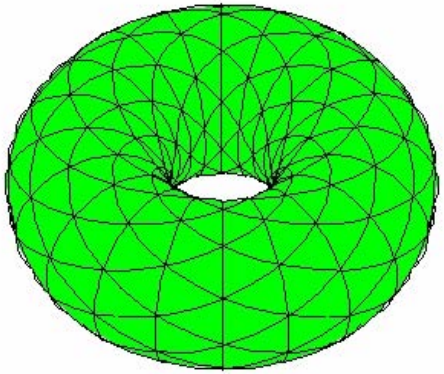
January 8, 1998

The Rose-Hulman Approach to Undergraduate Research - What Works for Us

Sean A Broughton



Friedman Cwatset

$\{000,110,101\}+110$
$= \{110,000,011\}$
$= \{000,110,101\}^{(1,2)}$

The Rose-Hulman Approach to Undergraduate Research

- What Works for Us -

S. Allen Broughton

Rose-Hulman Institute of Technology

DMS #9619714

Outline of Presentation

- Rose-Hulman Background
- REU History
- A Philosophy of Undergraduate Research
- Doable Problems: Geometry
- Can we Build it into the Program?
- Audience Questions

Rose-Hulman Background

- private, undergraduate college, 1600 mathematics, science and engineering students
- teaching paramount, scholarship expected
- 17 math faculty, pure and applied
- 50-75 majors, most are Math & CS majors
- year long sequence in discrete math, 50-70 students/year average
- abundant computing facilities

REU History

- *1988-1996* Gary Sherman, 6 students, computational group theory, developed REU tradition and philosophy
- *1997* Allen Broughton, 6 students, hyperbolic geometry and computational group theory,
- *1998-2000*, Allen B., Gary S., John Rickert, eight students, underlying focus of computational group theory and discrete math

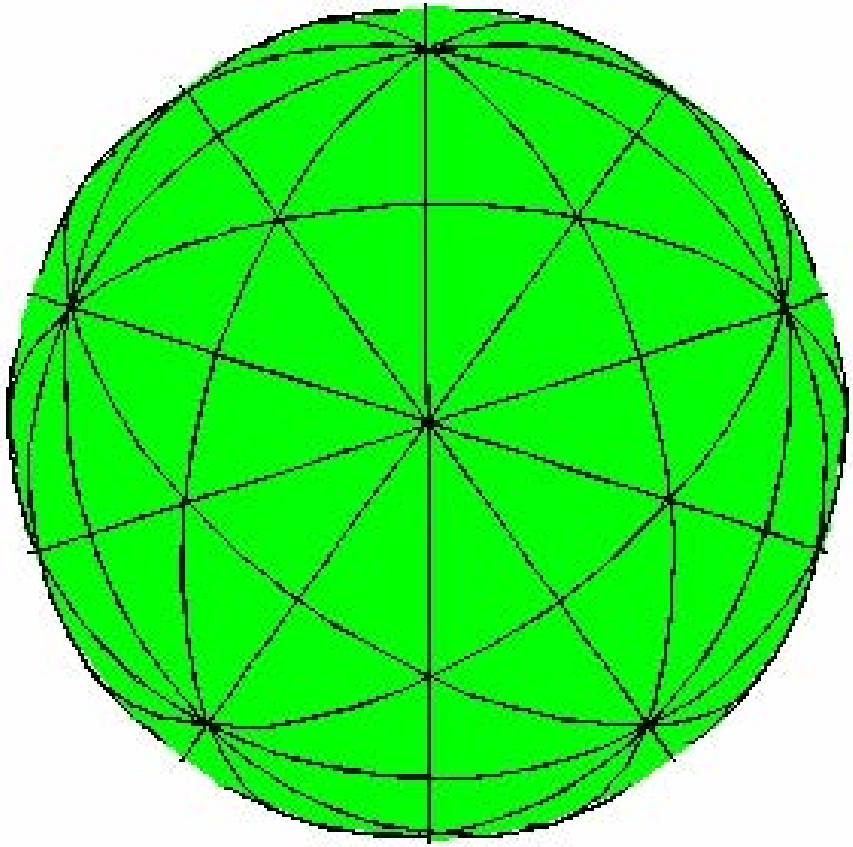
A Philosophy of Undergraduate Research

- "doable", interesting problems
- student - student & student -faculty collaboration
- computer experimentation (Magma, Maple)
- student presentations and writing
  - Undergrad Math Conference
  - Technical Report Series
- consistent, though loose focus

