Market-based Transfer Prices and Intracompany Discounts

(preliminary and incomplete draft)

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Abstract

Firms frequently value internal transactions at external market prices subject to an intracompany discount. These discounts are generally explained by cost differences between internal and external sales. In a model where the supplying division has monopoly power in the external market, we find that cost differences are neither necessary nor sufficient for intracompany discounts to be desirable. The imposition of discounts always increases the divisional profits of the buying division, but may also lower the divisional profits of the selling division. We derive conditions for discounts to enhance firm-wide profit. We also study the sensitivity of the optimal discount to cost differences between internal and external transactions. Under certain conditions, market-based transfer prices subject to optimally chosen discounts perform well. If the buying division sells its final product in a competitive market and if demand and cost parameters are positively correlated, then market-based transfer pricing induces the divisions to engage in transactions which are nearly efficient from the corporate perspective.

1 Introduction

It appears to be commonly accepted both in practice and in the management literature that firms should value intrafirm transactions at market prices, provided there is a competitive external market for the intermediate good in question. Cook (1955) and Hirshleifer (1956) argued many years ago that if a supplying division is a price taker for some intermediate good, i.e., it can sell an "unlimited" quantity of the good at a given price externally, then this price should also be the internal transfer price. In this paper, we analyze the effectiveness of market-based transfer prices when the external market for the intermediate good is imperfectly competitive in the sense that the internal seller is a monopolistic price setter externally.

Our model considers one selling and one buying division inside the firm. We suppose that the buyer cannot obtain the intermediate product from an external source. In contrast, the selling division can find other customers for the intermediate product, and it is assumed to have monopoly power in the external market. If the firm's central management adopts a policy of market-based transfer pricing, it entitles the buying division to purchase the intermediate product at whatever price the selling division charges externally. Anticipating the buyer's demand, the selling division will choose a monopoly price which maximizes its divisional profit given the combined internal and external demand. From the firm's overall perspective, this pricing mechanism has two drawbacks. First the selling division does not get to maximize its external profits. Secondly the internal transfer price is too high resulting in inefficiently low trade.

Surveys indicate that in practice many firms do not simply set the internal transfer price

at market price, but instead impose intracompany discounts.[1] The argument commonly given for these discounts is that the supplying division can avoid certain costs on internal transactions, such as selling expenses or provisions for bad debt. Absent any such cost differences, one might still conjecture that an intracompany discount is desirable because it ameliorates the problem of inefficiently low trade between the divisions. These internal trading gains, however, have to be weighed against potential losses resulting from the fact that, in response to the imposed discount, the selling division will increase the external sales price.

Our analysis establishes conditions under which market-based transfer prices should be subjected to an intracompany discount. A sufficient condition is that the unadjusted price the selling division would charge both externally and internally under market-based transfer pricing exceeds the internal monopoly price, i.e., the price that the selling division would impose if it only served the internal buyer and could act monopolistically towards that buyer. In that case, the tradeoff described above does not arise since the discount induces the selling division to choose an external sales price which is closer to the unrestrained monopoly price.

Under certain circumstances, an intracompany discount will be harmful. For instance, if the internal demand exhibits relatively high price elasticity and the internal monopoly price exceeds the external monopoly price, then the imposition of a discount will only have a small impact on intrafirm trade. Yet, in order to maintain the sales with the internal buyer, the selling division may have to move the external sales price even farther above the unconstrained optimum. An important special case we analyze is one where both the

[1] See, for instance, Price Waterhouse (1984) or Eccles and White (1988).

internal and external demand can be described by linear demand functions. It then turns out that a discount is always desirable. We derive the optimal internal discount and identify its sensitivity to any cost differences between external and internal sales.

In general, market-based transfer pricing remains suboptimal, even if the firm implements the optimal discount. First, there is an *information problem* as the corporate controller is uncertain about divisional costs and revenues when setting the discount. Secondly, there is an *incentive problem* in that the supplying division does not internalize the firm's profit function and tends to set the external price too high. However, we identify conditions under which both these problems become negligible. If the uncertainty is such that both demand functions and the supplier's marginal cost function are affected by perfectly correlated additive shocks, then the controller would implement the same discount for any realization of that shock. Moreover, if the buying division needs the input to manufacture a final product which it sells in a highly competitive market, then the goal conflict between the supplying division and the firm as a whole becomes negligible. Thus, if both these conditions hold, adjusted market-based transfer pricing comes close to the benchmark of first-best performance.

The information problem can be somewhat mitigated if the central office only prescribes a *minimum* intracompany discount. After the divisions observe the state of the world, the supplier may want to offer the buying division a higher discount if the internal demand turns out to be particularly low. By doing so, the supplier can more effectively extract the surplus from the external market. The firm will always benefit from this additional flexibility, and we show that the optimal minimum discount is lower than the optimal "fixed" discount if the divisions were not granted this flexibility.

The remaining paper is organized as follows: Section 2 introduces the basic model setup. In Section 3, we derive conditions for an intracompany discount to be desirable. In Section 4, efficiency properties of adjusted market-based transfer pricing are discussed. Section 5 addresses the case where the central office only fixes a minimum internal discount, and Section 6 concludes.

2 The Model

We consider a firm composed of two divisions. Division 1 is an upstream division which can sell its output to Division 2 (the downstream division) and to external customers. Division 1 incurs a constant unit variable cost $c(\theta)$ in producing its output. The production cost depends on a state variable $\theta \in \Theta$ which is viewed as random ex-ante by all parties. Prior to deciding on quantities and transfers, however, the division managers, but not the central office, observe the realization of θ and thereby also all revenue and cost information. Thus division managers are assumed to be symmetrically informed at all times.[2]

If Division 1 charges a price p externally, then it can sell $Q_e(p, \theta)$ units on that market. Division 2 can use the same product in its operations and earn a net revenue (net of production costs) of $R_i(q_i, \theta)$ provided q_i units of the intermediate product are transferred internally. Given an external price p, an internal transfer quantity q_i and a transfer price TP, divisional profits become

$$\pi_1 = p \cdot Q_e(p, \theta) + TP \cdot q_i - c(\theta) \cdot (Q_e(p, \theta) + q_i)$$

[2]This assumption can be relaxed somewhat. The state variable θ could have multiple components. While the manager of Division 1 needs to observe all components of θ, it would suffice for our model if the manager of Division 2 could observe only those components of the state variable θ that affect his division's valuation of the intermediate product.

for Division 1, and

$$\pi_2 = R_i(q_i, \theta) - TP \cdot q_i$$

for Division 2.

For technical reasons, we assume that the domain of θ is a compact subset of $\mathbb{R}^n$, and that the functions $c(\cdot)$, $R_i(q_i, \cdot)$ and $Q_e(p, \cdot)$ are continuous in θ, for any given values of q_i and p.

In our model of market-based transfer pricing, TP will be a function of the external price p. Furthermore, the selling division must satisfy the buying division's demand at the transfer price $TP(p)$. Let $Q_i(TP, \theta)$ denote Division 2's best quantity response given the transfer price TP and the state θ. Thus

$$Q_i(TP, \theta) \in \text{argmax}_{q_i} \{R_i(q_i, \theta) - TP \cdot q_i\} \ .$$

We assume that $R_i'(\cdot, \theta)$ is decreasing in q_i and ultimately falls to zero. Thus Division 2 always has a best response. Also, if there are multiple maximizers, we assume that Division 2 demands the maximum among them.

