  \hspace{1cm} (3)

with the fit parameters $f_0 = 216.5 \pm 0.5$ Hz and $\gamma = 231 \pm 5$ s$^{-1}$. These values are, within the error limits, in agreement with those from the amplitude fit. The offset term is a consequence of the reference chosen: when the filament is excited acoustically, the driving force arises from the sound pressure field. The pressure in an acoustic wave is phase shifted by $\pi/2$ with respect to the displacement of the gas particles. Far below resonance, the phase shift of the filament deflection respective to the sound pressure is expected to be zero. This corresponds to a phase shift of $\pi/2$ between the filament and driving plate motions. The retardation of the sound wave from the excitation source to the filament is of the order of a few microseconds and can be neglected. With increasing frequency, the phase shift respective to the pressure oscillations decreases continuously, reaching $-\pi$ for very high frequencies. Consequently, the phase shift of the filament motion with respect to the driving plate should pass zero at resonance and reach an asymptotic value of $-\pi/2$ at high frequencies. The phase of the filament motion with respect to the plate is described by the function $\phi_{offs}$ in eqn (3), in place of the tangent that would reflect the phase shift respective to the pressure variation. Strictly, these relations hold when the distance between driving plate and filament is very large compared to the oscillation amplitude of the plate. This condition is not quite fulfilled in our experiments, the filament is not far enough from the excitation source. This leads to the small offset of $\phi_{offs} = 0.06\pi$ in eqn (3), cf. Fig. 6 which is irrelevant for the determination of the resonance frequencies $f_0$.

Since the sound pressure drives the oscillations, the plate amplitude has to be scaled properly. We adjust a constant product of oscillation amplitude and frequency, which yields constant pressure amplitudes in the acoustic wave. When a broad frequency range is swept, however, this leads to the problem that the plate has to oscillate with very large amplitudes at low frequencies in order to maintain the pressure amplitude. This problem can be circumvented when a lower excitation amplitude is chosen and the filament oscillation is scaled with the reduced excitation amplitude. In Fig. 6, we have scaled all measured filament deflections by the product of driving plate amplitude and frequency ($A_{plate \times f_0}$).

These measurements have been performed for a large number of filaments with different (apparent) radii and lengths. Particular care has been taken that the filaments have uniform radii along their complete length. The attribute 'apparent' refers to the observation that the filaments are not always exact cylinders. A large number of our measurements have been performed before the cylindrical geometry could be checked with a rotatable setup (bullets). These results include measurements by electric excitation, taken from ref. 9 (open circles). Some new data obtained with electric excitation are included (crossed circles). The solid line results from a model described in section III C which assumes that the tension of the filaments arises exclusively from its surface tension. It is obvious that the experimental data deviate systematically to higher resonance frequencies.

The large scattering of the data led to the conclusion that the filament shape might often deviate from a circular cross-section. Therefore, we have reconstructed the setup so that the filament can be rotated before the oscillation measurement. The cylindrical cross-section was checked in each of the experiments represented by solid symbols.

Material 2 in the B7 phase gave qualitatively similar results. The frequencies represented in Fig. 8 are systematically much faster than predicted by the surface tension model. The deviations from the model increase with larger filament diameter. Peculiar for the B7 filaments is that if the acoustic excitation experiment is repeatedly performed over a long period (1 day), the pitch increases significantly. This is seen exemplarily in the triangles in

---

Fig. 6 Amplitudes (a) and phase shifts (b) of forced oscillation of a filament ($L = 4.3$ mm, apparent diameter $d = 41$ μm, same as in Fig. 4) of substance 1 in the PM-SmC phase at 150 °C under excitation with acoustic waves. The solid line is a fit with the parameters given in the text.
Fig. 8. These data have been obtained with one single filament during a time period of 1 day. The resonance frequency changes, while all other parameters of the filament remain constant. This means that the forces that restore the straight filament increase with time. We have no reasonable explanation for this effect in the B7 material. One suggestion is that defects anneal over very long periods in time. Such defects soften the fiber elasticity. When they disappear, the fiber may become stronger and the natural frequency will increase. However, we cannot exclude degradation of the material after one day at 150°C.