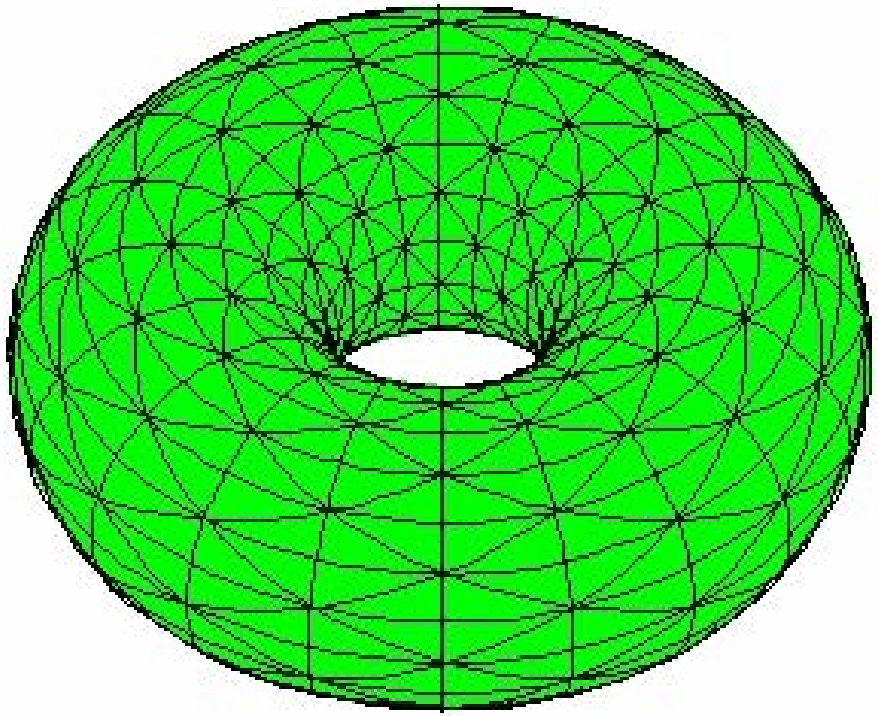
Doable Problems Hyperbolic Tilings

- show tilings
- the tiling group, link to computational group theory
- sample doable problems and results

