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Farshid Alambeigi$^{1*}$, Ali Zamani$^2$, Gholamreza Vossoughi$^3$, Mohammad Reza Zakerzadeh$^4$

$^1,^3,^4$ Department of Mechanical Engineering, Sharif University of Technology, Tehran, Iran
(Tel : +98-21-6616-5578; E-mail: alambeigi.farshid@gmail.com) * Corresponding author
$^2$ Department of Mechanical Engineering, K. N. Toosi University of Technology, Tehran, Iran
(Tel : +98-21-8867-4748; E-mail: ali.zamani1986@gmail.com)

Abstract: There has been great demand for shape memory alloy (SMA) wires as actuators for shape control of flexible structures. The experimental setup of this study consists of a flexible beam actuated by two active SMA actuators. The input applied to the SMA actuator in this setup is electrical current while the output is the strain or position. To control strain of the actuator, the SMA wire is heated resistively in order to reach the desired temperature calculated by inverse of the phenomenological model. In heating the SMA wire resistively, the controllable quantity is the heat input to the wire via an applied current. In controller design, changes of physical properties of SMA wires and the surrounding air due to temperature change must be taken into consideration. This adds uncertainty to the presented model. Furthermore, both wires must approach the desired temperature while maintaining the same temperature history. Moreover, a suitable shape control requires overdamped response. A 2-DOF robust controller is designed in this study in order to achieve all the above requirements. The robust controller by the DK iteration method is designed after modeling of the system uncertainties. Required simulations are performed for evaluation of the controller. Obtained results show the ability of controller against time variant uncertainties.

Keywords: DK iteration, temperature control, SMA actuator

1. INTRODUCTION

SMA actuators have several advantages such as excellent power-to-weight ratio, maintainability, light weight, and clean-noiseless actuation comparing to other actuators. Therefore, SMAs are used in a variety of different applications such as civil engineering, aeronautical fields, robotic surgical systems, and walking robots. However, they have some disadvantages such as nonlinearity, dependency to the hysteresis, and parameter uncertainties that make strain control of these actuators inaccurate.

SMAs are heated by passing an electrical current through the wires and can be cooled using air convection. Temperature variations during these processes change the physical properties of SMA wires and the surrounding air. Therefore, parametric uncertainties caused by these temperature changes must be taken into consideration in controller design.

Some literatures have proposed mathematical models to represent characteristics of shape memory alloys [1-3]. The thermo-mechanical behavior between the temperature and the SMA wire strain is usually modeled by phenomenological models like Preisach model or Prandtl-Ishlinskii [4, 5]. In addition, several researchers have focused on the control aspects of SMA actuators. Various types of linear PID controllers have been suggested in some literatures [6-8]. Also, fuzzy logic controller [9] and neuro-fuzzy logic model with a PD controller [10] have been used. In another work, feedback linearization method was used by Arei et al. [11]. Variable Structure Control method is another control approach that has been used for position control [12] and force control [13] of the differential type actuators using a linear model. Elahinia and Ashrafuon proposed a nonlinear robust control algorithm for accurate positioning of a single degree of freedom rotary manipulator actuated by shape memory alloy [14].

In this work, we have developed a two degree-of-freedom (2-DOF) robust control algorithm based on the DK iteration method for the temperature control of two SMA actuators. The setup of this study consists of a flexible beam actuated by two active SMA actuators. To control strain of the actuator, SMA wire is heated resistively in order to reach the desired temperature calculated by inverse of the phenomenological model. During the resistive heating of the SMA wires, heat input to the wire via an applied current is the controllable quantity. In controller design, physical properties changes of SMA wires and the surrounding air due to temperature variation must be taken into consideration. Furthermore, both wires must approach the desired temperature while maintaining the same temperature during the control process. Moreover, a suitable shape control requires overdamped response. To achieve all the above requirements, a 2-DOF robust controller with reference model is designed in this study. After modeling system uncertainties, the robust controller is designed by the DK iteration method and simulations are performed for evaluation of the controller. Obtained results show the ability of controller against time variant uncertainties.
2. SYSTEM MODELING

In this paper, an experimental setup with a Flexinol TM actuator wire, manufactured by Dynalloy Inc is used. The SMA wire is placed horizontally (parallel to the beam neutral axis) with one end fixed to the end of the beam and the other end to the base of the beam. The schematic of the cantilever flexible beam set-up actuated by a SMA wire is shown in Fig 1. The main properties of the SMA wire and the cantilever aluminum beam are presented in Table 1 and 2, respectively.

The input to the SMA actuator in this setup is applied current while the output is the strain or position. The hysteretic behavior between the temperature and its strain is usually modeled by phenomenological models like Preisach model or Prandtl-Ishlinskii model [4, 5]. The schematic of these actuator models is illustrated in Fig 2.

