2010

Would cheaper capital replace labour?

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July 22, 2009

Abstract

Left-leaning members of the ruling alliance should be careful what they wish for. By estimating elasticities of substitution and factor demand between capital and four labour types, we find micro-economic evidence that cheaper capital would reduce demand for labour. While capital and labour are substitutes, many but not all occupation types are themselves complements. These results allow for endogenous changes in output and apply to the vast majority of firms in our sample.

JEL Classification: E24 J21 J23 Key Words: Labour demand, elasticity of substitution

1 Introduction

According to the narrow definition, South Africa's unemployment rate is about 25% (Statistics South Africa, 2009). While the unemployment problem is complex, our focus is on the potential role of factor prices.

South Africa has strong bargaining institutions which together with the introduction of new labour legislation in 1995[1] may be raising the costs of labour. Together with these rising labour costs, the Congress of South African Trade Unions (COSATU) repeatedly lobbies for cheaper capital. These arguments have traditionally been made on the basis of standard macroeconomic transmission from monetary conditions to fixed investment and output expansion but have at times incorporated the potential role lower interest rates could have in weakening the currency and stimulating export demand.[2] Similar arguments have been advanced in a book funded by the United Nations Development Programme (Pollin et al., 2006).

The view that Jacob Zuma's victory at Polokwane is due to COSATU and South African Communist Party backing points to some leftward movement in economic policy (Piper & Matisonn, 2009). The magnitude and nature of any move is not clear, but one potential aspect might be a more flexible attitude towards inflation when it is above the target and hence more scope to reduce interest rates. However, within some circles of the African National Congress (ANC), there has been concern over potentially adverse employment effects of lower capital costs. An ANC discussion document (2005:23) states:

> *"Cheaper capital without reforms to reduce the relative cost of labour is likely to result in higher investment that displaces labour."*

[1] See Republic of South Africa (1995).

[2] There was an "angry" response to an apparent end to the easing cycle by the Reserve Bank in July 2009 (Mail & Guardian, 2009). For an example of the transmission argument, see COSATU (1998) and for an example of the exchange rate argument, see COSATU (2005).

This clearly presents a microeconomic substitution argument. While 2005 seems relatively distant in light of recent political events, this does not mean the argument is wrong. If correct, seemingly "pro-labour" policies may actually harm workers. They would certainly go against the World Bank's prescription of adopting wage restraint as a mechanism for employment creation (Fallon & Lucas, 1998). While some suggest factor costs can contribute to unemployment (Fedderke et. al., 2001), others are sceptical (Kaplinsky, 1995; Pollin et al, 2006).

We make a formal microeconomic contribution to the debate by estimating the elasticity of substitution as well as cross- and own-price elasticities between capital and four labour occupations. The signs of the elasticities can predict whether a (relative) fall in the cost of capital would lead to a (relative) rise or fall in labour demand. They would also indicate how higher wages may be affecting labour demand within and across occupations. Despite the importance of such measures for policy, few estimates of disaggregated labour demand elasticities exist for developing countries. Most appear to be for Latin America (Hamermesh, 1993).

This paper is unique because it presents estimates of uncompensated factor demand. While studies generally provide elasticities that hold output constant (or are unclear about what they claim to do), we can account for endogenous changes in the optimal level of output. This is important because, even if a fall in the price of one factor leads to a fall in demand for the other if we hold output constant, it may be that the fall in price leads to an expansion in output and demand for all factors such that the overall effect is a rise in demand for the other factor.

Section 2 discusses the theoretical background, including why and how we use a translog cost function to calculate elasticities. Section 3 describes the data, the the basis of which is the firm-level National Enterprise (NE) Survey. According to Fajnzylber & Malony (2001), only two of the nearly 200 studies surveyed by Hamermesh (1993) are at the establishment level for developing countries. Having a drawback that is common in these countries, the firm-level data does not contain wages. Therefore, data from an October Household Survey are used to predict wages for each firm according to characteristics that are common to both the enterprise and household surveys. Section 4 briefly reviews the empirical methodology, including our method for conducting inference on the non-linear elasticities.

Section 5 presents the regression results before discussing the elasticities. Capital and all forms of labour are substitutes, which indicates a rise in the price of labour relative to capital would lead to a relative fall in employment. Cheaper capital would replace labour. We find unskilled and skilled/artisanal workers are substitutes while unskilled and semi-skilled workers are complements; this would not have been possible without disaggregated occupation data. Uncompensated own-price elasticities range from –0.59 for skilled/artisanal occupations to –0.88 for semi-skilled employees. Section 6 discusses policy implications.

2 Theory

2.1 The elasticity of substitution

Our point of departure is production function $q = pf(x_1, x_{2,}..., x_n)$, where q is a measure of output like sales or value added. The dual cost function is $C = C(w_1, w_2, ..., w_n, q)$ where the price of factor x_i is $w_i = p\frac{\partial f}{dx_i}$. In the two-factor case, Robinson (1933) defines the elasticity of substitution between

x_i and x_j as

$$\sigma_{ij}^R = -\frac{d\log\frac{x_i}{x_j}}{d\log\frac{w_i}{w_j}}, \tag{1}$$

where she assumes output and the price of the other factor are constant. In the two-factor case, $\sigma_{ij}^R > 0$. Allen (1938) generalises the elasticity to n factors, where σ can be positive or negative. The Allen elasticity is expressed in terms of the production function but Uzawa (1962) uses the duality between the production and cost function to express the Allen elasticity of substitution as

$$\sigma_{ij} = \frac{CC_{ij}}{C_iC_j}, \tag{2}$$

where C_i, C_j are first derivatives with respect to the prices of factors i, j and C_{ij} is the cross partial derivative.

