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The Plea Bargain in Theory: A Behavioral Model of the Negotiated Guilty Plea*

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The negotiated guilty plea pervades our system of justice. Pleas of guilty entered by defendants prior to trial in exchange for assurances of leniency in sentencing account, by our best estimates, for well over ninety percent of all criminal convictions in the United States.[1] But while serious and difficult questions of ethical propriety surround the practice of plea bargaining and make its ubiquitous existence a source of continuing uneasiness in the law, perhaps no other aspect of the criminal process is so hidden and so little understood. Moreover, the need for systematic study of the nature and determinants of the plea bargaining process and the problems of constitutional values which it raises are not unrelated. The constitutionality of negotiated pleas turns largely on the question of whether the pressures of the bargaining situation are such that innocent defendants might be persuaded to accept a proffered plea bargain and subject themselves to punishment for crimes which they did not commit. Indeed, the United States Supreme Court, in approving the practice of plea bargaining, framed the issue in just this way:

> We would have serious doubts about this case if the encouragement of guilty pleas by offers of leniency substantially increased the likelihood that defendants, advised by competent counsel, would falsely condemn themselves.[2]

The purpose of this paper is to cast some light on these problems through the specification of a formal model of the plea negotiation process. In Section I, we extend earlier work in this area and seek to characterize the initial

*I am deeply indebted to Robert Pollak, Robert Inman, and Noel Edelson for invaluable advice and encouragement, although any errors which remain are, of course, the responsibility of the author alone.

1. See Newman [13, 3] and Task Force Report [16, 9–13].
2. Brady v. United States, 397 U.S. 742, 757–758 (1970).

decision as to whether to enter into negotiations at all or to insist upon a determination of guilt or innocence at trial, and to define for both prosecutor and defendant the range of bargaining outcomes preferred to the prospect of a full trial. Given the existence of a "contract zone," the set of bargaining outcomes seen by both parties as preferred alternatives to trial, we attempt in Section II to specify in principle the precise terms of the agreement, if any, reached as a result of the negotiation process and to present a comparative statics framework within which the model might be empirically tested. A principal product of the analysis is the suggestion that the structure of the plea negotiation process itself may encourage precisely the kind of result feared by the Supreme Court, for the introduction of time-related costs and their attendant incentives as significant factors in the decision of the defendant to plead guilty adds a dimension to the defendant's behavior which is wholly unrelated to his guilt or innocence. Even where they are supported by empirical evidence, however, the ultimate significance of such conclusions must be the subject of careful scrutiny and critical evaluation. The ostensible power of the arguments may be mitigated by endemic epistemological problems which arise at the interface of law and social science and serve to delimit sharply the relevance of formal economic analysis in many kinds of adjudicative settings. A brief discussion of these issues is contained in Section III.

I. The Initial Decision Model

In a provocative early paper, William Landes [11] has analyzed the preliminary decision of whether or not to enter into pretrial negotiations.[3] Portraying both prosecutor and defendant as rational maximizers, Landes fixes attention upon inputs of material resources into the case as the principal instrument variable in the hands of the parties. He then develops in a straightforward way the conditions under which pretrial bargaining is to be expected, although the precise terms of the resultant agreement are left indeterminate. Apart from this indeterminacy, however, Landes' treatment of the problem appears to leave other important questions unresolved. We observe, for instance, that the dockets of contemporary criminal courts, particularly in urban jurisdictions, are crowded with the cases of indigent defendants, each entitled under the Sixth and Fourteenth Amendments to fully subsidized legal counsel.[4] The Landes model predicts that policy measures designed to

3. Landes' model can be seen as a special case of the more general framework presented in Gould [9].

4. See Johnson v. Zerbst, 304 U.S. 458 (1938), assuring counsel to indigent criminal defendants in the federal courts, and Gideon v. Wainwright, 372 U.S. 335 (1963), extending the guarantee to defendants charged with felonies in the state courts. On the problems of the poor in the criminal justice system generally, see Blumberg [4], Task Force Report [16], and Oaks and Lehmann [14].

provide counsel to such defendants without cost to them will increase the proportion of trials demanded, and indeed he produces statistical evidence of a positive correlation between the subsidization of the legal expenses of indigent defendants and the incidence of trials in the United States District Courts. Yet in many urban jurisdictions, particularly at the state level, we see both a large proportion of defendants represented by court-appointed subsidized counsel *and* a very high rate of guilty pleas.[5] This apparent anomaly suggests that inputs of material resources by the defendant for trial preparation may not be the sole, or even the principal determinant of either party's decision to negotiate or insist upon a trial. The passage of time, however, is a factor not considered by Landes, and it may well be that the simultaneous use of time by both parties as a bargaining tool to impose pretrial costs upon the opposition is the most critical element of the initial decision problem.

