Concrete–Semiconcrete–Abstract (CSA) instruction: A Decision Rule for Improving Instructional Efficacy

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Abstract

A concrete-semiconcrete-abstract (CSA) instructional approach derived from discovery learning (DIS) was embedded in a direct instruction (DI) methodology to teach 8 elementary students with math disabilities. One-minute abstract-level probes were the primary metric used to assess student performance on subtraction problems (minuends 0-9). A single-subject, multiple baseline across subjects design was employed to identify the differential effects of discontinuing instruction at crossover (i.e., the point when the correct response rate exceeded the incorrect response rate). Results indicate that the commonly accepted practice of teaching an entire CSA unit of lessons may not be the most efficacious approach for classroom teachers. Instead, through daily data collection and application of the “crossover decision rule” (discontinue rule), teachers can selectively target those students appropriate for additional concrete- and/or semiconcrete-level instruction and those students for whom continued practice at the abstract level is more appropriate. Implications for teaching computational skills are examined.

Keywords: math disabilities, subtraction, direct instruction, instructional efficacy, data-based decision rules
Concrete-Semiconcrete-Abstract (CSA):

A Decision Rule for Improving Instructional Efficacy

In this era of publicized declining mathematics achievement and increasing pressure for accountability from mandates such as No Child Left Behind (NCLB, 2002), educators must institutionalize effective teaching strategies into their classrooms. These strategies must be high quality and scientifically based (Fuchs & Fuchs, 2006). In addition, student progress must be systematically monitored to obtain data that will drive instructional decisions (Kavale & Spalding, 2008; Mercer, Mercer & Pullen, 2011). These basic tenets are embedded in the reauthorization of the Individuals with Disabilities Education Improvement Act (2004).

Multiple studies, including the Third International Math and Science Study (TIMMS; Mullins et al., 2000), have reported that mathematical achievement in the United States continues to lag behind that of many industrialized countries. The results of the 2009 National Assessment of Educational Progress (NAEP) revealed that, although fourth graders’ mathematical skills have improved over the last 19 years, many children in elementary grades are failing to acquire skills in the basic operations and applications of mathematics. Unfamiliarity with basic number facts and the lack of procedural fluency play a major role in the math difficulties of students (Fosnot & Dolk, 2001; Funkhouser, 1995; Fuson, Grandau, & Sugiyama, 2001; Griffin, 2003; National Research Council, 2001). Moreover, Bouck, Kulkarni, & Johnson (2011) noted that current approaches to instruction may not be meeting the needs of many students. Many researchers have reported that students with learning disabilities lack proficiency in basic number facts and consequently are unable to retrieve answers to math facts efficiently.
In 1988, the National Council of Supervisors of Mathematics (NCSM) concluded that mathematics instruction must address critical thinking, conceptual understanding, computation, and application skills. The National Council of Teachers of Mathematics (NCTM) in their *Principles and Standards for School Mathematics* (2000) recognized that computational proficiency alone is not enough and basic arithmetic skills must include abstract abilities. Mathematics educators concur that operations, applications, and conceptual understanding must comprise the content of instruction. Maccini and Gagnon (2002) noted that the focus of mathematics standards is on conceptual understanding. However, considerable disagreement exists regarding just how to teach these skills (e.g., Battista, 1999; Campbell, Rowan, & Suarez, 1998; Jones, 2012; Kanu & Dominick, 1998; Karp & Voltz, 2000; Kroesbergen & Van Luit, 2003; Maccini & Gagnon, 2000; O’Brien, 1999; Swanson, 2001; Van de Walle, 2004; Yang, Liao, Ching, Chang, & Chan, 2010).

Educators generally agree that effective teaching includes the application of Bloom’s taxonomy of educational objectives, which addresses six cognitive processes that govern how students garner knowledge: remembering, understanding, applying, analyzing, evaluating, and creating information. Furthermore, Bloom’s taxonomy posits that knowledge is comprised of factual knowledge including terminology and facts, conceptual knowledge such as models and theories, procedural knowledge or the knowledge of how to carry out tasks, and metacognitive knowledge or awareness of one’s
own thought processes (Ormrod, 2008). All of these are areas of knowledge that mathematics educators must address when helping students learn mathematics. Despite agreement regarding the tenets of Bloom’s taxonomy, two prevailing methods of instruction have emerged: discovery learning (DIS), which is typically associated with constructivism, and direct instruction (DI), which is often associated with behaviorism (Jones, 2012; Reys, Lindquist, Lamdin, & Smith, 2012; Tipps, Johnson, & Kennedy, 2011).

