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Alpha+²⁸Si Cluster Structure as Solitons on the Nuclear Surface

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The characteristic alpha-particle spectra from the decay of intermediate states in $\alpha+^{28}\text{Si}$ (^{32}S) to the silicon ground state have been measured during the bombardment of thick targets. The α -spectra were measured in backward angles for the determination of the spins of the states formed in elastic alpha scattering. The level spectra have been investigated in terms of a nonlinear Lagrangean which gives a quantum analogue to the classical large amplitude collective motion in nuclei, successfully used for the evaluation of the preformation factor in alpha and cluster decays. The corresponding Hamiltonian becomes diagonal with a spectrum similar to a sum of harmonic oscillator spectra. In this way new quantum numbers are introduced. The present model not only reproduces the observed levels but predicts also positions and spins of other levels of the quasimolecular spectra. Both even and odd parities are reproduced using the same parameters.

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In the present paper we describe the resonances observed in the elastic scattering of alpha particles on light nuclei. Such states are observed in elastic alpha scattering from many light nuclei within a broad range of bombarding energies. In the scattering from ^{40}Ca , 3 and ^{28}Si , ⁴ they occur as fragments of broad resonances forming groups with a common angular momentum. In ^{28}Si this structure can be seen from 6.5 to 19 MeV in bombarding energy and 3 to 10 $\hbar$ in spin⁵. The resonance states form a mixed parity rotational band of excited states in ^{32}S . The expression $E_{exc} = 0.103I(I+1) + 12.6$ MeV gives a good fit to the experimental results⁶. The excitation function for elastic scattering of alpha particles by an angle 173 degrees from Si is shown in the upper part of Fig.1. It was measured at the Åbo Akademi University cyclotron and partly at the Uppsala University Van de Graaff ^{5,7}. The angular distribution measurements yielded the spin for 86 of the 113 well resolved narrow resonances seen in the figures. For further experimental details, see ^{8,9}.

By using a quantum nonlinear analogue of the classical hydrodynamic description of solitons on the nuclear surface we succeeded to obtain a Hamiltonian which, in the diagonal form, generates a spectrum in excellent agreement with the experimental data. It is well known that the shell model can not describe the cluster aspects of nuclei¹⁰. Microscopically,

in order to explain such effects, we have to introduce into the shell model a cluster-like component¹¹ or many-body correlations. We consider that only a collective description, in which the outside nucleons of a rigid core join together to form a cluster, is the most appropriate frame for the evaluation of the preformation factors. Recently^{12,13} we have shown that by introducing nonlinear terms in to the classical hydrodynamic equations, stable solitons exist on a circle. This model was shown to describe excellently the experimental preformation factors for alpha¹³ and cluster¹² decays. A similar cluster model was introduced microscopically in ref.¹¹. To explain the detailed structure of the resonances observed in the scattering of alpha particle on light nuclei (Fig.1 - upper part) one may try phenomenological models, which describe the states as rotational states fragmented by dipol¹⁴ or radial vibrations¹⁵, or by anharmonic quadrupole surface vibrations, analogous to β and γ vibrations in deformed nuclei¹⁶. These approaches lead to the the energy spectrum of a rotor plus an anharmonic vibrator:

$$E_{n,I} = E_0 + A\left(n + \frac{1}{2}\right) - B\left(n + \frac{1}{2}\right)^2 + CI(I + 1). \quad (1)$$

This expression was successfully used for the description of positive parity resonant states for two colliding carbon nuclei and for bound alpha cluster states in $\alpha + {}^{12}\text{C}$ and $\alpha + {}^{16}\text{O}$ ¹⁷. However the great number of states as well as their irregular appearance, and the relatively small moment of inertia of the $\alpha + {}^{28}\text{Si}$ -systems ($4.85\hbar^2$)⁶ call for other models.

A better dynamical description of these states is given by considering that in the intermediate states the alpha particle interacts with the nucleus as a soliton or a breather¹⁸. We introduce a Lagrangean depending on a complex scalar field, $u(\phi, t)$ defined on the nuclear surface

$$\mathcal{L} = \int_0^{2\pi} \left(\frac{R^2}{c^2} |u_t|^2 + |u_\phi|^2 - \frac{1}{4} |u|^4 + 4\beta^2 |u|^2 \right) d\phi \quad (2)$$

where ϕ is the angular polar coordinate, β is a real parameter and R is the radius of the nucleus. In the above expression the first term is the kinetic energy $\mathcal{T}(u, u^*)$ and the three last terms are the potential energy $\mathcal{U}(u, u^*)$. The corresponding Lagrange equation leads to the following nonlinear equation

$$\frac{R^2}{c^2} u_{tt} + u_{\phi\phi} = -\frac{1}{2} |u|^2 u + 4\beta^2 u. \quad (3)$$

