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Marshall's Rules with Aggregate Inputs

Alberto Behar, *University of Oxford*

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Alberto Behar Centre for the Study of African Economies and St Anne's College
Department of Economics, Manor Road, Oxford, OX13UQ; alberto.behar@economics.ox.ac.uk

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Abstract

We establish the formal link between the separability of inputs in a production function and the aggregate elasticity of demand for those inputs. This validates the implicit assumption used when calculating an aggregate elasticity with aggregated input prices and provides a practical approach to calculating an aggregate elasticity when one has disaggregated prices. We illustrate the approach to add to a thin empirical literature on labour demand elasticities in developing countries by using South African data.

1 Introduction

Economists are interested in studying labour demand elasticities for a number of reasons. For example, they help understand the employment effects of policies that affect labour costs, like minimum wage legislation or payroll taxes. Cross elasticities are also important; for example, they can measure whether higher capital costs increase or decrease the demand for labour. Based on early work by Marshall (1920) and Allen (1938), many studies have estimated own- and cross-elasticities of labour demand. See Hamermesh (1993) and Cahuc & Zylberberg (2004) for reviews.

In many cases, data are only available for one homogeneous group. Even if there are a number of labour categories, it is inevitable that these combine different workers within the category, so all studies are aggregating to some degree. To do so, studies implicitly assume some form of separability in the underlying production function. While Berndt & Christensen (1973a) formulate precise statements on separability and the legitimacy of aggregating factors to study the concept of the elasticity of substitution, no equivalent statements are available for the the elasticity of factor demand. The purpose of this paper is to produce such statements.

In particular, it establishes the relationship between the separability of inputs and the validity of Marshall's Rules for an aggregate of those inputs. This would legitimize the use of aggregated data to estimate an aggregate elasticity when we know that not all labour is the same. The relationship has practical uses when the researcher has disaggregated data but would nonetheless like to produce an aggregate summary measure. For example, many CGE or similar simulation-based exercises need one summary parameter as an input.

We start by reviewing the basic link beween separability and the elasticity of substitution. Our theoretical contribution is the application to factor demand. Therafter, we use a translog cost function to appy the theory to a dataset in which we have four occupation types plus capital and the objective is to produce one elasticity between labour types.

The application has its own merit. The data is from South Africa, which makes this study one of few for developing countries: according to Fajnzylber & Malony (2001), only two of the nearly 200 studies surveyed by Hamermesh (1993) use establishment data for developing countries. It is also a setting where unemployment is high and it is feared new labour legislation has raised the cost of labour. Our results yield labour demand elasticities of almost unity. While capital and both labour types are substitutes, More- and Less-skilled labour are complements.

2 Theory

The elasticity of substitution and separability The elasticity of substitution measures the percentage change in relative demand for two inputs in response to a change in relative factor prices. Our point of departure is homothetic production function $Q = f(x_1, x_2, ..., x_n)$, from which Allen (1938) developed an elasticity measure when there are more than two inputs. Uzawa (1962) uses the dual cost function $C = g(w_1, w_2, ..., w_n, q)$ to express this elasticity of substitution as

$$\sigma_{ij} \equiv -\frac{d\log\frac{x_i}{x_j}}{d\log\frac{w_i}{w_j}} = \frac{Cg_{ij}}{g_i g_j}, \tag{1}$$

where g_i, g_j are first derivatives with respect to the prices of factors i, j - w_i, w_j - and g_{ij} is the cross partial derivative. This partial measure assumes relative price of factors i and j change exogenously but the prices of all other factors and output remain constant. If $\sigma_{ij} > 0$, the factors are substitutes. If $\sigma_{ij} < 0$, they are complements.