Under a system of pure market-based transfer pricing, the firm's central office (controller) simply sets $TP(p) = p$. In contrast, a system of market-based pricing subject to intracompany discounts results in a transfer price of the form $TP(p) = p - \Delta(p)$ with $\Delta(p) \geq 0$. There are several ways such discounts are determined in practice. Probably the most common alternatives are a constant discount, i.e. $\Delta(p) \equiv \Delta$, and a proportional discount where $\Delta(p) = \delta \cdot p$ for some constant δ. For tractability reasons, we focus our analysis on constant discounts. For several of our main results though, we establish their validity for proportional discounts as well.

For a given constant discount, Δ, the selling division will choose the external sales price, $p(\Delta, \theta)$, so as to maximize its divisional profit which is

$$\pi_1(p, \Delta, \theta) = (p - c(\theta)) \cdot Q_e(p, \theta) + (p - \Delta - c(\theta)) \cdot Q_i(p - \Delta, \theta) \tag{1}$$

subject to:

$$Q_i(p - \Delta, \theta) \in \text{argmax}_{q_i}\{R_i(q_i, \theta) - (p - \Delta) \cdot q_i\} \ .$$

The problem for the central office is to select a discount, Δ, which maximizes the firm-wide profit on average. We denote by $E_\theta[\cdot]$ the expectation operator with respect to θ. The firm's expected profit can then be represented as:

$$\pi(\Delta) = E_\theta[(p(\Delta, \theta) - c(\theta)) \cdot Q_e(p(\Delta, \theta), \theta) + R_i(Q_i(p(\Delta, \theta) - \Delta, \theta), \theta) - c(\theta) \cdot Q_i(p(\Delta, \theta) - \Delta, \theta)]$$

where $p(\Delta, \theta)$ is a solution to the maximization problem in (1).

Throughout our analysis we suppress any moral hazard issues at the divisional level and assume that division managers seek to maximize their own division's profit.[3] This model specification seems reasonably descriptive of most multidivisional firms.

3 Intracompany Discounts

When the central office stipulates that internal sales are valued at external market prices less some discount, it enhances intrafirm trade. While this effect is unambiguously positive, (at least for discounts which induce a transfer price above the unit cost $c(\theta)$) there is also the secondary effect of inducing Division 1 to change its external sales price $p(\Delta)$. Without a discount, i.e., $\Delta = 0$, Division 1 will choose a price $p(0, \theta)$ which is an average of the

[3] Like in Edlin and Reichelstein (1995), it would be possible to add a divisional moral hazard problem.

two "pure" monopoly prices it would charge if it served exclusively either one of these markets. With mild regularity conditions, it can be shown that a positive discount will raise the external price such that the internal transfer price remains below the original average monopoly price $p(0, \theta)$. Formally:

$$p(\Delta, \theta) \geq p(0, \theta) \geq p(\Delta, \theta) - \Delta \ ,$$

for all θ provided Δ is "sufficiently" small. It then follows that a discount is beneficial to the firm if the price $p(\Delta, \theta)$ allows Division 1 to obtain higher external profits than the original price $p(0, \theta)$.

To state the result on the desirability of a discount formally, we introduce the following terminology. Consider the external and internal monopoly problems:

$$(p - c(\theta)) \cdot Q_e(p, \theta) \tag{2}$$

and

$$(p - c(\theta)) \cdot Q_i(p, \theta) \tag{3}$$

respectively. Let $\bar{p}$ be the lower of the two reservation prices associated with $Q_i(\cdot, \theta)$ and $Q_e(\cdot, \theta)$.

Assumption (A1): Both the internal and the external monopoly problem are strictly concave in p on the interval $[c(\theta), \bar{p}]$. For any θ, the unique maximizers $p_e^m(\theta)$ and $p_i^m(\theta)$, respectively, are in the interior of $[c(\theta), \bar{p}]$.

Proposition 1 *Given* $(A1)$*, suppose for all* θ*,* $p_e^m(\theta) > p_i^m(\theta)$*. Then the firm benefits from imposing an intracompany discount.*

All proofs can be found in the Appendix. The sequence of arguments in the proof of Proposition 1 mirrors the preceding discussion. If the external monopoly price exceeds the internal monopoly price, i.e., $p_e^m(\theta) > p_i^m(\theta)$, then absent a discount, Division 1 will set $p(0, \theta)$ in between $p_e^m(\theta)$ and $p_i^m(\theta)$. A sufficiently small discount, Δ, will push the external sales price, $p(\Delta, \theta)$, closer towards the external monopoly price. At the same time, the internal monopoly problem is mitigated since Division 2 buys the intermediate product at the transfer price $p(\Delta, \theta) - \Delta$ which is below $p(0, \theta)$. As a consequence of the imposed discount, both divisions achieve higher profits. In general, a discount may harm the profits of Division 1, but in the scenario considered in Proposition 1, a discount effectively allows Division 1 to engage in profit-increasing price discrimination.

As noted in the Introduction, surveys indicate that firms frequently impose intracompany discounts because of cost differences between internal and external transactions. Proposition 1 above shows that even with identical costs discounts may be desirable since they allow for beneficial price discrimination. At the same time, however, Proposition 1 does not rely on the assumption of identical costs. The reader may verify that the reasoning given above continues to hold without substantive changes if the model included internal and external unit variable costs $c_i(\theta)$ and $c_e(\theta)$ such that $c_i(\theta) \leq c_e(\theta)$.

How large a discount should the central office impose? Our analysis below computes the optimal discount for a specialized setting of linear demand curves. In the general setting, we can make use of the above arguments to identify a lower bound on Δ. Given the concavity assumptions stated in (A1), higher discounts will result in increased profits provided the resulting external sales price $p(\Delta, \theta)$ is not pushed above the monopoly price $p_e^m(\theta)$ and,

secondly, the internal transfer price does not fall below marginal cost.

Corollary to Proposition 1: *The optimal discount,* Δ^*, *satisfies*

$$\Delta^* \geq \min_{\theta}\{\min\{p_e^m(\theta) - p(0,\theta), p(0,\theta) - c(\theta)\}\} \tag{4}$$

The first expression inside the curly brackets reflects the requirement that $p(\Delta,\theta) \leq p_e^m(\theta)$ as well as the inequality $p(\Delta,\theta) - \Delta \leq p(0,\theta)$. On the other hand, since $p(\Delta,\theta) \geq p(0,\theta)$, the requirement that $p(\Delta,\theta) - \Delta \leq c(\theta)$ will be met provided $\Delta \leq p(0,\theta) - c(\theta)$.

Proposition 1 remains true if discounts are not constant but proportional to the external sales price. If $TP(p) = p \cdot (1-\delta)$, where δ is a constant (percentage) chosen by the central office, then parallel to the arguments given in the proof of Proposition 1, the external sales price $p(\delta,\theta)$ satisfies the inequalities:

$$p(\delta,\theta) \geq p(0,\theta) \geq (1-\delta) \cdot p(\delta,\theta)$$

for δ "sufficiently" small. Given these inequalities, the remainder of the proof of Proposition 1 is virtually unchanged. We next construct an example to demonstrate that a discount is not always desirable.

