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CLOSURE UNDER TRANSFINITE EXTENSIONS

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Dedicated to Christian Ulrik Jensen

Abstract. The closure under extensions of a class of objects in an abelian category is often an important property of that class. Recently the closure of such classes under transfinite extensions (both direct and inverse) has begun to play an important role in several areas of mathematics, for example, in Quillen's theory of model categories and in the theory of cotorsion pairs. In this paper we prove that several important classes are closed under transfinite extensions.

1. Definitions and basic results

Throughout this paper $\mathcal{A}$ will be a Grothendieck category with a fixed projective generator U. We will be concerned with direct and inverse limits of systems of objects of $\mathcal{A}$ indexed by the well ordered set of ordinals α, where $\alpha \leq \lambda$ (or $\alpha < \lambda$) for some ordinal λ. To simplify notation, we will denote such a system (direct or inverse) by $(X_\alpha \mid \alpha \leq \lambda)$ with the associated morphisms understood.

Definition 1.1. A direct (inverse) system $(X_\alpha \mid \alpha \leq \lambda)$ is said to be continuous if $X_0 = 0$ and if for each limit ordinal $\beta \leq \lambda$ we have $X_\beta = \varinjlim X_\alpha$ (or $X_\beta = \varprojlim X_\alpha$) with the limit over the $\alpha < \beta$. The direct (inverse) system $(X_\alpha \mid \alpha \leq \lambda)$ is said to be a system of monomorphisms (epimorphisms) if all the morphisms in the system are monomorphisms (epimorphisms).

In order for a continuous direct system $(X_\alpha \mid \alpha \leq \lambda)$ to be a system of monomorphisms it suffices that $X_\alpha \to X_{\alpha+1}$ be monomorphism whenever $\alpha + 1 \leq \lambda$. This follows from what is called the AB5 axiom of a Grothendieck category. If $(X_\alpha \mid \alpha \leq \lambda)$ is a continuous inverse system such that each $X_{\alpha+1} \to X_\alpha$ (when $\alpha + 1 \leq \lambda$) is an epimorphism, then $(X_\alpha \mid \alpha \leq \lambda)$ is a system of epimorphisms. This is a consequence of the existence of a projective generator U and the fact that $(\mathrm{Hom}(U, X_\alpha) \mid \alpha \leq \lambda)$ is a continuous

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inverse system of sets. The analogous claim for continuous inverse systems of sets is true. Since U is a projective generator, the fact that $\mathrm{Hom}(U, X_\beta) \to \mathrm{Hom}(U, X_\alpha)$ is surjective gives that $X_\beta \to X_\alpha$ is an epimorphism when $\alpha \leq \beta \leq \lambda$.

In the rest of the paper let $\mathcal{L}$ be a class of objects of $\mathcal{A}$ which is closed under isomorphisms.

Definition 1.2. An object X of $\mathcal{A}$ is said to be a direct (inverse) transfinite extension of objects of $\mathcal{L}$ if $X = \varinjlim X_\alpha$ (or $X = \varprojlim X_\alpha$) for a continuous direct (inverse) system $(X_\alpha \mid \alpha \leq \lambda)$ of monomorphisms (epimorphisms) such that

$$\mathrm{coker}(X_\alpha \to X_{\alpha+1}) \ (\ker(X_{\alpha+1} \to X_\alpha))$$

is in $\mathcal{L}$ whenever $\alpha + 1 \leq \lambda$. $\mathcal{L}$ is said to be closed under direct (inverse) extensions if each direct (inverse) transfinite extension of objects in $\mathcal{L}$ is also in $\mathcal{L}$.

We give several examples of such closures.

Theorem 1.3 (Auslander [Aus55], Proposition 3). *If $n \geq 0$, then the class of objects L with* proj. $\dim L \leq n$ *is closed under direct transfinite extensions.*

Auslander proved his theorem for categories of modules, but his proof carries over to this general setting. We will see below that Auslander's result is also a consequence of a theorem of Eklof.