The width of the resonance curve gives the damping coefficient $\gamma$, which reflects dissipative terms in the oscillator equation. It has been shown$^9$ that the damping coefficient is inversely proportional to the filament length. This indicates that dissipation occurs mainly at both ends of the filament, not within the filament itself. The longer the filament is, the lower the relative influence of friction is. Fig. 9 demonstrates this relation for a filament of substance 2 that has been expanded stepwise to different lengths.

The filament diameter dependence of the relaxation time $\tau = \gamma^{-1}$ is shown in Fig. 10 for filaments of comparable length.

Fig. 9 Inverse damping coefficient $\tau = \gamma^{-1}$ vs. filament length for material 2 at 150°C, the filament diameter was 35 μm. The dashed line marks a linear relationship $\tau \propto L$.

Fig. 10 Inverse damping coefficient $\tau = \gamma^{-1}$ vs. filament diameter. (a) PM SmCP filaments of substance 1 at 150°C, filament lengths between 4.2 and 4.3 mm. The time $\tau$ increases with filament diameter $d$ when the lengths of the filaments are approximately the same. Bullets represent data of exactly cylindrical filaments, open symbols are data from filaments where the cross-section shape has not been checked. (b) B7 filaments of substance 2 at the same temperature, filament lengths between 4.2 and 4.5 mm. Different symbols represent different filaments. The dashed parabolic lines are guides for the eye.

With larger filament diameters, the influence of friction becomes smaller, and $\tau$ increases. The dashed lines in the images are model-free and guides for the eye, they represent a dependence $\tau \propto d^2$. Even though the available data are not accurate enough to prove such a parabolic dependence, the similarity of the trend
of these curves to the measured diameter dependence may indicate that the relaxation time increases with the filament cross-section area. Oscillations in filaments of less than ~10 μm are damped very fast so that the relaxation time reaches the oscillation period. Such filaments do not perform free oscillations, they relax monotonously.

C. Filament tension model

A first approximation of the expected oscillation frequency of the filaments can be made when one assumes that the back-driving force \( T \) is related to the surface tension \( \sigma \). For cylindrical filaments of diameter \( d \), \( T = \sigma \pi d \). In combination with the mass density \( \rho \) and the specific mass (mass per length) of the filament \( \mu = \pi d \rho / 4 \), one obtains a velocity of capillary waves on the filament of

\[
c = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{4 \sigma}{d \rho}} \tag{4}
\]

This velocity contains the root of the ratio of filament circumference and cross-section area, \((4/d)\). When the surface tension of the material is assumed to be \( \sigma \approx 25 \text{ mN m}^{-1} \) (ref. 8) and the mass density as \( \rho = 1 \text{ g cm}^{-3} \), then one obtains the solid curves \( c \propto d^{-1/2} \) in Fig. 7 and Fig. 8. It is evident that the agreement with the experimental data is far from being satisfactory. All data except of one single datum point are far below the prediction. This discrepancy has been mentioned earlier,\(^8\) on the basis of relaxation data after electrical excitation. The restoring force must be larger than assumed in the surface tension model. If we take into account that the filament surface is actually corrugated (Fig. 11, middle), then the surface may be larger by a factor of about \( \pi/2 \), and \( c \) increases by \( \sqrt{\pi/2} \) (dashed lines in Fig. 7 and Fig. 8).

Thin filaments which have been checked to be cylindrical in good approximation are in reasonable quantitative agreement with the corrugated surface model predictions. Cylindrical filament shapes could be achieved only with radii of 50 μm and less. For the larger filaments, we find systematically too high resonance frequencies.

IV. Discussion and summary

The comparison of electric and acoustic excitation of oscillations of the same filament showed in all cases very good agreement between the two methods. While the electric experiment measures the Fourier transform of free oscillations, the acoustic experiment evaluates the resonance curves of amplitude and phase of forced oscillations. The small difference of the order of 5% in the resonance frequencies and the somewhat different damping coefficients may be a consequence of electric charges and their interactions with the grounded electrodes after the field is switched off. The results of the acoustic measurements are more reliable in that respect, and more accurate.