Example: The "fixed-quantity scenario." Consider a case where the buying division receives a one-time offer, which would require a fixed number of units of the intermediate product, $\bar{q}_i(\theta)$. Internal demand hence is inelastic up to a reservation price and perfectly elastic above that price:

$$Q_i(p,\theta) = \begin{cases} \bar{q}_i(\theta), & \text{if } p \leq \bar{p}_i(\theta), \\ 0, & \text{otherwise,} \end{cases}$$

where $\bar{p}_i(\theta)$ denotes the buying division's willingness-to-pay per unit. The supplier's profit from selling externally is again assumed to be strictly concave.

Proposition 2 *In the fixed-quantity scenario, an internal discount lowers expected firm profit, if $\bar{p}_i(\theta) \geq p_e^m(\theta)$, for all θ.*

The scenario is illustrated in Figure 1. In contrast to the conditions posited in Proposition 1, the external monopoly price is less than the internal one: $p_e^m(\theta) \leq p_i^m(\theta) \equiv \bar{p}_i(\theta)$.

— Insert figure 1 here —

Absent a discount, Division 1 will set a price $p(0,\theta) \in [p_e^m(\theta), \bar{p}_i(\theta)]$. If the central office stipulates that $TP(p) = p - \Delta$, Division 1 will choose $p(\Delta,\theta) \geq p(0,\theta) \geq p(\Delta,\theta) - \Delta$. As a consequence, the external sales price is driven further away from the monopoly price $p_e^m(\theta)$, without fostering internal trade, as $Q_i(p(\Delta,\theta) - \Delta, \theta) = Q_i(p(0,\theta),\theta) = \bar{q}_i(\theta)$. Hence, the firm-wide profit from internally traded intermediate goods remains constant, while the external profit decreases. In this situation, the firm would benefit from imposing a mark-up on internal transactions.

Proposition 2 can be extended in at least two directions. First, it does not matter whether discounts are constant or proportional to the external sales price. Either discounting method would force the selling division to charge a less profitable price externally. Second, the above example requires no modification if $c_i(\theta) < c_e(\theta)$. Thus internal cost advantages are not by themselves sufficient to warrant an intracompany discount. As shown in connection with Proposition 4 below, in situations where a discount is desirable with identical costs for

internal and external transactions, it will continue to do so when internal transactions are in fact cheaper.

For a more detailed understanding of the impact of discounts, we now consider the particular setting in which both internal and external market demand are linear functions. Thus,

$$Q_e(p,\theta) = \alpha_e(\theta) - \beta_e(\theta) \cdot p \qquad \text{and} \qquad Q_i(p,\theta) = \alpha_i(\theta) - \beta_i(\theta) \cdot p \ . \tag{5}$$

We denote the corresponding external willingness-to-pay curve by $p_e(q_e,\theta) = a_e(\theta) - b_e(\theta) \cdot q_e$ so that

$$a_e(\theta) = \frac{\alpha_e(\theta)}{\beta_e(\theta)} \qquad \text{and} \qquad b_e(\theta) = \frac{1}{\beta_e(\theta)} \ .$$

The linear specification in (5) implies that the buying division's net revenue, $R_i(q_i,\theta)$, is a quadratic function, such that

$$R_i'(q_i,\theta) = a_i(\theta) - b_i(\theta) \cdot q_i \ , \qquad a_i(\theta) = \frac{\alpha_i(\theta)}{\beta_i(\theta)} \qquad \text{and} \qquad b_i(\theta) = \frac{1}{\beta_i(\theta_i)} \ .$$

In the following analysis of the linear demand scenario, we will restrict attention to parameter configurations for which the price that Division 1 charges under unadjusted market-based transfer pricing (i.e., with no internal discount) is such that both markets will buy positive quantities. In particular, we impose the assumption:

Assumption (A2): For all θ,

$$p(0,\theta) = \frac{1}{2} \cdot \left[\frac{\alpha_e(\theta) + \alpha_i(\theta)}{\beta_e(\theta) + \beta_i(\theta)} + c(\theta) \right] < \min\left\{ a_e(\theta), a_i(\theta) \right\} \ .$$

The expression on the left-hand side of the above inequality represents the average monopoly

price in the linear scenario. This price will be optimal if it is less than the intercept of both the internal and the external willingness-to-pay curve.

Proposition 3 *Assume that both internal and the external demand are given by linear functions, such that* $(A2)$ *holds. Then the firm always benefits from imposing an intracompany discount.*

As argued above, if the external monopoly price exceeds the internal one, i.e., $p_e^m(\theta) > p_i^m(\theta)$, then a discount will allow both divisions to achieve higher profits. If $p_i^m(\theta) > p_e^m(\theta)$, a discount will induce Division 1 to charge an external sales price which is even further away from the unconstrained monopoly price. Yet, the buying division will continue to benefit from the discount. Proposition 3 asserts that the latter gains always outweigh the potential loss incurred by Division 1. The primary thrust of Proposition 3 is to demonstrate that the conditions stated in Proposition 1 are by no means necessary for an internal discount to be advantageous.

The derivative of firm-wide profit with respect to the discount Δ, for any θ, is given by:

$$\begin{aligned} \pi'(\Delta,\theta) &= [R_i'(Q_i(p(\Delta,\theta)-\Delta,\theta),\theta)-c(\theta)]\cdot Q_i'(p(\Delta,\theta)-\Delta,\theta)\cdot[p'(\Delta,\theta)-1] \\ &+ [R_e'(Q_i(p(\Delta,\theta)-\Delta,\theta),\theta)-c(\theta)]\cdot Q_e'(p(\Delta,\theta)-\Delta,\theta)\cdot p'(\Delta,\theta) \end{aligned} \tag{6}$$

With linear demand functions, expression (6) simplifies considerably since $Q_i'(\cdot) = -\beta_i(\theta)$, $Q_e'(\cdot) = -\beta(\theta)$ and $p'(\Delta,\theta) = \frac{\beta_i(\theta)}{\beta_e(\theta)+\beta_i(\theta)} \equiv \nu(\theta)$. As a consequence, $Q_e'(\cdot)\cdot p'(\cdot) = -[p'(\cdot)-1]Q_i'(\cdot) = \frac{1}{b_i+b_e}$, and $\pi'(\Delta,\theta)|_{\Delta=0} > 0$ if and only if

$$R_i'(Q_i(p(0,\theta),\theta),\theta) > R_e'(Q_e(p(0,\theta),\theta),\theta). \tag{7}$$

The buying division maximizes its profit given any price p by choosing $Q_i(p,\theta)$ in such a way that $R_i'(Q_i(\cdot),\theta) = p$. At the same time, note that $R_e'(\cdot)$ is the marginal revenue function associated with the inverse market demand function $Q_e^{-1}(\cdot)$ and, therefore, for any p, we have $R_e'(Q_e(p,\theta),\theta) < Q_e^{-1}(Q_e(p,\theta),\theta) \equiv p$. Combining these two insights validates inequality (7). We conjecture that a proportional rather than constant discount would again have no impact on the validity of Proposition 3 .

The next step in our analysis is to characterize the optimal discount. With linear demand curves, the optimal value of Δ can be calculated explicitly since the profit function $\pi(\Delta,\theta)$ is quadratic in Δ.