Eq.(3) admits localised finite energy solutions of the form

$$u(\phi, t) = \pm 2 \sqrt{\left(4\beta^2 + \frac{\alpha^2 R^2}{c^2}\right)} \text{sech} \left[\phi \sqrt{4\beta^2 + \frac{\alpha^2 R^2}{c^2}} \right] e^{i\alpha t}. \quad (4)$$

This solution yields $|u|^2$ as a static soliton and $\text{Re}(u)$ as a static breather-like soliton¹⁹. Eq.(3) has also more general solutions in the form of modulated traveling wave packets $u(\phi, t) = r_0 \text{sech}((\phi - Vt)L^{-1}) \exp(i(a\phi + bt))$, i.e. breathers with half-width L and speed V . By introducing this expression in eq.(3) and by differentiation the real part of the obtained equation with respect to ϕ , we obtain a modified Korteweg de Vries (MKdV) equation for $|u(\phi, t)|$. Then, identifying $|u(\phi, t)|$ with the unique one-soliton solution of the MKdV equation,¹⁹ and using also the imaginary part of eq.(3), we can determine the parameters

V , a and b as functions of r_0 , L and β only. In order to make the connection with the classical hydrodynamic approach^{12,13} we use the complex nonlinear Miura transformation¹⁹ $\eta(\phi, t) = u(\phi, t)^2 + iu_\phi(\phi, t)$ where η is a solution of the KdV equation which describes the classical soliton. Consequently the parameter β is correlated with the soliton geometry which overlaps the alpha volume.

In order to quantize the soliton solutions we use the well known non perturbative weak-coupling procedure which consists in finding the normal modes of oscillation about the lowest state of the classical soliton and then exactly quantizing these normal modes²⁰ We write the potential energy in eq.(2) as a functional Taylor expansion about the solutions which are its extrema, $\delta\mathcal{U}(u, u^*)/\delta u^* = 0$. These are also solutions of the stationary part of eq.(3). There are two classes of solutions: trivial static solutions $u_0^{vac}(\phi) = \sqrt{2(k^2 - 4\beta^2)}e^{ik\phi}$ depending on the label k , and non trivial ones $u_0(\phi, t) = 4\beta \text{sech}(2\beta\phi)$, connected with the solitons. We note that one can obtain the same results (with a corresponding change in the values of the free parameters) using instead of a static solution the traveling wave solution or the quasi-static solution given in eq.(4). The functional Taylor expansion in $u(\phi, t) = u_0(\phi, t) + \xi(\phi)$ for the potential $\mathcal{U}(u, u^*)$ in second order in ξ reads $\mathcal{U}(u, u^*) = \mathcal{U}(u_0, u_0^*) + \mathcal{U}_2(\xi, \xi^*)$ where, evidently, the first order term $\mathcal{U}_1(\xi, \xi^*)$ is zero due to the extremum conditions for the potential. The second order term $\mathcal{U}_2(\xi, \xi^*)$ has the form

$$\mathcal{U}_2(\xi, \xi^*) = \int \xi^* \Omega \xi d\phi + c.c. \quad (5)$$

where Ω is a hermitian operator

$$\Omega = \frac{1}{2} \frac{\partial^2}{\partial \phi^2} + \frac{1}{2} |u_0|^2 + \frac{1}{4} u_0^2 \mathcal{C} - 2\beta^2 \quad (6)$$

in which $\mathcal{C}$ denotes the complex conjugation operator. In order to find the fluctuations around the solutions u_0 and u_0^* we have to find the eigenvalues ω_k with $\omega_k^2 > 0$ and eigenfunctions ξ_k of the operator Ω , $\Omega \xi_k = \omega_k^2 \xi_k$. The trivial solutions describe the vacuum associated with this theory and its excitations. However the vacuum sector of these energies and solutions is unconnected with the sector of nontrivial solutions²⁰. In particular, the quantum soliton is stable against decay into mesons. In the following we consider only the nontrivial solutions u_0 . We look for ξ_k in the form

$$\xi_k(\phi) = \text{sech}^\gamma(2\beta\phi) F(2\beta\phi) \quad (7)$$

where γ_k is a parameter and F an arbitrary function. By introducing eq.(7) in the eigenvalues equation for Ω and by using the substitution $v = \frac{1}{2}[1 + \tanh(2\beta\phi)]$ we obtain a hypergeometric differential equation for F

$$v(v-1)F'' + \left[\left(2 + \frac{\gamma}{\beta}\right)v - \left(1 + \frac{\gamma}{2\beta}\right) \right] F' + \frac{\gamma(\gamma+1) - 6}{4\beta^2} F = 0 \quad (8)$$

with the eigenvalues given by $\omega_k^2 = 2\beta^2(\gamma_k^2 - 1)$. In order to have localized solutions (rapidly decreasing functions for large v), the asymptotic behavior of the hypergeometric function F