We can motivate this assertion in a largely qualitative way.[6] Consider first the case of the prosecution. It is convenient for our purposes to postulate the existence of a single individual whose task it is to formulate the overall policy of the prosecutor's office and to administer its budget; let us call this policymaker the District Attorney. We assume that at the outset of the budget period the District Attorney is faced with the problem of allocating the agency's total budget over an anticipated case load for the budget period. Associated with each case (or, alternatively, with each type of case) is a prosecutorial utility function defined over the range of possible sentences which might be imposed upon conviction in the case. At this preliminary stage, however, the District Attorney must account for the possibility that each defendant will exercise his right to trial and refuse to bargain. Thus, the outcome of each case is uncertain, and the District Attorney faces an expected utility function for each case of the form $\overline{U} = QU(S)$, where S is the expected sentence upon conviction at trial and Q is the probability of conviction. The District Attorney's problem is to allocate the agency's budget so as to maximize the sum of these expected utilities over the anticipated case load.

The agency's budget must cover two distinct kinds of expenditures. First, there are outlays which might generally be termed trial preparation costs; under this heading fall all expenditures which are productive of expected utility through increases in the conviction probability Q. A second kind of expenditure covers costs which are *not* productive of expected utility but which are built into the criminal process on a per case basis. These internal and systemic pressures emanate from the calendar; forces as diverse as the

5. It has been estimated that thirty to sixty per cent (depending on the jurisdiction) of all felony defendants cannot afford to employ counsel. See Silverstein in Task Force Report [16], Appendix D (estimating a median figure of 58% for all the states). In New York City, for example, some 70% of all defendants in the criminal courts were represented by the Legal Aid Society in 1971, while during that year, approximately 95% of all convictions were by guilty pleas. See Landes [12].

6. A fuller mathematical exposition of the initial decision model is presented in Adelstein [1].

police, the media, and the constitutional guarantee of a "speedy" trial confront the prosecutor and urge upon him the necessity of as quick a disposition of each case as possible, a point not lost on experienced defense counsel.[7] A principal source of costs of this kind, often overlooked in analyses of prosecutorial decisions,[8] are expenditures on pretrial detention of defendants not released on bail.[9]

Now let B_j be the total expenditure on the jth case, and assume that the agency's entire budget is spent on the m defendants in the case load, so that $\sum_{j=1}^{m} B_j = B$, the total prosecutorial budget. Then the solution of the District Attorney's optimization problem is a vector (B_j^*) which represents the optimal allocation of the total budget B over the case load. Since the conviction probability Q_j depends upon the level of trial preparation expenditures, this optimal allocation implies a total expected utility $\overline{U}^*$, which represents the "best" the District Attorney can do, in terms of total expected utility, in the "worst" possible situation, that in which each defendant demands a full trial. It should be clear, however, that if the District Attorney is able to plea bargain in some subset of the m cases, and thus spend less than B_j^* for each of these cases, amounts greater than B_j^* will be available for the pursuit of the remaining cases, and the total utility which results from this new budget allocation might well exceed $\overline{U}^*$.

Implicit in this argument is the notion that, for some cases, a positive utility increment can be realized by the prosecution by exchanging units of an uncertain expected punishment in a given case for the combination of a smaller but certain[10] punishment and resources saved on that case, thus freeing the conserved resources for transfer and use in the pursuit of another case. Should this situation obtain, we may visualize the actual allocation of the total budget B as a *sequential* process:

(1) At the outset of the budget period, the District Attorney solves the original allocation problem, yielding a vector of optimal case allocations (B_j^*).

7. See Alschuler [2, 56–57].

8. An exception in Abraham Blumberg: "[Detention costs are] a frequently overlooked element of internal pressure in the court system. Accused who are not released on bail are housed in the short-term prison; as a rule it is overpopulated in terms of its capacity. Each day a statistical resume of the jail population is sent to the prosecutor, the judges, and the probation division. Its message is not lost upon its recipients; faster production." Blumberg [4, 59].

9. We assume here that outlays for pretrial detention come directly from the prosecutor's total budget allocation, or, as is more likely, from the budget of another agency, say the corrections department, which competes with the prosecutor for operating revenue from a limited governmental allocation for pre-conviction criminal justice services. In either case, the more spent on "non-productive" pretrial detention, the less that remains for utility productive trial expenditures.

10. The prosecutor, of course, has no power to impose sentence upon conviction, and thus the "certainty" of sentence even after the entry of a guilty plea is somewhat illusory. Typically, the bargain takes the form of a sentence recommendation to the trial judge, or the reduction of offenses charged as a result of the criminal activity in question. In practice, recommendations of leniency are almost always respected by the sentencing authority, and the usual charge reduction narrows considerably the discretion of the trial judge in fixing sentences.

(2) Next, based upon the original utility functions $U_1, \ldots, U_m$, the District Attorney evaluates the possible punishment/savings tradeoffs and constructs a new set of utility functions $V_1, \ldots, V_m$, each of the form $V = V(QU(S),w)$. The variables Q and S are defined as before, while w is defined as dollars allocated for the case at hand but not in fact spent, and thus available for transfer to another case.