Following Berndt & Christensen (1973a), partition the n inputs in $f(\cdot)$ into R mutually exclusive and exhaustive subsets $[X^1, ..., X^R]$, which we call partition P. The production function is weakly separable with respect to partition P if the marginal rate of technical substitution between any pair of inputs x_i, x_j from any subset X^S is independent of the quantity of any input outside X^S. That is, $\frac{d}{dx_k}\left(\frac{f_i}{f_j}\right) = 0\ \forall i, j \epsilon X^S, k \notin X^S$, where f_i, f_j are the marginal products of inputs x_i, x_j. Differentiation gives:

$$f_j f_{ik} - f_i f_{jk} = 0\ \forall i, j \epsilon X^S, k \notin X^S \tag{2}$$

Weak separability is necessary and sufficient for $f(\cdot)$ to be legitimately written as $q = F[X_1, ..., X_R]$, where X_S is a function of the elements of X^S only. Lau (1969) shows weak separability in the production function with respect to partition P implies weak separability in the cost function (and vice versa), so $g(\cdot)$ can after partition P consist of R subsets. Then, the cost function can be written as $C = G[W_1, ..., W_R, q]$, where W_S is a function of the prices of the inputs in X^S, which comprise set W^S. The analogue to (2) is:

$$g_j g_{ik} - g_i g_{jk} = 0\ \forall i, j \epsilon X^S, k \notin X^S \tag{3}$$

Berndt & Christensen (1973a:407) build on these conditions to establish that separability of factors x_i and x_j from all others in the production function is equivalent to:

$$\sigma_{ik} = \sigma_{jk}\ \forall k : k \neq i, k \neq j \tag{4}$$

In other words, the elasticity of substitution between some aggregate of x_i and x_j, which we call X_I, and a third input x_k is $\sigma_{Ik} = \sigma_{ik} = \sigma_{jk}$. They also show this is equivalent to the legitimate construction of an aggregate index of factors x_i and x_j or their prices.

Aggregation and the elasticity of factor demand Based on Marshall's Rules (1920:383), we can write the compensated cross-elasticity of demand in terms of factor shares and the elasticity of substitution. When the prices of all other factors and output remain constant (Hamermesh, 1986),

$$\bar{\lambda}_{ij} = \frac{d\log x_i}{d\log w_j} = s_j \sigma_{ij}, \tag{5}$$

where s_j is the cost share of factor j. While Berndt & Christensen (1973ab, 1974) make statements about separability that allow us to produce aggregated elasticities of substitution (equation (4)), the same has not been said about the cross and own-elasticities of factor demand.

Assume each of the disaggregated input quantities change by the same proportion and that each of the disaggregated input prices also change by the same proportion. Informally, we can say that the elasticity of an aggregate of one set of input quantities with respect to an aggregate of another set of input prices is the *sum* of the elasticities of *one* of the input quantities with respect to each of the input prices. Equivalently, the elasticity of the aggregate of one set of input quantities with respect to an aggregate of another set of input prices is the elasticity of substitution between the aggregates multiplied by the cost share of the aggregate input whose price has changed.

Formally, let W_I be a legitimate aggregate of one or more input prices ie all $w_i \in W^I$ are weakly separable from $w_i \notin W^I$. Write $d\log w_j = \hat{w}\ \forall j \in W^J$ and $d\log x_i = \hat{x}\ \forall i \in X^I$. Define the constant output cross-elasticities as follows:

- $\bar{\lambda}_{ij} \equiv \frac{d\log x_i}{d\log w_j}$
- $\bar{\lambda}_{iJ} \equiv \frac{d\log x_i}{d\log W_J}$
- $\bar{\lambda}_{IJ} \equiv \frac{d\log X_I}{d\log W_J}$ (The aggregate elasticity of factor demand.)

$S_J \equiv \sum_j s_j\ \forall j \in W^J$ such that the factor share of an aggregate is the sum of the disaggregated shares. σ_{ij} is the disaggregated elasticity of substitution, σ_{IJ} is the aggregate elasticity of substitution and σ_{iJ} is the elasticity of substitution between a disaggregated input and an aggregated input.