Proposition 4 *Suppose that both internal and external demand are given by linear functions such that* $(A2)$ *holds. Then the optimal discount is:*

$$\Delta^* = \frac{E_\theta\{\beta_e(\theta)\cdot\nu(\theta)\cdot[a_e(\theta)-p(0,\theta)]\}}{E_\theta\{\beta_e(\theta)\cdot\nu(\theta)\cdot(1+\nu(\theta))\}} \tag{8}$$

The optimal discount attempts to force the selling division to grant the internal buyer a lower transfer price. Since the central office does not observe θ, the optimal discount has to be determined in expectation over all states of the world.

It is instructive to compare the formula in (8) with the lower bound on Δ derived in the Corollary to Proposition 1. As one might expect, the optimal Δ^* in (8) exceeds the bound given in (4). To see this, we note that since $0 < \nu(\theta) < 1$:

$$\Delta^* \geq \frac{E_\theta\{\beta_e(\theta)\cdot\nu(\theta)\cdot[a_e(\theta)-p(0,\theta)]\}}{E_\theta\{\beta_e(\theta)\cdot\nu(\theta)\cdot 2\}} \geq \frac{1}{2}\cdot\min_\theta\{a_e(\theta)-p(0,\theta)\}\ .$$

Therefore, Δ^* exceeds the bound in (4) if for all θ:

$$\tfrac{1}{2} \cdot [a_e(\theta) - p(0,\theta)] > p_e^m(\theta) - p(0,\theta) \ ,$$

this last inequality indeed holds because $p_e^m(\theta) = \frac{1}{2}[a_e(\theta) + c(\theta)]$ and $p(0,\theta) > c(\theta)$.

To conclude this section, we consider the impact of lower internal costs on the optimal discount. Suppose that, as before, the cost of providing the intermediate good to external customers is $c(\theta)$, but the internal cost is $c_i(\theta) = c(\theta) - \varepsilon(\theta)$, where $\varepsilon(\theta) \geq 0$. Let $p(\Delta, \varepsilon(\theta), \theta)$ denote the external price set by the seller, given an imposed discount of Δ and a cost differential of $\varepsilon(\theta)$.

Corollary to Proposition 4: *Given the assumptions stated in Proposition 4, suppose furthermore that $c_e(\theta) = c(\theta)$ and $c_i(\theta) = c(\theta) - \varepsilon(\theta) \leq c(\theta)$. Then:*

$$\Delta^{**} = \frac{E_\theta \left\{ \beta_e(\theta) \cdot \nu(\theta) \cdot \left[a_e(\theta) - p(0,0,\theta) + \left(1 + \frac{\nu(\theta)}{2}\right) \cdot \varepsilon(\theta) \right] \right\}}{E_\theta \left\{ \beta_e(\theta) \cdot \nu(\theta) \cdot (1 + \nu(\theta)) \right\}} \geq \Delta^* \tag{9}$$

For the particular case where $\beta_i(\theta) \equiv \beta_i$, $\beta_e(\theta) \equiv \beta_e$ and $\varepsilon(\theta) \equiv \varepsilon$, a comparison of (8) with (9) allows us to determine the rate at which the optimal discount increases in the internal cost reduction ε:

$$\Delta^{**} - \Delta^* \ = \ \left[1 - \frac{\beta_i}{2(2\beta_i + \beta_e)} \right] \cdot \varepsilon \ \in \ \left(\frac{3}{4}\varepsilon, \varepsilon \right).$$

The optimal discount increases in ε with a rate that is greater than $\frac{3}{4}$ but less than unity, depending on the relative values of β_i and β_e. As indicated above for the case of general demand functions, lower cost associated with internally shipped quantities are neither necessary nor sufficient for an internal discount to be desirable. However, if a discount is found to be desirable, then we can show that it increases in the cost differential.

4 Efficiency

This section examines efficiency properties of market-based transfer pricing, given that the optimal discount is employed as derived above. In particular, can one find conditions under which the expected firm profit comes close to the first-best level, i.e., the profit that could be realized if the central office would observe θ and if it would determine both quantities in a centralized fashion? To analyze this question, it is instructive to distinguish two potential sources of inefficiencies that may arise.

First, there is an *information problem* because the center does not know θ when determining the discount Δ: a discount that is optimal in expectation over all states of the world will likely be suboptimal for any specific realization of θ. This problem is particularly severe if the functions $a_i(\theta)$ and $a_e(\theta)$ are negatively correlated. Suppose, for instance, that $\theta \in \{\underline{\theta}, \overline{\theta}\}$ and $a_i(\underline{\theta}) < a_e(\underline{\theta})$ while $a_i(\overline{\theta}) > a_e(\overline{\theta})$, i.e., depending on θ, either of the reservation prices may be higher. If $\theta = \underline{\theta}$, then, ex post, the discount should be higher than if $\theta = \overline{\theta}$, all else equal. However, since Δ is set ex ante, it will take an intermediate value of the two discounts that would be optimal ex post. If, on the other hand, θ impacts the two demand functions and the seller's unit cost in a positively correlated fashion, then Proposition 5 below demonstrates that the information problem becomes less severe, as the same discount may work well in many states of the world.

Secondly, there is an *incentive problem* as the supplying division's objective will generally not be perfectly aligned with that of the firm. To illustrate, consider the following scenario: the center can observe θ, but for exogenous reasons it is confined to imposing an internal discount as opposed to implementing the optimal quantities or prices directly. The discount

merely prescribes the difference between external and internal price. However, for any Δ and θ, the supplier will charge an external price that is too high from a firm-wide perspective, because his divisional marginal revenues from internally transferred units exceed the firm-wide marginal revenues at $p(\Delta, \theta)$:[4]

$$\begin{aligned} \frac{\partial}{\partial p}\left[(p-\Delta)\cdot Q_i(p-\Delta,\theta)\right]\Big|_{p(\Delta,\theta)} &= \alpha_i(\theta) - 2\beta_i(\theta)\cdot[p(\Delta,\theta)-\Delta] \\ > \qquad \frac{\partial}{\partial p} R_i(Q_i(p-\Delta,\theta),\theta)\Big|_{p(\Delta,\theta)} &= -\beta_i(\theta)\cdot[p(\Delta,\theta)-\Delta]. \end{aligned}$$

The following assumption describes a setting where θ shifts the reservation prices and the marginal cost in an identical additive fashion (e.g., an additive price shock), without affecting the slopes of the demand functions.

Assumption (A3): $a_e(\theta) = a_e + \psi(\theta)$, $a_i(\theta) = a_i + \psi(\theta)$, $c(\theta) = c + \psi(\theta)$, $b_e(\theta) \equiv b_e$ and $b_i(\theta) \equiv b_i$ for some function $\psi : \Theta \to \mathbb{R}$.

Proposition 5 below states conditions under which both the information and the incentive problem become negligible, so that properly adjusted market-based transfer pricing approaches first-best performance. Let Π^* and Π^m, respectively, denote first-best expected firm profit and expected firm profit under market-based transfer pricing.

