Mechanics of Wrinkled Surface Adhesion

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Available at: https://works.bepress.com/alfred_crosby/6/
Mechanics of wrinkled surface adhesion†

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Received 28th January 2011, Accepted 29th March 2011
DOI: 10.1039/c1sm05146f

Surface buckling instabilities, particularly wrinkles, are spontaneously occurring surface patterns that can cover large areas and have the potential to modify the adhesion of surfaces in a systematic manner; however, the impact of these wrinkled features is not understood quantitatively. We utilize a novel fabrication process to form aligned wrinkles from polystyrene and polydimethylsiloxane and quantify their adhesion using contact adhesion tests. Wrinkle amplitudes range from 0.3 μm to 11.4 μm and wavelengths range from 6.2 μm to 74.0 μm, and these two parameters are tuned independently. The maximum separation force of a flat cylindrical probe from a wrinkled surface depends nonlinearly on the wrinkle geometry, as described by both amplitude and wavelength. Additionally, results are presented for a set of adhesion experiments conducted on single, macroscopic cylinders using small circular flat probes to mimic the contact of individual wrinkles. A simple scaling is presented that incorporates geometric parameters, testing geometry and materials properties to predict the separation force. This relationship is shown to be in good agreement with the experimental data.

Introduction

Control of surface properties through patterns is commonly found in Nature. Arachnids, insects, and several types of reptiles including skinks and geckos use patterns on their feet to tune adhesion.1–3 Plants, such as the sacred lotus, use periodic surface roughness to control the wetting properties of their leaves.4 Inspired by these examples, patterns have been used to fine tune the surface properties of various synthetic materials. Whether the patterns consist of chemical variations or topographic features, the effects on the wetting, friction, and adhesive properties of engineered surfaces have been characterized and studied.5,6

In most instances, complex and expensive fabrication techniques ranging from lithography to carbon nanotube processes have been used to create these synthetic analogs to naturally occurring surfaces.7,8 However, wrinkled surfaces, which are spontaneous or self-forming and attractive for large scale, inexpensive pattern production, have recently been explored as an alternative patterning method to these more conventional techniques. Wrinkles are caused by a compressive strain on a surface and by altering the strain conditions through materials properties, system geometry, and processing conditions, researchers have been able to demonstrate great control over wrinkle wavelength (λ), amplitude (b), and orientation over large surface areas.9

As control over wrinkle morphology has progressed, groups have studied the ability to use these structures to control surface properties, such as friction and adhesion.10,11 Lin and coworkers have shown that wrinkle aspect ratios (b/λ) can be used to control the maximum separation force of a rigid sphere on a wrinkled surface.12 Further, Chan and coworkers have shown that decreases in wrinkle wavelength can lead to enhanced adhesion of a rigid, flat punch on a wrinkled surface.13 More recently, the impact of both wavelength and amplitude on the adhesion of a curved surface has been reported.14 These works have demonstrated the impact of wrinkled topography on the apparent adhesion of a surface; however, there are still many fundamental questions of wrinkle adhesion that have yet to be understood.

In this work, we investigate the adhesion of aligned wrinkles consisting of a glassy polymer film attached to an elastomeric substrate using a rigid, flat punch under normal adhesion conditions. Since surface buckling instabilities are developed by the application of a compressive strain to a confined surface, a residual stress exists inherently within each sample. By taking advantage of the curing kinetics of the elastomeric substrate, we utilize wrinkles where the residual stress of the system is diminished, allowing the wrinkle kinetics of the elastomeric substrate, we utilize wrinkles where the residual stress of the system is diminished, allowing the wrinkle geometry effects to be studied independently from the impact of residual stress. Furthermore, we choose wrinkled materials with known elastic moduli and interfacial properties to allow us to develop a confirmed scaling relationship between the adherence force, materials properties, and wrinkle geometry.14,15

Experimental

Wrinkle formation

The process used to form the aligned wrinkles on the surface of the smooth elastomeric substrates yields uniform, aligned
wrinkles with an almost perfectly sinusoidal cross-sectional profile (Fig. 1d). This technique has been reported previously by Miquelard-Garnier and coworkers.\textsuperscript{16} Fig. 1a contains a schematic of this process.