In a nondirective or discovery learning (DIS) approach, the teacher acts as a facilitator, arranging the environment to enable students to “discover” mathematical principles (e.g., Bruner, 1961, 2004; Dienes, 1961; Joyce, Weil, & Callahan, 2006; Kamii, Kirkland, & Lewis, 2001; Piaget, 1973; Taba, 1966; Van de Walle, Karp, & Bay-Williams, 2010). DIS is inductive in nature in that the teacher presents or arranges for the student to observe instances of a rule. Through the manipulation of objects and/or symbols, the student comes to understand relations and solve problems involving mathematical principles. One of the most familiar features of DIS in teaching mathematics to elementary students is incorporation of the “three stages of learning,” concrete, semiconcrete or representational, and abstract (Piaget, 1973), into the teaching process. Kamii and Rummelsburg (2008) stressed that educators must be cognizant of Piaget’s levels of development and ensure the use of manipulatives and other models before the formal introduction of algorithms. The National Research Council (2001) encourages using a variety of representations. A representation, according to the National Council of Teachers of Mathematics, is “the act of capturing a mathematical concept or relation” (2000, p. 67). The effective use of a representation can bridge the gap between
conceptual knowledge and procedural knowledge, between understanding and the steps to the algorithm. Supporters of DIS contend that children learn more, better understand the problem-solving process, and generalize learned information more readily to novel settings than peers not involved in DIS (Bruner, 1961; Moreno, 2004; Rogers, 1969). The case has been made that reasoning skills can be improved through discovery learning (Bruner, 2004; Dewey, 1997; Joyce et al., 2006). Students of DIS purportedly become intrinsically motivated and enjoy learning more than their nondiscovery learning peers (Bruner, 1961; Dienes, 1961; Rogers, 1969). A major criticism of DIS is that it is ambiguous and too imprecise to be used meaningfully and predictably by educators (Ausubel, 1968; Keislar & Shulman, 1966; Kirschner, Sweller, & Clark, 2006; Mayer, 2004; Scandura, 1964/1969; Tuovinen & Sweller, 1999). Another criticism is that teachers may not have the experience or aptitude for this approach (Tamir, 1995).

Conversely, direct instruction (DI), first introduced by researchers at the University of Oregon in the 1960s, is deductive, systematic, and rule governed (e.g., Engelmann, 1968; Englert, 1984; Rebar, 2007). DI incorporates the following three key instructional design criteria (e.g., Crawford, Engelmann, & Engelmann, 2009; Jitendra, Salmento, & Haydt, 1999): (a) Lessons should have clear objectives and prerequisite skills must be taught, (b) rules and multiple exemplars of those rules are taught in the context of structured lessons, and (c) students are shown overtly how examples conform with or violate the rules. DI is an effective instructional approach for teaching basic or isolated skills (Maccini & Gagnon, 2000). Ball et al. (2005) asserted that computational fluency of whole numbers is so vital that practice should be conducted to the point of automaticity.
Four teaching techniques associated with a typical direct instructional lesson include a carefully organized presentation of rules and exemplars with explicit teacher demonstration and modeling, immediate feedback on student performance, guided practice to avoid or reduce errors, and independent practice as students move toward mastery (Brophy & Good, 1986; Gersten, Carnine, & Woodward, 1987; Murphy, Weil, & McGreal, 1986; Stevens & Rosenshine, 1981). Research indicates that teacher-directed instruction has a number of positive outcomes, including increased academically engaged time (i.e., the time the student spends in performing an academic task), reduced error rates, and achievement gains (Hendrickson & Frank, 1992). Advocates of the DI method also contend that students with learning and behavioral difficulties need the structure of DI to optimally benefit from available instructional time.

Kroesbergen and Van Luit (2003) concluded that DI is the optimum instructional method for teaching mathematics skills to students with disabilities. There are, however, concerns and cautions raised regarding DI. Silbert, Carnine, and Stein (1981), for example, cautioned that DI may result in students participating in redundant explanations of previously mastered skills, thereby reducing functional learning time and fostering boredom. Slavin (1991) warned that DI deemphasizes learners’ autonomy, may inhibit the creative thought of learners, and may change the quality of their understanding and use of invented strategies (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Villasenor & Kepner, 1993).