As has been shown in the previous section, the oscillation frequencies show a systematic deviation from the predicted model curve to large filament diameters. Since it seems unlikely that the density of the liquid crystal is an error source (densities of conventional liquid crystals do not deviate by more than 5 to 10% from the density of water) we have to focus on the filament shape and the forces connected with filament bend and filament elongation. A simple explanation of the observed discrepancies between measurement and model predictions, in particular for thicker filaments would be the assumption that thick filaments may have irregular shapes. Therefore we have tested the actual filament shape by rotating the object successively. It turns out that, very often, filaments are indeed not cylindrical after preparation. An example is shown in Fig. 12. The apparent filament diameter varies between approximately 50 μm and 80 μm. The stack shows a selection of filament images, taken in 10° rotation steps. Since it has been demonstrated\(^6\) that twists of fiber bundles can remain stable for days, filaments with non-cylindrical bundle shape might persist as well for a long time. When the filament diameter is determined from a single image (as it has been done in all previous studies of liquid crystalline filaments), then this apparent diameter might not be representative. If the cross-section is an ellipse with half axes \( a, b \), one could in principle measure all possible projections in the range \( 2a-2b \) onto the viewing plane (Fig. 12, right).

As a consequence, the specific mass could either be larger or smaller than assumed, as could the circumference. The quotient of both, which enters the velocity (eqn (4)), can differ from the cylinder value by ~10%, e.g., when the axes ratio is 1:1.2, or by about 30% when the axes ratio is 1:1.6 as in the example shown in Fig. 12. Probably the only way to circumvent this geometrical

![Fig. 11 Schematic plot of cross-sections assumed in the filament tension models, labelled with circumferences. The circumference of the corrugated filament can be estimated as \( \pi/2 \) times the circumference of the cylinder. The circumference of filaments with other shapes may be larger or smaller than derived from the apparent diameter and an assumed cylinder shape.](image1)

![Fig. 12 Typical thick filament observed from different viewing angles. Images of the central part of the filament, taken during stepwise rotation by 10° from 0 to 90° are stacked, the measured apparent diameters in μm are given for each image.](image2)
The filament tension in the form:

\[ T = \frac{4 \sigma}{\rho} \times \sqrt{1 + \frac{3d}{4\sigma}} \]  

(5)

where \( \beta \) is a coefficient that comprises the bulk contributions to the filament tension in the form:

\[ T = \sigma \pi d + \beta \pi d^2 \]  

(6)

The inclusion of this phenomenological coefficient \( \beta \) in eqn (5) and eqn (6) is model independent. It contains any interactions that are related to volume forces. In the model of Bailey et al.,\(^{16} \beta \) is related to the layer compression constant \( B \) by \( \beta = B/4 \). Alternatively, one could attribute this bulk correction term to defects in the regions between the thin fibrils of a bundle. Those defects would contribute to the elastic free energy, and they would lead to a bulk force that restores the straight filament.

In Fig. 7, we have included a dotted graph according to eqn (5), with a choice of \( \varepsilon \beta = 12 \mu m \). This would correspond to a value of \( B \approx 10^4 \text{Nm}^{-2} \) that is suggested by Bailey et al.\(^{16} \) The graph in Fig. 7 describes the small diameter filaments, in particular those with established cylinder shape, satisfactorily within experimental uncertainty. The trend of the large diameter filaments is qualitatively reproduced. The data scatter around this curve mainly because of deviations from the cylindrical shape. Note that the values derived from electrically excited filaments still are systematically too small (they have higher resonance frequencies than predicted). This indicates that other interactions are present there, e.g. Coulomb forces due to bulk charges.

The friction forces that damp the filament oscillations are reflected by the damping time constant \( \tau \). This parameter increases linearly with the filament length. Consequently, we can assume that the energy dissipation is practically independent of the filament length, the dissipation processes are very likely restricted to the ends of the filaments. The relaxation times can be satisfactorily fitted in both substances with a quadratic dependence on the filament diameter.

Finally, we mention that in all experiments with individual filaments that have been drawn to different lengths, the eigen-frequency of the ground mode was found to scale with \( 1/L \). The length dependence of the filament resonances is in excellent agreement with the liquid chord model.

Acknowledgements

This work was partially supported by the Deutsche Forschungsgemeinschaft under Grant STA 425/20, by BMWT under project OASIS-Co, by NSF FRG under Contract No. DMS-0456221, and by NSF IRES OISE-0727185. W. Weissflog is gratefully acknowledged for supply of the mesogenic material.

References