To control the strain of the actuator, the inverse of the phenomenological model that maps the desired strain to desired temperature must be obtained. Then there should be a temperature controller to map the temperature obtained from the inverse of the phenomenological model to the desired current. The controller of this research is designed to do the second task. The schematic of the whole controller is shown in Fig 3.

During the resistive heating of the SMA wires, the controllable quantity is not the temperature; it is the heat input into the wire via an applied current. To create a temperature control law using current as the input, we consider two assumptions about the SMA wire actuator:

**Assumption (a):** the wire is relatively long with a small cross-sectional area.

**Assumption (b):** there is uniform temperature distribution along the length of the wire (neglecting any temperature deviations at the clamped ends.)

The governing equation for the heat transfer problem of an SMA wire under constant applied load is [15]:

$$ C_v \left( \frac{dT(t)}{dt} \right) = -\frac{4h(T, D)}{D}(T(x, t) - T_\infty) + \rho_c i^2 $$

Where:
- $T(x, t)$ = temperature at time $t$,
- $D$ = diameter of the SMA wire,
- $T_\infty$ = surrounding bulk temperature,
- $h(T, D)$ = rate of convective heat transfer along the sides of the SMA wire,
- $\rho_c$ = electrical current resistivity, and
- $i$ = magnitude of the current.

Solving Eq. (1) for $\dot{T} = \frac{dT}{dt}$ yields the equation,

$$ \dot{T} = -\frac{4h}{C_vD}(T - T_\infty) + \frac{\rho_c i^2}{C_v} $$

The three coefficients of Eq. (2) can be lumped into general parameters:

### Table 1 Parameters of flexible aluminum beam

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Length</th>
<th>Thickness</th>
<th>Width</th>
<th>Young Modulus</th>
<th>Yield Stress</th>
<th>SMA Wire Offset Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>400 mm</td>
<td>1.27 mm</td>
<td>25 mm</td>
<td>70 GPa</td>
<td>410 MPa</td>
<td>10 mm</td>
</tr>
</tbody>
</table>

### Table 2 Thermo-mechanical parameters of SMA wire actuator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Martensite Finish Temperature</th>
<th>Martensite Start Temperature</th>
<th>Austenite Start Temperature</th>
<th>Austenite Finish Temperature</th>
<th>Maximum Recoverable Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>43.9 °C</td>
<td>48.4 °C</td>
<td>68 °C</td>
<td>73.75 °C</td>
<td>4.10 %</td>
</tr>
</tbody>
</table>

Fig. 1 Schematic of the cantilever flexible beam set-up actuated by a SMA wire

Fig. 2 SMA model

Fig. 3 SMA position controller
\[ a = \frac{4h}{C_v D}, \quad b = \frac{\rho}{C_v}, \quad \text{and} \quad c = \frac{4h}{C_v D} T_w \]

Eq. (2) takes the form
\[ \dot{T} = -aT + bu + c \]
where \( u = i^2 \) is the control variable. For two SMA wires there is a couple of equations:
\[ \begin{align*}
T_1 &= -a_1 T_1 + b_1 u_1 + c_1 \\
T_2 &= -a_2 T_2 + b_2 u_2 + c_2
\end{align*} \]

Where the coefficients according to Tables 1 and 2 have the following nominal values:
\[ \begin{align*}
a_1 &= a_2 = 0.8 \text{ (1/s)} \\
b_1 &= b_2 = 150 \text{ (K/s.A^2)} \\
c_1 &= c_2 = 16 \text{ (K/s)}
\end{align*} \]

A control strategy should be designed (control inputs \( u_1 \) and \( u_2 \)) to force \( T_1 \) and \( T_2 \) (both of them) have set point tracking with satisfying the following conditions:
- \( T_i (t) = T_i (t) \) at all times.
- \( T_1 \) and \( T_2 \) have overdamped responses. It means that the overshoot is not allowable for them.
- The input current should not be exceeded from 1.5 A.