For the study reported here, the glassy thin films are high molecular weight, atactic polystyrene (PS) (\(M_n = 1100\) kg mol\(^{-1}\), \(\text{PDI} = 1.15\), Polymer Source, Inc.) spincast from toluene (Fisher Scientific) onto ultraviolet/ozone (Jelight 342 UVO system) treated silicon wafers (University Wafer). By varying the concentration and spin speed, film thicknesses ranging from 20 nm to 300 nm as measured by white light interferometry (Filmetrics) are obtained. The substrates are polydimethylsiloxane (PDMS) (Dow Corning Sylgard 184) prepared using an oligomer to curing agent ratio of 10 : 1 by weight. The PDMS mixtures are degassed in a reduced pressure vacuum environment for 30 min, poured into glass molds measuring 30 \(\times\) 50 mm to form a flat layer approximately 3 mm thick, and cured at 70 °C for 20 min (resulting in a tacky, partially cured rubber). The modulus of each partially crosslinked substrate is determined independently using the custom built contact mechanics device described in the Adhesion characterization section below.

After wrinkling the PS film on the PDMS, the samples are allowed to finish curing either in a 70 °C oven for 24 h or at room temperature (25 °C) for 120 h. Wrinkle wavelength, amplitude, and substrate modulus are monitored over the course of the curing process using optical profilometry (Zygo NewView 7300) and contact mechanics testing (see Adhesion characterization below). While the wrinkle geometry does not change as a result of the curing process, the substrate modulus increases as expected until a final, consistent modulus value of \(E^* = 1.25\) MPa (120 h at 25 °C) or \(E^* = 2.00\) MPa (24 h at 70 °C) is reached (see ESI 1†).

The wrinkle fabrication technique covers large areas (several cm\(^2\)) with well-aligned wrinkles over the course of a few minutes. Fig. 1b contains optical micrographs of three different samples prepared with films of increasing thickness, showing the wrinkle alignment and giving some sense of the long range order achievable with this method.

Wrinkle wavelength and amplitude can be controlled independently through materials properties, sample geometry, and process parameters. The wavelength of each wrinkled surface is determined by the effective elastic modulus mismatch between the stiff film and soft substrate (\(E_f^*/E_s^*\)) and the thickness of the film (\(t\)) according to\textsuperscript{15,17,18}:

\[
\lambda = 2\pi t \left(\frac{E_f^*}{3E_s^*}\right)^{\frac{1}{3}}
\]

In the present study, the glassy film and elastic substrate are held constant and the wavelength is tuned by controlling the film thickness. The modulus of the film is constant for all samples (\(E_f^* = 3.00\) GPa). Fig. 1c shows the linear wavelength dependence on film thickness, with the slope of the line representing the modulus mismatch and yielding an effective modulus of \(E_s^* = 0.43\) MPa for the partially-cured PDMS substrate. This modulus value is in good agreement with the results of the contact mechanics measurements performed on the partially-cured elastomers.

While the wrinkle wavelength is controlled through materials properties and film geometry, the wrinkle amplitude is determined by the processing parameters, specifically the processing velocity during wrinkle formation. Details on the effect of velocity on the amplitude have been published previously.\textsuperscript{16}
Essentially, the slower the processing velocity, the larger the strain (e) applied to the substrate during the formation of the wrinkles, resulting in higher wrinkle amplitudes through the relationship:

\[
\frac{b}{\lambda} \sim \sqrt{e}
\]  

(2)

Additional parameters that impact the applied strain are the surface tension (γ), and the adhesion energy (G) between the glassy film and soft substrate. In Fig. 1d, it can be seen that changing the processing velocity by an order of magnitude results in a marked change in the amplitude of the wrinkled structures with minimal effects on the wavelength.

By applying the film prior to final curing of the substrate, the wrinkled surfaces have lower residual stress states than surfaces with films applied to fully crosslinked substrates. A more practical advantage of the two step curing process is that after crosslinking the substrate, the film can be removed by dissolving it away with an appropriate solvent, allowing the wrinkled substrate to be characterized with or without the capping layer. Removal of the film from the wrinkled substrates processed in two thermal curing steps does not have any impact on the aspect ratio of the surface patterns; the wrinkles remain the same size and shape. For wrinkles created on fully crosslinked substrates, removal of the flat films results in relaxation of the surface and disappearance of the wrinkles.