2.2 The choice of cost function

To estimate σ, Binswanger (1974a) lists why we in general prefer using cost functions for (2) to production functions based on the original Allen Elasticity. First, the calculation is much more tractable. Second, to satisfy a necessary condition for optimizing behaviour, cost functions must exhibit homogeneity of degree one in prices, which can be imposed to improve estimates without recourse to technological assumptions. Third, cost functions are more consistent with the view that wages are exogenous. Because we are trying to understand the impacts of factor prices on factor quantities, cost functions match the exogeneity assumptions underlying the elasticities we estimate.

A Cobb Douglas function is used in a macroeconomic model of skilled and unskilled labour demand and supply by Du Toit & Koekemoer (2003). The implication that $\sigma = 1$ would completely circumvent the aims of the paper. Constant Elasticity of Substitution technologies yield relative factor demand functions that can be estimated and interpreted easily. These allow the elasticity of substitution to differ from one, but the elasticity of substitution is the same between all input pairs (Chung, 1994), which is still a major restriction. Edwards (2003) estimates an equation for the demand for skilled relative to unskilled labour as a function of relative wages, trade and technology variables for the Gauteng Province.

If the elasticities of substitution are not the key parameters of interest but wages need to be included as controls, this specification can be appropriate, especially if one is willing to consider only two factors. Adding factors requires more complex non-linear techniques or step-wise regression to estimate the production function directly (Fallon & Verry, 1988; Hamermesh, 1993). Fallon & Lucas (1998) include capital in their CES function to estimate, with non-linear three-stage-least-squares and calibration techniques, demand for black and white labour as proxies for unskilled and skilled labour. The restriction is also particularly problematic as all factors will necessarily be found to be substitutes. Again, this would defeat the purpose of this study.

More flexible functional forms do not impose a priori technological assumptions like equality of elasticities between all factor pairs. An example is the Transcendental Logarithmic (translog) function developed by Christensen, Jorgenson & Lau (1973). Teal (2000) estimates elasticities of substitution

in Ghana using a translog cost function like

$$\begin{aligned}\log C &= \log a_0 + \sum_i a_i \log w_i + \frac{1}{2}\sum_i\sum_j B_{ij}\log w_i \log w_j + a_q \log q \\ &\quad + B_q \log^2 q + \sum_i B_{iq}\log q \log w_i.\end{aligned} \tag{3}$$

Following Chung (1994), $\frac{d\log C}{d\log w_i} = a_i + \sum_j B_{ij}\ln w_j + B_{iq}\ln q$, but $\frac{d\log C}{d\log w_i} = \frac{w_i}{C}\frac{dC}{dw_i} = \frac{w_i x_i}{C}$ (by Shephard's Lemma). Furthermore, we can define the cost share of factor i as $s_i \equiv \frac{w_x x_i}{C}$. Therefore,

$$s_i = a_i + \sum_j B_{ij}\ln w_j + B_{iq}\ln q. \tag{4}$$

Consistent with cost minimizing behaviour (Berndt & Khaled, 1979), Slutsky symmetry requires

$$B_{ij} = B_{ji} \tag{5}$$

while inspection of the cost function reveals linear price homogeneity requires

$$\sum_i B_{ij} = 0 \tag{6a}$$

$$\sum_j B_{ij} = 0 \tag{6b}$$

$$\sum_i a_i = 1 \tag{6c}$$

$$\sum_i B_{iq} = 0. \tag{6d}$$

These restrictions will be imposed in estimation. In addition, technological assumptions can be tested for and if applicable imposed on the cost function and share equations. By differentiating the cost function with respect to $\log q$, it can be shown that $B_{iq} = 0\ \forall i$ implies homotheticity (returns to scale that are independent of factor prices). This can also be observed in (4), where the factor share is no longer a function of output. If homothetic, the cost function is homogeneous of degree r if $B_q = 0$, with $r = \frac{1}{a_q}$. $a_q = 1$ corresponds to constant returns to scale.

2.3 Factor demand with a translog cost function

The constant-output elasticity of factor demand ($\bar{\lambda}_{ij}$) is the responsiveness of the quantity of factor i to a change in the price of factor j, holding output and all other factor prices constant. Using the fact that $\bar{\lambda}_{ij} \equiv \frac{w_j}{x_i}\frac{\partial x_i}{\partial w_j}$ and $s_i \equiv \frac{w_i x_i}{C}$,

$$\bar{\lambda}_{ij} = \frac{w_j}{x_i}\frac{\partial}{\partial w_j}\left(C\frac{s_i}{w_i}\right) \tag{7a}$$

$$= \frac{w_j}{x_i}\left(\frac{CB_{ij}}{w_i w_j} + \frac{x_j s_i}{w_i}\right) \tag{7b}$$

$$= \frac{B_{ij}}{s_i} + s_j \tag{7c}$$

We know from Marshall's Rules (1920) that $\bar{\lambda}_{ij} = s_j \sigma_{ij}$ (see for example Hamermesh, 1986), so

$$\sigma_{ij} = \frac{B_{ij}}{s_i s_j} + 1. \tag{8}$$

If $\sigma_{ij} > 0$, the factors are said to be p-substitutes. This implies that a rise in the relative price of factor j would lead to a rise in the relative demand for factor i (cf equation (1)). Analagously (Binswanger, 1974a), the own-elasticity of factor demand is:[3]

$$\bar{\lambda}_{ii} = \frac{B_{ii}}{s_i} + s_i - 1 \tag{9}$$

To measure the uncompensated elasticity of factor demand (λ), we must consider output effects. The optimum choice of output is a function of the industry product price and the industry input prices. In a competitive industry with a homothetic technology, a unit fall in the price of one factor will lower average and marginal cost by that factor's share (Estrin & Laidler, 1995). For a fall in the price of factor x_j, profit-maximising industry output will rise and so will demand for all factors, by their share. However, as industry output rises, the product price falls, which lowers the value marginal product of each factor and mitigates the increase in demand for all factors. For all constant returns to scale technologies, $s_j|\eta|$ captures the scale effect in the equation below:

$$\lambda_{ij} = \frac{B_{ij}}{s_i} + s_j(1 - |\eta|), \tag{10}$$

where $|\eta|$ is the elasticity of product market demand. So, if two factors are substitutes such that $\sigma_{ij} > 0$ and hence $\bar{\lambda}_{ij} > 0$, it may be the case that $\lambda_{ij} < 0$. Furthermore,

$$\lambda_{ii} = \frac{B_{ii}}{s_i} + s_i(1 - |\eta|) - 1. \tag{11}$$

The size of $|\eta|$ can affect the sign of λ. We will use existing estimates of $|\eta|$, but we estimate the cost function to predict B and factor shares. In the next section, we describe the data used for this purpose.