(3) Now case 1 is considered, and the prosecution maximizes $V_1(Q_1,w_1)$ subject to the constraint $B_1 + w_1 < B_1^*$. Only if $w_1 = 0$ at the optimal (Q,w) pair will the case be pursued to trial. If $w_1 > 0$, the prosecution will seek to realize this "surplus" through plea negotiations.

(4) Cases 2 through m are dealt with sequentially in the same way, where now the budget available for some cases is likely to be greater than B_j^*.[11]

Let us turn now to a consideration of the behavior of the District Attorney or his delegate (whom we shall now simply call the "prosecutor") with regard to each defendant in the case load. Because the usual criminal transaction involves the commission of more than one statutory offense, the defendant is generally charged at arraignment with a number of separate offenses; thus, we replace the single probability of conviction variable Q with a set of probability variables corresponding to the collection of charges against the defendant. Then if the defendant is charged with b offenses, the total expected punishment at trial is given by $\bar{P} = \sum_{k=1}^{b} q_k S_k$, where S_k is the anticipated sentence administered upon conviction at trial of charge k.[12] For each defendant, then, we have a prosecutorial expected utility function of the form $V(q_1U(S_1),\ldots,q_bU(S_b),w)$ or, more compactly, $V(q_k,w)$. We have assumed that the prosecutor can invest material resources in the case so as to increase q_k. Let the level of these trial preparation expenditures be R. Then we can specify $q_k = q_k(R)$, with $\partial q_k/\partial R > 0$ and $\partial^2 q_k/\partial R^2 < 0$. Thus, the expenditure R is a "public good" with respect to the set of b offenses, and is allocated by the prosecutor over the m cases as outlined above.[13]

11. This process will not, in general, lead to a resource allocation which in fact *maximizes* prosecutorial utility. After solving for w_1, the District Attorney immediately allocates these savings over the remaining cases before he solves for w_2. Having done this, he never returns to case 1 to recalculate w, in light of new budgets (or actual outcomes) determined in cases 2, ..., n. He thus aborts an iterative optimization process after just one iteration and so while the allocation reached will certainly result in utility greater than $\bar{U}^*$, it cannot generally be characterized as optimal. Certainly an optimization process of greater mathematical complexity could, in general, lead the District Attorney to an allocation which in fact maximizes utility. The present formulation seems preferable, however, for in addition to predicting the existence of a contract zone, it captures more realistically the sequential prosecutorial decision process given the District Attorney's uncertainty as to which defendants on the roster will ultimately decide to negotiate and his inability to reallocate resources back to previously completed cases once this information becomes available to him.

12. Note here that the dropping of charge k by the prosecutor simply reduces q_k to zero.

13. We assume here that R is defined such that there exist no external effects between cases, *i.e.*,

$$\partial q_k{}^j/\partial q_k{}^h = 0, \; h, j = 1,\ldots,m \text{ and } k = 1,\ldots,b.$$

The introduction of time as a relevant variable suggests two distinct kinds of costs which the prosecution must bear:

(1) As the actual criminal transaction recedes into the past, the memory of witnesses becomes less sharp, physical evidence loses its probity and degenerates as a fact-finding tool; the prosecution's case "stales" with time, making conviction at trial less likely. Defining t as the number of days elapsed from the initiation of criminal proceedings, we represent this "evidence effect" as $q_k = q_k(R,t)$, where $\partial q_k/\partial t < 0$ and $\partial^2 q_k/\partial t^2 > 0$. Note that if T is the number of days from the outset of proceedings to the trial date, a constant known to the prosecutor, then $t \leqslant T$.

(2) If the defendant is incarcerated prior to the disposition of his case, a detention expense accrues with time, given by $D = \int_0^t D(z)dz$, where $D(\cdot)$ is a function yielding detention costs per day.

Given the instrument variables, R, t, and w, then, the prosecutor's problem is to maximize $V(q_k(R,t),w)$ subject to the constraints $R + w + \int_0^t D(z)dz \leqslant B$ and $t \leqslant T$. The optimal triple (R^*,t^*,w^*) must, of course, satisfy the Kuhn-Tucker conditions and may occur at an interior point in the opportunity set, in which case each component is strictly positive, or at a boundary point, in which case one or more of the components vanishes. As we have suggested, a choice by the prosecutor to pursue a trial to the full extent of available resources corresponds to a boundary solution in which $w^* = 0$, $t^* = T$ and thus $R^* = B - D(T)$. These values in turn imply a maximum value for the expected punishment at trial, which we may call P_{max}. Moreover, it is easy to show that for every interior solution (*i.e.*, $w^* > 0$), there exists an expected trial punishment $P^* < P_{max}$. If we impose the appropriate concavity conditions on the function $V(P,w)$,[14] then an interior solution (P^*,w^*) has the property that $V(P,w)$ is decreasing in every direction away from $V(P^*,w^*)$ within the opportunity set. It follows that there must be some subset of points (P,w) within the larger set $S = \{(P,w): 0 < P < P_{max} \text{ and } 0 < w < w^*\}$ such that $V(P^*,w^*) > V(P,w) > V(P,0)$, and it is this collection of punishment/surplus pairs which defines the prosecutor's bargaining set. The precise contours of the bargaining set depend, of course, upon the particular form of the utility function $V(\cdot)$ and upon the budget and time parameters which constrain the opportunity set.