Lemma 1 *Weak separability with respect to partition P implies $\bar{\lambda}_{iJ} = \sum_{j\in W^J} \bar{\lambda}_{ij}$.*

Proof. By equation (5), $d\log x_i = \sum_j s_j\sigma_{ij} d\log w_j$ when output is constant. In particular, if only the prices in the aggregate W_J change, $d\log x_i = \sum_{j\in W^J} s_j\sigma_{ij} d\log w_j$. However, $d\log w_j = \hat{w}\ \forall j \in W^J \rightarrow \sum_{j\in W^J} d\log w_j = \frac{\sum_{j\in W^J} dw_j}{\sum_{j\in W^J} w_j} = \hat{w} = d\log W_J$. Therefore $\bar{\lambda}_{iJ} = \sum_{j\in W^J} s_j\sigma_{ij} = \sum_{j\in W^J} \bar{\lambda}_{ij}$. ■

Lemma 2 *Weak separability with respect to the partition P implies $\bar{\lambda}_{iJ} = S_J\sigma_{iJ}$.*

Proof. As shown in Berndt & Christensen (1973a), $\sigma_{ij} = \sigma_{iJ}\ \forall j \in W^J$. By Lemma 1, $\bar{\lambda}_{iJ} = \sum_{j\in W^J} s_j\sigma_{iJ}$. Therefore $\bar{\lambda}_{iJ} = S_J\sigma_{iJ}$ ■

Lemma 3 *Weak separability with respect to the partition P implies $\bar{\lambda}_{iJ} = S_J\sigma_{IJ}$.*

Proof. Using $\sigma_{ij} = \sigma_{iJ} \forall\ j \in W^J$, this follows trivially from Lemma 2. ■

Lemma 4 *Weak separability with respect to the partition P implies $\bar{\lambda}_{IJ} = S_J\sigma_{IJ}$.*

Proof. If only the prices in W_J change, by equation (5), $\sum_{i\in X^I} d\log x_i = \sum_{i\in X^I}\sum_{j\in W^J} s_j\sigma_{ij} d\log w_j$. By Lemma 2, $\sum_{i\in X^I} d\log x_i = \sum_{i\in X^I} S_J\sigma_{iJ} d\log W_J$. But $d\log x_i = \hat{x}\ \forall i \in X^I \rightarrow \sum_{i\in X^I} d\log x_i = \frac{\sum_{i\in X^I} dx_i}{\sum_{i\in X^I} x_i} = \hat{x} = d\log X_I$. Therefore $\bar{\lambda}_{IJ} = S_J\sigma_{iJ}$. It follows from Berndt & Christensen (1973a) that $\sigma_{iJ} = \sigma_{IJ}\ \forall i \in X^I$ and therefore $\bar{\lambda}_{IJ} = S_J\sigma_{IJ}$. ■

The results are summarised as follows:

Proposition 1 *Weak separability with respect to the partition P implies $\bar{\lambda}_{IJ} = \bar{\lambda}_{iJ} = S_J \sigma_{IJ} = S_J \sigma_{iJ} = \sum_{j \in W^J} \bar{\lambda}_{ij}$.*

Proof. This follows from the lemmata. ■

The proposition presents Marshall's Rules for aggregate inputs and shows the aggregate elasticity can be calculated by summing the disggregated elasticities. We can also make a statement about aggregated own-price elasticities:

Corollary 1 *Weak separability with respect to the partition P implies $\bar{\lambda}_{II} = -\sum_{J:J \neq I} \bar{\lambda}_{IJ}$.*

Proof. Using linear price homogeneity, Sato & Koizumi (1973) show $\sum_j \bar{\lambda}_{ij} = 0$. Dividing variables w_j into those that are together with w_i in aggregate W_I and those that are not, we have $\sum_{j \in W^I} \bar{\lambda}_{ij} = -\sum_{j \notin W^I} \bar{\lambda}_{ij}$. By Proposition 1, $\sum_{j \in W^I} \bar{\lambda}_{ij} = -\lambda_{II}$ and $\sum_{j \notin W^I} \bar{\lambda}_{ij} = -\lambda_{IJ}$. Therefore$\bar{\lambda}_{II} = -\sum_{J:J \neq I} \bar{\lambda}_{IJ}$. ■

3 Empirics

Data and background South African unemployment has been "*literally off the charts*" compared to other developing countries (Nattrass, 2004:90). Those unemployed under the narrow ILO definition comprise about 25% of the labour force and as many as 40% are unemployed according to the expanded definition (Statistics South Africa, 2005, 2009). Commentators fear South Africa's wage bargaining institutions and the introduction of new labour legislation in 1995 may be raising the costs of labour and contributing to unemployment (Fedderke et. al., 2001). South Africa is therefore an appropriate setting in which to gauge the potential impact of labour costs on employment.