3 Data

The core data set is the NE survey of firms. While the dataset has rich disaggregated information on occupation quantities, it does not have wage data. This is a common problem for many developing country datasets.[4] Therefore, appropriate wage information is transplanted from household data based on the October Household Surveys. Characteristics common to both the NE and OHS surveys are used to predict wages by occupation for each firm. This section briefly describes the NE data. It justifies the procedure for transplanting wage data and describes it in more detail. Thereafter, it describes other aspects of data construction.

[3] Humphrey & Wolkowitz (1976) suggest $\sigma_{ii} = \frac{B_{ii}}{s_i s_i} + 1 - s_i$ can be interpreted as the change in a factor's demand responsiveness after a change in its own price.

[4] Many of the World Bank "ICA" datasets suffer from this ommission.

Table 1: Descriptive statistics for turnover and input demand by firm

Variable	Mean	Median	Std. Deviation
Turnover (R million)	90	9	331
Capacity adjusted fixed capital stock (R million)	42	3	203
Managerial/Professional employees	18	4	67
Skilled/Artisanal employees	23	4	73
Semi-skilled employees	99	13	537
Unskilled employees	72	10	335

Note: Employee numbers include part time workers with a weighting of 0.5

3.1 Data from the firm-level manufacturing survey

The dataset used is from the National Enterprise Manufacturing Survey (NE survey) covering the period of 1998. After adjusting for non-response and outliers, there are about 300 firms with the appropriate variables. Unlike the Greater Johannesburg Metropolitan Council Survey used by Edwards (2003), the NE survey is national in coverage. For a thorough analysis of the data, see Bhorat & Lundall (2002), but key statistics are shown in Table 1.

The dataset is a single cross section, so variables are required to control for firm-specific effects and avoid omitted variable bias. Fortunately, the NE dataset has a rich set of variables for the purpose. There are nine industries and nine provinces. There is information on whether the firm is a member of a bargaining council or otherwise subject to a bargaining council agreement. There are also ordinal variables for how much difficulty firms have recruiting workers within each occupation, which may capture aspects of the firm's activity and have non-price effects on factor choice. Other variables include the percentage of sales that a firm exports, the age of the firm's equipment, the manager's satisfaction with productivity, the percentage of raw materials the firm imports and the percentage of assets invested in computers.

The key explanatory variables for the cost function are factor costs (the cost of capital and wages for the occupation groups[5] in Table 1) and value added.

3.2 Factor costs

Why firm-level wages can be represented by supra-firm data In South Africa, average wages by industry and occupation are a good approximation to those faced by firms. Nattrass (2000) reports that the main wage setting institutions are industrial level bargaining councils (BC), noting that 65% of manufacturing workers are covered by a BC. Furthermore, the Minister of Labour is obliged to extend BC agreements to non-members. Nattrass concludes that extension to non-members is at the core of wage setting in an industry. Moll (1996) shows how extensions of bargaining council agreements make some firms become more capital intensive and other firms, which tend to be small and labour intensive, leave the industry. This leads to convergence in technologies and wages in the industry.

The NE survey provides data on whether the firm is subject to collective bargaining and/or a BC agreement. On average, over 70% of firms are subject to an agreement. Firms with fewer than 50 employees are almost 100% covered while large firms vary from 32% to 61% by industry in coverage.

[5] There is also a fifth group for Sales/Clerical workers. Including this group produces many nonsensical results. We conjecture this is due in part to a large degree of heterogeneity within this group, which consequently means the wages constructed were not accurate for this group. Results are available on request.

There is therefore support for convergence of wages in industries and justification for wages being calculated at a supra-firm level.[6] Before proceeding, we note that Teal (2000) predicts wages for firms using the characteristics observed in their workers.

Using household data for wages We restrict the 1997 October Household Survey[7] sample to those 3 500 people working for somebody else in formal manufacturing industries. Definitional correspondence to the NE survey in terms of industry, province and occupation is good. As will be explained, the correspondence regarding union membership / collective bargaining is not. Details of the dataset and survey methodology are available in Statistics South Africa (1998). People were interviewed in geographical clusters and stratified by magisterial district while the sample surveyed is not fully representative of the population. We take survey design effects like these into account.[8] This study accounts for probability weights and clustering but only partially adjusts for stratification.[9] For each occupation, the characteristics available in both the firm and household data are:

- economic activity (broken down into nine industries)
- province group (the nine provinces were ex post broken down into two groups with similar wages)
- individual trade union membership (household data); collective bargaining and bargaining council membership (firm data)

Construction entails calculating the survey-adjusted means for groupings of people for each occupation. Preliminary work constructed a number of alternative wage series. One classified wages by industry, location and trade-union membership. There are nine industries and nine provinces, meaning that, together with a trade-union membership dummy, there are potentially 162 different wages. However, while some means were calculated using a comfortable number of observations, others were based on few data points, sometimes only one. This means the standard errors on the wage estimates would be high (or non-existent). To mitigate this, the nine provinces are divided into two groups, as variation within each of the two groups is low. Furthermore, more precise estimates can be achieved by combining some locations and industries and/or not distinguishing by trade-union membership in cases where wages do not differ substantially. Before discussing the process undertaken to determine classification, it may be helpful to look at one example. Table 2 presents six of the fifteen composite groups the skilled/artisan wages are divided into and the associated estimates.

