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Abstract

The bias of forward exchange rates as a predictor of future spot rates is typically explained or decomposed as (1) a risk premium and (2) a convexity term which accounts for the fact that, when there is stochastic inflation, nominal gains from forward currency speculation are higher than real ones and correspondingly losses are smaller. We use Nalebuff's envelope puzzle to explain a third source of bias which involves real profits from foreign exchange speculation. Both the "real profit" bias and stochastic inflation bias arise from convexity of g(s)=1/s and so derive from Jensen's inequality as observed by Siegel (1972).

1. Introduction

This paper is largely pedagogical, exploring the strong parallels between Nalebuff's envelope puzzle and Siegel's paradox in foreign exchange (Nalebuff (1988,1989) and Siegel (1972,1975)). We explain that Siegel's paradox is richer than Nalebuff's, although by frequent interpretation it is trivialized to what amounts to Nalebuff's envelope puzzle. In particular, the point made in this paper is that if the variation in exchange rates is driven by stochastic inflation, then Nalebuff's envelope puzzle is quite parallel to the situation of foreign exchange, and then Siegel trading profits are not real. On the other hand, a "fallacious" argument in Nalefuff's puzzle emerges as correct in a general equilibrium model of foreign exchange if exchange rates are driven by factors that create real disparities in domestic purchasing power that depend upon the fraction of portfolio holdings that were in foreign assets. This added richness is one potentential explanation for the bias of forward foreign exchange rates as a predictor of future spot exchange rates.

Kemp and Sinn (1989,2000) reopened an old debate about the Siegel paradox, by demonstrating in a well-specified general equilibrium model that Siegel profits can be real, and that speculation in pursuit of such profits can in fact lower welfare. Here, we argue that Siegel traders can cause part or all of forward discount bias and should therefore play a role in a full understanding of that bias. Sinn (1989) made this point in a partial equilibrium model.

Siegel (1972) introduced a foreign exchange paradox in which two players whose utility depends upon money, and who do not have divergent priors, seek to trade their assets. Importantly, their trade is not motivated by risk-sharing.

To understand Siegel's paradox, consider two countries and the rate of exchange between their currencies. It follows from Jensen's inequality that for non-degenerate distributions of exchange rates, the arithmetic mean of the future spot exchange rate, s, exceeds the harmonic mean: i.e., provided $Var(s)>0$, the harmonic mean $(1/E(1/s))$ is less than the arithmetic mean $E(s)$. Therefore, whatever the forward exchange rate, people from at least one country will gain in expected value terms from exchange, and it is possible that both do.[1] This is Siegel's "paradox".

Siegel's paradox is frequently explained away as either an artifact of an ad-hoc model or as a reflection that nominal gains do not reflect real gains when there is stochastic inflation. Boyer (1975), Engel (1984), and Adler and Dumas (1983) interpret the Siegel paradox as a monetary illusion—a confusion between nominal gains and real gains—while McCulloch (1975) argued that if the model were fuller, i.e., if foreign accounts were forced to balance, then the returns would dissipate.[2] Siegel (1975), himself, became convinced that Siegel gains are not real.

Nalebuff's envelope paradox is analogous to the discussion of Siegel's paradox and provides perfect entree to the issue. Nalebuff (1988,1989) presents a puzzle in which "the other envelope is always greener". Analyzing his problem and its relationship to Siegel will show exactly what is right about the money illusion criticisms or interpretations of Siegel's paradox. It will also reveal how they can go wrong, and why in the end a real Siegel effect is both possible and realistic. The bearing of this analysis upon the decomposition of forward discount bias is explained in sections 3 and 4.

In a Mundell-Fleming model with perfect capital flows, the forward exchange rate equals the expected value of the future spot rates, or put alternatively, the forward discount equals the expected depreciation of the currency or change of the spot exchange rate (eqn. <1> of Macklem (1991)). A large empirical literature, however, rejects the idea that the forward rate is an unbiased predictor of future spot rates.[3] Froot and Frankel (1989) and Lewis (1995, p. 1922) see two possible sources of the bias: (1) a risk

[1] Let f represent the forward rate in DM/$, and s the uncertain future spot rate. Risk-neutral Germans concern themselves with whether $E(s/f)$ exceeds unity; risk-neutral Americans with whether $E(f/s)$ does. But their product exceeds unity unless future spot rates are certain. So, with any non-degenerate distribution of s, either one or both must exceed unity, depending upon f.