The two sources of data are manufacturing data for about 300 firms from the National Enterprise Survey conducted in 1998, which has been merged with data from the 1999 October Household Survey. We use four occupation types from the firm-level data, namely the Managerial/Professional and Skilled/Artisanal occupations (More skilled) and the Semiskilled and Unskilled occupations (Less skilled). For further motivation and description of the procedure, see Behar (2009).

Translog functions With origins due to Christensen, Jorgenson & Lau (1973), we follow Teal (2000) and represent $g(\cdot)$ by means of a translog cost function,

$$\begin{aligned} \log C &= \log a_0 + \sum_i a_i \log w_i + \frac{1}{2} \sum_i \sum_j B_{ij} \log w_i \log w_j + a_q \log q \\ &\quad + B_q \log^2 q + \sum_i B_{iq} \log q \log w_i, \end{aligned} \tag{6}$$

which will be estimated together with the associated factor share equations to improve efficiency using a seemingly unrelated regression method:

$$s_i = a_i + \sum_j B_{ij} \ln w_j + B_{iq} \ln q \tag{7}$$

See Berndt (1991) for details. We impose restrictions on the coefficients consistent with cost minimizing behaviour (Berndt & Khaled, 1979). Slutsky symmetry requires $B_{ij} = B_{ji}$ while linear price homogeneity requires $\sum_i B_{ij} = 0, \sum_j B_{ij} = 0, \sum_i a_i = 1$ and $\sum_i B_{iq} = 0$. The elasticities of factor demand are (Binswanger, 1974):

$$\bar{\lambda}_{ij} = \frac{B_{ij}}{s_i} + s_j \tag{8}$$

$$\bar{\lambda}_{ii} = \frac{B_{ii}}{s_i} + s_i - 1 \tag{9}$$

By applying (2) or (3) to a translog function, one can test for separability of factors x_i and x_j from all others by means of the following restrictions (Berndt & Christensen, 1974):

$$s_i B_{jk} - s_j B_{ik} = 0 \forall k : k \neq i, k \neq j \tag{10}$$

Separability implies the existence of a valid price index, but it doesn't solve the problem of how best to perform the aggregation. One might conjecture that an average of the prices, weighted in some way by their relative shares, would be appropriate. This corresponds to the conditions for separability in equation (10).Thus, we operationalise separability by running a *disaggregated* regression while imposing these restrictions. For simplicity, we use the sample average of s for the restrictions and for the elasticity calculations.

3.1 Results

Regressions This section presents the results from estimating (6) with appropriate restrictions of the form (10) imposed. As a preliminary step, we ran an unrestricted model in which we did not impose separability / aggregate the inputs and, using Wald tests of equation (10), failed to reject the restrictions. The restrictions imply our disaggregated cost function $C = g(w_1, w_2, w_3, w_4, w_5, q)$ can be written as $C = G(W_M, W_L, W_5, q)$, where W_5 is capital, W_M is more skilled labour and W_L is less skilled labour. The regression results (with restrictions imposed) are presented in Table 1. The overall fit of the regression is good. Our specification also rejects homotheticity, so our results are only valid on the assumption of a locally homothetic technology. We find the B_{ij} jointly significant at 10%, which rejects the null hypothesis of a Cobb-Douglas technology.