Despite the cautions and concerns, researchers have used DI to teach skills with positive results. Mercer and Miller (1992) developed an instructional sequence by pulling effective components from discovery learning and combining them with the
teacher-directed, empirically documented instructional tactics of DI. This approach uses
the widely accepted concrete-semiconcrete-abstract (CSA) approach to teaching basic
skills that has been found to be effective in teaching math skills to students with learning
problems (Bley, 1994; Bley & Thornton, 1995; Hudson, Peterson, Mercer, & McLeod,
1988; Mercer & Miller, 1992). The CSA sequence employs the tenets of direct
instruction by including a demonstration and model sequence, guided practice,
independent practice, and feedback. Instruction of the target skill begins with the
concrete stage in which three 30-min lessons using manipulatives are paired with the
abstract symbols. Progression through each of the three concrete lessons is contingent on
mastery as measured by daily, independent practice sheets. The instructional sequence
continues with three lessons at the semi-concrete level (also known as the
representational level) and three lessons at the abstract level for a total of nine lessons.
Mercer and Miller (1992) reported that students taught using the CSA sequence acquired
the target skills, demonstrated an understanding of the respective operation, and were
able to maintain the skills over time. In another study, Witzel, Mercer, and Miller (2003)
showed that students who learned how to perform algebra transformation equations
through a concrete-to representational-to abstract (CRA) sequence outperformed peers
receiving traditional instruction during both post-instruction and follow-up tests.

Noteworthy in the data generated by Hudson et al. (1988) and Mercer and Miller
(1992) was the evidence of skill acquisition as demonstrated by a crossover phenomenon
(the point at which the correct response rate becomes higher than the error rate). This
crossover or “ah-ha” effect was observed in most students; however, there was no
consistent pattern as to when in the instructional sequence crossover occurred. Some
students demonstrated crossover of corrects and errors during the concrete lessons whereas others crossed over during the semiconcrete lessons. All students were taught each of the nine lessons regardless of where in the instructional sequence crossover occurred.

For classroom teachers to make data-based decisions and thereby deliver the most efficacious CSA instruction to students with math disabilities, it is advisable to monitor student progress and collect daily data as recommended by Mercer and Miller (1992). However, for daily data on math probes to be useful in instructional decision making, teachers need benchmarks for determining whether to modify, continue, or discontinue instruction. To date there is no empirical basis upon which to build decision rules for teachers using a co-joined DIS and DI approach to remediating math disabilities.

Students who may have mastered the target skill(s) computationally and conceptually are not differentiated from those who are still acquiring the skill(s).

The purpose of this investigation was to establish a preliminary data base from which one or more decision rules could be generated for improving the efficacy of CSA instruction with students with math disabilities. For the purpose of this study, crossover was defined as 2 consecutive days (versus a single day) in which the number of corrects exceeded the number of errors on 1-min abstract-level probes. This definition was selected to reduce the chance of random event(s) causing the rate of corrects to be higher than the rate of errors. The study was designed to answer the question: What effect does discontinuance of CSA instruction at the point of crossover have on the acquisition, maintenance, and generalization of subtraction facts (minuends 0 – 9)? More
specifically, does discontinuance of instruction adversely affect students, or is student learning positively and sufficiently impacted to warrant termination of instruction?

**Method**

**Participants**

Eight first- and second-grade students with high-incidence disabilities who were classified as having an emotional/behavioral disorder (EBD) (N=2) and/or with a specific learning disability (SLD) (N=6) served as participants. Table 1 presents descriptive data on each participant. The seven boys and one girl ranged in age from 6 years 1 month to 8 years 6 months. Four students were white and four were African American. All students had IQ scores falling within the average range and were served in resource rooms for fewer than two hours per day. For the remainder of the day, students were served in general education classrooms.

Insert Table 1 about here

As a precondition, each student was required to orally name and independently write the numerals 0-9 with 100% accuracy. In addition, each student who participated exhibited a higher frequency of errors than corrects when given a 24-item subtraction (minuends 0-9) worksheet.

Three special education resource room teachers at three different elementary schools served as instructional agents. They averaged 3 years of experience teaching students with special needs. All were working on a master’s degree in special education.

**Setting**

Student participants were enrolled in one of three special education resource room programs in three counties in a southeastern U.S. state. Typically, three to eight children
were present during any given class period. Each teaching and assessment activity associated with the study was conducted in one-on-one sessions with a teacher and took place at a table in the math area of each classroom. Other students present in the classroom worked with either a paraprofessional or an adult classroom volunteer. All classrooms were portable units divided so that the teacher had approximately a 30 ft x 25 ft room for class activities. Lap boards and/or a chalkboard were available during the study. A video camera was set up on a tripod approximately 6 ft from the teacher and student(s).