[6] The NE survey does not reveal which occupations within a firm are subject to BC wages. One could argue that more skilled wages are less likely to be influenced by collective bargaining, but the household data show that the proportion of trade union membership does not vary materially by occupation. Even if not influenced by bargaining, more skilled people tend to be mobile, which standardises wages across firms through ordinary market processes.

[7] The 1998 survey was much smaller due to funding problems. This and an allowance for adjustment lags make the 1997 survey the preferred edition.

[8] People in the 10 households interviewed per geographical cluster don't have independent characteristics, being more likely to have similar features. Therefore, the survey sample variance of the wage would be lower than would be the case in a random sample. Failing to account for clustering often results in standard error estimates that are half what they should be. In contrast, stratification guarantees that this similarity will not happen across strata. It mitigates the chances of there being a non-representative sample and therefore standard error estimates should (correctly) be lower than in the absence of stratification Deaton (1997).

[9] The reason for this is that many magisterial districts (strata) have only one cluster – many have only one observation – and at least two are needed for variance estimates. A standard procedure for dealing with this is to collapse or merge strata (Statacorp, 2003), but the number of cases to collapse is high in this study. Therefore, compromise stratification by province, which sometimes has close to 100 magisterial districts, is carried out. An aggregate estimate of monthly salary has a Deff statistic, which is the ratio of the estimated variance accounting for survey design to the unadjusted variance, of 2.44. This indicates the importance of dealing with the survey design effects.

Table 2: Selection of categories for Skilled/Artisan wages

Group	Estimate	Std. error
Food & Beverages	1562	161
Wood, Pulp & Paper - Prov0	1116	229
Wood, Pulp & Paper - Prov1	1993	169
Chemicals, Rubber & Plastic - Prov0, not unionised	786	152
Chemicals, Rubber & Plastic - Prov0, unionised	2316	264
Chemicals, Rubber & Plastic - Prov1	2067	284

Source: own calculations based on October Household Survey data

The first row contains wages for all skilled/artisanal workers in the Food & Beverages industry, regardless of location or union membership. The Wood Pulp & Paper industry is subdivided by province group but not union membership (rows 2 and 3). Wages in the Chemicals, Rubber and Plastic industries are subdivided by province group. One group of provinces is further divided into unionised and non-unionised workers (rows 4 and 5) while the other group is not (row 6). In some cases, industries are combined, with the possibility of disaggregation by other criteria.

Classifying the wages involves a number of trade-offs. While averages across two or more groups are different, the standard errors may be large, resulting in imprecise estimates. This is often because of a small sample size for that group. One way to proceed is to separate all groups with statistically significant differences in means. However, this is imperfect. An extreme but not infrequent occurrence is that of one observation per group, which generates no standard error and is also outside the confidence interval of another group. Similarly, inference based on very few observations is not reliable. On the other hand, some estimates, even if based on few observations, are so radically different that the groups should be classified separately. The aim is to produce a set of estimates per group with better precision characteristics but sufficient variation to represent the firm-level data. To do this, various combinations are carefully inspected. Factors considered are differences in log wages, the number of observations, and comparisons of the standard errors and confidence intervals of the separate and combined groups. Of course, all the criteria are related.

Comparing the confidence intervals of two groups is naturally akin to performing a two-sample t-test. However, visual inspection is quicker for all the combinations and allows for analysis in conjunction with the other criteria. The choice of confidence interval is a matter of taste in this application, so 85% bands are used. To augment this procedure more formally, standard t-tests, regressions and non-parametric procedures[10] are performed on certain groups.

Going through the above procedure on a case-by case basis therefore produces a set of wages, for each occupation, which partially disaggregates each industry by location and/or trade union membership in a way that optimizes the trade-off between achieving representative wage estimates and having precise estimates. Depending on the occupation, the number of categories ranges from 7 to 15, with the average number of observations per category ranging from 16.9 to 43.7. Data from TIPS shows wages rose by approximately 15% between the time of the household survey and the time of the NE survey. Wage measures are raised by this percentage. Next, we adjust wages for firm size.

[10] Tests of median equality are performed, but they do not factor in survey design. The results do not indicate material differences in classification. Another useful way to compare specific groups is to use Anova and Scheffe's method of comparing the means of each group to those of all the others. This method is used but there is also no readily available way to adjust for survey design.

Table 3: Estimates of relationship between firm size and wages

Managers	Professional/ technological	Craft	Operators	Labourers	Total
0.089	0.076	0.096	0.094	0.031	0.065

Source: Bhorat & Lundall (2002)

Adjusting wages for firm size The appendix shows analytically why failing to account for firm size can lead to poor results. Informally, failure to account for firm size leads to a conflation of wage effects and output effects in the cost function. A simple way to adjust wages is through the linear function

$$\ln w_i = \ln \hat{w}_i + \gamma_i \ln q, \tag{12}$$

where w_i is the wage adjusted for firm size and $\hat{w}_i$ is unadjusted. There is no information on the size of the firms which individuals in the household survey work for. We therefore attach values of γ_i to the wage series. Using GJMC data, Bhorat & Lundall (2002) estimate manufacturing firm-size wage effects, which are shown in Table 3. Their estimates are rudimentary but the authors claim similarity to the US study of Doms, Dunne & Troske (1997). Assuming the unadjusted wages represent those for an average-sized firm, the wages transplanted from the household sata are inflated/deflated accordingly.

Costs of capital We use the expression from an industry-level study of capital in South Africa (Fedderke et al., 2001):

$$c = (r - \pi) + \delta + \tau \tag{13}$$

For the real interest rate $(r - \pi)$, we use the average prime lending rate and consumer price inflation for 1999. They calculate industry-level values for δ, the depreciation rate.[11] Fedderke et al. use the nominal corporate tax rate for τ, which was 35% for the relevant fiscal year (RSA, 1998). They state it would be ideal to have the effective rates of taxation by industry. Negash (1999) calculates effective tax rates to be about 15% below nominal rates for the 1990s, so a 20% average effective rate is applied to all firms. Furthermore, we adjust the interest rate to account for risk. Adjustments range from –2% for large (>50 employees) and old (> 5 years) firms to + 5% for new small firms.[12] The resulting range of costs of capital is 40-53%.