Elasticities The elasticities were confirmed to be exactly equal for separable inputs, for example $\sigma_{15} = \sigma_{25} = \sigma_{M5}$. We present the three aggregated elasticities in Table 2. Capital and both skill types are found to be roughly equally substitutable such that a rise in the cost of labour relative to capital would lead to a relative fall in its employment quantity. More-skilled and Less-skilled labour are complements with an elasticity of substitution of -1.71. Table 3 reports the compensated elasticities of factor demand. Concurring with the review in Hamermesh (1993), more skilled labour demand is less elastic than less skilled labour demand. These aggregate own-elasticities suggest wage push would have contributed to decreased employment levels. Furthermore, the cross elasticities of factor price of -0.45 and 0.58 imply a rise in wages for less skilled workers reduces demand for their skilled counterparts and increases demand for capital.

4 Conclusion

Our substantive contribution has been to estimate elasticities of demand for Capital, More-skilled and Less-skilled labour for South Africa; estimates which are scarce for developing countries. Labour demand elasticities are almost unity and the two labour types are complements. To do this using disaggregated data, we imposed the relevant restrictions implied by separability when estimating a translog function. This is only legitimate because we confirmed the equivalence of the separability of two inputs with respect to other inputs and the calculation of an aggregate elasticity for those inputs. Our result also justifies the notion of a homogeneous elasticity when labour is not homogeneous.

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Table 1: Cost Function Parameter Estimates and System Diagnostics

Dependent variable: Cost

Variable	Coefficient	p-value	Variable	Coefficient	p-value
Capital	0.09	0.90	value added	0.28	0.00
Man/Prof	0.26	0.42	0.5*(value added)^2	.013	0.00
Skil/Art	0.15	0.42	(value added)*Cap	0.01	0.47
Semi	0.30	0.46	(value added)*Man/Prof	-0.02	0.00
Un	0.20	0.41	(value added)*Skil/Art	-0.004	0.45
value added	0.28	0.00	(value added)*Semi	0.004	0.62
0.5*Capital^2	-0.27	0.10	(value added)*Un	0.005	0.48
Capital*Man/Prof	0.06	0.19	Observations		307
Capital*Skil/Art	0.05	0.19	RMSE		0.54
Capital*Semi	0.10	0.17	"R^2"		0.85
Capital*Un	0.07	0.17	χ^2 for regression significance		2018.55
0.5*Man/Prof^2	0.03	0.24	p value		0.00
Man/Prof*Skil/Art	-0.03	0.09	Joint significance of $B_{ij} = 0$		0.09
Man/Prof*Semi	-0.03	0.06	Homotheticity p value		0.00
Man/Prof*Un	-0.02	0.06	Constant and controls for exports as a share of output, raw		
0.5*Skil/Art^2	0.03	0.12	materials as a share of cost, ease of recruitment, training		
Skil/Art*Semi	-0.03	0.06	expenditure, a market conditions index, a large-firm dummy,		
Skil/Art*Un	-0.02	0.06	computer investment as a share of output, a dummy for		
0.5*Semi^2	0.02	0.74	owner-managed firms, a productivity dissatisfaction		
Semi*Un	-0.05	0.26	measure, firm age, province and location dummies and an		
0.5*Un^2	0.03	0.51	indicator for technology intensity are not presented.		

System Diagnostics

Share Equation	Obs	RMSE	"R^2"	χ^2	p
Managerial/Professional	307	0.06	0.43	236.06	0.00
Skilled/Artisanal	307	0.08	0.17	71.06	0.00
Semiskilled	307	0.13	0.16	60.85	0.00
Unskilled	307	0.11	0.11	41	0.01

Table 2: Elasticities of Substitution

σ_{IJ}	
Factor pairing	Elasticity
Capital / More	2.40
Capital / Less	2.19
More / Less	-1.71

Table 3: Elasticities of Factor Demand

$\bar{\lambda}_{IJ}$		J		
		Capital	More	Less
	Capital	-0.94	0.35	0.58
I	More	1.26	-0.80	-0.45
	Less	1.14	-0.25	-0.90

Additional Data information (not intended for publication)

The core data set is the National Enterprise survey of firms. While the dataset has rich disaggregated information on occupation quantities, it does not have wage data. This is a common problem for many developing country datasets. Therefore, appropriate wage information is transplanted from household data based on the October Household Surveys. Characteristics common to both surveys are used to predict wages by occupation for each firm.