**Teacher Training**

The three teachers who volunteered to participate in the study attended three 1-hr group training sessions led by one of the authors. The teacher-directed activities were evaluated in the third session using a *Yes/No* checklist composed of the following: adhering to the sequence of the script, fluency of instructional pacing, and using the correct materials. Each teacher earned a *Yes* for the three components measured. Additional checks of procedural integrity were assessed during the study and are reported at the end of this section.

**Assessment Instruments/Procedure**

To assess student progress, a pre- and post-test, daily abstract-level probes, and generalization tests were constructed as follows. A 24-item abstract-level (i.e., written numerals and symbols only) pre- and post-test consisting of randomly selected subtraction problems (minuends 0-9) was constructed using a worksheet format. This pretest-posttest used see/write as the input and output modalities. The 24-item test was administered as an untimed test. To create the daily abstract-level probes, a worksheet
containing 60 see/write subtraction problems (minuends 0-9) was constructed, and three alternate forms of the worksheet were computer generated. These three forms were used for daily 1-min timings/probes to assess student learning across all conditions of the study. Correct digits per minute (corrects) were reported over incorrect digits per minute (errors).

To assess generalization, three equivalent forms of a five-item listen/write word-problem test were constructed. The minuend 0-9 word problems were presented orally on a one-to-one basis by the classroom teacher prior to instruction and after the last maintenance probe was administered.

**Teaching Materials**

Using a Direct Instruction format, scripted lessons were developed for teaching minuends 0-9. The entire teaching unit consisted of nine lessons. The first three lessons required the learner to solve problems using manipulative objects (the concrete stage). In the next three lessons, the learner was taught to solve problems using worksheets with illustrations of manipulative objects (the semiconcrete or representational stage), and in the final three lessons, the learner solved problems relying solely on arithmetic symbols (the abstract stage).

Each lesson had four sections and was formatted in the same manner. The sections included (a) an advance organizer, which provided a review of past information and introduced the current lesson; (b) demonstration and prompting, where the teacher modeled the skill to the student and prompted desired responses; (c) guided practice, where the student performed the target skills with corrective feedback; and (d) an independent worksheet, where the student practiced and demonstrated his/her
understanding of the skill in the absence of teacher assistance. In addition, each lesson contained an untimed mastery test consisting of 10 problems (items). Ninety-percent accuracy was required on the mastery test before a student advanced to the next lesson. Student responses during instruction were consequated with specific and general praise. Teaching an entire lesson required approximately 15 minutes.

Experimental Design

Researchers (e.g., Barlow & Hersen, 1984; Best & Kahn, 1998; Gall, Borg, & Gall, 1996) have noted that when attempting to answer questions regarding the relation between an intervention (the independent variable) and a corresponding change in the behavior of the individual (as measured by the dependent variable), single-subject experiments or single-case research methodology is the most appropriate choice. To determine whether a functional relation existed between teaching using a CSA format up to the point of crossover (independent variable) and performance on an abstract minuends 0-9 probe (dependent variable), a single-subject, multiple baseline across subjects design (Sealander, 2003) was used to assess student learning. Three teachers in three different settings participated. Daily data were collected on 1-min see/write subtraction problems (minuends 0-9) across the experimental conditions of baseline, intervention, and maintenance.

Procedure

Prebaseline, baseline, intervention (CSA instruction), and maintenance activities were conducted at a table in the back of the resource classrooms. Lessons were taught on a one-to-one basis by the three participating teachers who provided specific and general praise and corrective feedback on an intermittent basis only during instruction. No
feedback or praise was delivered during the 1-min abstract-level probes. Specific descriptions of these activities follow.

*Prebaseline:* The 24-item untimed pretest on see/write abstract subtraction problems (minuends 0-9) was administered to each student on a one-to-one basis.

*Baseline:* One of three 1-min abstract-level probes was administered on a rotating basis for a minimum of 3 days or until stability (i.e., rate of errors remained higher than corrects) was achieved. An every 2-day skip or rest was implemented for those students who completed 5 days of baseline. This was done in an effort to reduce potential frustration or satiation with the procedure. As noted, no feedback regarding corrects or errors was given to students on their performance during the abstract-level probe.