Since the parameters \( a, b, \) and \( c \) in Eq. (4) are temperature dependant, they have uncertainties. Therefore, we considered uncertainty of 20% of the nominal values. Besides, we supposed that there is a constant disturbance \( d = 1(A) \), where we also considered an uncertainty of 20% of the nominal value. Regarding four inputs (two disturbances and two control inputs) and two outputs (temperature of wires), state space matrices are given by:
\[ \begin{align*}
A &= \begin{bmatrix}
-a_1 & 0 \\
0 & -a_2
\end{bmatrix}, \quad B = \begin{bmatrix}
b_1 & 0 & c_1 & 0 \\
0 & b_2 & 0 & c_2
\end{bmatrix} \\
C &= \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{bmatrix}, \quad D = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\end{align*} \]

3. \( \mu \) SYNTHESIS AND DK ITERATION

The structured singular value \( \mu \) for a loop shown in Fig. 4 and \( M \in C^{n \times n} \) is defined by:
\[ \mu_\Delta^{-1} (M) = \min \{ \sigma (\Delta) \mid \Delta \in \Delta, \det (1 - M\Delta) = 0 \} \]

Unless no \( \Delta \in \Delta \) makes \( I - M\Delta \) singular, in which case \( \mu_\Delta^{-1} (M) = 0 \). Where
\[ A = \begin{bmatrix}
d_1 & \cdots & d_n \\
\delta & \cdots & \delta
\end{bmatrix} \]

\[ \begin{bmatrix}
d_1 & \cdots & d_n \\
\delta & \cdots & \delta
\end{bmatrix} \]

For consistency among all the dimensions, we must have
\[ \sum_{i=1}^n a_i + \sum_{j=1}^n m_j = n \]

For the purpose of controller design, the controller \( K \) from the transfer function matrix \( M(s) \) in Fig. 4 had been extracted. So, Fig. 4 can be reshown in Fig. 5. Transfer function matrix \( M \) is:
\[ \begin{align*}
M(P, K) &= \begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix} + \begin{bmatrix}
P_{31} & 0 \\
0 & P_{32}
\end{bmatrix} K \begin{bmatrix} I - P_{33} K \end{bmatrix}^{-1} \begin{bmatrix} P_{31} & P_{32} \end{bmatrix}
\end{align*} \]

Where \( P \) is the nominal, open-loop interconnected transfer function matrix. For the optimal robust stability and robust performance design, a stabilizing controller \( K \) must be found such that:
\[ \inf \sup_{\omega \in \mathbb{R}} \sigma \left[ M(P, K) \right] \]

A reasonable approach is iteratively solving the following equation for \( K \) and \( D \):
\[ \inf \sup_{\omega \in \mathbb{R}} \sigma \left[ DM(P, K) D^{-1} \right] \]

This approach is called D-K iteration \( \mu \)-synthesis method. The scaling matrix \( D(s) \) is chosen such that \( D(s)\Delta(s) = \Delta(s)D(s) \). In this approach, two parameters are sequentially minimized: first the minimization of \( K \) with \( D \) fixed, then the minimization of \( D \) with \( K \) fixed, successively. The D-K iterative \( \mu \)-synthesis algorithm is as follow [16]:

**Step 1:** Fix an initial estimate of the scaling matrix \( D \), usually set \( D = I \).

**Step 2:** Fix \( D \) and solve the \( H_\infty \) optimisation for \( K \).
Fig. 6 The block diagram of the closed-loop system

\[ K = \arg \inf_{K} \| M(\hat{P}, K) \| \]  

(14)

**Step 3:** Fix \( K \) and solve the following convex optimization problem for \( D \) at each frequency over a selected frequency range,

\[ D(j\omega) = \arg \inf_{D(j\omega) \in \mathcal{D}} \mathbb{E} \left[ DM(P, K)D^{-1}(j\omega) \right] \]  

(15)

**Step 4:** Curve fit \( D(j\omega) \) to get a stable, minimum-phase \( D(s) \); go to Step 2 and repeat, until a prespecified convergence tolerance is achieved, or a prespecified maximum iteration number is reached.

### 4. MODEL UNCERTAINTY

Fig. 6 illustrates the block diagram of the closed-loop system including the design requirements consideration shown by weights. We can realize the desired dynamics of the closed-loop system by implementing an appropriately selected model \( T_{ref} \). By using this model, the design specifications for producing the desired response of the system can be implemented. In order to achieve the mentioned control objectives, the outputs must have overdamped response. Consequently, two second order transfer functions with these coefficients \((\omega_n = 0.7, \zeta = 0.97)\) are considered. The off-diagonal elements of the \( T_{ref} \) matrix are chosen as zeros to minimize the interaction between the channels.

The performance weighting function \( W_p \) is used to close the system dynamics to the desired model dynamics \( T_{ref} \) over the objective control frequency range. Furthermore, the control weighting function \( W_u \) is used for limiting the magnitude of input current to the wires. The weight \( W_{ref} \) scales the magnitude of the SMA wires temperature, assuming that the maximum temperature is 15°C.