3.3 Value added, total costs and cost shares

Factor prices and quantities are used in the determination of cost shares and total costs. Labour costs are obtained by multiplying labour quantities by the constructed wage for each occupation. Capital costs are the cost of capital percentage multiplied by the capacity-adjusted capital stock. Total factor cost (C_f) is the sum of factor costs. To calculate each factor share, we multiply the factor's wage by its quantity and divide it by the sum of the factors' costs; that is: $s_i = \frac{w_i x_i}{\sum_i w_i x_i}$. The shares are

[11] We thank Prof Fedderke for providing the data.

[12] This amendment can be accused of confusing Jorgenson's notion of the cost of capital with the weighted average cost of capital due to Modigliani & Miller (Lau, 2000). Bergström & Panas (1992) use a weighted average cost of capital measure in their study, while Teal (2000) constructs predicted profit rates as a percentage of the capital stock with regressions containing firm- and industry-specific variables. In equilibrium, these two costs should be equal (Lau, 2000). To the extent that this equilibrium does not hold realistically, and because of the practical reality that smaller and newer firms are likely to have higher borrowing costs, accounting for these risk premia is necessary.

Table 4: Calculated factor shares

Capital	Managerial/Professional	Skilled/Artisan	Semiskilled	Unskilled
54.7%	9.4%	7.4%	16.4%	11.6%

Sources: NE and OHS surveys, author's calculations

presented in Table 4. Combining Managerial/Professional and Skilled/Artisan yields a share of 17% and combining Semiskilled and Unskilled yields 28%. These calculations are similar to those of Teal (2000), where capital comprises 60%, skilled labour 11% and unskilled labour 29%.

There is information in the data on what percentage of total costs is comprised of raw materials costs, but there is no data on total costs or on raw materials costs. To derive a measure of raw materials costs, it is necessary to assume that turnover equals total costs. Then raw materials as a percentage (p) are multiplied by turnover (q) to get a measure of raw materials costs. Value added is constructed as sales minus the constructed raw materials so that $v = (1 - p)q$. [13]

4 Empirical methodology

Estimation The stochastic specification of the cost function and cost share equations assumes that the firms make random errors in their input selection. We will estimate the cost function together with the system of share equations. We assume the errors have mean vector $\mathbf{0}$ and variance matrix $\mathbf{\Omega}$. It is likely that errors in input selection will be correlated across the share equations and the cost function such that $\mathbf{\Omega}$ is not diagonal. Single equation estimation is therefore inefficient (Berndt, 1991). The cost share equations will therefore be estimated together as a system with the cost function as a seemingly unrelated regression, which exploits correlations between the errors in each of the share equations to improve efficiency. Cross equation restrictions allow for further efficiency improvements (Greene, 2003). Many restrictions exist because the cost shares are derivatives of the cost function while Slutsky symmetry conditions also imply cross equation restrictions. Additional restrictions consistent with profit-maximizing behaviour and technological features can also be imposed (cf Section 2.2).

By construction, the sums of the a_i coefficients across the factor share equations equal unity for each observation. Therefore, the residual cross product and disturbance covariance matrices are singular and prevent estimation (Berndt, 1991). A common response is to impose price homogeneity on the cost function and hence across the share equations. Using (6c), let $a_k = 1 - \sum_l a_l$ where k refers to capital and l to the labour inputs. This allows for the capital equation to be dropped and the remaining share equations for each of the labour inputs to be estimated as

$$s_i = a_i + \sum_j B_{ij} \ln \frac{w_j}{w_k} + B_{iq} \ln q + \omega_i, \tag{14}$$

where ω_i is a stochastic error term. We drop the capital equation but the choice is arbitrary because the Zellner iterated efficient (IZEF) procedure is used (Berndt, 1991).

[13] As a check, we calculated an alternative measure using a completely different method based on p and total factor costs, where total costs are calculated using the factor quantities and prices. The correlation between the two measures was 0.90.

Inference For indications of the significance of coefficients and for tests of price homogeneity, technological restrictions, and separability, it is standard to use Wald tests. We also use such tests but with the caveat that they may not be valid if the residuals are not multivariate normally distributed. The literature appears to have little difficulty with the assumption of normality. However, the values of factor shares are bounded between 0 and 1 and Jarque-Bera tests reject normality at 1% for all shares.

Significant regression coefficients (B_{ij}) neither imply nor are necessary for significant elasticities (Anderson & Thursby, 1986). "Significant" can refer to rejecting a null hypothesis of the elasticity being zero, in which case we can be confident the factors are complements or substitutes. The difficulty lies in the fact that the elasticity estimates are highly non-linear combinations of the coefficients and data. Anderson & Thursby present conditions under which Allen-Uzawa elasticities of substitution asymptotically follow the normal or ratio-of-normals distribution. They are not appropriate under many alternative calculations found in the literature and we do not have the option to use their result as we use predicted not actual factor shares.

Reviews of empirical work on elasticities of substitution make no mention of significance (Chung, 1994; Hamermesh, 1993). Some studies do not report confidence intervals for the estimators at all (Bergström & Panas, 1992; Teal, 2000). Others (Binswanger, 1974b) regard the factor shares as fixed and treat the coefficient as the only variable with a confidence interval, incorrectly inferring the elasticity significance from a t-statistic.

An alternative is to use non-linear approximation procedures like the Delta method to conduct inference (see Greene, 2003). These are particularly sensitive to non-normality so we present an informal method by indicating how the elasticities vary across the sample of firms. Besides, it can be more informative to find the elasticity is positive for most firms than to know what the average is. Furthermore, elasticity estimates might be precise at the centre of the data, but could be badly behaved elsewhere. Therefore, the elasticity for each firm is calculated using the coefficient estimates and each firm's input quantity. We will have a distribution of elasticities. If an elasticity is positive down to the 5th percentile, we can say 95% of firms have positive elasticities. If an elasticity is negative up to the 95th percentile, we can indicate 95% of firms have a negative elasticity.