²¹ demands the condition:

$$\gamma_k = -\frac{1}{2} - 2k\beta \pm \sqrt{\frac{25}{4} - 4k\beta^2 + 2k\beta} \quad (9)$$

where $k = 0, 1, \dots$ such that $\omega_k^2 > 0$, $\gamma_k > 0$. The remaining spectrum of Ω is continuous. Due to the fact that the eigenfunctions ξ_k are orthogonal we can write the expansion

$$\xi(\phi, t) = \sum_k C_k(t) \xi_k(\phi) \quad (10)$$

where $C_k(t)$ are Hermitian quantum operators ²⁰. With this choice for ξ , the Hamiltonian assumes a simple form (in the second order):

$$\mathcal{H}(u_0 + \xi, c.c.) = \mathcal{T} + \mathcal{U} = \mathcal{U}(u_0, u_0^*) + \frac{1}{2} \sum_k \left(\left| \frac{dC_k}{dt} \right|^2 + \omega_k^2 \frac{c^2}{R^2} |C_k|^2 \right) \quad (11)$$

which reduces the problem to a set of independent harmonic oscillators with the energy spectrum given by:

$$E = \mathcal{U}(u_0, u_0^*) + \sum_k \hbar \tilde{\omega}_k \left(N_k + \frac{1}{2} \right) \quad (12)$$

where $\tilde{\omega}_k = c\omega_k/R$ and N_k are new quantum numbers associated with the excited states of the soliton. We stress that the eigenvalues ω_k are uniquely determined by the soliton geometry (r_0, L) through β . The states of the theory, $\Psi_{N_k, k}^{sol}(\phi, t)$, are defined as eigenstates of the Hamiltonian associated with eq.(11) and by the occupation numbers k of the various modes. The wavefunctions $u_k(\phi, t)$ have the form:

$$u_k(\phi, t) = u_0(\phi) + \sum_k \text{sech}^{\gamma_k}(2\beta\phi) F_k(a, bc\nu) \quad (13)$$

where F_k is the hypergeometric function in two arguments: $a = -k$, $b = (6 - \gamma(\gamma + 1))/4\beta^2 k$, $c = 1 + \gamma/2\beta$.

In the following we apply the above quantum nonlinear approach of clusters in nuclei for the description of the states observed in alpha elastic scattering by silicon. In this case the total wave function for the $\alpha + {}^{28}\text{Si}$ -system, taking into account the global rotation of the system $(\tilde{\theta}, \tilde{\phi})$ and the radial oscillations labeled by the quantum number n , is $\Psi_{I, N_k, k, n}(\tilde{\theta}, \tilde{\phi}, \phi, r, t) = \Psi_I^{rot}(\tilde{\theta}, \tilde{\phi}, t) \Psi_{N_k, k}^{sol}(\phi, t) \Phi_n^{vibr}(\rho, t)$ where $\Phi_n^{vibr}(\rho, t)$ is assumed to be the wave function associated with the radial vibration responsible for the anharmonic terms in energy, eq.(1). and depending on some radial coordinate ρ .

In the present model the α -particle in its cluster states is a soliton with a height $r_0 R$ and an angular half-width L . According to the dynamics of the soliton ^{12,13} the α -cluster moves on a shallow liquid layer around a rigid spherical core, with a depth $h < R$. The parameters r_0 and L are fixed by imposing two conditions: the mass of the soliton must be equal with the mass of the α -particle and a maximum overlap between the soliton shape and a sphere of radius $R_\alpha = 1.3A_\alpha^{1/3}$ fm must be realised. In this way r_0 and L are not considered as free parameters. The reciprocal moment of inertia $C = \hbar^2/2\mathcal{I}(r_0, L, h)$ can

also be extracted from the soliton geometry by considering the rotation of the soliton plus the layer about the center of mass of the system. By introducing the solution eq.(13) in the Hamiltonian, eq.(11), we obtain the energy density profile of the α -cluster considered as a quantum extended particle²⁰, characterized by a half-width $L_{en}(\beta)$. We fix the parameter β and, consequently, the range of the quantum number k and the sign $+$ in eq.(18), by the equation $L_{en} = L$ with L obtained from the geometry of the soliton, as shown above. In the above conditions we obtain the following values for the parameters: $r_0 = 0.41$, $L = 0.546$, $\beta = 0.262$ and $n = 0, 1, 2$. With these values we obtain the corresponding values for the soliton excitation energy, $\hbar\omega_k = 0.801, 24.39$ and 34.15 MeV for the corresponding $k = 2, 1$ and 0 . We note that for other cluster configurations (i.e. larger mass of the colliding particle or of the target) a larger number of solitonic bands (ω 's) $k > 2$, is allowed. In this case we consider that only the $k = 2$ solitonic mode is excited (the other two modes introduce too high energies) and the total spectrum is limited to:

$$E_{n,I,N} = E_0 + A\left(n + \frac{1}{2}\right) - B\left(n + \frac{1}{2}\right)^2 + CI(I + 1) + \hbar\omega_2\left(N + \frac{1}{2}\right) \quad (14)$$

where we denoted $N = N_2$. The solitonic model, based on this nuclear molecular behaviour, succeeds to reduce the number of the parameters to only four: A , B , E_0 and h . The other parameters occurring in the theory (i.e. β , r_0 and L) are obtained from the geometry of the system. By fitting the theoretical spectrum, eq.(14), with the experimental resonances we obtain, for the values $A = 0.23$ MeV, $B = 0.015$ MeV $E_0 = 5.102$ MeV and $h = 0.17$, an excellent agreement, shown in Fig.2. The theoretical levels are indicated by horizontal lines and the experimental ones by circles. However, the nonlinear degree of freedom which is related to the soliton quantum number N is essential to reproduce the appearance of irregularly spaced levels with the same spin I . A surprising agreement is obtained and it must be emphasized that no additional fragmentations of levels due to the neglected collective levels are necessary like in the traditional models. In fact, the present description accounts for virtually all the observed states. We note that in this case the corresponding mass of the rigid core is 16.01 and of the layer 11.98 in a.m.u. Consequently this description implies that the α -cluster as a soliton plus ^{28}Si system could be interpreted as a swell of a α -particle in a liquid of sd-shell nucleons circumfloating a ^{16}O core.

Thus, the solitonic description reproduces excellently the observed experimental energies and spins of the intermediate states of $\alpha + ^{28}\text{Si}$ system. The present calculations predicts also positions and spins of other levels (which can be seen as shoulders, broadening and/or weak bumps in the excitation function as one can see from Fig.1). In the lower part of Fig.1 we present the theoretical spectrum (up-ward lines represent the fit and down-ward lines represent the theoretical predictions of the present model). It should be thus further noted that the predicted states then almost completely exhaust the experimental states, i.e. very little of the experimental cross section can be attributed to other kinds of states. There is further strong evidence for the fragmentated α -molecular structure is seen in other sd-shell nuclei than ^{28}Si . Narrow resonances similar to those shown in Fig.1 are observed in elastic scattering from ^{24}Mg ,²² and $^{29,30}\text{Si}$,²³, both elastic and inelastic scattering from $^{34,36}\text{S}$,²⁴ and also inelastic from ^{28}Si ⁸. The scattering from lighter nuclei does not show such a fragmentation.

We stress that both even and odd parities are reproduced using the same parameters. The present results also imply that the odd-even parity splitting of the band members predicted by other models like RGM and OCM, etc., is not needed nor supported by neither the present experimental data or this nonlinear model. The even and odd states form mixed parity bands which implies that the rotating mass has an asymmetric shape as it is natural for the $\alpha + {}^{28}\text{Si}$ -system. This theoretical description could imply that the alpha cluster as a soliton could be viewed as a new kind of nuclear deformation. Evidently the transitions between the states of the same rotational band, between the states with different vibrational numbers and especially between states belonging to a breather state and other states could confirm or infirm such a description. Such data could be obtained by detecting also the gamma ray spectra of a given resonance with a crystal ball. At present, and quite immediately a number of angular distributions of some nonidentified resonances can be compared with our spin predictions.

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Figure captions

Fig.1-Upper curve- The experimental excitation function of alpha particle elastic scattering on ^{28}Si . Resonances are plotted against the excitation energy, the upper panel for the energy range 6.5 MeV to 12.5 MeV, and the lower from 12.5 MeV to 19.5 MeV. The numbers indicate the values of the corresponding measured angular momentum.

Fig.1-Lower spectra- The theoretical spectrum. The up-ward lines represent the fit with the experimental resonances with measured angular momentum. The down-ward lines represent the theoretical predictions for the spin for other observed resonances. The value I of the spin is written next to each line.

Fig.2. The predicted resonances in the elastic scattering of alpha particles on ^{28}Si presented separately for even (Fig.2a) and odd (Fig.2b) I -values. Five groups of rotational bands depending of soliton quantum numbers indicated below each group (N) are plotted. Each group contains 7 rotational bands with the same moment of inertia identified by different vibrational numbers ($n = 0$ up to $n = 6$). The experimentally observed resonances [6,7b] are indicated by circles (full dots for resonances with identified spins and empty circles for resonances with unidentified spins - the spins are theoretically predicted). We can see a surprising agreement with 84 observed resonance resonances. Other 68 measured resonances without spin identification are also included in this scheme (open circles). Only the coefficients E_0 , A , B and h connected with the absolute value of the energy, the anharmonic vibrations and the depth of the layer are fitted for all these 152 states.