[2] These and other similar opinions are surveyed by Kemp and Sinn (1989).

[3] See, e.g., Hansen and Hodrick (1980) for a sample of this literature and Hodrick (1988) for a survey.

premium; and (2) a violation of rational expectations.[4] Froot and Frankel reject the hypothesis that a risk premium fully explains the bias and cannot reject the hypothesis that systematically irrational expectations do. Macklem (1991) and Silbert (1989) decompose the bias into two effects: the convexity term and the risk premium. Macklem (1991, p. 375) explains that with risk neutrality the convexity term "just compensates the agent for the fact that stochastic inflation tends to reduce the real value of nominal profits to forward speculation while magnifying the real value of nominal losses." Silbert (1989) challenges the conventional wisdom that this convexity term is small by illustrating in an OLG model with logarithmic preferences larger convexity terms than risk premium.

We explain below how with risk neutral agents who have rational expectations so that the two Froot and Frankel explanations are eliminated, we can still get bias. This bias can be from *either* stochastic inflation which creates a wedge between real profits and nominal profits—this "Nalebuff term" is perfectly analogous to gains in Nalebuff's envelope puzzle; or real Siegel profits which are almost analogous to a losing argument from the envelope puzzle. On the one hand, the Siegel effect can sometimes merely reflect the fact that nominal returns may be positive even when real returns are zero, i.e., the money illusion seen in Boyer (1975), Engel (1984), Adler and Dumas (1983) or Macklem (1991); or on the other hand, it can reflect real gains.

For a full understanding of biases in forward exchange as a predictor of future spot rates, Siegel-traders seeking (and getting) real excess returns should be included in the analysis. However, as the conclusion suggests, doing so will not necessarily make the empirical findings that constitute the "Forward Discount Puzzle" any less puzzling – Siegel traders could aggrevate the puzzle.

2. Nalebuff's Envelope Problem: an Allegory for Siegel's Paradox

2.1. The Allegory:

Nalebuff's (1988, 1989) puzzle begins with Ali being given at random one of two envelopes, while Baba gets the other.[5] They have no idea how much money is in the envelopes, but know that one envelope contains twice what the other does. Ali is asked whether he would like to switch envelopes (NB: both Ali and Baba are risk neutral).

Ali is initially struck by indifference. But then he reasons that if his envelope has $X in it, the other has a 1/2 chance of having $2X (if his envelope is low) and a 1/2 chance of having $X/2 (if his envelope is high). This yields an expected value of $5/4X in the other envelope. Trade apparently profits Ali. This seems implausible, though, since by identical reasoning it should profit Baba.

Is it really possible that both get more expected dollars from the trade? To an outside observer, undoubtedly the expected amount of money in the two envelopes cannot be increased by the trade, and since the players' information is the same as this observer, their perspectives should not differ.

In fact, Baba may reason quite soberly that if there is $Y in the low envelope and $2Y in the high one, then switching envelopes has a 1/2 chance of yielding a gain of Y and a 1/2 chance of a loss of Y—depending upon which envelope he was initially given. Trade then has no returns. Intuition insists that *this argument* must be the right one. But, then what could be wrong with Ali's argument? If the reader has not read Nalebuff's papers, she might pause here to try to answer this herself, before the fun is spoiled.

Ali mistakenly believes that it is valid to keep the amount in his envelope fixed at X while entertaining the possibility that the envelope may be low or high. Baba's calculation does not allow that.

Ali might defend his position by maintaining that he can fix X by looking in his envelope and counting the money. And, this point is incontrovertible. After X is fixed, Ali can assert the merit of his calculation. Further, if he will do this after he looks in his envelope irrespective of what he finds, surely he need not look in his envelope to see the virtue of switching.

But, after looking in the envelope, Ali's previous calculation becomes loaded. It was previously innocuous for him to state, "There is a 1/2 chance that my envelope is the high one and a 1/2 chance that it is the low one." After opening the envelope, however, this statement implicitly carries likelihood ratios

[4]For a theoretical setup explaining the nature of the risk premium, see Stockman (1978) or Grauer *et al*. (1976).

[5]See Nalebuff for the puzzle's origins and an expanded discussion.

about the possible absolute amounts in the low envelope.[6] Ali was loosely thinking that the fact that he did not know what was in the envelopes meant that the low envelope was drawn from a uniform density over $[0,\infty)$. This distribution is improper and Ali's thinking about the value of switching envelopes is consequently wrong-headed. Once the importance of priors is realized, it should be observed that no distribution of the amount in the low envelope will always allow Ali to make the above statement. Therefore, he certainly cannot justify his calculation before opening the envelope.