Thus, the concurrent pressures of prosecutorial risk aversion and the steady accrual of "nonproductive" detention costs might well motivate the prosecution to seek a plea agreement within a conceptually well-defined bargaining set. Moreover, our analysis has suggested an explanation for the failure, in some cases, of negotiations even in the presence of a bargaining set, for the attractiveness of negotiated sentences within the set depends

14. Since expected trial punishment P is simply a positive linear transformation of the q_k, the change of variables in the utility function can be made to clarify the analysis. The requisite concavity conditions imposed upon the utility function are developed in Adelstein [1].

directly upon the size of the surplus associated with the punishment, and the magnitude of this surplus decreases with time as detention costs accumulate. Thus, the time taken to reach a pretrial settlement appears to be as important an element in the prosecutor's initial decision problem as the terms of the settlement itself.

The defendant's initial decision problem yields to a similar analysis. We assume that, at the outset of proceedings against him, the defendant has allocated some portion of his total wealth as a budget for the defense of his case. Subject to this budget constraint, the defendant is taken here to minimize an expected loss function $L(\cdot)$, defined over the range of punishments which might be imposed upon conviction.[15] As before, we let the defendant be charged with b offenses, with the probability of conviction at trial of the kth charge defined as q_k.[16]

These conviction probabilities are assumed to be responsive to inputs of material resources by the defendant; if x is the defendant's trial preparation expenditure, we have $q_k = q_k(x)$, with $\partial q_k/\partial x < 0$ and $\partial^2 q_k/\partial x^2 > 0$, and x a "public good" with regard to the set of b offenses charged. The passage of time, however, is seen to affect the defendant's problem in two ways. On the one hand, the defendant benefits from the "evidence effect" outlined above, so that $q_k = q_k(x,t)$, with $\partial q_k/\partial t < 0$, $\partial^2 q_k/\partial t^2 > 0$, and t constrained by the trial date, $t \leqslant T$. On the other hand, however, the passage of time inflicts serious direct disutility upon the defendant in the period between the initiation of proceedings and the disposition of the case. If the defendant is incarcerated during this period, these losses take the form of foregone potential earnings coupled with presumably severe psychic costs associated with pretrial confinement. Given the overcrowding and dearth of supportive services which characterize many urban detention facilities, these direct time costs are likely to be quite substantial in the event of pretrial incarceration.[17] Alternatively, should he be free on bail, the defendant incurs psychic losses con-

15. The assumption $\partial L/\partial P > 0$ is a natural one, and the desired properties of the solution to the optimization problem require the further convexity assumption $\partial^2 L/\partial P^2 > 0$, *i.e.*, that the defendant is risk averse with respect to potential punishment. The overwhelming prevalence of negotiated pleas in American courts suggests that defendants must, to some degree, be averse to the risks of trial and willing to accept a certain bargained settlement rather than face the vagaries of trial and the possibility of more severe punishment, and the descriptive literature in this area is unanimous in its support of this proposition. See Casper [5, 185–205], Newman [13, 97]. The seminal analysis of Becker [3], however, contains a suggestion that, at the earlier moment when individuals decide whether or not to engage in unlawful activity, potential criminals may be risk preferrers.

16. These probabilities, and the corresponding probabilities in the prosecutor's problem, are, of course, subjective probabilities, and we need not assume that the two estimates are equal in a given case.

17. At least one commentator has noted that the prosecution may not hesitate to use this state of affairs to "soften" defendants who otherwise would be reluctant to bargain. See Blumberg [4, 59]. We should note generally that these direct time effects upon the defendant are analytically distinct from losses imposed by punishment *after* conviction, and that these effects are independent of one another.

nected with the accusation against him in addition to possibly significant dollar costs resulting from economic manifestations of his uncertain legal status.