**Adhesion characterization**

The normal adhesion of each wrinkled surface is measured with a custom-built contact mechanics testing device. A PS-coated silica cylindrical probe is mounted on a load cell which is subsequently attached to a piezo-controlled linear actuator (Burleigh InChworm Nanopositioner) over the objective of an inverted microscope (Zeiss Axiovert 200M). The sample is then placed between the testing equipment and the microscope. All three components of the test are monitored simultaneously using a custom computer program (National Instruments Labview)

![Diagram](image-url)

**Fig. 2** Adhesion testing procedure and initial results. (a) Schematic of the contact adhesion measurement setup. (b) Force (P) as a function of time with significant points labelled corresponding to: P₀, before the probe contacts the sample surface; P₁, initial contact; Pₘ, maximum compression; and Pₛ, separation. (c) Typical force versus displacement (d) plots where P < 0 is compression and P > 0 is tension. (d) Optical micrographs of the contact areas for wrinkled samples of various wavelengths throughout an adhesion test. Insets are 100 × 100 μm. All results presented here are for wrinkles with an aspect ratio of b/λ~0.07 and approach and retract velocities are 1.0μm/s.
8.5). Fig. 2a is a schematic of the experimental setup. This testing method allows for the simultaneous measurement of the vertical displacement ($d$) of the probe, the normal force ($P$), and the area ($A$) of the interface over the course of a test.

An adhesion test consists of the probe being brought into contact with the sample, compressed at a fixed displacement rate until a specified, arbitrary maximum compressive force is achieved, and then the displacement is reversed until separation occurs. The contact area and force exerted on the probe are recorded throughout the test. In this study, tests have been performed at crosshead velocities ranging from 0.1 to 1.0 m/s and maximum compressive force values of $P_m = 10$ mN to $P_m = 50$ mN. However, within these ranges, little variation was observed in the value of maximum separation force, or adherence force, $P_s$ (Fig. 3). Therefore, all wrinkle adhesion data reported here are for $P_m = 10$ mN and $d\delta/dt = \pm 1.0$ m/s. Each data point represents an average of three tests.

Fig. 3  Force versus displacement data for adhesion tests run at (a) various maximum compressive force ($P_m$) values on the same wrinkled substrate ($\lambda = 22.0 \mu m$, $b = 2.92 \mu m$, $E_s^* = 1.25$ MPa) and (b) various displacement rates on the same wrinkled substrate ($\lambda = 21.9 \mu m$, $b = 1.53 \mu m$, $E_s^* = 2.00$ MPa).

Fig. 4  Effect of wrinkle amplitude ($b$) at a constant wavelength ($\lambda \sim 29 \mu m$) on adhesive behavior. (a) Typical force versus displacement curves for various amplitude wrinkled surfaces during adhesion testing where $P < 0$ is compression. (b) Optical micrograph contact images over the course of a test for various amplitude wrinkled surfaces. The circle in each low magnification image is the cylindrical probe face (radius $c = 0.50$ mm) and the black circle outline in the $P = P_i$ image for $b = 0.69 \mu m$ is added as a guide to the eye. Higher magnification images are $125 \times 125 \mu m$. For amplitudes of 1.92 \mu m and 3.16 \mu m only the darkest lines in the insets are areas in contact with the probe. The intermediate gray regions are a result of optical effects of imaging the sides of each wrinkle.
For all tests, unless otherwise stated, a cylindrical probe with a radius ($c$) of 500 µm is used. The flat, circular face of this probe is coated with a 250 nm PS film and annealed at 160 °C for 5 min to ensure conformal contact and prevent delamination between the cylindrical probe surface and the film during testing. The PS coated probe has an RMS surface roughness of approximately 15 nm over a lateral distance of 1mm, measured using optical profilometry over the entire face of the probe. Optical positioning equipment (Newport) is employed to ensure good parallel alignment between the face of the probe and the sample surface prior to each test.

The same polystyrene film is used to coat both the surface of the wrinkled samples and the probe used for adhesion testing. Since both surfaces have identical chemistry, the work of adhesion, $w_{adh} = 2\gamma$ where $\gamma$ is the surface energy of each contacting surface.19 At equilibrium, $w_{adh} = G_c$, the critical strain energy release rate for the interface, as modeled using linear elastic fracture mechanics.20

Though crack propagation velocity can impact $G_c$, the tests presented here are performed relatively slowly and $G_c$ is taken as a constant.21 This assumption is supported by the relative testing rate independence of $P_s$ measured here (Fig. 3b). By performing contact adhesion tests on smooth, unpatterned PS films attached to a PDMS substrate with the PS-coated cylindrical probe, $G_c$ has been measured and is found to be 0.048 J m$^{-2}$ for the PS-PS interface. This value for the critical strain energy release rate is in excellent agreement with values from literature.22 Representative force versus displacement curves and an explanation of the $G_c$ measurement on smooth surfaces is provided in ESI 2†.