Nonetheless, the astute reader may realize that Ali can form priors so that after opening the envelope he may always make the following true statement almost like the one above: "There is at most *1/(2-ε)* chance that the other envelope is the low one and at least a *(1-ε)/(2-ε))* chance that the other envelope is the high one". This requires that the probability density of the amount *y* in the low envelope, $f(y):[1,\infty)\rightarrow\Re_+$, decline geometrically at a rate 1-ε along any sequence $\{y_n\}_{n=1}^{\infty}$ proportional to *{1,2,4,8,...}*. No matter what Ali sees in his envelope he will strictly prefer to switch. However, it no longer follows that he strictly prefers to switch before looking from the fact that he will strictly prefer to switch after looking. After all, the expected utility of each envelope before opening is infinite.[7]

2.2. The Moral:

Ali is correct that the expected purchasing power of Baba's envelope relative to his own is 5/4. But his own envelope is not a sensible numeraire for Ali to base his utility upon. For, in the state of the world where Ali doubles his money by switching, his money was not worth very much because he initially held a low envelope. He doubles half as much when he doubles his money as when he halves it.

Siegel's paradox is analogous. To be concrete, if the exchange rate tomorrow will be either 1/2 or 2 with equal probability, exchanging $1 for 1 DM (in the forward markets) yields an expected value of $5/4 tomorrow. Exchanging 1 DM for $1 yields 5/4 DM. Indeed, as was explained in the introduction, because of the convexity of *g(s)=1/s*, regardless of the forward rate or the distribution of future spot rates between two countries, it nominally profits those from at least one country to speculate through exchange of currencies.

Are these profits purely nominal and illusory as that of Ali switching envelopes, or can they be real as Ali initially thought? The answer depends entirely upon what drives exchange rates. Siegel's critics are all correct to think that such gains *may* be illusory:[8] Siegel may be taken as an example of the envelope problem. To be concrete, if the exchange rate will go up or down as a result of an inflation in one country or the other with the imposition of purchasing power parity, then all gains are purely nominal and illusory just as Engel (1984) and Adler and Dumas (1983) suggest. Macklem's (1991) decomposition is apt in this circumstance. This fact is fairly well understood, see e.g. Engel (1984), but we will put it into the present context for expository purposes.

To see how this convexity bias from the nominal Nalebuff profits works, consider the following two period example. Exchange rates are driven by the nominal price level. First period price levels and spot rates are all unity: i.e., $p_{\$}=1$, $p_{DM}=1$, $s_0=1$. We don't know which of two countries will inflate: in period 2, either the US or Germany will have price level 2, while the other will have price level 1.[9] Thus, the state of the world $\omega\in\Omega$ is drawn in period 2 from two states with equal probability weights (each state being given by the two price levels): $\Omega=\{(1,2),(2,1)\}$, $Prob\{(1,2)\}=Prob\{(2,1)\}=1/2$. Thus future spot rates *s* are given in \$/DM by

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[6] In particular, if $100 is observed, from Bayes' rule that statement is seen to imply that the prior probability that the low envelope contained $50 equals the prior probability that it contained $100.

[7] It is no secret or surprise that a sum which is infinite can be arranged so that it is smaller term by term than another divergent sum. But this in no sense makes the first sum smaller than the second. Whenever the first moment of the distribution of the amount in the low envelope is finite, it *cannot* be that Ali will always want to switch after looking in his envelope.

[8] Some go too far, as we will see, by asserting that the gains *must* be illusory.

[9] We could instead presume each nation's price level is an independent draw rather than negatively correlated. This would only add the possibility that future spot exchange rates are 1; it would not change the substance of the analysis.

$$s = \begin{cases} 1/2 & \text{if } \omega \equiv (U.S.\ \text{price level } p_{\$},\ \text{German price level } p_{DM}) = (1,2) \\ 2 & \text{if } \omega \equiv (U.S.\ \text{price level } p_{\$},\ \text{German price level } p_{DM}) = (2,1) \end{cases} \quad (1)$$