5 Results

We perform a brief diagnostic on the regression before turning to the parameters of direct interest: the elasticities. We present elasticities of substitution before proceeding to constant output labour demand. Thereafter, we discuss the implications of adjusting for non-constant output.

5.1 Cost function and share estimates

We estimate the system (14) together with the stochastic version of cost function (3).[14] The cost equation is in Table 5. We fail to reject the hypothesis that all $B_{ij} = 0$. Overall, however, the regression fit is good, with a pseudo-R^2 of 0.85 for the cost equation. Presenting the share equations would reveal very little additional information but basic diagnostics for the system are presented in Table 6. Notably, all the share equations are highly significant. In Table 5, we reject the assumption of homotheticity, so

[14] Our results are robust to the choice of control variable. However, our results are unreliable if we use alternative wage measures which do not account for firm size. Furthermore, including a fifth occupation (sales/clerical workers) produced

Table 5: Cost Function Parameter Estimates

Dependent variable: Cost

Variable	coefficient	p	Variable	coefficient	p
Constant	4.11	0.04	ind2	0.20	0.41
Capital	0.25	0.73	ind3	0.50	0.05
Man/Prof	0.27	0.44	ind4	-0.25	0.19
Skil/Art	0.08	0.72	ind5	-0.05	0.79
Semi	0.17	0.73	ind6	0.42	0.15
Un	0.25	0.37	ind7	0.11	0.63
value added	0.29	0.00	ind8	0.07	0.81
0.5*Capital^2	-0.33	0.07	ind9	-0.31	0.05
Capital*Man/Prof	0.06	0.28	loc2	0.21	0.43
Capital*Skil/Art	0.08	0.20	loc3	-0.33	0.08
Capital*Semi	0.15	0.17	loc4	-0.29	0.12
Capital*Un	0.05	0.48	loc5	-0.75	0.00
0.5*Man/Prof^2	0.03	0.41	loc6	0.74	0.08
Man/Prof*Skil/Art	-0.03	0.08	loc7	0.70	0.04
Man/Prof*Semi	-0.03	0.43	loc8	-0.23	0.58
Man/Prof*Un	-0.03	0.34	loc9	-0.29	0.11
0.5*Skil/Art^2	0.02	0.37	exports / output %	0.24	0.25
Skil/Art*Semi	-0.07	0.05	raw materials / cost %	0.01	0.00
Skil/Art*Un	0.01	0.84	recruitment ease Man/Prof	0.1	0.05
0.5*Semi^2	0.01	0.94	recruitment ease Sale/Cler	-0.05	0.23
Semi*Un	-0.05	0.26	recruitment ease Skil/Art	-0.07	0.11
0.5*Un^2	0.03	0.58	recruitment ease Semi	0.01	0.82
0.5*(value added)^2	0.13	0.00	recruitment ease Un	0.02	0.81
(value added)*Cap	0.01	0.78	training expenditure	0.00	0.01
(value added)*Man/Prof	-0.02	0.00	market conditions index	-0.01	0.17
(value added)*Skil/Art	0.00	0.96	firm size > 50 employees	0.37	0.00
(value added)*Semi	0.01	0.43	computer investment / output %	-3.33	0.00
(value added)*Un	0.00	0.76	ownermanaged	-0.61	0.00
Observations	307		productivity dissatisfaction	0.052	0.02
"R^2"	0.85		collective bargaining	0.00	0.96
Homotheticity (p value)		0.02	firm age	0.04	0.09
Joint significance of B_{ij} (p value)		0.31	capital intensity indicator	1.40	0.00

Table 6: System Diagnostics

Share Equation	Obs	RMSE	"R^2"	χ^2	p
Managerial/Professional	307	0.06	0.43	232	0.00
Skilled/Artisanal	307	0.08	0.18	72	0.00
Semiskilled	307	0.13	0.16	62	0.00
Unskilled	307	0.11	0.11	39	0.02

Table 7: Elasticities of Substitution

$\sigma_{ij} = \sigma_{ji}$		j				
		Capital	Man/Prof	Skil/Art	Semi	Un
	Capital	-1.62*	2.19*	2.91*	2.73*	1.74*
	Man/Prof	2.19*	-5.96	-5.77	-1.46	-2.04
i	Skil/Art	2.91*	-5.77	-7.53	-7.28*	1.79*
	Semi	2.73*	-1.46	-7.28*	-5.78*	-2.44*
	Un	1.74*	-2.04	1.79*	-2.44*	-5.94*

* indicates sign consistent for at least 95% of firms in sample

the constraints are not imposed. One possible explanation is that the wage adjustment is not accurate enough and poor data are erroneously causing false rejections of homotheticity. In particular, the firm-size effects on wages imposed in construction may not be large enough. Söderbom & Teal (2004) produce firm-size effect estimates for African firms of up to 0.15, which are more than twice the average we took from Bhorat & Lundall (2002). Another is that factor shares really are a function of output. Bigger firms have cheaper capital and may therefore may employ more of it, or it could be a genuine technological feature.

The variables controlling for firm specific effects are entered in levels and not interacted with any of the input prices. As they are not of direct interest, we keep the discussion brief. Of the included variables that are significant, the indicator of raw materials as a percentage of costs is highly positive, which suggests firms using a large component of their inputs might be less efficient. Firms with older equipment and which themselves are older also tend to have high costs, which might be expected. Firms incurring higher training expenditure tend to have higher costs, but this can be a natural correlation for big firms. The firm size dummy is also significant. We find the dummy for owner managed firms, which we take to be those with only one manager, is negative. Firms that invest more in computers tend to be those with lower costs while the indicator of the capital labour ratio is positive.