The effective modulus ($E'$) for each substrate is determined independently through contact adhesion measurements on a flat portion of the PDMS substrate. In these experiments, $E'$ is calculated from the stiffness ($K = dP/da$) of the elastomer according to classical contact mechanics:23

$$E' = \left(\frac{1}{2\nu}\frac{\partial P}{\partial a}\right)$$  \hspace{1cm} (3)

where $E'$ is related to the Young’s modulus, $E$, and Poisson’s ratio, $\nu$, by $E' = E(1 - \nu^2)$.

Wrinkle adhesion results

The normal adhesion is characterized for a library of wrinkle samples. Fig. 2 shows the effect of wavelength on the force versus displacement history for typical contact adhesion measurements of wrinkled surfaces. Here, the aspect ratio is held fixed while wrinkle wavelengths are changed, allowing a direct comparison of wrinkles which are geometrically similar. From these tests, specific information about the sample stiffness, $G_c$, and maximum separation force ($P_s$) can be determined. In this study, the principle metric employed is the separation force. For the tests in Fig. 2, all samples are created using the same processing velocity ($V$) to yield wrinkles that have the same aspect ratio with various wavelengths.

Fig. 4a shows the adhesive behavior of samples created with the same film on the same PDMS substrate but cast at different velocities. Thus, all three samples have the same wavelength but varied amplitudes. The lowest amplitude wrinkles have the highest separation force. The contact images captured for each of the samples can be seen in Fig. 4b and provide some insight into the changes. As the amplitude increases, the contact and separation mechanism of the wrinkles changes. For the low amplitude wrinkles, the crests of each wrinkle come into contact and some of the troughs also contact the probe surface. Upon separation, these lower aspect ratio wrinkles separate in a more concerted way from the probe. However, for the two higher amplitude wrinkles, the contact with the probe is formed only with the top of each wrinkle and the width of each individual contact line ($2a$) grows and decreases in a less coupled manner.24 The higher magnification images in Fig. 4b for the higher amplitude samples show these narrower contact lines.

![Figure 5](image)

Fig. 5  (a) Separation force as a function of wrinkle wavelength where $E_s' = 1.25$ MPa (•) and $E_s' = 2.00$ MPa (○) for a constant aspect ratio of $b/l = \lambda^{-0.07}$. (b) Separation force as a function of wrinkle amplitude for a fixed wavelength of $\lambda \sim 29 \mu m$. The lines represent the separation forces for a flat substrate with a modulus of $E_s' = 1.25$ MPa (solid line) and $E_s' = 2.00$ MPa (dashed line).
In general, the separation force, $P_s$, decreases with increasing wrinkle wavelength for a set aspect ratio ($b/\lambda$) (Fig. 5a). $P_s$ also scales inversely with wrinkle amplitude for a given wavelength (Fig. 5b). Changes in both $b$ and $\lambda$ are shown to have a significant impact on the measured adherence force; however, the controlling balance between wrinkle geometry and materials properties is not evident.

**Stiffness effects**

The thickness of the glassy thin film could lead to changes in the effective stiffness ($K$) of these bilayer systems, possibly changing the separation force for the flat probe contact adhesion experiments. To address this possibility, the stiffness of each surface is determined experimentally. Using the force versus displacement results obtained from each adhesion test, the slope of the compressive region is used to measure the stiffness: $K = dP/d\delta$. Fig. 6a shows the relationship between the film thickness and the measured stiffness. Over the range of thin film thicknesses used in our adhesion measurements, the stiffness is independent of film thickness. Additionally, in Fig. 6b it can be seen that there is no discernible trend between the stiffness and wavelength of the wrinkled surfaces for a constant wrinkle aspect ratio ($b/\lambda$~0.07) meaning that the geometric features are not impacting the measured system stiffness.

The independence of $K$ with the film thickness can be explained by contact mechanics arguments of bilayer or composite systems.²⁵⁻²⁷ It has been shown that a stiff, thin layer on top of a soft substrate does not impact the effective modulus of the system if the contact area or probe dimensions are much larger than the stiff layer thickness. For the systems tested here, $10^{-5}<t/c<10^{-4}$ for all PS films. Therefore, the expected stiffness independence with thickness is verified.

**Discussion**

To predict the separation force ($P_s$) of a wrinkled surface as a function of wrinkle geometry ($b$ and $\lambda$) and materials properties ($E_s$ and $G_c$), the surface is considered as an array of aligned cylinders acting simultaneously but independently on the probe face. This total separation force can be modeled as a summation of the individual separation forces per unit length ($P_i$) exerted by each wrinkle:

$$P_s \sim \sum_{i=1}^{N} P_i l_i$$  \hspace{1cm} (4)

where $l_i$ is the length of an individual wrinkle. To determine the appropriate scaling for $P_i$, two possible contact separation mechanisms are considered.