The performance and control action weighting functions are chosen as:

\[ W_p = \begin{bmatrix} s + 1.96 & 0 \\ 0.98s + 0.00196 & s + 1.96 \end{bmatrix} \]  

(16)

\[ W_u = \begin{bmatrix} 0.444s & 0 \\ 0.1592s + 1 & 0.444s \\ 0 & 0.1592s + 1 \end{bmatrix} \]  

(17)

\[ W_{ref} = \begin{bmatrix} 150 & 0 \\ 0 & 150 \end{bmatrix} \]  

(18)

The inverse of performance weighting function \( W_p \) and the control weighting function \( W_u \) are shown in Fig. 7.

### 5. DK ITERATION

Fig. 8 illustrates the block diagram of the closed-loop system used in the \( \mu \)-synthesis. Let us consider the transfer function matrix of input, output open-loop system consisting of the SMA model and the actuator and weighting functions as \( P(s) \). Define a block structure of uncertainty \( \Delta_P \) as:

\[ \Delta_P = \begin{bmatrix} \Delta \quad 0 \\ 0 \quad \Delta_f \end{bmatrix} : \Delta \in \mathbb{R}^{6 \times 4}, \Delta_f \in \mathbb{C}^{6 \times 4} \]  

(19)

In this matrix, the uncertainty block \( \Delta \) represents the uncertainties in the SMA model. The weighted error signals \( Z_1 \) and \( Z_2 \) are the inputs to the block \( \Delta \), a fictitious uncertainty block representing the robust performance objective, while the exogenous signals (two reference inputs and two unit disturbance) are the outputs from it.

With partitioning the scaled, two-degree-of-freedom controller as \( K(s) = [K_f(s) \ K_f(s)] \), where \( K_f \) is the prefilter...
Fig. 8 The block diagram of the closed-loop system used in the $\mu$-synthesis part of the controller and $K_2$ is the feedback part, the controller must be determined so that the structured singular value satisfies the following condition at each frequency $\omega \in [0, \infty]$:

$$\mu_{\lambda_1} \left[ F_L(P, K)(j\omega) \right] < 1$$

(20)

The $\mu$-synthesis is carried out by using the `dksyn` command from robust control toolbox in MATLAB software. Table 3 shows the accomplishment of the D-K iteration. As it can be seen from Table 3, the maximum value of $\mu$ is equal to 0.899 after the three iterations. To prove the stability of the system under perturbations, the maximum value of the $\mu$ must be under 1. According to Table 3, this condition has been satisfied.

$$\frac{1}{0.899} < 1$$

(21)

The closed-loop system achieves robust performance because the maximum value of $\mu$ is equal to 0.806.

Fig. 9 also illustrates the structured singular value versus frequency for confirmation of robust stability.

6. RESULTS

For evaluating the controller designed based on the proposed approach, we generate 20 random samples of the plant and plot the corresponding transient responses (temperatures of two wires) of the open-loop and closed-loop system for a step reference signal. A desired 100 $^\circ$C temperature is considered for simulation of the controller. According to Figs. 10-11, solid lines are open loop responses of 20 randomly chosen uncertain plants while dash lines represent their corresponding closed loop responses. As these figures show, all of the 20 closed-loop responses are exactly coincident with each other and good tracking of the reference input has been obtained. This means that the designed controller is robust against parameter uncertainty due to temperature change. Besides, the responses to the corresponding references have no overshoot. Since another control objective was to force wires to have equivalent temperatures at all times, difference between transient responses of two wires should be evaluated. For the proposed controller, this difference is near to zero as illustrated in Fig. 12. Last control objective is to consider saturation of the input current (1.5A). Figs. 13-14 represent the control input applied to the wires during simulation, which are smaller than allowable saturation limit. Considering all the results, three control objectives have been accomplished.

Table 3 DK-Iteration summary for the controller

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Controller order</th>
<th>Maximum value of $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>13.599</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>1.201</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>0.899</td>
</tr>
</tbody>
</table>

Fig. 9 The structured singular value versus frequency

Fig. 10 Transient responses of the open-loop and closed-loop system of wire no.1 for a step reference signal.

Fig. 11 Transient responses of the open-loop and closed-loop system of wire no.2 for a step reference signal.

7. CONCLUSION

The main purpose of this research was to design a 2-DOF robust controller by DK iteration for two SMA actuators. To this end, the mathematical model of the system was first obtained. Then $\mu$ synthesis method was applied to the system. Since the characteristics of the material used in SMA actuators are highly temperature dependent, we considered uncertainty of 20% of the nominal values. Then robust controller was designed with considering three control objectives:
Temperatures of two SMA wires must be same at all the times.

Overshoot response is not allowable.

The input current should not be exceeded from 1.5A.

Obtained simulation results showed that the performance of the controller in terms of the tracking errors was improved even with exerted uncertainties. Therefore, using the proposed approach is more promising for practical implementations.

REFERENCES