Data from the firm-level manufacturing survey

The National Enterprise Survey (NE survey) covers the period of 1998. After adjusting for non-response and outliers, there are about 300 firms with the appropriate variables. For a thorough analysis of the data and descriptive statistics, see Bhorat & Lundall (2002). The dataset is a single cross section, so variables are required to control for firm-specific effects and avoid omitted variable bias. The NE dataset has a rich set of variables for the purpose. There are nine industries and nine provinces. There is information on whether the firm is a member of a bargaining council or otherwise subject to a bargaining council agreement. Consistent with the view that trade unions are more likely to survive in some industries than others (Booth, 1995) and that they may have a non-price effect on factor quantity, this is useful information.

The key variables for the cost function are the cost of capital and and wages by occupation group. The four groups used are:

- Managerial/Professional
- Skilled/Artisan (technicians, welders)
- Semi-skilled (machinery operators)
- Unskilled (labourers, security guards)

Factor Costs

Why firm-level wages can realistically be represented by supra-firm data Predicting wage data has precedence. Teal (2000) generates predicted values for firm-level wages using characteristics of the employees at the firm. Average wages by industry and occupation are a good approximation to those faced by firms in South Africa. Nattrass (2000) reports that the main wage setting institutions are industrial level bargaining councils (BC), noting that 65% of manufacturing workers are covered by a BC. Furthermore, the Minister of Labour is obliged to extend BC agreements to non-members. Nattrass concludes that extension to non-members is at the core of wage setting in an industry. The NE survey provides data on whether the firm is subject to collective bargaining and/or a BC agreement. On average, over 70% of firms are subject to a BC agreement. Firms with fewer than 50 employees are almost 100% covered while large firms vary from 32% to 61% by industry in coverage. There is therefore support for convergence of wages in industries and justification for wages being calculated at a supra-firm level.

Table 4: Selection of categories into which wages for skilled/artisanal workers have been placed

Mean Monthly Salary: Skilled/Artisan (Rands)		
	Estimate	Std Error
Food & Beverages	1562	161
Wood, Pulp & Paper - Prov0	1116	229
Wood, Pulp & Paper - Prov1	1993	169
Chemicals, Rubber & Plastic - Prov0, not unionised	786	152
Chemicals, Rubber & Plastic - Prov0, unionised	2316	264
Chemicals, Rubber & Plastic - Prov1	2067	284
Source: own calculations based on October Household Survey data		

Using household data for wages We reduced the 1997 October Household Survey sample to include only those 3 500 people working for somebody else in formal manufacturing industries. Definitional correspondence to the NE survey in terms of industry, province and occupation is good, but, as will be explained, the correspondence regarding union membership / collective bargaining is not.

In the survey, people were interviewed in geographical clusters and stratified by magisterial district. The sample surveyed is not fully representative of the population. We take survey design effects like these into account.

This study accounts for probability weights and clustering but only partially adjusts for stratification. For each occupation, the characteristics available in both data sources are:

- economic activity (broken down into nine industries)
- province group (the nine provinces were ex post broken down into two groups with similar wages)
- individual trade union membership (household data); collective bargaining and bargaining council membership (firm data)

Construction entails calculating the survey-adjusted means for groupings of people for each occupation. Preliminary work constructed a number of alternative wage series. One classified wages by industry, location and trade-union membership to generate. There are nine industries and nine provinces, meaning that, together with a trade-union membership dummy, there are potentially 162 different wages. However, while some means are calculated using a comfortable number of observations, others are based on few data points, sometimes only one. This means the standard errors on the wage estimates can be high (or non-existent). To mitigate this, the nine provinces are divided into two groups, as variation within each of the two groups is low. However, more precise estimates can be achieved by combining some locations and industries and/or not distinguishing by trade-union membership in cases where wages do not differ substantially. Before discussing the process undertaken to determine classification, it may be helpful to look at one example. Table 4 presents six of the fifteen composite groups the skilled/artisan wages are divided into and the associated estimates.