The defendant, then, seeks to minimize the loss function $L(q_k(x,t),t)$ subject to constraints which depend upon his pretrial status with respect to bail. Should he be incarcerated prior to disposition, he is constrained to choose $x \leqslant B$ and $t \leqslant T$, where B is his defense budget. Alternatively, should he be free on bail, he incurs a positive dollar cost for bail expenses, C_B, but is able to supplement his budget through a daily earning function $E(t)$. In this case, the defense budget is a function of time, $B(t) = x - C_B + \int_0^t E(z)dz$, which is constrained to be no greater than $B_{max} = B - C_B + \int_0^T E(z)dz$. The solution to either of these optimization problems is a pair (x^*,t^*) which satisfies the Kuhn-Tucker conditions and which can be interpreted analogously to the prosecutor's situation. A choice by the defendant to insist upon a trial corresponds to a boundary solution in which $t^* = T$, and let us call the expected trial punishment associated with a decision to invest the full extent of the defendant's resources in a trial P_{min}. Then for every solution in which $t^* < T$, the convexity of the loss function ensures the existence of a subset of feasible points (P,t) such that $L(P^*,t^*) < L(P,t) < L(P_{min},0)$, and this subset of punishment/pretrial time pairs defines the limits of the defendant's bargaining set. Should the bargaining sets of the prosecution and defense overlap to form a "contract zone," the parties can be expected to enter plea negotiations and seek a pretrial settlement of the case. Note once again the significance to both parties of the time taken to reach a pretrial settlement as well as the punishment agreed upon in the initial decision problem.[18]

II. Specifying the Bargain: A Dynamic Model

We seek in this section to set forth a model of bargaining behavior which will enable us to portray the actual progress of plea negotiation itself and to specify the eventual outcome of negotiations in cases where there exists a non-empty contract zone. The model presented here is an adaptation of a more general dynamic behavioral model of the bargaining process developed by John Cross [7], which is itself an example of that class of behavioral theories which have been termed "Decision/Expectation/Adjustment" (DEA)

18. One interesting policy relevant implication of this model should be noted here. It can be shown that, *ceteris paribus*, expenditures designed to improve the generally deplorable services and physical conditions in jails and pretrial detention facilities can be expected to shift the bargaining sets of both parties so as to confine the set of punishments in the contract zone to generally smaller values, resulting in settlements on terms more favorable to defendants than had been the case prior to the rise in detention expenditures. Further, the model preserves Landes' conclusion that, *ceteris paribus*, defendants incarcerated prior to trial can be expected to enter into a greater proportion of plea bargains than those released on bail. See Landes [11].

models.[19] The central conceptual feature of DEA models is that they are both dynamic, in that they depict the bargaining process as one in which the passage of time plays an important role in altering the expectations and behavior of the bargainers, and deterministic, in that they specify, in principle at least, the precise outcome of the negotiation process.

Two important elements of the Cross model will serve to focus our discussion. The first is the impact of time costs upon the bargaining behavior of the parties, in terms of both the manner in which each accommodates his own behavior to the incidence of these costs upon him and the way each assesses, at a given moment in time, the effect of his opponent's time costs on the opponent's behavior. These considerations point to a second element, the notion of "time-dependent" optimality; if each bargainer fashions his own optimal behavior at any instant based upon his current assessment of his adversary's likely response, and if these appraisals are themselves functions of time, then each bargainer's optimal behavior will be a function of time as well.

Assuming the existence of a prosecutorial bargaining set, let us qualitatively examine the prosecutor's behavior. His problem is to select and offer to the defendant that settlement which he finds optimal, taking into account both the utility derived from the negotiated punishment itself and the effects of the time costs which are imposed upon him with each passing day in which no bargain is struck. Since the value of a given settlement depends upon the time at which it would be reached, and thus upon the time required for the defendant to concede to an agreement at that punishment, the prosecutor, in optimizing his offer, must account for the relative willingness of the defendant to concede toward an agreement by estimating the defendant's concession rate over the difference between their offers. This optimum, therefore, will be such that the marginal utility derived from an increment in the punishment offered is just offset by the marginal cost of the estimated incremental time delay which results from the higher offer. But as negotiations proceed, the prosecutor has the opportunity to observe the *actual* rate at which the defendant is (or is not) conceding toward the prosecutor's offer, and thus his optimal offer may change over time as these observations moti-

19. See Coddington [6]. Coddington also provides an interesting and relevant discussion of the comparative strengths of DEA models and more traditional game theoretic models, particularly with regard to the informational problems which confront bargainers. A major point in favor of DEA models is their relaxation of the strong assumptions concerning foreknowledge of the participants which characterize most game theoretic formulations. While the DEA bargainer is a rational optimizer, his decisions are optimal only at the time they are made, for they are based upon expectations of his adversary's behavior which are themselves altered by time and the acquisition of new information. Thus, while the DEA bargainer might formally be described as choosing a given strategy in the game theoretic sense, a more natural interpretation is one which fixes attention upon the series of short run choices which, taken collectively over the period of bargaining, reveal at the end an optimal "strategy," but which capture more completely the elements of imperfect and mistaken information which characterize most bargaining situations.

vate revisions in his estimate of the defendant's concession rate. If, for mathematical convenience, we collapse the cyclical nature of the bargaining process into a continuum of offer and counteroffer, the prosecutor's bargaining behavior can be modelled as a pair of related time functions: $P_p(t)$, the utility maximizing punishment offered by the prosecutor at any moment in time, and $r_d(t)$, the prosecutor's *estimate* of the defendant's concession rate, itself a function of time.