*Intervention:* When the first student in each of the three classrooms obtained a stable (rate of errors remained higher than corrects) baseline, intervention (i.e., the teaching lesson sequence) was initiated. Instruction was delivered individually to each student by the teacher. In this phase, each student was taught to subtract using the scripted lesson, which began with concrete instruction (object manipulation) and moved through semiconcrete- and abstract-level instruction, or until crossover occurred. Each lesson concluded with the 10-item mastery test for that lesson. Students who received 90% correct or better on the mastery test were advanced to the next lesson in the sequence. Students who did not reach the 90% mastery criterion were instructed individually in the same lesson later in the school day or prior to the next the teaching session. If a student failed to reach mastery after three repetitions of the same lesson, the data decision rule was that he or she was to be advanced to the next lesson to reduce redundancy and or frustration.
As during baseline, each student was given one of three 1-min abstract probes, and no feedback was provided to the student during this probe. When student data showed crossover, that is, 2 days with the frequency of correct responses greater than the frequency of errors, instruction was discontinued.

**Maintenance:** This condition assessed whether the student continued to exhibit the skill in the absence of direct instruction and to determine if any growth occurred. As during baseline, the student was administered one of three abstract-level probes. Neither additional instruction nor opportunities to practice subtraction facts 0-9 outside of the probe administration took place in this phase. At the beginning of the maintenance phase one of the three 1-min abstract probes was administered daily for 5 instructional days. Thereafter, the student was given a probe on an every-other-day basis for 6 school days (i.e., three probes). Last, the student was not probed again for 5 school days, and then he or she was assessed once a week for 2 weeks. No-chance days (i.e., those days where there was no opportunity to administer the maintenance probe due to pupil or teacher absence, schedule conflicts, or school holidays) were noted and the probe was administered the next instructional day. Maintenance of student performance on abstract probes was assessed across approximately 4 weeks.

**Posttest:** The same procedure used to administer the pretest was applied the last day of maintenance.

**Generalization:** A listen/write word-problem test consisting of five questions was administered to each student on a one-to-one basis. This occurred at three points in time: during baseline and on the first and last days of the maintenance condition.
Inter-observer Agreement

Procedures used to determine reliability of the measurement and treatment procedures included (a) inter-observer agreement on assessment probes and (b) inter-observer agreement on the procedural integrity with which instruction was delivered. Two independent observers scored all abstract-level probe sheets for each student. The sheets and scores were compared on a digit-by-digit, problem-by-problem basis. Two disagreements were observed across the entire study, which included 157 assessment probes. The average inter-rater agreement score was 98.7% using the formula: number of agreements divided by the number of agreements plus the number of disagreements multiplied by 100.

An independent, trained observer viewed one to three randomly selected videotaped instructional sessions for each teacher to assess the integrity with which the procedures were applied. All three teachers followed the instructional procedure with high fidelity. That is, Teacher 1 averaged 98% correct adherence to the assessment and instructional procedures, Teacher 2 averaged 99% correct procedural adherence, and Teacher 3 averaged 97% correct on all steps of the assessment and instruction processes.

Results

Figures 1, 2, and 3 contain daily probe data for Students 1 and 2 who were taught by Teacher 1; Students 3, 4, and 5 who were taught by Teacher 2; and Students 6, 7, and 8 who were taught by Teacher 3.

As shown in the figures, during baseline, all eight students exhibited a higher rate of errors than corrects on the abstract-level subtraction probes, suggesting that none of
the students learned the subtraction facts until after the intervention was initiated. All students reached crossover (2 days with a higher frequency of corrects than errors) before the entire CSA teaching sequence was completed. None required teaching at the abstract lesson level.

On average, 4.38 lessons (range 3-6) were required for students to reach crossover. Three of the students (Students 3, 4, and 5) had to repeat lessons. Student 3 repeated concrete lesson 1, while Students 4 and 5 repeated both concrete lessons 1 and 2.

Figures 1, 2, and 3 reveal that, following the termination of instruction, all of the students’ correct response rates on the abstract-level probes either maintained or actually increased (see the maintenance condition). During maintenance, seven of the students maintained average error rates below those observed during instruction. That is, in the absence of direct instruction, error rates continued to decline and all students’ correct responses increased during instruction/intervention and maintenance phases in comparison to their baseline rates.

More specifically, Student 1 reached crossover following the third concrete lesson, and Student 2 achieved crossover after the second semiconcrete lesson. Student 1 was taught a total of three lessons for a cumulative instructional time of 45 min. Student 2 was taught a total of five lessons for a cumulative instructional time of 75 min.