The first row contains wages for all skilled/artisans in the Food & Beverages industry, regardless of location or union membership. The Wood Pulp & Paper industry is subdivided by province group but not union membership (rows 2 and 3). Wages in the Chemicals, Rubber and Plastic industries are subdivided by province group. One group of provinces is further divided into unionised and non-unionised workers (rows 4 and 5) while the other group is not (row 6). In some cases, industries are combined, with the possibility of disaggregation by other criteria.

Classifying the wages involves a number of trade-offs. While averages across two or more groups are different, the standard errors may be large, resulting in imprecise estimates. This is often because of a small sample size for that group. One way to proceed is to separate all groups with statistically significant differences in means. However, this is imperfect. An extreme but not infrequent occurrence is that of one observation per group, which generates no standard error and is also outside the confidence interval of another group. Similarly, inference based on very few observations is not reliable. On the other hand, some estimates, even if based on few observations, are so radically different that the groups should be classified separately. The aim is to produce a set of estimates per group with better precision characteristics but sufficient variation to represent the firm-level data. To do this, various combinations are carefully inspected. Factors considered are differences in log wages, the number of observations, and comparisons of the standard errors and confidence intervals of the separate and combined groups. Of course, all the criteria are related.

Comparing the confidence intervals of two groups is naturally akin to performing a two-sample t-test. However, visual inspection is quicker for all the combinations and allows for analysis in conjunction with the other criteria. The choice of confidence interval is a matter of taste in this application, so 85% bands are used. To augment this procedure more formally, standard t-tests, regressions and non-parametric procedures are performed on certain groups.

Going through the above procedure on a case-by case basis therefore produces a set of wages, for each occupation, which partially disaggregates each industry by location and/or trade union membership in a way that optimizes the trade-off between achieving representative wage estimates and having precise estimates. Depending on the occupation, the number of categories ranges from 7 to 15, with the average number of observations per category ranging from 16.9 to 43.7. Data from the TIPS shows wages rose by approximately 15% between the time of the household survey and the time of the NE survey. Wage measures are raised by this percentage.

Adjusting wages for firm size Failing to account for firm size can lead to poor results Informally, failure to account for firm size effecst in wages leads to a conflation of wage effects and output effects in the cost function. A simple way to adjust wages is through the linear function

$$\ln w_i = \ln \hat{w}_i + \gamma_i \ln q, \tag{11}$$

where w_i is the wage adjusted for firm size and $\hat{w}_i$ is unadjusted. There is no information on the size of the firms which individuals in the household survey work for. We attach values of γ_i to the wage series estimated by.Bhorat & Lundall (2002). They claim similarity to the US study of Doms, Dunne & Troske (1997). Assuming the unadjusted wages represent those

Table 5: Estimates of factor shares

Capital	Man/Prof	Skil/Art	Semi	Un
54.7%	9.4%	7.4%	16.4%	11.6%

for an average-sized firm, the wages transplanted from the household sata are inflated/deflated accordingly.

Costs of capital We use the expression from an industry-level study of capital in South Africa (Fedderke et al., 2001):

$$c = (r - \pi) + \delta + \tau \tag{12}$$

For the real interest rate $(r - \pi)$, we use the average prime lending rate and consumer price inflation for 1999. They calculate industry-level values for δ, the depreciation rate, ranging from 11% to 16%. Fedderke et al. (2001) use the nominal corporate tax rate for τ, which was 35% for the fiscal year starting early in 1998 (RSA, 1998). They state it would be ideal to have the effective rates of taxation by industry. Negash (1999) calculates effective tax rates to be about 15% below nominal rates for the 1990s, so a 20% average effective rate is applied to all firms. Furthermore, we adjust the interest rate to account for risk. Adjustments range from –2% for large (>50 employees) and old (> 5 years) firms to + 5% for new small firms.