Definition 1.4. For the class $\mathcal{L}$ we define $\mathcal{L}^\perp$ (or ${}^\perp\mathcal{L}$) to be the class of objects Y such that $\mathrm{Ext}^1(L, Y) = 0$ (or $\mathrm{Ext}^1(Y, L) = 0$) for all objects $L \in \mathcal{L}$.

Theorem 1.5 (Eklof [Ekl77], Theorem 1.2). *For any $\mathcal{L}$ the class ${}^\perp\mathcal{L}$ is closed under direct transfinite extensions.*

Theorem 1.6 (Trlifaj [Trl03], Lemma 2.3). *For any $\mathcal{L}$ the class $\mathcal{L}^\perp$ is closed under inverse transfinite extensions.*

We now show that Auslander's theorem above follows from Eklof's theorem.

Corollary 1.7. *If $n \geq 0$ and if $\mathcal{L}$ is the class of objects L such that* proj. $\dim L \leq n$ *(*proj. $\dim L \leq n$*), then $\mathcal{L}$ is closed under direct (inverse) transfinite extensions.*

Proof. Let $\mathcal{C}$ be the class of all n-th cosyzygies of objects of $\mathcal{A}$. Then $L \in {}^\perp\mathcal{C}$ if and only if $\mathrm{Ext}^{n+1}(L, -) = 0$ so if and only if proj. $\dim L \leq n$. By Eklof's theorem $\mathcal{L} = {}^\perp\mathcal{C}$ is closed under direct transfinite extensions. This gives the claim for the class $\mathcal{L}$ of L with proj. $\dim L \leq n$. A dual argument gives the claim for the class of L with proj. $\dim L \leq n$. $\square$

2. Categories of complexes

We now consider the category $\mathcal{C}(\mathcal{A})$ of complexes of objects of $\mathcal{A}$. If C is an object of $\mathcal{C}(\mathcal{A})$, we will write $C = (C^n)(n \in Z)$. We will use subscripts to distinguish objects of $\mathcal{C}(\mathcal{A})$.

For the terminology in the next result and its corollaries see [AF91] or [EJX96].

THEOREM 2.1 ([EJX96], Theorem). *If $\mathcal{L}$ is the class of exact complexes in $\mathcal{C}(\mathcal{A})$, then ${}^{\perp}\mathcal{L}$ is the class of DG-projective complexes and $\mathcal{L}^{\perp}$ is the class of DG-injective complexes. Furthermore $({}^{\perp}\mathcal{L})^{\perp} = \mathcal{L} = {}^{\perp}(\mathcal{L}^{\perp})$.*

We have the following applications. Both are immediate applications of this theorem and of Theorems 1.5 and 1.6 above.

COROLLARY 2.2. *The class of DG-projective complexes is closed under direct transfinite extensions and the class of DG-injective complexes is closed under inverse transfinite extensions.*

COROLLARY 2.3. *The class of exact complexes is closed under both direct and inverse transfinite extensions.*

We note that in fact the class of exact complexes is closed under arbitrary direct limits since $\mathcal{A}$ satisfies AB5. The fact that this class is closed under inverse transfinite extension is known as the Mittag-Leffler theorem when the inverse systems in question are indexed by $n < \omega$, where ω is the first infinite ordinal. So we have a generalization of the Mittag-Leffler theorem. We will need this generalization in what follows.

DEFINITION 2.4. A complex P in $\mathcal{C}(\mathcal{A})$ is said to be a complete projective resolution if each P^n is projective, if P is exact and if for every projective object Q of $\mathcal{A}$, the complex $\mathrm{Hom}(P, Q)$ of abelian groups is exact. An object X of $\mathcal{A}$ is said to be Gorenstein projective if there is a complete projective resolution P such that $X = \ker(P^0 \to P^1)$. In this case we say P is a complete projective resolution of X.

The dual notions are those of a complete injective resolution and a Gorenstein injective object.