5.2 Elasticities of substitution

Elasticities of substitution are calculated for each firm using (8). The median values are presented in Table 7.

We indicate with an asterisk values that have that sign for at least 95% of firms in the sample. A positive coefficient denotes a pair of factors are substitutes. A fall in the cost of one factor relative to the cost of the other will lead to a rise in its relative quantity. A negative coefficient denotes a pair of complements.

The first row/column suggests capital is a substitute for all occupations. A relative fall in the cost of capital will lead to a fall in employment relative to utilization of capital. This holds across at least 95% of the firms in the sample. There is no indication of the Griliches (1969) capital-skill complementarity hypothesis in these statistics. It does not appear to be the case that capital and less skilled occupations are more substitutable than the more skilled occupations.

The suggestion that all forms of labour seem roughly equally substitutable for capital has two methodological implications (Berndt & Christensen, 1973a). First, studies of labour/capital substitution do not incur a great cost by aggregating various forms of heterogeneous labour. Second, should

nonsensical results.

Table 8: Compensated Elasticities of Factor Demand

$\bar{\lambda}_{ij}$		j				
		Capital	Man/Prof	Skil/Art	Semi	Un
	Capital	-0.96*	0.18*	0.18*	0.40*	0.19*
	Man/Prof	1.28*	-0.56	-0.32*	-0.20	-0.20
i	Skil/Art	1.77*	-0.42*	-0.56	-0.99*	0.19*
	Semi	1.60*	-0.12*	-0.43*	-0.80*	-0.26*
	Un	1.03*	-0.16*	0.12*	-0.34*	-0.65*

* indicates sign consistent for at least 95% of firms in sample

data constraints prevent the use of costs of capital in studies of intra-labour elasticities, omitting capital would not affect the estimates badly.

Most occupations share a common substitute – capital – but are themselves complements. This finding is important because two-factor studies or those using standard CES technologies would find skilled and unskilled labour to be substitutes by construction. While the previous paragraph suggested some simplifications to the model need not be overly damaging, others can be very misleading. Furthermore, our combination of multiple inputs and flexible functional form allow us to find that not all skill types are complements. This will be revealed in the next section.

5.3 Elasticities of factor demand

Compensated Elasticities Table 8 presents the median conditional elasticities of factor demand, which are calculated using equations (7) and (9).

Own-price elasticities are negative for all occupations. The fact that the own-elasticity is not negative for 100% of firms suggests some of the predicted factor shares are outside of a firm's feasible production set (see Berndt & Christensen, 1973b). The fact that the negative numbers are constant-output elasticities verifies that this is consistent with cost minimizing behaviour and that the negative effect is not artificially created by a scale effect. Holding output constant, a 1% rise in the unskilled wage will lead to a 0.65% fall in unskilled employment. The results also imply that, in general, a rise in the wage of one labour type has negative employment consequences for the other labour types. For example, a 1% rise in semi-skilled wages would lead to a 0.34% fall in unskilled employment. The exception to this finding is skilled/artisanal workers, where a 1% rise in skilled/artisan wages would lead to a 1.2% rise in unskilled employment, holding output constant. This demonstrates the usefulness of disaggregation.

Uncompensated Elasticities The results so far do not allow for output effects. On the assumption of locally constant returns, we can take these into account. Inspection of equations (10) and (11) shows the scale or output effect is $-s_j|\eta|$. With the exception of capital, the factor shares are all quite small, which suggests that the scale effect will tend to be quite small for labour.

To allow for scale effects, we need measures of $|\eta|$. Selvanathan & Selvanathan (2003) produce estimates for a variety of industries in manufacturing and other sectors. Two manufacturing industries that overlap with the industry definitions in our dataset are clothing and furniture. The elasticity value for clothing is $|\eta| = 0.423$, which is relatively inelastic. Furniture demand is the most elastic at $|\eta| = 0.947$. When we allow for output effects in these industries, not one of the signs is changed from the uncompensated elasticities. Because the numbers are not materially different to the constant

Table 9: Uncompensated Elasticities of Factor Demand

λ_{ij}		j				
		Capital	Man/Prof	Skil/Art	Semi	Un
	Capital	-1.25*	0.14*	0.15*	0.32*	0.14*
	Man/Prof	0.99*	-0.60	-0.36*	-0.28*	-0.25*
i	Skil/Art	1.48*	-0.46*	-0.59	-1.07*	0.13*
	Semi	1.3*	-0.16*	-0.45*	-0.88*	-0.31*
	Un	0.74*	-0.19*	0.08*	-0.42*	-0.70*

* indicates sign consistent for at least 95% of firms in sample

output elasticities, we omit the results.

Instead, we take a simple average of the industry elasticities in Selvanathan & Selvanathan (2003) - even though they are not all straightforwardly classified as being in the manufacturing sector - to obtain an average elasticity of $|\eta| = 0.5$ for manufacturing. The corresponding uncompensated elasticities are presented in Table 9.

Unsurprisingly, the own-elasticities are more negative than in Table 8, though not by much. These can be taken as evidence the factor costs would affect unemployment. All the signs from the uncompensated elasticities are preserved. This is particularly important in the case of labour and capital, where the sign is positive. This means the substitution effect is not outweighed by the output effect. In other words, despite leading to a rise in manufacturing output, a fall in the price of capital would lead to a fall in demand for all labour types. The Skilled/Artisanal and Unskilled groups are substitutes: a fall in the price of skilled/artisanal workers would lead to a fall in unskilled employment, despite positive output effects.

6 Summary and policy implications

This paper estimates elasticities of substitution between four occupations and capital. It also presents results of conditional (constant-output) and unconditional factor demand elasticities. Estimates of such phenomena are rare for developing countries and this paper appears to be the first to present unconditional elasticities. While the shortage of establishment-level wage data prevails for developing countries, this study presents an option for supplementing it with data from a household survey.