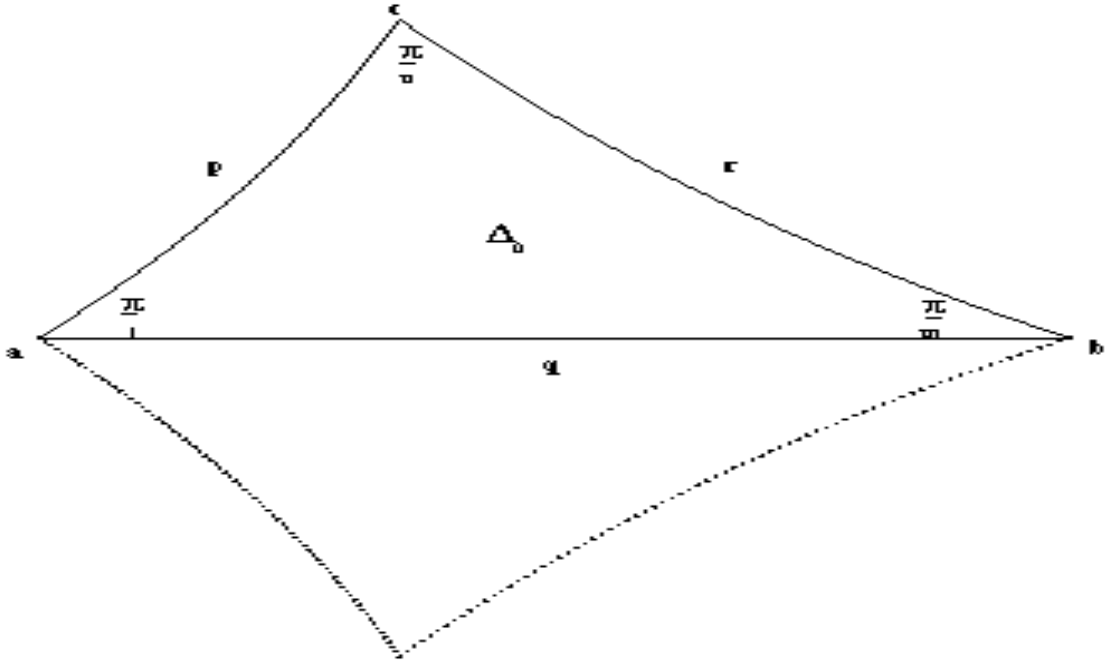
Icosahedral-Dodecahedral ((2,3,5), spherical geometry)



Tiling of the Torus ((2,4,4), Euclidean geometry)



The Master Tile (hyperbolic when genus > 1)



The Tiling Group & Relations

Tiling Group (a finite group)

$$G^* = < p, q, r >$$

Group Relations

$$p^2 = q^2 = r^2 = 1.$$

$$(pq)^l = (qr)^m = (rp)^n = 1.$$

Riemann-Hurwitz Equation

Let S be a surface of genus s with tiling group G^* then:

$$\frac{2s-2}{|G^*|/2} = 1 - \frac{1}{l} - \frac{1}{m} - \frac{1}{n}$$

$$\frac{2\times 0-2}{120/2} = 1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{5}$$

The Tiling Theorem

A surface S of genus s has a tiling with tiling group:

$$G^* = < p, q, r >$$

if and only if

- the group relations hold, and
- the Riemann Hurwitz equation holds.

Therefore Tiling Problems can be solved via group computation.

Doable Tiling Problems

- Tilings of low genus (Ryan Vinroot)
- Divisible tilings: surfaces simultaneous tiled compatible tilings of triangles and quadrilaterals, e.g., (2,4,4) tiling of torus (Dawn Haney & Lori McKeough)
- Oval intersection problems (Dennis Schmidt)

Sample Results
Divisible Tilings

- Show pictures

Sample Results
Divisible Tilings

A group theoretic surprise - 1

- Haney and McKeough have a found (3,7,3,7) tiling of the hyperbolic plane subdivided by the divisible by the (2,3,7) tiling
- For the surface S of smallest genus with this divisible tiling we have

$$|G^*| = 2357200374260265501327360000$$

$$s = 1403095460869205655552000 1$$

A group theoretic surprise - 2

$$|G^*| = 2^{21} \cdot 22! \text{ and}$$

$$1 \to Z_2^{21} \to G^* \to \Sigma_{22} \to 1$$

Building Student Research into the Regular Program

- need faculty support and interest
- need institutional support
- a career preparation
  - traditional student research for grad school bound students
  - industrial consulting projects for industry bound students

Thank You for listening!

Questions???

Shameless RHIT Promotion Slide

- Rose-Hulman Mathematics Dept
  - **http://www.rose-hulman.edu/Class/ma/HTML**
- Undergrad Math Conference March 13-14
  - **http://www.rose-hulman.edu/Class/ma/HTML/Conf/UndergradConf.html**
- NSF-REU
  - **http://www.rose-hulman.edu/Class/ma/HTML/REU/NSF-REU.html**