Proposition 5 *Suppose both demand functions are linear, there are no cost differentials, and (A2) and (A3) hold. Then for any $\varepsilon > 0$, there exists a value a_i sufficiently close to c, so that $\Pi^m = \Pi^* - \varepsilon$.*

[4] Roughly speaking, the divergence of the objectives between supplying division and the firm arises from the fact that, for a given discount Δ, the supplier will set the external price p so as to move p close to $p_e^m(\theta)$ and the transfer price $p - \Delta$ close to $p_i^m(\theta)$. From the firm-wide perspective, p should be set as close as possible to $p_e^m(\theta)$, but the transfer price should approximate $c(\theta)$, where $c(\theta) < p_i^m(\theta)$.

Assumption (A3) ensures that the internal discount is sufficient to overcome the uncertainty problem, as θ shifts all relevant functions in a similar fashion.[5]

— Insert figure 2 here —

Furthermore, for any internal reservation price a_i close to c, there exists a corresponding value b_i small enough (or, equivalently, $\beta_i = Q_i'(\cdot)$ sufficiently large) to ensure that (A2) is satisfied. In the limit, as ε approaches zero, $\beta_i \to \infty$, and one might think of a scenario where the buying division sells a final product in a highly competitive market. The unit net realizable value for this final product is very small, but the quantity that can be sold is sufficiently high, so that serving this market is still in the firm's best interest. The discount then allows the selling division to move the external price close to $p_e^m(\theta)$ while charging a transfer price close to $p_i^m(\theta)$ with $p_i^m(\theta) \to c(\theta)$. As a consequence, the incentive problem becomes negligible in the limit.

The preceding analysis has shown that an unconditional discount in general does not result in profit-maximizing decisions. One might conjecture that a discount $\Delta(p)$, which is itself a function of the price charged externally, might achieve this benchmark. However, in general this turns out not to be the case. Consider a simple example where (a_e, a_i, b_e, b_i) are all independent of θ, i.e., there is only cost uncertainty as expressed by $c(\theta)$.[6] Given an optimal discount function $\Delta(p)$, the following conditions have to be satisfied simultaneously

[5] As is shown on the proof of Proposition 5, the optimal discount for given θ, denoted by $\Delta^*(\theta)$, is independent of θ. Therefore, $\Delta^*(\theta) \equiv \Delta^*$, where Δ^* is the value derived in Proposition 4.

[6] In this simple example, the price $p(\Delta(\cdot), \theta)$ turns out to be invertible with respect to θ for any function $\Delta(\cdot)$. Note that this is generally not the case, as different vectors $(a_e(\theta), a_i(\theta), b_e(\theta), b_i(\theta), c(\theta))$ can induce the supplier to charge an identical external price, $p(\Delta(\cdot), \theta)$, given a discount function $\Delta(\cdot)$. This is despite the fact that each of these vectors would call for a different discount.

to ensure profit maximization: for any θ,

$$p(\Delta(\cdot),\theta) \in \arg\max_p \{(p-\Delta(p)-c(\theta))\cdot Q_i(p-\Delta(p))+(p-c(\theta))\cdot Q_e(p)\}, \quad (10)$$

$$\text{while} \quad : \quad p = p_e^m(\theta) \quad \text{and} \quad p-\Delta(p) = c(\theta). \quad (11)$$

Using (11), the necessary first-order condition for the supplier's objective function in (10) can be reduced to

$$[1-\Delta'(p(\Delta(\cdot),\theta))]\cdot(a_i - c(\theta)) = 0.$$

For this to be the case, $\Delta'(p(\cdot)) = 1$ must hold. At the same time, however, if marginal cost $c(\theta)$ increases by an amount of \$1, due to a variation in θ, the optimal transfer price increases by \$1, whereas the external monopoly price, $p_e^m(\theta) = \frac{a_e+c(\theta)}{2}$ increases by 50 cents. Since $\Delta(p) = p - TP(p)$, profit maximization requires $\Delta'(p) = -\frac{1}{2}$, a contradiction.

5 Minimum Discounts

As argued above, the ex-ante optimal discount Δ^* will often severely distort trade for many realizations of θ, if $a_i(\theta)$ and $a_e(\theta)$ are negatively correlated. The firm can to some extent mitigate this information problem by having the corporate controller fix a *minimum discount*. The supplying division then has the flexibility to unilaterally grant the internal buyer a greater, discretionary, discount after θ has been realized. For any realization of θ, the discount that maximizes the supplier's profit equals

$$\Delta_s(\theta) = p_e^m(\theta) - p_i^m(\theta) = \frac{1}{2}[a_e(\theta) - a_i(\theta)].$$

For any given ex-ante discount Δ and θ, the seller will exercise his option to grant the buyer the greater discount whenever $\Delta_s(\theta) > \Delta$. Let $\Theta(\Delta) \equiv \{\theta | \Delta_s(\theta) < \Delta\}$. The ex-post

discount then equals $\Delta^{flex}(\Delta^{***}, \theta) = \max\{\Delta^{***}, \Delta_s(\theta)\}$, where[7]

$$\Delta^{***} \in \arg\max_{\Delta} \quad \left\{ \int_{\Theta(\Delta)} \left[R_e(Q_e(p(\Delta,\theta),\theta),\theta) + R_i(Q_i(p(\Delta,\theta)-\Delta,\theta),\theta) \right] f(\theta) d\theta \right.$$
$$\left. + \int_{\Theta \setminus \Theta(\Delta)} \left[R_e(Q_e(p_e^m(\theta),\theta),\theta) + R_i(Q_i(p_i^m(\theta),\theta),\theta) \right] f(\theta) d\theta \right\}.$$

If the state of the world is such that the internal reservation price $a_i(\theta)$ is sufficiently low so that $\theta \notin \Theta(\Delta^{***})$, then the supplier will want to exercise the option to grant the internal buyer a discount $\Delta_s(\theta) > \Delta^{***}$, thereby being able to better extract the external customers' surplus. We will refer to the optimal discount derived in Proposition 4 as the "optimal *fixed* discount".

Proposition 6 *Suppose both demand functions are linear, there are no cost differentials, and (A2) holds. Then the optimal minimum discount is lower than the optimal fixed discount:* $\Delta^{***} \leq \Delta^{*}$.

The optimal fixed discount Δ^* derived in Proposition 4 trades off the risk of having too high a discount in case that, ex post, the external demand turns out to be low, versus the risk of having too low a discount, in case that the internal demand is low. If the supplier has the flexibility to grant the buying division a higher discretionary discount, then the latter risk becomes less severe, so that $\Delta^{***} \leq \Delta^*$.

Corollary to Proposition 6: *The expected firm profit is higher if the central office determines a minimum discount than if it determines a fixed discount.*

[7] As above, we can ignore the effect a discount has on total production cost, since the total production quantity does not vary in Δ.

The corollary is an immediate consequence of the above discussion: if the central office simply sticks to the optimal fixed discount Δ^*, then both divisions would be better off if the supplier has the option to offer a greater discount. By revealed preference, this a fortiori holds for the optimal minimum discount Δ^{***}.