First, the contact of a long cylinder with a flat surface is considered in the context of scaling for $P_i$. As Barquins proposed²⁴ and Chaudhury has confirmed experimentally,²⁸ the separation force per unit length for a long cylinder ($P_{cyl}$) is:

$$P_{cyl} = \frac{3}{16} \pi E_s G_c R$$  \hspace{1cm} (5)

where $R$ is the radius of curvature of the cylinder. For this geometry, the contact persists along the entire length of the cylinder while the contact width decreases as separation occurs. Eqn (5) is appropriately applied to “infinitely” long cylinders contacting “infinitely” laterally extensive probes, where end effects are neglected.

For finite cylinders, end effects play an important role, leading to contact areas that are predominantly elliptical. Johnson developed a model to describe the separation force of an elliptical contact area through the modification of the well-known relationship for a sphere contacting a flat surface.²³,²⁹

![Fig. 6](image-url) The stiffness ($K$) values obtained experimentally through adhesion testing for various wrinkled substrates. Stiffness as a function of (a) thickness ($t$) and (b) wavelength ($\lambda$) for wrinkles with a constant aspect ratio of $b/\lambda$~0.07.
Ps = \frac{1}{\sqrt{\pi}} \left( \frac{E^* G c A^3}{b^2} \right)^{1/2}

where the contact area of an ellipse at separation is

A_s = \pi a_s c. In our wrinkle system, the contact width (2a) of each wrinkle is consistent with the contact width of a cylinder predicted by Barquins. Therefore, the half contact width of each wrinkle at the point of separation is:

a_s = \left( \frac{2GcR}{\pi E^*} \right)^{1/3}

Combining eqn (6) and 7 with the contact area of an ellipsoid at the point of separation \( A_s \) results in the prediction of the maximum separation force for a finite cylinder:

\[
P_{s_{ell}} = \left( \frac{2\pi G^* E^* R^3}{c^{3/4}} \right)^{1/4} \]

To help determine whether the relationships presented in eqns (5) or (8) are more appropriate in the context of a flat circular probe separating from an “infinitely long” cylinder, we conducted macro-scale, single cylinder adhesion experiments. ESI 3† contains a summary of these cylinder adhesion experimental results, where it is clearly shown that the scaling presented in eqn (8) is consistent with the presented results.

Therefore, utilizing eqn (8), the separation force of a wrinkled surface can be modeled by considering the surface as an array of independent elliptical contacts acting on the probe. Using the geometry of the wrinkles and testing setup, the \( P_s \) for a wrinkled surface can be determined for \( n \) wrinkles contacting the probe:

\[
P_s \sim n \frac{P_{s_{ell}}}{2c} l_{avg}
\]

The value of \( n \) is determined by \( c \) and \( \lambda \):

\[
n \equiv \frac{2c}{\lambda}
\]

To account for the variations in length of the wrinkles resulting from the circular shape of the flat probe, \( P_{s_{ell}} \) is normalized by the probe diameter and multiplied by the average length of the wrinkles \( (l_{avg}) \) contacting the probe:

\[
l_{avg} \equiv \frac{\pi c^2}{2c} \equiv \frac{\pi c}{2}
\]

Additionally, the radius of curvature \( (R) \) of each wrinkle can be estimated from the measured wavelength \( (\lambda) \) and amplitude \( (b) \) of each array of wrinkles, such that \( R \approx \lambda^2/2\pi b \) for small strain surface instabilities \( (e \approx 0.10) \). The relationship between \( R, b \) and \( \lambda \) has been published previously and confirmed for the wrinkles characterized here by measuring the radius of curvature of several wrinkles using optical profilometry.

Application of these geometric corrections to the scaling presented in eqn (9) leads to a prediction for the separation force of a wrinkled surface:

\[
P_s = \left( \frac{\pi E^* G^* c^4}{2^{3/2} b^2} \right)^{1/4} \left( \frac{\pi E^* G^* c^4}{2^{3/2} c} \right)^{1/4}
\]

From this relationship it can be seen that \( P_s \) is a function of materials properties \( (G_c \text{ and } E^*) \), wrinkle geometry \( (b \text{ and } \lambda) \), and probe size \( (c) \). Eqn (12) is presented in two ways, recalling the relationship between \( \lambda \) and \( b \) through the strain given in eqn (2).