Total costs, cost shares and value added

Wages are also used in the determination of cost shares and total costs. Labour costs are obtained by multiplying labour quantities by the constructed wage for each occupation. Capital costs are the cost of capital percentage multiplied by the capacity-adjusted capital stock. Total factor cost (C_f) is the sum of factor costs. To calculate each factor share, we multiply the factor's wage by its quantity and divide it by the sum of the factors' costs; that is: $s_i = \frac{w_i x_i}{\sum_i w_i x_i}$. Combining the Man/Prof and Skil/Art groups yields a share of 16.8% and combining the Semiskilled and Unskilled yields 28%. These estimates are similar to those of Teal (2000), where Capital comprises 60%, skilled labour 11% and unskilled labour 29%.There is information in the data on what percentage of total costs is comprised of raw materials costs, but there is no data on total costs or on raw materials costs. To derive a measure of raw materials costs, it is necessary to assume that turnover equals total costs. Then raw materials as a percentage (p) are multiplied by turnover (q) to get a measure of raw materials costs. Value added is constructed as sales minus the constructed raw materials[1] so that $v = (1 - p)q$.

Table 6: Cost Function Parameter Estimates

Dependent variable: Cost Variable	coefficient	p	Variable	coefficient	p
Constant	4.11	0.04	ind2	0.20	0.41
Capital	0.25	0.73	ind3	0.50	0.05
Man/Prof	0.27	0.44	ind4	-0.25	0.19
Skil/Art	0.08	0.72	ind5	-0.05	0.79
Semi	0.17	0.73	ind6	0.42	0.15
Un	0.25	0.37	ind7	0.11	0.63
value added	0.29	0.00	ind8	0.07	0.81
0.5*Capital^2	-0.33	0.07	ind9	-0.31	0.05
Capital*Man/Prof	0.06	0.28	loc2	0.21	0.43
Capital*Skil/Art	0.08	0.20	loc3	-0.33	0.08
Capital*Semi	0.15	0.17	loc4	-0.29	0.12
Capital*Un	0.05	0.48	loc5	-0.75	0.00
0.5*Man/Prof^2	0.03	0.41	loc6	0.74	0.08
Man/Prof*Skil/Art	-0.03	0.08	loc7	0.70	0.04
Man/Prof*Semi	-0.03	0.43	loc8	-0.23	0.58
Man/Prof*Un	-0.03	0.34	loc9	-0.29	0.11
0.5*Skil/Art^2	0.02	0.37	exports / output %	0.24	0.25
Skil/Art*Semi	-0.07	0.05	raw materials / cost %	0.01	0.00
Skil/Art*Un	0.01	0.84	recruitment ease Man/Prof	0.1	0.05
0.5*Semi^2	0.01	0.94	recruitment ease Sale/Cler	-0.05	0.23
Semi*Un	-0.05	0.26	recruitment ease Skil/Art	-0.07	0.11
0.5*Un^2	0.03	0.58	recruitment ease Semi	0.01	0.82
0.5*(value added)^2	0.13	0.00	recruitment ease Un	0.02	0.81
(value added)*Cap	0.01	0.78	training expenditure	0.00	0.01
(value added)*Man/Prof	-0.02	0.00	market conditions index	-0.01	0.17
(value added)*Skil/Art	0.00	0.96	firm size > 50 employees	0.37	0.00
(value added)*Semi	0.01	0.43	computer investment / output %	-3.33	0.00
(value added)*Un	0.00	0.76	ownermanaged	-0.61	0.00
Observations	307		productivity dissatisfaction	0.052	0.02
"R^2"	0.85		collective bargaining	0.00	0.96
Homotheticity		0.02	firm age	0.04	0.09
Joint significance of B_{ij}		0.31	cap/lab ratio indicator	1.40	0.00

Additional Regression with unrestricted coefficient (not intended for publication)

Presenting the share equations would reveal very little additional information. A formal separability testing procedure would stop at our failure to reject the null hypothesis that $B_{ij} = 0 \ \forall i, j$ in Table 6, which is a failure to reject so-called strong global separability of all inputs (this is a feature of the Cobb Douglas Function). In other words, the testing procedure would stop before investigating the separability of particular sets of inputs as in equation (10). For our purposes, it legitimizes the aggregation of labour types by imposing the relevant restictions.

[1] As a check, we calculated an alternative measure using a completely different method based on p and total input costs, where total costs are calculated using the input quantities and prices. The correlation between the two measures was 0.90.

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