LEMMA 2.5. *The class of complete projective (injective) resolutions in $\mathcal{C}(\mathcal{A})$ is closed under extensions.*

Proof. Let $0 \to P' \to P \to P'' \to 0$ be an exact sequence in $\mathcal{C}(\mathcal{A})$ with P' and P'' complete projective resolutions. Since $0 \to (P')^n \to P^n \to (P'')^n \to 0$ is exact for each $n \in Z$ and since by hypothesis $(P')^n$ and $(P'')^n$ are projective, we get that P^n is projective. Both P' and P'' are exact, so P is an exact

complex. Now suppose that Q is a projective object of $\mathcal{A}$. Since each $0 \to (P')^n \to P^n \to (P'')^n \to 0$ is split exact, we get that

$$0 \to \mathrm{Hom}(P'', Q) \to \mathrm{Hom}(P, Q) \to \mathrm{Hom}(P', Q) \to 0$$

is an exact sequence of complexes. Since by hypothesis each of $\mathrm{Hom}(P', Q)$ and $\mathrm{Hom}(P'', Q)$ is exact, we get that $\mathrm{Hom}(P, Q)$ is also exact. Hence P is also a complete projective resolution.

The argument for complete injective resolutions is dual to this one. □

THEOREM 2.6. *The class of complete projective (injective) resolutions in $\mathcal{C}(\mathcal{A})$ is closed under direct (inverse) transfinite extensions.*

Proof. We let $(C_\alpha \mid \alpha \leq \lambda)$ be a continuous direct system of monomorphisms in $\mathcal{C}(\mathcal{A})$ such that $C_{\alpha+1}/C_\alpha$ is a complete projective resolution when $\alpha + 1 \leq \lambda$. Then for each $n \in Z$, $(C_\alpha^n \mid \alpha \leq \lambda)$ is a continuous direct system of monomorphisms in $\mathcal{A}$ with $C_{\alpha+1}^n/C_\alpha^n = (C_{\alpha+1}/C_\alpha)^n$ projective. Hence by Theorem 1.3 we get that C_λ^n is projective. By Corollary 2.3 C_λ is exact. Now let Q be a projective object of $\mathcal{A}$. The inverse system $(\mathrm{Hom}(C_\alpha, Q) \mid \alpha \leq \lambda)$ is continuous. If $\alpha + 1 \leq \lambda$, then the exact sequence

$$0 \to C_\alpha \to C_{\alpha+1} \to \frac{C_{\alpha+1}}{C_\alpha} \to 0$$

"splits at the object level," i.e., for each $n \in \mathbb{Z}$,

$$0 \to C_\alpha^n \to C_{\alpha+1}^n \to \left(\frac{C_{\alpha+1}}{C_\alpha}\right)^n \to 0$$

is split exact. This follows from the fact that $C_{\alpha+1}/C_\alpha$ is a complete projective resolution and so each $(C_{\alpha+1}/C_\alpha)^n$ is projective. Hence

$$0 \to \mathrm{Hom}\left(\left(\frac{C_{\alpha+1}}{C_\alpha}\right)^n, Q\right) \to \mathrm{Hom}(C_{\alpha+1}^n, Q) \to \mathrm{Hom}(C_\alpha^n, Q) \to 0$$

is exact for every projective object Q of $\mathcal{A}$. But then

$$0 \to \mathrm{Hom}\left(\frac{C_{\alpha+1}}{C_\alpha}, Q\right) \to \mathrm{Hom}(C_{\alpha+1}, Q) \to \mathrm{Hom}(C_\alpha, Q) \to 0$$

is an exact sequence of complexes. Since $\mathrm{Hom}\,(C_{\alpha+1}/C_\alpha, Q)$ is exact for each $\alpha + 1 \leq \lambda$, we see that $\mathrm{Hom}(C_\lambda, Q)$ is an inverse transfinite extension of exact complexes of abelian groups. So by Corollary 2.3 we get that C_λ is a complete projective resolution.

A dual argument gives the result for complete injective resolutions. □

3. The Gorenstein version of a result of Auslander

In his thesis [Hol04] Henrik Holm states the following metatheorem: Every result in classical homological algebra has a counterpart in Gorenstein homological algebra. The results in this section support his claim by proving the Gorenstein version of Theorem 1.3.