If we assume the marginal trader is risk neutral, then the forward rate f =1 is a market clearing price. The forward discount, $(f-s_0)/s_0$, equals 0 and there are no real gains to forward exchange in either direction. However, the nominal expected return measured in dollars of exchanging one dollar for Deutsche Marks in the forward markets and then exchanging back in the future spot markets is 5/4, the mean of 2 and 1/2. The same is of course true for Deutsche Marks. Accepting Macklem's decomposition here we have no forward discount and the convexity term *(1-5/4)* is exactly balanced by the expected appreciation in the spot rate.[10]

When dollars are worth little $(p_{\$}=2)$, the speculator gets \$2 and when they are precious $(p_{\$}=1)$, he gets \$1/2. The American who speculates thinking that he will profit is deceived. He merely exchanges risk with the German. Using the dollar as a fixed frame of reference is as deluded as Ali viewing X as fixed, but allowing its highness or lowness to vary. If the consumption good is instead the frame of reference, then the American can save in dollars or Deutsche Marks and expect to buy the same number of goods tomorrow. A risk neutral investor is indifferent between his prospects, and a risk averse one will diversify to spread risk.

As we will see below, however, this analysis does not comprehend the richness of Siegel's paradox. Not all Siegel profits are illusory Nalebuff profits. Suppose Ali actually cared about the ratio of the amount in the other envelope to the amount in his own; suppose his utility function were a function not of money, but of this ratio. Or in the case of foreign exchange, if utility were not a function of purchasing power (in whatever is exchanged) but of money, then exchanging currency (like envelopes) would be profitable. 5/4 becomes a relevant number if Ali derives utility from dollars; in that case, he will strictly prefer to exchange his currency. For some people, it is undoubtedly reasonable to think that dollars are the right numeraire to consider.

3. A Simple General Equilibrium Model with Real Siegel Profits

3.1 Real Siegel Profits, but Siegel traders don't influence forward rates:

In both this section and section 3.2, we model risk neutral agents so there will be no risk premia. Siegel profits will be quite real, unlike the illusory envelope switching profits of the Nalebuff variety above. The model avoids money to steer far clear of Boyer (1975) who thought Siegel profits are all from money illusion, and we have a general equilibrium model with all markets clearing to answer those who suggest that real Siegel profits do not occur if foreign accounts are forced to balance. Although there will be real Siegel profits in this subsection, the reader will have to wait until subsection 3.2 to see how these may bias forward rates.

The only consumption good is wine, which has two varieties—American and German. The economy has only two periods, and consumption is in the second period. Every American is endowed with 10 bottles of American wine which they consider to be their currency; every German in endowed with 10 bottles of German which they consider to be their currency. Wine matures in the second period when consumption takes place. Most people in both nations are fickle and derive twice the utility from consuming fashionable wine, W_F, as they would from consuming unfashionable wine, W_U: a fickle utility function is

$$U^f = 2W_F + W_U \quad (2)$$

A few Americans, however, are loyal citizens. Their preferences do not flip-flop and they would as soon have 1 bottle of American wine as 10 of German no matter what the fickle experts think: They maximize their utility given by

$$U^l = 10A + G \quad (4)$$

where A *(resp. G.)* represents the number of bottles of American (resp. German) wine that the loyal American consumes. The reverse is true for some loyal Germans.

[10]Put in other words, the bias in the forward rate as a predictor of the future spot rates is given solely by the convexity term since the marginal traders are risk neutral and so the risk premium equals 0.

With probability .5, American wine is fashionable so $W_F=A$ and with probability .5 German wine is, so $W_F=G$ Since the fickle agents are quite numerous, they are the marginal traders who determine both the future spot terms of exchange and the forward rates. We will now explain why it is an equilibrium for $f = 1$ American bottle/1 German bottle, and for future spot rates s to follow the trends and equal 1/2 when $W_F=A$ and 2 when $W_F=G$.

$$s = \begin{cases} 1/2 & \text{if } W_F = A \\ 2 & \text{if } W_F = G \end{cases} \tag{5}$$

