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A model of a multilateral proxy war with spillovers

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Abstract

Motivated by the war in Syria and the ascension of ISIS, this paper models a proxy war with three sponsors and three combatants as a dynamic game. Sponsors are leaders that provide resources for combatants to fight each other. Sponsors 1 and 2 have strong aversion to sponsor 3's proxy, but not against each other. Three pure strategy equilibria exist in the game. When the *ex post* value of winning is small, all players fight in equilibrium. However, when the *ex post* value of winning is large, in equilibrium either sponsors 1 and 2 coordinate their actions, with one of them staying out of the contest, or sponsor 3 does not participate. The probability of winning and the sponsors' payoffs depend on a spillover effect. We find that no unique way of characterizing the comparative statics of the spillover effect emerges and that the answer varies from one equilibrium to another. Finally, we identify conditions under which sponsors 1 and 2 would want to form an alliance.

1 Introduction

The purpose of this study is to use a conflict model to understand the growth and evolution of multilateral proxy wars, such as in Syria. The story of that war undoubtedly was dominated by ISIS (Dodd 2016).[1] It arguably is the most powerful Islamic terrorist group the world has ever seen.[2] ISIS emerged from the Syrian civil war, spread to Iraq and grew rapidly into a powerful army of combatants, massacring enemies and amassing billions of dollars selling oil from its conquered territories. Its human and financial resources are now being used to launch terrorist attacks in Europe (e.g., in France and Belgium) and the United States (Dodd 2016).[3] The actual and potential costs of ISIS are considerable.[4] Understanding the conflict and milieu that generated ISIS is crucial to formulating a winning strategy against it and to prevent similar recurrences in the future.[5] We hope that the insights of this paper will be useful to countries such as the United States as they deliberate on the best ways to intervene in the Syrian conflict. Those considerations are particularly important at this moment because of the recent change in the US administration.

The Syrian civil war is not confined to two opposing sides – pro and anti-government. Syria's conflict involves multiple agents, separated by religious, ethnic and political differences (and at a deeper level it is related to family and patronage loyalties). It involves three major confessional groups – two Islamic (Sunni, who comprise 74% of the population; Shia – and minor sects, such as

[1] On terrorist spectaculars, see Arce (2010), Hoffman (2006) and Enders and Sandler (2012).

[2] On the growth of Islamic terrorism, see Enders and Sandler (2000) and Barros and Proenca (2005).

[3] On ISIS, see Wood (2015).

[4] For an estimate of the economic impact of terrorism and conflicts on income per capita growth, see Gaibulloev and Sandler (2009). For the macroeconomic impacts of terrorism, see Blomberg et al. (2004) and Tavares (2004).

[5] Phares (2005) discusses the information war on Jihadism in academia and media, wherein Jihadists and their supporters actively participate, attempting to divert it, camouflage it, and move it in different directions.

Alawite and Druze, who comprise 16% of the population), plus Christians (10% of the population); different ethnic groups – both Arab (90%) and non-Arab (10%);[6] and three broad political groups – the pro-establishment dictatorship of the Baath party, democrats, and Islamic fundamentalists. Each of those combatant groups has one or more sponsors: the United States and western allies, supporting political opposition to the Syrian Assad regime (who they assume think like western liberals); Russia and Iran, supporting the Assad regime (which is thought to represent Syrian Shia, Allawite, Druze, and Christians); and, finally, a loose army of international Muslim Sunni volunteers (an important part composed of Muslim Europeans), supporting and fighting for ISIS.

Syria clearly is plagued by communal cleavages. Throughout its history Syria never was an independent country with a uniform ethnic-religious-political composition. After its independence from France in 1946 it struggled to find an identity. The Baath party gave Syria a much needed identity, with an ideology that is a mixture of Arab nationalism and socialism. It has a Bolshevik type of organization that permeates all regions, cities, villages, institutions and groups of Syrian society. The Baath party and the army are the only institutions that allowed a significant share of Syrian society the opportunity for social mobility, in particular for religious minorities, such as the Allawites, which ended up forming the Army's elite.