6 Concluding Remarks

Appendix

Proof of Proposition 1:

We first demonstrate that

$$(i) \qquad p(\Delta,\theta) \geq p(0,\theta) \geq p(\Delta,\theta) - \Delta \ .$$

By definition, $p(0,\theta)$ maximizes Division 1's profit:

$$\pi_1(p,\Delta{=}0,\theta) \equiv \pi_i(p,\theta) + \pi_e(p,\theta) \equiv (p - c(\theta)) \cdot Q_i(p,\theta) + (p - c(\theta)) \cdot Q_e(p,\theta) \ .$$

The corresponding first-order condition is

$$(ii) \qquad \pi_i'(p(0,\theta),\theta) + \pi_i'(p(0,\theta),\theta) = 0 \ .$$

Since $p_e^m(\theta) > p_i^m(\theta)$ for all θ, and

$$\pi_i'(p_i^m(\theta),\theta) = \pi_e'(p_e^m(\theta),\theta) = 0 \ ,$$

concavity of $\pi_i(\cdot,\theta)$ and $\pi_e(\cdot,\theta)$ by (A1) implies

$$(iii) \qquad p_i^m(\theta) < p(0,\theta) < p_e^m(\theta)$$

and

$$(iv) \qquad \pi_e'(p(0,\theta),\theta) > 0 > \pi_i'(p(0,\theta),\theta) \ .$$

Consider now a discount $\Delta \leq k$ and suppose that, contrary to the claim in *(i)*, $p(0,\theta) \geq p(\Delta,\theta) > p(\Delta,\theta) - \Delta$. By definition, $p(\Delta,\theta)$ maximizes

$$(p - \Delta - c) \cdot Q_i(p - \Delta,\theta) + (p - c) \cdot Q_e(p,\theta),$$

and, therefore,

$$(v) \qquad \pi_i'(s(\Delta,\theta),\theta) + \pi_e'(p(\Delta,\theta),\theta) = 0$$

where $s(\Delta,\theta) \equiv p(\Delta,\theta) - \Delta$. If it were true that $p(0,\theta) > p(\Delta,\theta) > s(\Delta,\theta)$, then the concavity of $\pi_i(\cdot,\theta)$ and $\pi_e(\cdot,\theta)$ would imply

$$\pi_i'(s(\Delta,\theta),\theta) + \pi_e'(p(\Delta,\theta),\theta) < \pi_i'(p(0,\theta),\theta) + \pi_e'(p(0,\theta),\theta) = 0.$$

That, however, would contradict *(v)*. A parallel argument shows that it would be impossible to have $p(\Delta,\theta) > p(\Delta,\theta) - \Delta \geq p(0,\theta)$. Thus, we have established *(i)*.

From *(i)* it follows that a marginal internal discount makes the buyer weakly better off, because the transfer price he actually pays is non-increasing in Δ, as expressed in the second inequality in *(i)*. To show that the supplying division also benefits from an internal discount, we differentiate the seller's profit function

$$\pi_1(\Delta,\theta) \equiv \pi_1(p(\Delta,\theta),\Delta,\theta) \equiv \pi_i(p(\Delta,\theta) - \Delta,\theta) + \pi_e(p(\Delta,\theta),\theta)$$

with respect to Δ at $\Delta = 0$. Using the Envelope Theorem, we find

$$\left.\frac{d\pi_1}{d\Delta}\right|_{\Delta=0} = -\pi_i'(p(0,\theta),\theta).$$

Using *(iii)* and *(iv)* now yields $\pi_i'(p(0,\theta),\theta) \leq 0$, completing the proof of Proposition 1.

■

Proof of Proposition 2: In the absence of an internal discount, the supplier will set a price $p(0,\theta) \in [p_e^m(\theta), \bar{p}_i(\theta)]$, if $p_e^m(\theta) \leq \bar{p}_i(\theta)$, for all θ. (Notice that the supplier's profit from selling internally, $(p - c(\theta)) \cdot Q_i(p,\theta)$, is weakly concave.) Following the same arguments

as in the proof of Proposition 1, for any small positive discount Δ, the resulting external price set by the seller satisfies $p(\Delta,\theta) \geq p(0,\theta) \geq p(\Delta,\theta) - \Delta$. Thus, the internal quantity remains constant at $Q_i(p(\Delta,\theta) - \Delta, \theta) = Q_i(p(0,\theta),\theta) = \bar{q}_i(\theta)$, whereas the external price is pushed farther above the profit-maximizing level. By strict concavity of the external profit function, $(p - c(\theta)) \cdot Q_e(p,\theta)$, the firm-wide profit thus decreases in Δ, for any θ. ■

Proof of Proposition 3: We first show that in the linear case

$$p(\Delta,\theta) = p(0,\theta) + \nu(\theta) \cdot \Delta \ ,$$

where $\nu(\theta) = \frac{\beta_i(\theta)}{\beta_i(\theta)+\beta_e(\theta)}$. Given any discount Δ, the selling divisions solves the problem

$$\max_P \{(p - \Delta - c(\theta)) \cdot Q_i(p - \Delta, \theta) + (p - c(\theta)) \cdot Q_e(p,\theta)\} \ .$$

The optimal $p(\Delta,\theta)$ satisfies:

$$\begin{aligned} \alpha_i(\theta) - \beta_i(\theta) \cdot [p(\Delta,\theta) - \Delta] - \beta_i(\theta) \cdot [p(\Delta,\theta) - \Delta - c(\theta)] & \\ +\alpha_e(\theta) - \beta_e(\theta) \cdot p(\Delta,\theta) - \beta_e(\theta) \cdot [p(\Delta,\theta) - c(\theta)] &= 0 \ . \end{aligned}$$

Solving for $p(\Delta,\theta)$ yields:

$$p(\Delta,\theta) = \frac{1}{2}\left[\frac{\alpha_i(\theta) + \alpha_e(\theta)}{\beta_i(\theta) + \beta_e(\theta)} + c(\theta)\right] + \frac{\beta_i(\theta)}{\beta_e(\theta) + \beta_i(\theta)} \cdot \Delta \equiv p(0,\theta) + \nu(\theta) \cdot \Delta \ .$$

To demonstrate the desirability of a positive discount, we note that firm-wide profit is given by

$$\begin{aligned} \pi(\Delta,\theta) &= R_i(Q_i(p(\Delta,\theta) - \Delta,\theta),\theta) - c(\theta) \cdot Q_i(p(\Delta,\theta) - \Delta,\theta) \\ &\quad + R_e(Q_e(p(\Delta,\theta),\theta),\theta) - c(\theta) \cdot Q_e(p(\Delta,\theta),\theta) \ . \end{aligned}$$

It suffices to show that

$$\pi'(\Delta,\theta)|_{\Delta=0} > 0$$

for all θ. From equation (5) in the text, we know that in the linear setting

$$\begin{aligned}\pi'(0,\theta) &= [R_i'(Q_i(p(0,\theta),\theta),\theta) - c(\theta)] \cdot [-\beta_i(\theta)] \cdot [\nu(\theta) - 1] \\ &\quad + [R_e'(Q_e(p(0,\theta),\theta),\theta) - c(\theta)] \cdot [-\beta_e(\theta)] \cdot \nu(\theta) \ .\end{aligned}$$

Equivalently,

$$\pi'(0,\theta) = \beta_e(\theta) \cdot \nu(\theta) \cdot [R_i'(Q_i(p(0,\theta),\theta),\theta) - R_e'(Q_e(p(0,\theta),\theta),\theta)]$$