For forward rates of $f=1$, fickle people are indifferent to which form of wine they hold (from their vantage holding a bottle of either wine is just as holding a dollar or Deutsche Mark was in the previous example—it is a lottery that either allows them to purchase 1 util or 1/2 util in the future depending upon "inflation"). At $f=1$, loyal citizens can expect to get more bottles of domestic wine tomorrow if they hold foreign wine (which in this institutional setup is equivalent to taking a forward position in foreign currency) than if they hold domestic wine; they therefore exchange as much wine as possible on the forward markets. As loyal citizens speculate and trade, they each get real gains. Despite the fact that loyal citizens are risk neutral and expect real gains from their forward position their demand for forward contracts is not infinite. Since they must meet their forward contracts in both states of the world, they are constrained by the state of affairs with the unfavorable resolution of exchange rates. When the exchange rate is unfavorable at $s=1A/2G$, an American's budget constraint will be $2A+G = 20-2A^f + A^f / f$ where A,G are quantities consumed and A^f is the number of bottles of American wine to be delivered for German, i.e., the number sold in the forward markets. From this, we see that the loyal American who actively seeks a forward position in German wine is restricted in his forward position by $A,G \geq 0$ *to* $A^f \leq 20/\left(2-f^{-1}\right)$.

Because exchange is profitable, risk-neutral loyal Americans will speculate to a corner: they can at best promise to deliver 20 bottles of American wine for German tomorrow.[11] Because we have assumed that there are many more fickle people than loyal citizens, and fickle people are indifferent to trade at $f=1$, the forward markets clear at $f = 1$. In the future spot market, prices are set by the preferences of the fickle people and the loyal citizens are free to exchange any wealth they are left with back into domestic currency (wine). Thus, all markets clear and we have a general equilibrium. Trade is possible despite common prices because people maximize the expected value of their wealth measured in different units—and unlike Ali, they do so reasonably. However, in this subsection the Siegel profiteers add no new biases to the analysis because they are not sufficiently numerous to affect forward rates (given the fact that there is no curvature to the indifference curves of the fickle people).

3.2. Siegel Traders drive Forward Rates:

We now show how these real Siegel profits can add an additional bias to forward exchange rates. This requires that these traders play some part in determining the forward rates. This section shows explicitly an example where expectations are rational, all agents are risk neutral and markets clear. Still, forward rates are biased. This is possible because of the real Siegel effect—Ali's losing argument is made telling here.

Consider an economy with 0 loyal Americans, 20 loyal Germans, 10 fickle Americans and a sufficient number of fickle Germans to soak up demand in the future spot markets (and to determine future spot prices). Since there are a large number of fickle Germans, future spot rates are as before*: $s=2$ or $1/2$* with equal likelihood. But what forward rate will clear markets?

If $f \geq 4$ American bottles/5 German bottles, the loyal Germans will want to take a forward position in American wine, promising to deliver German wine for American during period 2, because in that case for every bottle of German wine they sell in the forward markets, they receive f American. When they trade

[11] An alternative way to find this constraint at the realization of exchange, $s=1A/2G$, is to think about the American coming to meet his commitments and trading his first 10 bottles; for these he gets 10 German worth 5 American. He can then fulfill 5 more of his forward contracts and is left with 5/2 American. The limit of this process will exhaust his wealth and he can deliver a maximum of 20 bottles of American wine!

back in the spot markets at rate *s* to get German, they expect to receive $E[f/s] = f(\frac{1}{2}\cdot 2 + \frac{1}{2}\cdot\frac{1}{2}) = \frac{5}{4} f$, which is greater than or equal to unity whenever $f \geq \frac{5}{4}$. When this foreign forward position works out poorly, and period 2 spot prices are s=2A/1G each loyal German's budget set is

$$A + 2G = 20 + A^f - 2A^f / f \tag{6}$$

where *A,G* are quantities consumed and A^f is the number of bottles of American wine to be received from forward contracts, i.e., the number bought in the forward markets. From this, we see that the loyal German who actively seeks a forward position in American wine is restricted in his forward position by $A,G \geq 0$ to $A^f \leq 20/(2/f - 1)$. The loyal German will trade up to this constraint as long as $f \geq 4/5$. By replacing *f* with its inverse, we see that the fickle Americans who are willing to trade whenever $f \geq 1$ (and worry about the period 2 spot realization *s=1A/2G)* are able to receive at most *20/(2f-1)* bottles of German wine or equivalently sell $A^f = 20f/(2f-1)$ forward contracts to Germans.