General Hafiz al-Assad's 30-year dictatorship (1970-2000) was able to consolidate power because Assad had political influence inside the Baath party and control of the Army (Hinnebusch 2014). The only major segment of Syrian society that was not under governmental influence was the poor Sunni majority, profoundly influenced by the Islamic fundamentalism of the Muslim Brotherhood (Zollner 2009) and at odds with the secular and socialist outlook of the regime. Hafiz al-Assad

[6] On conflict in dual population lands, see Levy and Faria (2007).

fought and prevailed over the Islamic fundamentalist uprising during the 1978-1982 period. In spite of timid reforms, the regime of his son Bashar al-Assad, was not able to accommodate either liberal or Islamic fundamentalist opposition. Tensions escalated until the civil war broke out in 2011.

With the spread of the Syrian conflict to Iraq, it is important to recognize that Iraq also is a divided country with different religious confessions – Sunni and Shia – and significant non-Arab minority populations (e.g., Kurds). The sponsors in Iraq are Iran (for the Shia) and Saudi Arabia (Arab Sunni). Kurds and Christian minorities apparently have only the goodwill and wishful thinking of western powers on which to rely. Therefore, this conflict involves multiple combatants and sponsors.

The rise of ISIS and the civil conflict in Syria and Iraq illustrate some important facts about conflicts, as stressed by Salehyan (2010). First, international war is a rare event. Warfare conducted directly by antagonist states, with foreign aggression in the form of military attacks by state armies, constitutes a small fraction of interstate conflicts. Second, governments substitute rebel patronage for direct fighting in such conflicts. External support for rebel organizations is more common than direct state-to state fighting. Third, several civil wars involve agents with explicit ties to foreign powers as well as the use of militias by governments, which offer logistical and political benefits in the form of reduced liability for violence (Carey et al. 2015; Staniland 2015).[7]

These facts about international conflict and civil war have several important consequences. One of them is that factors normally considered to limit conflict, such as international trade, democratic institutions and international organizations (e.g., Bueno de Mesquita et al. 1999; Russett and Oneal 2001) play smaller roles. Another important consequence is that the conflict models based on

[7]For studies on armed groups switching sides during civil wars see Staniland (2012) and Otto (2017).

strategic interactions among pairs of states and incentives to engage in war (e.g., Fey and Ramsay 2007; Filson and Werner 2002) lose relevance.

Last, but not least, the facts enumerated above shed new light on civil war, highlighting additional factors, namely Islamic terrorism and foreign power sponsorship, which have not yet been considered in depth. It is well known that civil war is a powerful engine of poverty creation: it plagues poor countries and reduces income (Collier and Sambanis 2002),[8] which warrant the importance of its study and understanding. Most of the recent literature on civil war has emphasized grievance or greed (e.g, Collier and Hoeffler 2002; Reynal-Querol 2002) as the cause of violence; informational and commitment problems to explain the lengths of conflict (e.g., Azam 2002; Genicot and Skaperdas 2002); the conflict trap that sees civil violence as path-dependent, where past conflicts cause present and future ones (e.g., Murdoch and Sandler 2002a and 2002b; Blomberg and Hess 2002); and targeting ethnic minorities as a common strategy (e.g., Sambanis 2001; Fearon and Laitin 2003; Blimes 2006; Tir and Jasinski 2008).

In the Syrian and Iraqi civil wars, all of those factors are important. Greed is associated with oil production and oil rents, grievance with the repression of groups without political voice; and a desperate need for state building with a combination of laws, formal and informal institutions to solve commitment problems. Both countries have recent histories of past unsolved conflicts that linger on and histories of deeply rooted ethnic strife. However, the role of sponsor states and the rise of Islamic terrorism are in our view the main elements that provoked and fueled the actual

[8] A microeconomic study by Collier and Duponchel (2013) shows that during conflict, violence affects production through a form of technical regress and demand through a reduction in income. For an overview of disaggregated studies and the micro-dynamics of individual civil wars, see Cederman and Gleditsch (2009).

conflicts. This paper argues that any prospect of peace should address and understand their roles and investigate possible scenarios with or without their participation.[9]

The objective of this paper is to formulate and analyze a dynamic game with multiple combatants and sponsors. With the conflict in Syria as motivation, the model includes three combatants and three sponsors. The game is a proxy war in which the combatants fight on behalf of their sponsors. Given the prominence of ISIS, we assume that sponsors 1 and 2 have a strong aversion to sponsor 3's proxy. Because of that strong aversion, we assume the existence of positive spillovers between sponsors 1 and 2. Our theoretical results identify conflict scenarios and suggest strategies to achieve peace.