The estimates suggest occupations share a common substitute – capital. This holds despite allowing for output effects. Our results therefore support the concern that policies which lower the cost of capital relative to labour will lead to a fall in employment in favour of capital acquisition. This microeconomic contribution based on manufacturing evidence suggests that, despite potential scale effects, calls for a lowering of interest rates by trade unions may be against the interests of their constituents.

Taken literally, the corollary to these implications is that, to raise employment, there should be a rise in interest rates! This microeconomic contribution is not designed to confront standard macroeconomic transmission arguments for output expansion under the assumption of insufficient aggregate demand, so we by no means wish to make such a strong claim. We do not consider any productivity benefits from investment. Furthermore, the potential effects of technological change are not analysed. However, the results at the very least suggest some caution is called for by those advancing lower interest rates as a solution to unemployment. Furthermore, other policies besides interest rates can affect the cost of capital. Measures to increase the cost of capital relative to labour,

such as ending generous depreciation allowances on machinery (see Fedderke et. al., 2001) have been implemented. On the basis of these results, such measures would have positive employment consequences.

Furthermore, exploiting our ability to calculate disaggregated elasticities with a flexible cost function, we reveal that unskilled labour complements semi-skilled workers but substitutes for skilled/artisanal labour. These elasticities suggest that a wage subsidy for unskilled workers would adversely affect demand for skilled/artisanal workers but raise demand for semi-skilled workers, besides increasing demand for unskilled labour itself. A further implication is that wage restraint by semi-skilled labour would have positive employment benefits for unskilled workers and vice versa. This introduces the possibility of a coordination problem. To what extent it is addressed by trade unions representing both unskilled and semi-skilled workers is a matter of further empirical and theoretical research.

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Appendix

Oi & Idson (1999) discuss the positive relationship between firm size and wages. This may be due to inherent firm characteristics or unobservable attributes of workers in those firms. Regardless of the reason, the following paragraphs explain what impact ignoring this relationship may have on translog estimates, concluding that the estimations are more likely to (falsely) reject homotheticity and linear price homogeneity and to overstate returns to scale. Abstracting from individuals' characteristics, wages for occupation i can be seen as a simple function of firm size measured according to value added (q) and a vector of variables available from the household survey ($\mathbf{x}$):

$$\begin{aligned} \ln w_i &= \boldsymbol{\beta}_i \ln \mathbf{x} + \gamma_i \ln q, \gamma_i > 0 && (15) \\ &= \ln \hat{w}_i + \gamma_i \ln q && (16) \end{aligned}$$

In other words, (15) is the wage for an individual with characteristics $(\mathbf{x}, q)$ in occupation i while $\hat{w}_i$ is the (perfectly) predicted wage for that individual based on data from the household survey. Algebra shows a translog cost function such as (3) that does not account for firm effects on wages is the same as,

$$\log C = \sum_i a_i \ln \hat{w}_i + \log q + \frac{1}{2} \sum_i \sum_j B_{ij} \log \hat{w}_i \log \hat{w}_j + \Phi \log^2 q + \Omega \log \hat{w}_i \ln q, \qquad (17)$$

where $\Gamma = \sum_i a_i\gamma_i + a_q; \Phi = \frac{1}{2}\sum_i\sum_j B_{ij}\gamma_i\gamma_j + \sum_i B_{iq}\gamma_i + B_{qq}; \Omega = \sum_i\sum_j B_{ij}\gamma_j + \sum_i B_{iq}$. The coefficients containing value added may be vastly different to what they are supposed to be. To gauge the likely nature of the bias in a simple setting, we assume the coefficients are correctly estimated. Furthermore, on the convenient assumption that linear price homogeneity and constant returns to scale $\left(\sum_i a_i = 1; \sum_i B_{ij} = \sum_j B_{ij} = 0; B_{iq} = B_{qq} = 0; a_q = 1\right)$ hold in the true cost function:

$$\Gamma = \sum_i a_i\gamma_i + 1 \tag{18}$$

$$\Phi = \frac{1}{2}\sum_i\sum_j B_{ij}\gamma_i\gamma_j \tag{19}$$

$$\Omega = \sum_i\sum_j B_{ij}\gamma_j \tag{20}$$

We can't be sure $\sum_i a_i\gamma_i > 0$ as it is not necessarily the case that $a_i > 0$ for all i. Therefore, we cannot be sure $\Gamma > 1$, which would be the coefficient on $\ln q$ under constant returns. However, linearly homogeneous prices imply that, if all the values of γ_i for each occupation are close enough to the average across occupations, the result will tend to be an upward bias on the value added coefficient. If the firm-size effect is equal for all occupations, the bias is γ.

It is not possible to tell what direction the bias will be for Φ. However, if there is an equal firm-size effect for all occupations, price homogeneity implies this will be zero and in fact not biased. If the firm-size effect is not equal for each occupation, there is the possibility of Φ being found significant when it actually is not. This would falsely reject a homogeneous technology. A similar analysis concludes the coefficient on Ω may be found significant and therefore falsely reject homotheticity or that linear price homogeneity is rejected by distorted coefficient values.

To understand the likely effects on returns to scale, assume for simplicity a common firm-size effect across all occupations. The assumption of a homogeneous technology is relaxed but homotheticity and price homogeneity are maintained. Returns to scale are given by

$$\left[\frac{\partial C}{\partial q}\right]^{-1} = [\gamma + a_q + B_{qq}\log q]^{-1} \tag{21}$$

One can gauge that omitting the firm size variable will underestimate the denominator by γ on average, so returns to scale will be overestimated. This is intuitive: if wages rise for bigger firms, the returns to scale are less than otherwise. Therefore, including a measure of γ will reduce the estimated returns to scale.

We ran a number of specifications that did not adjust wages for firm size. Homotheticity was rejected but, if imposed, returns to scale were found to be implausibly large. More generally, the diagnostics on the regressions and the associated elasticities were not robust or reliable. Details are available on request.