Therefore the forward rate $f = 1$ which cleared markets in section 3.1 will not work here. We see that the 20 loyal Germans who are Siegel traders are each able to trade $A^f = 20$ for a total market demand of 400. However, supply is restricted to $A^f = 20$ by each fickle American for a total market supply of 200. Since there is so much demand for America's forward wine by the Siegel traders, they drive the price up—*f* falls until markets clear. In fact, *f* falls until *f*=4 American bottles /5 German bottles.[12] The loyal German Siegel traders have biased the forward exchange rates by their efforts to seek foreign exchange. In fact because there were so many of them they biased forward rates so disadvantageously for themselves that in the end they are indifferent between keeping their rights to 10 bottles of German wine and trading them in the forward markets for the rights to 8 bottles of American wine. If there had been only 19 loyal Germans, some of the real Siegel profits would have remained.[13] These Siegel traders like to "switch envelopes" because their numeraire is really the envelope they are given to begin with. Ali's argument is quite sound here even though it was wrong in the envelope puzzle.

4. Conclusions

Not all Siegel profits from speculation in foreign exchange are of the Nalebuff envelope variety (i.e., nominal ones from stochastic inflation which are not real). Real Siegel profits can exist because what was a losing argument for Ali in the envelope puzzle proves to be sound in forward foreign exchange markets.

Forward traders seeking these profits may provide a source of bias in forward rates additional to the biases from stochastic inflation, risk premia, or irrational expectations. While the discussion has been somewhat fanciful, the issue is realistic. Exchanging money today only to re exchange it in the future at spot rates carries considerable real risk. And the variance from this risk may be much larger than that from stochastic inflation. Whenever this is the case, traders like Ali can view the numeraire of their own currency as bringing fairly constant utility.[14] Therefore, because of Jensen's inequality, as we have already

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[12] Then total demand by the 20 loyal German Siegel traders is $A^f = 800/3$ which equals the supply of the fickle Americans.

[13] Since loyal Germans are indifferent to trade at these prices, the forward market would clear at these same prices if there were more loyal Germans as well.

[14] While it is true that high returns are earned when one's own currency is weak, and prices of imports are high; thinking as Ali does that domestic currency is a good measure of value is perfectly sensible if one plans to buy some domestic non-tradable goods (e.g., hair cuts) or if alternatively as in this model, preferences (for exchange) do not sway in concert with the marginal trader's valuations. This situation can easily arise when preferences vary between people and between countries, with or without changes in fashion such as here (Kemp and Sinn, 1989). It is not paradoxical that a person can profit from exchanging on the forward markets when she is not the marginal trader who is indifferent between exchange at the equilibrium forward rates.

observed it is advantageous for one side or the other to speculate in foreign exchange whatever the distribution of forward rates. The situation need not of course be as extreme as in the example in this paper. If these Siegel traders are risk averse then they will not go to the corner where their wealth goes to zero under unfavorable exchange rate fluctuations. These traders will take a forward position in foreign exchange until the risk this creates equals their Siegel gains.

Empirical research has found that forward exchange rates are biased predictors of future spot rates. See, for example, Flood and Rose [1997] and Lewis [1995]. In particular, high interest rate currency tends to appreciate, though the forward rates implicit in the interest rate differential would suggest that the currency is expected to depreciate. This observation constitutes the "forward discount puzzle". Our model points out that traders pursuing real Siegel profits can bias forward rates as a predictor of future spot rates, but this bias could be in either direction and there is no obvious reason that the direction would be correlated with high interest rate currency's, for example. Any Siegel biases seem as likely to aggravate as to attenuate the bias and so Siegel traders do not stand out as the likely cause of the forward discount puzzle. Nonetheless it is interesting to understand if one is attempting to decompose the discount bias that some of the bias in the convexity term may be caused by Siegel traders seeking real profits. Moreover, as Kemp and Sinn (2000) point out, Siegel speculation may actually lower overall welfare in a world with transaction costs.

All that is necessary for profits from speculation not to be illusory (from some vantage) is that Ali's X be fixed, not move too much, or move the right way. Sufficiently various goods and objectives exist that this must be true for many people in practice. Whenever it is the case that the variance in inflation is low but there are high variances in exchange rates, some people will realize real profits from speculating in foreign currency.

If speculation brings real returns, its risk may be (imperfectly) hedged. A loyal American who wants to buy domestic non-tradable goods whose prices will not swing as wildly as exchange rates would do well to borrow money, buy an importer of Mercedes and take a forward position in Deutsche Marks. The currency speculation yields positive expected real returns; and, when the dollar becomes devalued so that the speculation is a bust, Mercedes will do booming business in the U.S..

It is an interesting empirical question how large are real Siegel profits, how well they are taken advantage of and what their effect is upon the bias in the rate of forward foreign exchange when compared with nominal Nalebuff profits, and risk premia.

5. Colophon

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