Given the transfer price $p(0,\theta)$, Division 2 will demand the quantity $Q_i(p(0,\theta),\theta)$ which satisfies

$$R_i'(Q_i(p(0,\theta),\theta),\theta) = p(0,\theta).$$

Therefore, $\pi'(0,\theta) > 0$, if:

$$p(0,\theta) - R_e'(Q_e(p(0,\theta),\theta),\theta) > 0.$$

By definition

$$\begin{aligned}R_e'(Q_e(p(0,\theta),\theta),\theta) &= a_e(\theta) - 2b_e(\theta) \cdot [\alpha_e(\theta) - \beta_e(\theta) \cdot p(0,\theta)] \\ &= a_e(\theta) - 2a_e(\theta) + 2p(0,\theta) \ .\end{aligned}$$

Thus,

$$\pi'(0,\theta) = \beta_e(\theta) \cdot \nu(\theta)[a_e(\theta) + p(0,\theta)]$$

which is always positive due to the assumption that both markets will be served, i.e., assumption (A2). That completes the proof of Proposition 3. ■

Proof of Proposition 4: Proceeding exactly as in the proof of Proposition 3, we find that for a given θ, $\pi'(\Delta, \theta) = 0$ whenever

$$R_i'(Q_i(s(\Delta, \theta), \theta), \theta) = R_e'(Q_e(p(\Delta, \theta), \theta), \theta)$$

with $s(\Delta, \theta) = p(\Delta, \theta) - \Delta$. Since in equilibrium the buying division chooses $Q_i(\cdot)$ so that $R_i'(Q_i(s(\Delta, \theta), \theta), \theta) = s(\Delta, \theta)$, it follows that

$$s(\Delta, \theta) = -a_e(\theta) + 2 \cdot p(\Delta, \theta).$$

From the proof of Proposition 3, we recall that: $p(\Delta, \theta) = p(0, \theta) + \nu(\theta) \cdot \Delta$, with $\nu(\theta) = \frac{\beta_i(\theta)}{\beta_i(\theta) + \beta_e(\theta)}$. Furthermore, $\pi'(\Delta, \theta)$ is linear in Δ with:

$$\pi'(\Delta, \theta) = \beta_e(\theta) \cdot \nu(\theta) \cdot [a_e(\theta) - p(0, \theta) - (1 + \nu(\theta)) \cdot \Delta] \ .$$

Since the optimal discount Δ^* satisfies[8]

$$E_\theta[\pi'(\Delta^*, \theta)] = 0.$$

Therefore,

$$\Delta^* = \frac{E_\theta[\phi_1(\theta)]}{E_\theta[\phi_2(\theta)]} \ ,$$

where $\phi_1(\theta) \equiv \beta_e(\theta) \cdot \nu(\theta) \cdot [a_e(\theta) - p(0, \theta)]$ and $\phi_2(\theta) \equiv \beta_e(\theta) \cdot \nu(\theta) \cdot (1 + \nu(\theta))$. ■

(Sketch of) Proof of Proposition 5:

Let $\Delta^*(\theta)$ denote the ex-post optimal discount from a firm-wide perspective, i.e., the discount the corporate controller would implement, if he could observe θ. Proposition 5 is proved in two steps:

[8] We are making the implicit assumption here that it is possible to differentiate "under the integral." One sufficient condition is that the probability distribution over the set of states θ has a continuous density function.

Claim 1: *If (A3) holds, then $\Delta^*(\theta) \equiv \Delta^*$, where Δ^* is the value derived in Proposition 4.*

Proof of Claim 1: Since $\Delta^*(\theta)$ is the "point-wise version" of the ex-ante optimal discount Δ^* derived in Proposition 4, we have

$$(vi) \qquad \Delta^*(\theta) = \frac{\beta_i \cdot \nu \cdot [a_e(\theta) - p(0,\theta)]}{\beta_i \cdot \nu \cdot (1+\nu)} = \frac{a_e(\theta) - p(0,\theta)}{1+\nu},$$

where assumption (A3) ensures that ν is independent of θ with $\nu = \frac{\beta_i}{\beta_i+\beta_e} = \frac{b_e}{b_i+b_e}$. The weighted monopoly price $p(0,\theta)$ equals

$$p(0,\theta) = \frac{b_e a_i(\theta) + b_i a_e(\theta) + (b_i+b_e)c(\theta)}{2(b_i+b_e)} = \frac{b_e a_i + b_i a_e + (b_i+b_e)c}{2(b_i+b_e)} + \psi(\theta).$$

Using this expression, the ex-post optimal discount turns out to be independent of θ:

$$(vii) \qquad \Delta^*(\theta) = \frac{(b_i+2b_e)a_e - b_e a_i - (b_i+b_e)c}{2(b_i+2b_e)} \equiv \Delta^*.$$

Hence, given (A3), there is no loss in basing the internal discount on the prior distribution of θ alone.

Claim 2: *Suppose that (A2) and (A3) hold and that the optimal discount Δ^* is implemented. Then $\Pi^m \to \Pi^*$, as $a_i \to c$.*

Proof of Claim 2: Two conditions must be satisfied for Claim 2 to hold: in the limit,

$$(viii) \qquad \lim_{a_i \to c} \Delta^* = p_e^m(\theta) - c(\theta) = \frac{a_e - c}{2}$$

and

$$(ix) \qquad \lim_{a_i \to c} p(\Delta^*, \theta) = p_e^m(\theta) = \frac{a_e + c}{2} + \psi(\theta)$$

First note that, for any a_i close to c, there exists a value b_i sufficiently small, so that assumption (A2) is satisfied. This critical value is obtained by rewriting (A2), which requires

that $p(0,\theta) < a_i(\theta)$, for all θ, in the following fashion:

$$b_i < \hat{b}_i(a_i) = \frac{b_e(a_i(\theta) - c(\theta))}{a_e(\theta) - 2a_i(\theta) + c(\theta)} = \frac{b_e(a_i - c)}{a_e - 2a_i + c}.$$

For $a_i \to c$, we find that $\hat{b}_i(a_i) \to 0$, $\nu \to 1$, and, using *(vii)*, we obtain $\Delta^* \to \frac{a_e - c}{2}$. That is, *(viii)* is satisfied. Finally:

$$\begin{aligned}\lim_{a_i \to c} p(\Delta^*, \theta) &= \lim_{a_i \to c}[p(0,\theta) + \nu \cdot \Delta^*] = \lim_{a_i \to c} p(0,\theta) + \lim_{a_i \to c} \Delta^* \\ &= c + \psi(\theta) + \frac{a_e - c}{2} = \frac{a_e + c}{2} + \psi(\theta) = p_e^m(\theta).\end{aligned}$$

Hence, condition *(ix)* is satisfied as well, which completes the proof of Proposition 5. ■

Proof of Proposition 6: By definition, Δ^* maximizes the expected firm profit, if the supplying division has to stick to the ex-ante determined discount:

$$(x) \qquad E_\theta\{\pi'(\Delta^*, \theta)\} = E_\theta\left\{\frac{1}{b_e(\theta) + b_i(\theta)}[a_e(\theta) - p(0,\theta) - (1 + \nu(\theta)) \cdot \Delta^*]\right\} = 0.$$