The defendant's problem is quite symmetrical. He too must optimize his settlement offer in terms of both the disutility associated with the punishment itself and the time required for the prosecutor to concede to an agreement, and this requires that the defendant estimate the prosecution's concession rate over the range of disagreement. As above, however, the defendant's observations of his adversary's actual concession rate form a continuously changing input into the optimizing decision, and thus we can depict the defendant's behavior as well through a pair of time functions: $P_d(t)$, the defendant's loss minimizing settlement offer at a given moment, and $r_p(t)$, the defendant's instantaneous estimate of the prosecutor's concession rate. Where, as might be the usual case in practice, the actual rates of concession of both parties are non-negative, that is, where $\partial P_d/\partial t \geqslant 0$ and $-\partial P_p/\partial t \geqslant 0$ (note that prosecutorial concessions represent *decreases* in P_p), and where at least one of these rates is strictly positive, the bargaining process will converge to a solution for some finite $t = t^0$. The process will terminate in an agreement at t^0 at the intersection of the offer functions, $P_p(t^0) = P_d(t^0)$, and this common value P^0 represents the terms of the plea bargain.[20]

The formalization of these arguments is straightforward. Consider first the prosecutor, and suppose $U(P_p)$ is the utility derived by the prosecutor from a given negotiated punishment in the case at hand, defined as in the budget allocation problem of Section I, so that $U' > 0$ and $U'' < 0$, and that $D(\cdot)$ is the per diem detention cost function. Then the value of a given plea agreement to the prosecutor may be written

$$\hat{U}(P_p,v) = U(P_p) - \int_0^v D(t)dt$$

where v is the prosecutor's estimate of the time required to reach the settlement P_p. This time, of course, is determined by the prosecutor's observation of the difference, at any moment in time, between his offer and that of the defendant, and by his estimate $r_d(t)$ of the defendant's concession rate. Thus,

$$v = \left(P_p(t) - P_d(t)\right) / r_d(t).$$

For simplicity, assume that per diem detention costs are constant, so that

20. This formulation makes clearer why negotiations may not result in agreement even given a non-empty contract zone. If t^0 exceeds T, the time between the initiation of proceedings against the defendant and the actual trial date, the bargaining process, while "progressing" toward a solution, will be terminated by the trial itself prior to the agreement point.

$D(t) = D$. Then to find the prosecutor's optimal *initial* offer, we write

$$U(P_p,0) = U(P_p) - D\left[(P_p(0) - P_d(0)) / r_d(0)\right].$$

Maximization of this expression with respect to P_p yields an optimal offer which satisfies

$$U'(P_p) = D/r_d(0).$$

At this initial optimum, the marginal utility of the punishment offered is just offset by the marginal cost imposed upon the prosecutor by the offer. It is interesting to note that this initial offer is independent of the defendant's initial offer, although this independence is quite transitory, for $r_d(t)$ depends directly upon the observed bargaining behavior of the defendant.[21]

Let us examine more closely the way in which this optimal offer changes over time. As we have argued, the prosecutor must revise his expectations with respect to r_d should he observe that his estimate differs from dP_d/dt, the defendant's actual rate of concession. It is clear that a learning model of this kind should have at least the following properties:

$$\text{if } dP_d/dt > r_d \text{ then } dr_d/dt > 0$$
$$\text{if } dP_d/dt = r_d \text{ then } dr_d/dt = 0$$
$$\text{if } dP_d/dt < r_d \text{ then } dr_d/dt < 0$$

Thus, for example, if the defendant concedes more rapidly than anticipated, the prosecutor will increase his estimate of the defendant's concession rate. Further, we make the reasonable assumption that the greater the discrepancy between the defendant's observed concession rate and the prosecutor's estimate of that rate, the faster the prosecution's expectations will change. Thus,

$$(d(dr_d/dt))/d\epsilon > 0,$$

where $\epsilon = dP_d/dt - r_d$, a measure of the prosecutor's "expectation error." We choose the simplest learning function $r_d(t)$ which satisfies all of these criteria:

$$dr_d/dt = \alpha\epsilon = \alpha(dP_d/dt - r_d)$$

where the learning parameter α is some positive constant.