A summary of the wrinkle adhesion separation force results is presented in Fig. 7a. The linear fit applied to the data demonstrates that the prediction for \( P_s \) shown in eqn (12) is a good descriptor of the separation force for the wrinkled surfaces tested here. The slope of this line yields a \( G_c \) value for a PS-PS interface of 0.131 J m\(^{-2}\), which is close to the reported and measured value.
of 0.045 J m\(^{-2}\) considering the approximations used to develop the scaling relationship.\(^{22}\)

Wrinkles or patterns are used to modify the adhesion behavior of a surface compared with a “flat” surface; therefore, it is informative to normalize the wrinkle separation force (\(P_f\)) by the separation force of a flat probe on a flat sample (\(P_s\)). The separation force relationship for a flat-on-flat contact geometry is:\(^{19}\)

\[
P_f = (8\pi G_c E^c c^3)^{1/2}
\]

Combining this relationship with eqn (10):

\[
P_s \equiv \left(G_c E^c \frac{h}{E b^4}\right)^{1/2} \cong 0.112 \left(G_c E^c \frac{c^3}{h^4}\right)^{1/2}
\]

The application of this relatively simple scaling to the wrinkled systems measured here can be seen in Fig. 7b. When plotted in this manner, the data falls onto a line, indicating good agreement between the measured values and our scaling relationship. It can also be seen that achieving enhancement of adhesion (\(P_f/P_s > 1\)) is dependent on the probe size relative to the wrinkle dimensions as well as the materials-defined length scale given by \(G_c/E^c\). Additionally, a similar dependence on materials properties and amplitude has recently been shown to work for randomly oriented wrinkles tested with a spherical indenter. The only differences in this case arise from the testing geometry.\(^{16}\)

**Conclusions**

In this work, we demonstrate that both the wavelength and amplitude of a wrinkled surface impact the adhesive properties, specifically the separation force. To investigate the two parameters independently, a novel sample preparation technique is employed. This same technique allows for the removal of residual stress stored in the elastically buckled surfaces of our samples by crosslinking the substrate after wrinkle formation. Furthermore, materials with well-characterized surface and bulk properties are used to create model examples for understanding the contact adhesion of interfaces between flat surfaces and surfaces with aligned wrinkles.

Several additional experiments are presented here that validate the assumptions used in the development of the wrinkle adhesion relationship. Single cylinder tests with finite flat probes are conducted and a model for the adhesion of finite length cylinders is developed. An investigation of the measured stiffness as a function of film thickness and wrinkle feature size verifies that the effective stiffness is not affected by the thin film or surface features. The significance of this work lies in the development of a model wrinkle system that allows contact mechanics and adhesion theory to be applied to a wrinkled surface. By understanding the impact of wrinkle geometry on the separation mechanisms, new material systems can be engineered with specifically-tuned adhesive properties. Additionally, the analysis presented here will enable natural adhesive systems to be more easily studied and compared.

**Symbols**

- \(a\) half contact width
- \(a_s\) half contact width at separation
- \(A\) contact area
- \(A_s\) contact area at separation
- \(b\) amplitude
- \(c\) probe radius
- \(\delta\) vertical displacement
- \(\varepsilon\) strain
- \(E^*\) effective modulus
- \(E_f\) film Young’s modulus
- \(E_s\) substrate Young’s modulus
- \(\gamma\) surface tension
- \(G_c\) critical strain energy release rate
- \(h\) substrate thickness
- \(K\) stiffness
- \(\lambda\) wavelength
- \(l_{avg}\) average wrinkle length
- \(l_i\) individual wrinkle length
- \(n\) number of wrinkles contacting probe
- \(P\) normal force
- \(P_{cyl}\) maximum separation force per unit length of cylinder
- \(P_f\) maximum separation force of flat surface
- \(P_i\) maximum separation force per unit length of single feature
- \(P_m\) maximum compression force
- \(P_{sep}\) maximum separation force
- \(P_{sep}^{ell}\) maximum separation force of elliptical contact
- \(R\) radius of curvature
- \(s\) film thickness
- \(\theta\) angle
- \(V\) processing velocity
- \(w_{adh}\) work of adhesion

**Acknowledgements**

The authors would like to thank M. Bartlett, D. Breid, and S. Kundu for thoughtful discussions and suggestions. We also acknowledge the NSF-IGERT in Nanotechnology and NSF-PIRE Korean Exchange NSF-0730243 for financial support of this project.

**References**