Note that $\pi''(\Delta, \theta) < 0$, for any Δ and θ, i.e., the firm profit is strictly concave in Δ. At the same time, Δ^{***} maximizes the firm's expected profit, given that the supplier can grant a higher discount. Formally, $\Delta^{***} \in \arg\max_\Delta E_\theta\{\pi(\Delta^{flex}(\Delta,\theta),\theta)\}$, where

$$\begin{aligned}E_\theta\{\pi(\Delta^{flex}(\Delta,\theta),\theta)\} \equiv & \int_{\Theta(\Delta)} [R_e(Q_e(p(\Delta,\theta),\theta),\theta) + R_i(Q_i(p(\Delta,\theta) - \Delta,\theta),\theta) \\ & -c(\theta) \cdot [Q_e(p(\Delta,\theta),\theta) + Q_i(p(\Delta,\theta) - \Delta,\theta)]]\, f(\theta) d\theta \\ & + \int_{\Theta \setminus \Theta(\Delta)} [R_e(Q_e(p_e^m(\theta),\theta),\theta) + R_i(Q_i(p_i^m(\theta),\theta),\theta) \\ & -c(\theta) \cdot [Q_e(p_e^m(\theta),\theta) + Q_i(p_i^m(\theta),\theta)]]\, f(\theta) d\theta,\end{aligned}$$

with $\Delta^{flex}(\Delta,\theta) = \max\{\Delta, \Delta_s(\theta)\}$ and $\Theta(\Delta) = \left\{\theta \,\middle|\, \Delta_s(\theta) = \frac{1}{2}[a_e(\theta) - a_i(\theta) < \Delta\right\}$. That is,

$$(xi) \qquad \left.\frac{\partial E_\theta\left\{\pi(\Delta^{flex}(\Delta,\theta),\theta)\right\}}{\partial \Delta}\right|_{\Delta = \Delta^{***}} = 0.$$

Now suppose that, contrary to our claim, the minimum discount Δ^{***} would exceed Δ^{*}. Then, $\Delta^{flex}(\Delta^{***}, \theta) \geq \Delta^{***} > \Delta^{*}$, for all θ. Thus, by strict concavity of $\pi(\Delta, \theta)$, it follows that

$$\pi'(\Delta^{*}, \theta) > \left. \frac{\partial \pi(\Delta^{flex}(\Delta, \theta), \theta)}{\partial \Delta} \right|_{\Delta = \Delta^{***}},$$

point-wise for all θ. Therefore, using (x), we have

$$0 = E_{\theta}\{\pi'(\Delta^{*}, \theta)\} > \left. \frac{\partial E_{\theta} \left\{ \pi(\Delta^{flex}(\Delta, \theta), \theta) \right\}}{\partial \Delta} \right|_{\Delta = \Delta^{***}},$$

which contradicts (xi). This completes the proof of Proposition 6. ■

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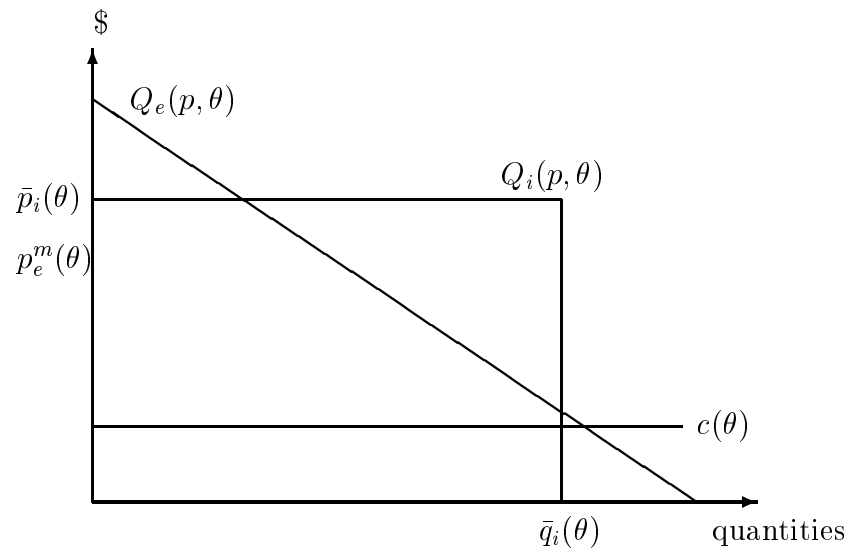

Figure 1: The "fixed-quantity scenario"

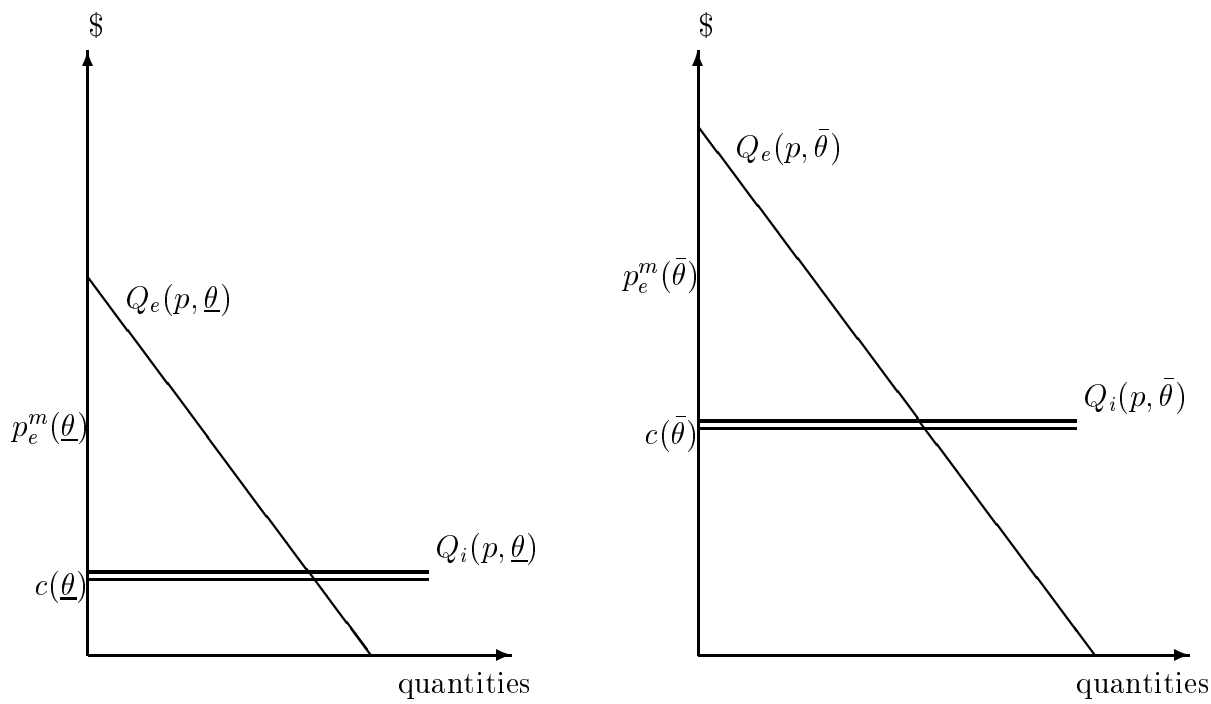

Figure 2: Discount approximates efficient trade ($\theta \in \{\underline{\theta}, \bar{\theta}\}$)