Specifying the estimated concession rate as a function of time makes clear that the prosecutor's optimal offer, that offer which maximizes $\hat{U}(P_p,t)$, is also time dependent. We have, at the optimum,

$$U'(P_p(t)) = D/r_d(t)$$

so that

21. Note also that $d^2\hat{U}_2/dP_p^2 = U''(P_p) < 0$, so that the first order condition locates a maximum of the prosecutor's value function.

$$[r_d(t)]\,[U'(P_p(t))] = D.$$

Differentiating this expression with respect to time yields

$$r_d[U''(P_p)\cdot(dP_p/dt)] + (dr_d/dt)[U'(P_p)] = 0.$$

Rearranging terms, we have

$$dP_p/dt = (-U'(P_p)/U''(P_p))\cdot(1/r_d(t))\cdot(dr_d/dt) \tag{1}$$

where, rewriting our earlier expression for the learning function $r_d(t)$,

$$dr_d/dt + \alpha r_d = \alpha(dP_d/dt). \tag{2}$$

The simultaneous differential equations (1) and (2) specify the prosecutor's bargaining behavior during plea negotiations, and highlight the interdependence of the actual rates of concession of prosecutor and defendant. More precisely, we observe that if $r_d(t)$ is positive, the concavity of the utility function $U(P_p)$ implies that dP_p/dt has the same sign as dr_d/dt. Thus, for example, should the prosecutor determine that the defendant is conceding less rapidly than expected, the settlement punishment which he offers will decrease with time. On the other hand, if the defendant concedes more rapidly than anticipated, the prosecutor's offer will *increase* with time, probably lengthening the bargaining process and possibly threatening the eventual consummation of an agreement.

Analysis of the defendant's bargaining behavior is precisely analogous,[22] and yields

$$dP_d/dt = (L'(P_d)/L''(P_d))\cdot(1/r_p(t))\cdot(dr_p/dt) \tag{3}$$

with

$$(dr_p/dt) + \beta r_p = -\beta(dP_p/dt) \tag{4}$$

where $L(\cdot)$ is the defendant's loss function, $r_p(t)$ his estimate of the prosecutor's concession rate, and β a positive constant representing his learning parameter. The simultaneous equations (1)–(4) along with the condition which defines agreement and the termination of the bargaining process,

$$P_p(t^0) = P_d(t^0) \text{ at agreement} \tag{5}$$

completely specify the dynamic plea bargaining model.

22. The total disutility to the defendant of a given negotiated settlement may be written

$$\hat{L}(P_d,t) = L(P_d) + \int_0^y J(t)dt$$

where $L(P_d)$ is the loss imposed upon the defendant by a given punishment, defined as in Section I, $J(t)$ is a per diem cost function representing the pretrial time costs experienced by the defendant, and y is the defendant's estimate of the time required to reach the settlement P_d, given by

$$y = (P_p(t) - P_d(t))/r_p(t).$$

If we assume that per diem pretrial costs are constant and equal to J, then the defendant is seen

It is, however, important to note that the Cross bargaining model, and our adaptation of it here, ultimately fails to resolve the classical indeterminacy which has characterized economic analysis of the problem of interdependent behavior in bilateral monopoly or duopolistic competition. While the model does in fact generate a precise solution to the bargaining problem, it must ultimately rest on one of two implicit assumptions. Either the negotiation process in question will take place only once for both parties, or the bargainers involved are "naive" in the sense that they are unable to discern patterns in their adversary's bargaining behavior and alter their own positions in response. Particularly in the plea bargaining situation, it may well be difficult to argue that neither prosecutor nor defense counsel will recognize these behavioral patterns and structure his subsequent behavior to take fullest advantage of them.

Still, for purposes of empirical analysis, the Cross model can be seen as a valuable tool. It attains one more level of specificity in the analysis of the bargaining problem than do the older duopoly models, and it provides a behavioral theory which is at once reasonable and empirically useful. In the sense that it models bargaining behavior at all and "postpones" for an analytical stage the classical indeterminacy, it represents a valuable improvement over models which fail to account for the behavior of individuals in a bargaining context or do so in a simplistic or unsatisfactory way.

Having specified our model of the plea bargaining process by the set of simultaneous differential equations (1)–(4) and the equilibrium condition (5), let us turn briefly to the problems involved with empirical testing of the model. Certainly, direct estimation of such a system is a most formidable endeavor. Apart from the mathematical difficulties to be expected in the solution of a system of differential equations,[23] direct estimation would require the assignment of values to the learning parameters α and β and the assumption of arbitrary functional forms for the utility and loss functions $U(P_p)$ and $L(P_d)$. These problems can be attenuated while important empirical tests of the model are developed by a careful investigation of the comparative statics properties of the system (1)–(4). Implicit in this approach is the recognition that the system itself is not invariant from case to case; its functional forms and the outcomes it implies are defined within the context of a given case and can be expected to vary with the identity of each defendant and the circumstances surrounding his particular case. More precisely, the components of the system and the equilibrium values P^0 and t^0 which are generated by it are in general functions of a vector of variables $(y_1, \ldots, y_n)$ representing a set of legal and socio-economic parameters defined exogenously to the model and which assume different values for each defendant. As this exogenous envi-

to minimize $L(P_d,t) = L(P_d) + J.y$. The remainder of the analysis is analogous to that of the prosecutor's behavior.