The paper is organized as follows. We review the literature in Section 2. We then present a two-stage dynamic game in Section 3. The solution of the model appears in Sections 4 and 5. Concluding remarks appear in Section 6.

2 Related literature

Research on proxy wars evolved gradually from the study of civil wars after it was observed that many civil wars in the modern era are not strictly domestic conflicts because of the involvement of foreign governments or groups. A common observation is that some civil wars have strong contagion effects. Gleditsch, Salehyan and Schultz (2008) provide an explanation of a circumstance in which a civil war mutates into an international war. Suppose a civil war breaks out in one country

[9] Sorli et al.'s (2005) study of Middle East conflict finds ethnic dominance to be a significant determinant of violence. In addition, the authors find that economic development and economic growth, in addition to longer periods of peace, generally reduce the likelihood of conflict.

over an issue that affects a foreign country. In that case, it is natural for the foreign country to try to affect the outcome of the civil war and this leads to a conflict between the two nations. Gleditsch and Beardsley (2004) examine the influence of foreign actors on civil wars in three Latin American countries and find that foreign actors can affect the level of cooperation between domestic adversaries.

An important question about proxy wars is why countries sometimes use proxies instead of fighting among themselves directly. That question was examined in Salehyan (2010). A country that is involved in a dispute with another country may use violence to improve its bargaining power in any future negotiations. Using proxies is a relatively cheap way of fomenting violence in a foreign country. Several reasons for using proxies can be given: First, the sponsor country does not suffer any casualties. Second, the sponsor can deny its involvement and escape international sanctions. Third, proxies may have better local knowledge of the terrain or the people living there. In a subsequent paper, Salehyan, Gleditsch and Cunningham (2011) ask the reverse question: Why is it that some rebel groups receive help from abroad and others do not? They show that the answer hinges on the characteristics of the rebel groups, such as their strength and their linkages with foreign constituencies.

Since prior research already has explained why a proxy war is preferable to direct participation in a conflict; we therefore do not pursue that point in our paper. However, we find that understanding how a proxy war might culminate contains a large gap. The end stage of such conflicts is the focus of the present paper. We show that several different outcomes are possible, depending on the parameter values and also depending on the characteristics of the equilibrium. Another aspect of civil wars, such as the one in Syria, is important, but has not been studied in the literature, namely, that a particular combatant may not have the same degree of aversion towards the other combatants. Suppose there are three combatants, $1, 2$ and 3. It is possible that 1 dislikes 3 more than 2. Hence, it would rather lose to 2 than to 3. Under some circumstances, that asymmetry opens the possibility of cooperation (maybe tacit) between 1 and 2.

War often is modelled as a bargaining problem (e.g., Anderton and Carter 2009; Baliga and Sjostrom 2013), following Schelling's (1966) observation that military power is a special case of bargaining power. According to Brito and Intrilligator (1985), conflicts happen because of incomplete information, which prevents rational agents from negotiating side payments. Jackson and Morelli (2007) show that cases exist in which political bias leads to war, regardless of the sizes of any transfer payments. Corchon and Yildizparlak (2013) study a war game in which conflict can occur even with little asymmetric information. In our context, if one considers ISIS to be an aggressor, then no doubt exists that ISIS is not bluffing – appeasement is not an option, but neither are transfer payments. Every player knows ISIS's nature and the type of combatant it is. War is unavoidable. Thus, our model is one of complete information, in which the distribution of resources is heterogeneous.[10]

A contest success function (CSF) (see, for example, Hwang 2012) is an essential tool for analyzing conflicts.[11] We consider a CSF of Tullock (1980) ratio form (Choudhury and Sheremeta (2011), in which the winning probability is a function of the aggregate level of effective efforts in the contest – that is, it depends on combatants' efforts and sponsors' resources. As a consequence, it is not a simple lottery CSF, but is in line with the generalization of a CSF (e.g., Dixit 1987;

[10] Bevia and Corchon (2010) examine a complete information war game in which the initial distribution of resources is heterogeneous.