Students’ average rate of corrects and incorrects (errors) on the abstract probe during all three phases (baseline, intervention, and maintenance) were as follows:

Student 1:  
Baseline – 3 correct/12 incorrect  
Intervention – 6 correct/7 incorrect
Maintenance – 6 correct/2 incorrect

Student 2: Baseline – 3 correct/14 incorrect

   Intervention – 5 correct/7 incorrect

   Maintenance – 8 correct/1 incorrect

Comparison of the average number of corrects to errors from the intervention phase to the maintenance phase showed that Student 1’s correct rate was maintained while errors continued to decline. Student 2 showed an increase in corrects and a decrease in errors.

Students 3, 4, and 5 were taught by Teacher 2. In spite of not reaching mastery on the first lesson (as measured by the 10-item mastery test) the lesson was retaught (concrete lesson 1). Student 3 reached crossover after the third lesson at the concrete level (a total of four lessons and cumulative instructional time of 60 min). Students 4 and 5 required reteaching of concrete lessons 1 and 2. Student 4 reached crossover after the second semiconcrete lesson, whereas Student 5 reached crossover following the first semiconcrete lesson. To reach crossover, six lessons were required for Student 4 (cumulative instructional time of 90 min) and five lessons for Student 5 (cumulative instructional time of 75 min).

Students’ average rate of corrects and incorrects (errors) on the abstract probe during all three phases (baseline, intervention, and maintenance) were as follows:

Student 3: Baseline – 6 correct/24 incorrect

   Intervention - 5 correct/4 incorrect

   Maintenance – 8 correct/3 incorrect

Student 4: Baseline – 1 correct/4 incorrect
Intervention – 3 correct/4 incorrect

Maintenance – 11 correct/4 incorrect

Student 5: Baseline – 7 correct/13 incorrect

Intervention – 5 correct/6 incorrect

Maintenance – 14 correct/4 incorrect

Based on the average number of errors and corrects between the intervention and maintenance phases, Students 3 and 5 demonstrated an increase in the average number of corrects while the errors continued to decline. Student 4’s correct rate continued to increase while the errors remained the same.

Teacher 3 taught Students 6, 7, and 8, and none required reteaching of any lessons. Student 6 crossed over after semiconcrete lesson 2 (a total of five lessons and cumulative instructional time of 75 min), whereas Student 7 crossed over after concrete lesson 3 and Student 8 crossed over after the first semiconcrete lesson. Student 7 reached crossover after a total of three lessons (cumulative instructional time of 45 min) and Student 8 required four lessons (cumulative instructional time of 60 min).

Students’ average rate of corrects and incorrects (errors) on the abstract probe during all three phases (baseline, intervention, and maintenance) were as follows:

Student 6: Baseline – 1 correct/6 incorrect

Intervention – 5 correct/8 incorrect

Maintenance – 19 correct/3 incorrect

Student 7: Baseline – 3 correct/11 incorrect

Intervention – 9 correct/4 incorrect

Maintenance – 23 correct/2 incorrect
Student 8: Baseline – 4 correct/27 incorrect

Intervention – 5 correct/4 incorrect

Maintenance – 24 correct/2 incorrect

All students taught by Teacher 3 showed a substantial increase in corrects and decrease in errors when the intervention phase was compared to the maintenance phase.

Insert Table 2 about here

Untimed Pretest and Posttest Scores

As reported in Table 2, five of eight students were unable to answer any of the 24 subtraction problems on the untimed listen/write pretest. Two of the remaining three students answered 20% of the items correctly. The remaining student scored 25%. On the posttest, three of the eight students scored between 94% and 96% correct, one student scored 80% correct, and four students had 100% correct. The difference in percent correct from pretest to posttest ranged from 60% to 100%.

Insert Table 3 about here

Generalization

In the absence of any direct instruction to teach the students to solve word problems, all students improved substantially in solving word problems (minuends 0-9). As shown in Table 3, Students 1 and 2 answered four of five (80%) of the word problems accurately on the five-item generalization pretest; Student 3 answered three (60%) of the word problems correctly; Students 4 and 5 answered one (20%) of the word problems correctly; and Students 6, 7, and 8 did not answer any problems correctly. On the final generalization check (check 2), Students 1, 2, 3, 6, 7, and 8 reached 100% accuracy whereas Students 4 and 5 answered with 80% accuracy.
Discontinuance of instruction at the point of crossover did not adversely affect learner performance. Indeed, by the last day of the maintenance phase, all students either maintained or increased a positive trend, with six of the eight students demonstrating both an increase in the number of corrects per minute and a decrease in the number of errors per minute when presented with the 1-min abstract-level probes. Additional measures used in the study (comparison of pretest-posttest scores) support this conclusion. Moreover, all students demonstrated the ability to solve word problems in the absence of direct instruction using the CSA teaching sequence.