23. On the solution of such systems generally, see Kaplan [10].

ronment is changed, the outcome of the bargaining process will generally change as well, and the object of the comparative statics analysis is to determine the responsiveness of the equilibrium values P^0 and t^0 to small changes in the y_i. The product of the analysis is a collection of predictions, which we might call "secondary hypotheses," concerning the algebraic sign of each derivative $\partial P^0/\partial y_i$ and $\partial t^0/\partial y_i$. This $(2 \times n)$ matrix of secondary hypotheses can then be tested directly against a data sample which includes observations on P^0, t^0, and the exogenous parameters y_i.[24]

Before this set of secondary hypotheses can be articulated, however, a collection of assumptions regarding the behavior of the bargainers and the structural relationships between the variables endogenous to the model must be set forth. These "primary hypotheses" generally are motivated either by the descriptive literature dealing with the plea bargaining process or by appeals to the economist's behavioral intuition. Because they are not tested directly against sample data, the set of primary hypotheses is the most problematic aspect of a comparative statics analysis. Their veracity as a group is tested only indirectly, but if the set of secondary hypotheses is supported by empirical observation, it is not unreasonable to infer the consistency of the set of primary hypotheses with the given data.

III. The Relevance of Formal Models

The scientific and social scientific disciplines tend to place greater stock in the usefulness of formal modelling techniques than does the law, which is able to adapt its outcomes to nuance and detail which cannot be captured in general theories and abstractions. The law, however, deals ultimately with questions of fact, and it is the purpose of modelling, indeed of scientific inquiry in general, to transform hypotheses into knowledge. Because important decisions in the law are often based upon speculation and hypothesis, the value of structured analytic inquiry to the legal process would appear at first glance to be self-evident. But the questions which social scientists are able to answer and the questions which courts find necessary to ask are not always the same.

24. The derivation of these hypotheses is straightforward and follows Samuelson [15, 7–20]. The equilibrium equations are generated by combining equations (1) and (2) for the prosecutor and (3) and (4) for the defendant. For the prosecutor, this yields

$$f_p(P_p, P_d, t, \alpha; y) = \{\alpha - [(\alpha dP_d/dt)/r_d(t)]\}\{U'(P_p)/U''(P_p)\} - dP_p/dt = 0$$

and for the defendant

$$f_d(P_p, P_d, t, \beta; y) = \{\beta + [(\beta dP_p/dt)/r_p(t)]\}\{L'(P_d)/L''(P_d)\} - dP_d/dt = 0.$$

These equations are differentiated totally with respect to y_i and the resultant system of two equations solved for $\partial P^0/\partial y_i$ and $\partial t^0/\partial y_i$. Given the sensitive nature of the plea bargaining process, data appropriate for testing hypotheses of this kind are not easily gathered. While substantial efforts in this direction are currently being made (see, for example, Georgetown University Law Center [8]), the results of tentative tests on extant data are reported in Adelstein [1].

Nowhere is this tension more clear than in the area of plea bargaining. As we saw at the outset of this paper, the constitutionality of negotiated pleas depends to a large extent on the actual motivations and behavioral patterns of the bargainers and the structure of the bargaining situation itself. The present analysis seems particularly relevant to this question and further empirical work might well suggest that the bargaining process in fact encourages the type of result which would cast substantial doubt on the constitutionality of negotiated pleas. But it must be recognized that, typically, plea bargains are not the result of sustained and formal negotiation procedures; rather, they are often reached after whispered and hurried conversations between prosecutor and defense counsel, and defense counsel and defendant, in the corridors of the courthouse on the day the defendant is scheduled to appear for trial. This means that, for most instances, the formal and precise framework developed here is in the nature of an "as if" behavioral model, and what empirical veracity it may have indicates not that plea bargains are struck according to the procedure described here, but simply that observed agreements correspond to some degree to the results which would be expected if the actual bargainers behaved as do the model's hypothetical prosecutor and defendant. Thus, the set of assumptions and primary hypotheses which lie at the base of the structure may contain an element of truth, and the logical propositions which derive from them may describe, to some extent and with inevitable distortion, the actual network of human interactions which combine to produce a given observed outcome. But in the final analysis, unless these fundamental building blocks of the model are verified directly, only "as if" conclusions may be drawn. This epistemological issue is, of course, familiar to economists, and we certainly do not mean to deny that models such as the one presented here can be of substantial value for many kinds of important policy decisions. Yet, in the only sense which is meaningful in constitutional litigation and adjudication, to demonstrate that plea agreements can be explained on the basis of bargainers behaving as if they were subject to a given set of pressures is not to show conclusively that bargainers are in fact making their decisions in this way. As a result, formal behavioral models, whatever their usefulness in executive or legislative policy analysis, may be seen by courts merely as guides to the judicial behavioral intuition which will continue to lie at the base of many kinds of constitutional adjudication. Formal modes of economic analysis may fit only imperfectly into the framework of litigation, and the role of the social sciences in the law may be circumscribed by the differing notions of proof and empirical verification which characterize the two disciplines. How this important and difficult issue will finally, if ever, be resolved is unclear. What is certain is that the courts will continue to cast a suspicious eye toward the social sciences until it is.

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