[11] For surveys of contest games, see Corchon (2007) and Corchon and Serena (2018).

Hirshleifer 1989; Skaperdas 1996) in which the CSF is weighted (e.g., Dahm and Porteiro 2008; Brown 2011) by sponsors' resources.

The game is a dynamic contest (see Konrad 2012), albeit a simple one played in only two periods. Generally, two-period dynamic contest games consider scenarios wherein the first period's effort has a positive effect on the probability of winning in the second period (Sela 2011; Moller 2012; Beviá and Corchón 2013). In our setup, the sequential contest is played by different players in periods 1 and 2. In the first period, it is played by the leaders (i.e., the sponsors) and in the second period by the followers (i.e., the combatants). As a result, combatants exert effort as a function of the sponsors' resources. In our game, sponsors and combatants form a team.[12]

In our model, because sponsors 1 and 2 have strong aversion to sponsor 3's proxy, we assume that they generate positive spillovers between them. If the proxy of sponsor 1 loses, it would prefer combatant 2 to win rather than 3. Similarly, if sponsor 2's proxy loses, it would prefer combatant 1 to win rather than 3. Externalities recently have been a focus of the conflict literature. Identity dependent externalities have been considered in the case of Tullock contest by Linster (1993) and in the case of all-pay auctions by Klose and Kovenock (2015). Faria et al. (2017) analyze two types of externalities – temporal and spatial – in a dynamic game between two national governments that fight a common terrorist organization. They show that when governments take into account terrorists' reactions both domestically and abroad, both terrorism and (costly) counterterror policies are reduced, irrespective of the nature of the policy externality. Oliveira et al. (2018) examine the link between coalition formation, counterterrorism (CT) and spillovers. In a symmetric model,

[12]Katz et al. (1990) analyze group contests wherein groups vary in their numbers of members. They find that when the members are identical, all groups exert the same aggregate effort regardless of asymmetries in group size.

CT and terrorism decline with the size of the externality regardless of the degree of cooperation between nations. In the asymmetric model, as the externality of the "smaller" nation increases, the "larger" nations reduce their efforts, and the smaller nation reacts by increasing its own efforts.

In our model the probability of winning and the sponsors' payoffs vary as functions of the sizes of the spillover effect. However, no unique way of characterizing the comparative statics of the spillover effect can be found; it varies from one equilibrium to another.

The contest prize in our model is victory, which is a unique prize. In the existing literature the prize in a winner-take-all contest generally leads players to display the highest possible level of effort, while a multiple-prize contest induces a general increase in players' efforts (e.g., Moldovanu and Sela 2001). In our setup the value of winning, whether it is large or small, is crucial for determining which kind of equilibrium holds.

3 Model

Consider a contest with hree sponsors s_i and three combatants c_i, for $i = 1, 2, 3$. Sponsor s_i supports combatant c_i. The game is played in two periods. In period 1, each sponsor s_i $(i = 1, 2, 3)$ decides simultaneously and independently how many resources $r_i \geq 0$ to provide to his corresponding combatant c_i. Examples of such resources include arms and training. The total cost of providing resources is given by

$$tr_i^2;\, t > 0.$$

In period 2, each combatant observes (r_1, r_2, r_3) and simultaneously and independently chooses his effort x_i in the contest. The *effective effort* that combatant i exerts is $r_i x_i$. Notice that for a combatant, the resources provided by the sponsor are a complementary input because it changes the productivity of the combatant. If s_i chooses $r_i = 0$, then the effective effort of c_i will be 0 for any chosen value of $x_i \geq 0$.