LEMMA 3.1. *If $0 \to L' \to L \to L'' \to 0$ is an exact sequence in $\mathcal{A}$ and if P' and P'' are complete projective resolutions of L' and L'', respectively, then there is an exact sequence $0 \to P' \to P \to P'' \to 0$ in $\mathcal{C}(\mathcal{A})$, where P is a complete projective resolution of L, such that $0 \to P' \to P \to P'' \to 0$ induces the original exact sequence $0 \to L' \to L \to L'' \to 0$. Consequently, if L' and L'' are Gorenstein projective, so is L. The dual result for Gorenstein injective objects also holds.*

Proof. This is just the horseshoe lemma. The usual version is for projective resolutions, but carries over to complete projective resolutions once we observe that

$$\begin{tikzcd} L' \arrow[r] \arrow[d] & L \\ (P')^0 \arrow[ur, dotted] \arrow[ur, phantom, "\lrcorner"] & \end{tikzcd}$$

can be completed to a commutative diagram since $\mathrm{Ext}^1(L'', (P')^0) = 0$. The dual of this argument gives the result for Gorenstein injective objects. $\square$

We now prove:

THEOREM 3.2. *The class $\mathcal{L}$ of Gorenstein projective (injective) objects of $\mathcal{A}$ is closed under direct (inverse) transfinite extensions.*

Proof. Let $\mathcal{L}$ be the class of Gorenstein projective objects in $\mathcal{A}$. Let $L = \varinjlim L_\alpha (\alpha \leq \lambda)$, where $(L_\alpha \mid \alpha \leq \lambda)$ is a continuous direct system of monomorphisms in $\mathcal{A}$ such that $L_{\alpha+1}/L_\alpha$ is Gorenstein projective whenever $\alpha + 1 \leq \lambda$. We must argue that L is Gorenstein projective. We do so by producing a complete projective resolution P of L. Our P will be of the form $\varinjlim P_\alpha$ such that (i) $(P_\alpha \mid \alpha \leq \lambda)$ is a continuous direct system of monomorphisms in $\mathcal{C}(\mathcal{A})$, (ii) each P_α is a complete projective resolution of L_α, (iii) $L_\alpha = \ker(P^0_\alpha \to P^1_\alpha)$, and (iv) $P_\alpha \to P_{\alpha'}$ induces our given morphism $L_\alpha \to L_{\alpha'}$ for $\alpha \leq \alpha' \leq \lambda$.

We use a transfinite construction to construct the continuous direct system $(P_\alpha \mid \alpha \leq \lambda)$ of monomorphisms in $\mathcal{C}(\mathcal{A})$ satisfying our conditions.

We start by letting $P_0 = 0$. If we have constructed P_α and if $\alpha + 1 \leq \lambda$, we construct $P_{\alpha+1}$ and the morphism $P_\alpha \to P_{\alpha+1}$ as follows:

We have the exact sequence

$$0 \to L_\alpha \to L_{\alpha+1} \to \frac{L_{\alpha+1}}{L_\alpha} \to 0.$$

By hypothesis $L_{\alpha+1}/L_\alpha$ is Gorenstein projective. Let P'_α be a complete projective resolution of $L_{\alpha+1}/L_\alpha$. Then by Lemma 3.1 there is an exact sequence

$$0 \to P_\alpha \to P_{\alpha+1} \to P'_\alpha \to 0$$

in $\mathcal{C}(\mathcal{A})$ with $P_{\alpha+1}$ a complete projective resolution of $L_{\alpha+1}$ and which induces the sequence

$$0 \to L_\alpha \to L_{\alpha+1} \to \frac{L_{\alpha+1}}{L_\alpha} \to 0.$$

This gives us both $P_{\alpha+1}$ and the monomorphism $P_\alpha \to P_{\alpha+1}$.