Discussion

Each of the eight primary-age students with learning difficulties in computational arithmetic demonstrated mastery of target subtraction skills as measured by the direct daily assessment. Pretest-posttest gain scores that ranged from 60% to 100% demonstrated skill maintenance in the absence of direct instruction. Six of the eight students obtained gains of 80% or better.

As might be expected, the greatest skill growth occurred for most students during the intervention condition. The maintenance condition revealed that learning continued in the absence of direct instruction, albeit at a slower rate. In all, discontinuance of the CSA sequence at the point of crossover did not hamper student progress, but resulted in both an increase in the frequency of correct responses and a decrease in the frequency of incorrect responses for six of the eight students (Students 2, 3, 5, 6, 7, and 8). Of the remaining two students, one exhibited an increase in the frequency of correct responses whereas the other demonstrated a decrease in error frequency. Student 4 demonstrated an increase in corrects but errors remained the same as during intervention. Student 1
exhibited a decrease in errors but corrects remained the same as during intervention. Therefore, when teachers use the CSA sequence coupled with direct assessment, a data-based decision rule can be used to ascertain whether to continue the teaching sequence or to discontinue the sequence and move to a practice-only format.

Important to note is that no student in the study had to progress through the entire CSA sequence (nine lessons). Three of the eight students (Students 1, 3, and 7) needed only the three concrete lessons; three students (Students 4, 5, and 8) needed the three concrete lessons plus the semiconcrete lesson 1. The remaining two students (Students 2 and 6) required the three concrete lessons plus semiconcrete lessons 1 and 2. None of the students needed semiconcrete lesson 3 or any of the three abstract lessons. Despite not receiving these lessons, all students continued to improve or maintained their skill accuracy. Across all students, the total time spent in direct instruction ranged from a total of 45 to 90 min. The practice-only sequence took less than 5 min per day.

When generalizability was examined (whether students would be able to apply the newly acquired skill of minuends 0-9 to the untaught skill of solving word problems containing minuends 0-9), all of the students demonstrated the ability to generalize to the higher level skill. This was evidenced by the generalization scores obtained during the maintenance condition (see Table 3). Posttest scores earned during this condition were 80% to 100%.

Implications for Practice

Although there is agreement over what to teach as it relates to mathematics instruction (e.g., Maccini & Gagnon, 2002; NCSM, 1998; NCTM, 2000), there is disagreement over how to teach (e.g., Battista, 1999; Campbell et al., 1998; Jones, 2012;
The findings of this study not only address the *how to teach*, but also support that pairing scientifically based effective teaching procedures (i.e., DIS and DI) along with monitoring student progress and using data to drive instructional decisions allows the teacher to make informed and individualized choices for the students. By using the CSA sequence and the discontinue rule as articulated in this study, the teacher instructs until the student has acquired the skill as demonstrated by the data and then moves to a practice-only format. This sequence has four important outcomes that address concerns noted in the literature: (a) increased teacher efficiency, (b) reallocation of valuable instructional time, (c) reduction of potentially redundant instruction while keeping students academically engaged, and (d) student maintenance of the target skill.

In an era of publicized declining mathematics achievement (Mullins et al., 2000; NAEP 2009), increased demands on teachers, the impact of standards-based education especially as it relates to students with disabilities (Fuchs & Fuchs, 2001), and increasing pressure for accountability (e.g., NCLB), it is important to employ instructional methods that are both efficient and data-based (Kavale & Spalding, 2008; Mercer et al., 2011). Using data to drive instructional decisions allows teachers to develop data-based decision rules (Hamilton, et al., 2009). This in turn can result in increased efficiency while promoting student skill acquisition and serves to build the bridge between the *what* and the *how* to teach.

As this study informs practice, it also provides opportunities for further research. The data in this study have implications for improving efficiency in teaching mathematics
skills using a one-on-one or small-group instructional sequence. A logical extension of this investigation is to examine the possibility of adapting the present procedures from small-group to large-group instruction. It remains to be determined if feedback on the rate of correct and incorrect responses serves to increase accuracy and speed.

Another area to explore is the effect of introducing a new skill (e.g., minuends 10-18) while building proficiency on a previously learned skill (minuends 0-9). At this time, no empirical data were found that would indicate whether it is more advantageous to teach these skills sequentially or simultaneously. Practitioners are encouraged to apply both approaches and observe the impact on their students’ progress.