Let p_i be the probability that s_i and c_i together win the contest, ndetermined as follows:

$$p_i = \frac{r_i x_i}{\theta}$$

where

$$\theta = \sum_{i=1}^{3} r_i x_i \tag{1}$$

is the aggregate effective effort in the contest. Notice that p_i is not defined if for all i either $r_i = 0$ or $x_i = 0$. In such a case, the probability that each sponsor wins is assumed to be $\frac{1}{3}$. The payoff to c_i is given by the following function:

$$U_i = p_i - x_i. \tag{2}$$

Our specification of the payoff functions of the combatants is complementary to the approach of Boudreau, Rentschler and Sanders (2017). We assume that the combatants have different productivities of effort, but the same cost, while Boudreau et al. assume that the combatants have the same productivities but different effort costs.

In the contest at hand, each sponsor prefers to win. However, the interesting aspect of the proposed scenario is that sponsors do not have the same degree of aversion to the proxy agents of their rivals. In particular, s_1 and s_2 do not dislike each other's proxy as much as they dislike c_3. Therefore, if s_1's proxy loses, it would prefer c_2 to win rather than c_3. Similarly, if s_2's proxy loses, it would prefer c_1 to win rather than c_3. Sponsor s_3, however, cares only about whether its proxy wins or loses.

Bevia and Corchon (2013) consider a two-period contest between two players and endogenize the strength of a player in a given period. We also endogenize strength and, following Bevia and Corchon (2013), r_i can be interpreted as combatant i's strength in our model. However, we allow sponsors to act in the first period and combatants to act in the next one. Consequently, the strength of a combatant depends on its sponsor's past actions. In contrast, in Bevia and Corchon (2013), a combatant's strength depends on its own actions and the past actions of its rival.

This model mimics the case of the Syrian civil war. Combatants c_1, c_2 and c_3 are, respectively, similar to the Assad regime, the opposition known as the Syrian National Coalition, and ISIS (Zorthian 2015). The Assad regime is supported by Russia and Iran, who therefore are analogous to s_1. The Syrian National Coalition is supported by the United States and other western nations, who therefore can be though of as s_2. Regarding ISIS, the group is partly self-financed (Gause 2014) and partly supported by private donations (Rogin 2014).[13] Therefore s_3 is similar to a set of private donors.

Among the belligerents, ISIS is the most hated and feared group. According to Gause (2014), ISIS ".has the unique ability to unite most of the players in the new Middle East cold war against it. Iran and Iran's allies detest it because of its fiercely anti-Shia ideology. The Saudis fear it as a potential domestic threat, turning Salafism into a revolutionary political ideology rather than the pro-regime bulwark it has usually been in Saudi Arabia. Turkey, the Kurds, the United States, the EU and Russia all stand to lose if ISIS wins." In order to capture that feature of the Syrian civil

[13] It has been alleged that ISIS also has state sponsors, but to the best of our knowledge that conjecture has not been established clearly. We thereforeavoid that issue.

war, we assume that the payoff of sponsor s_i is given by the following function:

$$W_i = (p_i + \beta p_j)\, V - tr_i^2 \text{ if } i,\, j = 1, 2 \tag{3}$$

and by

$$W_3 = p_3 V - tr_3^2 \tag{4}$$

for sponsor s_3, with $\beta \in [0, 1]$. Note that s_1 and s_2 earn their lowest payoffs of 0 when s_3 wins.[14] The parameter V is the *ex post* value of winning to a sponsor. Finally, the parameter β captures the positive externality – or spillover effect – that s_i enjoys if s_j wins the contest, for $i, j = 1, 2$, $i \neq j$. The spillover effect implies that s_i $(i = 1, 2)$ can earn a positive payoff even if it does not participate in the contest. Thus, one impact of the spillover effect is that it reduces the incentives for s_1 and s_2 to participate in the contest.

We solve the game using backward induction, beginning with period 2. The notations used in the paper are summarized in Table A.1 in the Appendix.

4 Period 2: Combat stage

In period 2, the combatants determine their effort levels. At that stage, c_i solves the following problem:

$$\max \quad U_i.$$

$$x_i$$

Two kinds of equilibria possible in stage 2, depending up r_1, r_2 and r_3.