If $\beta \leq \lambda$ is a limit ordinal and if we have constructed a continuous system $(P_\alpha \mid \alpha < \beta)$ of monomorphisms in $\mathcal{C}(\mathcal{A})$ with each P_α a complete projective resolution of L_α and which induces the system $(L_\alpha \mid \alpha < \beta)$, we let $P_\beta = \varinjlim P_\alpha$ $(\alpha < \beta)$. Then by Lemma 2.5 we have that P_β is a complete projective resolution. Also, it is a complete projective resolution of $\varinjlim L_\alpha = L_\beta$. This gives us the desired system $(P_\alpha \mid \alpha \leq \beta)$. So finally we get the system $(P_\alpha \mid \alpha \leq \lambda)$ with P_λ a complete projective resolution of $L_\lambda = L$. So L is Gorenstein projective. A dual argument gives the claim concerning Gorenstein injective objects. □

Definition 3.3. If X is an object of $\mathcal{A}$, the Gorenstein projective dimension of X is defined as the least $n \geq 0$ (if it exists) such that there is a partial projective resolution

$$0 \to C \to P_{n-1} \to \cdots \to P_0 \to X \to 0$$

with C Gorenstein projective. If there is no such n we say this dimension is infinite. We use $\mathrm{Gpd}(X)$ to denote this dimension.

The Gorenstein injective dimension is defined dually and is denoted $\mathrm{Gid}(X)$.

The next result gives the Gorenstein version of Auslander's Theorem 1.3 above.

Theorem 3.4. *If $n \geq 0$ and if $\mathcal{L}$ is the class of objects L of $\mathcal{A}$ such that $\mathrm{Gpd}(L) \leq n$ ($\mathrm{Gid}(L) \leq n$), then $\mathcal{L}$ is closed under direct (inverse) transfinite extensions.*

Proof. If $n = 0$ we have the claim by Theorem 3.2. The induction step then is essentially that given by Auslander (p. 69 of [Aus55]). We now give our version of Auslander's argument. Let $(L_\alpha \mid \alpha \leq \lambda)$ be a continuous system of monomorphisms. Since U is a projective generator of $\mathcal{A}$, the evaluation morphism $U^{(\mathrm{Hom}(U,L_\alpha))} \to L_\alpha$ is an epimorphism and $U^{(\mathrm{Hom}(U,L_\alpha))}$ is projective. The system $(U^{(\mathrm{Hom}(U,L_\alpha))} \mid \alpha \leq \lambda)$ is a system of monomorphisms, but may

fail to be continuous. We remedy this by defining the system $(P_\alpha \mid \alpha \leq \lambda)$, where $P_\alpha = U^{(\mathrm{Hom}(U,L_\alpha))}$ if α is not a limit ordinal, and $P_\beta = \varinjlim U^{(\mathrm{Hom}(U,L_\alpha))}$ $(\alpha < \beta)$ if β is a limit ordinal. Since $L_\beta = \varinjlim L_\alpha$ $(\alpha < \beta)$ for such β, we can still use the evaluation morphisms and get an epimorphims $P_\beta \to L_\beta$. Also P_β, as a coproduct of copies of U, is projective. For $\alpha \leq \beta \leq \lambda$, $P_\alpha \to P_\beta$ is a monomorphism. So $(P_\alpha \mid \alpha \leq \lambda)$ is a continuous system of monomorphisms. If $\alpha + 1 \leq \lambda$, then $P_{\alpha+1}/P_\alpha$ is a coproduct of copies of U and so is projective.

For each $\alpha \leq \lambda$ let $0 \to K_\alpha \to P_\alpha \to L_\alpha \to 0$ be exact. Then $(K_\alpha \mid \alpha \leq \lambda)$ is a continuous system of monomorphisms. Since we are in a Grothendieck category, $0 \to \varinjlim K_\alpha \to \varinjlim P_\alpha \to \varinjlim L_\alpha \to 0$ is also an exact sequence. Now suppose that $\mathrm{Gpd}(L_\alpha) \leq n$, where $n \geq 1$. Then $\mathrm{Gpd}(K_\alpha) \leq n-1$ for each α. So if $n = 1$ each K_α is Gorenstein projective. By Theorem 3.2 $\varinjlim K_\alpha$ is Gorenstein projective. By Theorem 1.3 $\varinjlim P_\alpha$ is projective. Hence $\mathrm{Gpd}(\varinjlim L_\alpha) \leq 1$. This gives our claim for $n = 1$. Then we finish the argument by induction on n using the analogous induction step.