Finally, research comparing the long-term effect of teaching the entire sequence versus discontinuing the sequence at the point of crossover is needed to identify which procedures better promote skill retention. Practitioners, in their routine data collection, can determine how abbreviating the teaching sequence impacts retention and student motivation (Cates et al., 2003). Teacher practice, as well as carefully designed research investigations, can contribute to identifying data benchmarks that increase the soundness of instructional decisions.

Given the positive results of this study, educators are encouraged to revise their instructional practices to include a CSA instructional approach and data-based decision rules. In mathematics instruction, content expertise is not enough; empirically based practices that promote higher levels of achievement are also necessary. Closing the mathematics achievement gap between the United States and other industrialized nations requires data-based instruction and decision making.
References

Allsopp, D., Lovin, L., Green, G., & Savage-Davis, E. (2003). Why students with special needs have difficulty with learning mathematics and what teachers can do to help. *Mathematics Teaching in the Middle School, 8*(6), 308.


Table 1

*Description of Students*

<table>
<thead>
<tr>
<th>Student</th>
<th>Gender</th>
<th>Chronological Age (yr/mo)</th>
<th>Grade Level</th>
<th>IEP Services</th>
<th>Ethnicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M</td>
<td>7.5</td>
<td>1</td>
<td>SLD/SL</td>
<td>w</td>
</tr>
<tr>
<td>2</td>
<td>M</td>
<td>7.1</td>
<td>1</td>
<td>SLD</td>
<td>w</td>
</tr>
<tr>
<td>3</td>
<td>M</td>
<td>6.1</td>
<td>1</td>
<td>EBD</td>
<td>w</td>
</tr>
<tr>
<td>4</td>
<td>F</td>
<td>7.1</td>
<td>2</td>
<td>EBD/SL/OT/PT</td>
<td>w</td>
</tr>
<tr>
<td>5</td>
<td>M</td>
<td>7.9</td>
<td>2</td>
<td>EBD/SL</td>
<td>A</td>
</tr>
<tr>
<td>6</td>
<td>M</td>
<td>8.6</td>
<td>2</td>
<td>SLD</td>
<td>A</td>
</tr>
<tr>
<td>7</td>
<td>M</td>
<td>8.3</td>
<td>2</td>
<td>SLD</td>
<td>A</td>
</tr>
<tr>
<td>8</td>
<td>M</td>
<td>8.0</td>
<td>2</td>
<td>SLD/SL</td>
<td>A</td>
</tr>
</tbody>
</table>

*Note.* M = male; F = female; SLD = specific learning disability; SL = speech and language; EBD = emotional/behavioral disorder; OT = occupational therapy; PT = physical therapy; w = white; A = African American.
Table 2

*Untimed Pretest and Posttest Scores*

<table>
<thead>
<tr>
<th>Student</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Gain&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>2</td>
<td>0%</td>
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<td>100%</td>
</tr>
<tr>
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<td>94%</td>
</tr>
<tr>
<td>4</td>
<td>20%</td>
<td>80%</td>
<td>60%</td>
</tr>
<tr>
<td>4</td>
<td>25%</td>
<td>96%</td>
<td>71%</td>
</tr>
<tr>
<td>6</td>
<td>0%</td>
<td>95%</td>
<td>95%</td>
</tr>
<tr>
<td>7</td>
<td>20%</td>
<td>100%</td>
<td>80%</td>
</tr>
<tr>
<td>8</td>
<td>0%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

<sup>a</sup>Raw percentage point difference
### Table 3

*Generalizability of Student Acquisition as Measured by Word Problem Pre-test Post-test*

<table>
<thead>
<tr>
<th>Student</th>
<th>Pretest</th>
<th>Check 1</th>
<th>Check 2</th>
<th>Gain&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80%</td>
<td>100%</td>
<td>100%</td>
<td>20%</td>
</tr>
<tr>
<td>2</td>
<td>80%</td>
<td>100%</td>
<td>100%</td>
<td>20%</td>
</tr>
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<td>80%</td>
<td>100%</td>
<td>40%</td>
</tr>
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<td>20%</td>
<td>80%</td>
<td>80%</td>
<td>60%</td>
</tr>
<tr>
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<td>20%</td>
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<td>0%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>7</td>
<td>0%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>8</td>
<td>0%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

<sup>Note</sup>. Check 1 = administered first day of maintenance; Check 2 = Administered on last day of maintenance

<sup>a</sup>Raw percentage point difference from pre-test to check 2
Figure 1. Teacher 1, Students 1 and 2 daily probe data
Figure 2. Teacher 2, Students 3, 4 and 5 daily probe data
Figure 3. Teacher 3, Students 6, 7 and 8 daily probe data