We now consider the Gorenstein injective case.

To dualize Auslander's argument we need to show that if $(X_\alpha \mid \alpha \leq \lambda)$ is a continuous inverse system of epimorphisms, then we can find a continuous inverse system $(E_\alpha \mid \alpha \leq \lambda)$ of epimorphisms of injective objects with $X_\alpha \subset E_\alpha$ for all α such that $(X_\alpha \mid \alpha \leq \lambda)$ is a subsystem of the system $(E_\alpha \mid \alpha \leq \lambda)$. This means not only that $X_\alpha \subset E_\alpha$, but also that $E_\beta \to E_\alpha$ agrees with $X_\beta \to X_\alpha$ whenever $\alpha \leq \beta \leq \lambda$. This is also done by a transfinite construction. We start with $E_0 = 0$. Having constructed E_α and, of course, all E_ξ with $\xi \leq \alpha$ along with the appropriate epimorphisms, we need to show how to construct $E_{\alpha+1}$ along with an epimorphism $E_{\alpha+1} \to E_\alpha$ when $\alpha + 1 \leq \lambda$. To do this, we use the horseshoe lemma associated with the diagram

$$\begin{array}{ccccccc} 0 \to & K & \to X_{\alpha+1} \to & X_\alpha & \to 0, \\ & \bigcap & & \bigcap & \\ & E' & & E_\alpha & \end{array}$$

where E' is injective. This gives an exact sequence $0 \to E' \to E_{\alpha+1} \to E_\alpha \to 0$ and so gives both $E_{\alpha+1}$ and the required epimorphism along with a morphism $X_{\alpha+1} \to E_{\alpha+1}$.

If $\beta \leq \lambda$ is a limit ordinal and if we have the appropriate system $(E_\alpha \mid \alpha < \beta)$, we want to expand to a system $(E_\alpha \mid \alpha \leq \beta)$. We let $E_\beta = \varprojlim E_\alpha$ $(\alpha < \beta)$. E_β is injective by Corollary 1.7 with $n = 0$. Continuing the process we finally have the continuous system $(E_\alpha \mid \alpha \leq \lambda)$. It is a system of epimorphisms by the comments at the beginning of the paper and it has the system $(X_\alpha \mid \alpha \leq \beta)$ as a subsystem. So we have our desired system.

Now we consider the system $(E_\alpha/X_\alpha \mid \alpha \leq \lambda)$. It is a system of epimorphisms. Also $E_0/X_0 = 0$. To get that it is continuous we appeal to a result of Jensen ([Jen72], Proposition 1.6 on p. 7).

With these observations we have the beginning of the induction step dual to the induction step used by Auslander. The rest of the argument is a straight-forward dual to his. □

The global Gorenstein projective dimension of $\mathcal{A}$ is defined to be the supremum of all $\mathrm{Gpd}(X)$, where X ranges over the objects of $\mathcal{A}$.

PROPOSITION 3.5. *The global Gorenstein projective dimension of $\mathcal{A}$ is the supremum of* $\mathrm{Gpd}(U/S)$*, where $S \subset U$ ranges over all subobjects of the projective generator U of $\mathcal{A}$.*

Proof. We only need to argue that every object X of $\mathcal{A}$ is a direct transfinite extension of objects of the form U/S. Then the result will follow from Theorem 3.4. Since U is a projective generator, we have an epimorphism $U^{(\lambda)} \to X$ for some ordinal λ. Here $U^{(\lambda)}$ denotes the coproduct of copies of U indexed by λ as a set. If $\beta \leq \lambda$, we identify $U^{(\beta)}$ with a subobject of $U^{(\lambda)}$. We let X_β be the image of $U^{(\beta)}$ in X under the map $U^{(\lambda)} \to X$. Then by the AB5 axiom of a Grothendieck category we have that X_β is the union of the X_α for $\alpha < \beta$ when β is a limit ordinal. If $\alpha + 1 \leq \lambda$, then by an application of the snake lemma $X_{\alpha+1}/X_\alpha$ is a quotient of U, so is of the form U/S. So we get X as the desired transfinite extension. □

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