[14]In that case, it is assumed that s_1 and s_2 do not provide any resources to the combatants.

4.1 All three combatants compete actively

An equilibrium in which all three combatants participate is described by the following proposition.

Proposition 1 *Suppose that r_1, r_2 and r_3 satisfy the following inequalities:*

$$r_i \geq \frac{r_j r_k}{r_j + r_k};\ i, j, k = 1, 2, 3;\ i \neq j \text{ or } k. \tag{5}$$

In equilibrium, the effort levels of the combatants in period 2 are

$$x_i^* \left(r_1, r_2, r_3\right) = \frac{\left(r_i - \theta^*\right) \theta^*}{r_i^2}, \tag{6}$$

and the aggregate effective effort is

$$\theta^* = \frac{2 r_i r_j r_k}{r_i r_j + r_j r_k + r_i r_k}. \tag{7}$$

Proof. See the Appendix. ■

In this equilibrium, (5) guarantees that $x_i^* \left(r_1, r_2, r_3\right) \geq 0$ and $U_i \geq 0$ for $i = 1, 2, 3$. Notice that (5) can be rewritten as follows:

$$\frac{1}{r_j} + \frac{1}{r_k} \geq \frac{1}{r_i}. \tag{8}$$

Equation (8) means that given r_j and r_k, the value of r_i cannot be too small for it to be the case that all three combatants compete actively.

Let us consider how the aggregate effective effort θ^* changes when $r_i$increaases. It follows from (7) that

$$\frac{\partial \theta^*}{\partial r_i} = \frac{1}{2 r_i^2} \theta^{*2} > 0. \tag{9}$$

That is, everything else remaining constant, an increase in r_i increases the aggregate effective effort. Notice from Proposition 1 that the probability of winning of c_i $(i = 1, 2)$ is

$$p_i^* = 1 - \frac{\theta^*}{r_i}. \tag{10}$$

Hence, it can also be shown that

$$\frac{\partial p_i^*}{\partial r_i} = -\frac{\partial}{\partial r_i}\left(\frac{\theta^*}{r_i}\right) = \frac{1}{r_i}\left(\frac{r_j + r_k}{r_i r_j + r_j r_k + r_i r_k}\right)\theta^* > 0. \tag{11}$$

We now examine the effect of a change in r_i on the effort levels of the combatants and their winning probabilities. In order to do so, let us define the elasticity of combat effort with respect to the amounts of resources supplied by sponsors as follows:

$$E_{x,r} = \frac{\partial x_i^*}{\partial r_i}\frac{r_i}{x_i^*}.$$

We now have the following result:

Corollary 1 *Consider the subgame in which all contestants participate. In equilibrium, the elasticity of combat effort with respect to resources is given by the following:*

$$E_{x,r} = \left(1 - 2p_i^*\right).$$

Proof. See the Appendix. ■

An important point to note from Corollary 1 is that an increase in r_i does not always increase c_i's effort level x_i^*. When sponsor s_i allocates more resources to its proxy, then c_i responds with more effort when its probability of winning the contest is small enough. However, c_i chooses to cut back on its effort x_i^* when its probability of winning is high enough.

4.2 One combatant drops out (and the other two remain)

In this subsection, we consider equilibria of the subgame in which combatant k drops out while i and j remain active. Such equilibria are described by the following proposition.

Proposition 2 *Suppose that r_i, r_j and r_k satisfy the following inequalities:*

$$r_k < \frac{r_i r_j}{r_i + r_j}. \tag{12}$$

In equilibrium, the effort levels of the combatants in period 2 are:

$$x_i^{**} = x_j^{**} = \frac{r_i r_j}{(r_i + r_j)^2} \text{ for } i = 1, 2,$$

and

$$x_k^{**} = 0,$$

and the aggregate effective effort is

$$\theta^{**} = \frac{r_i r_j}{r_i + r_j}.$$

Proof. See the Appendix. ■

Notice from Proposition 2 that the probability of winning for c_i ($i = 1, 2, 3$) is

$$p_i^{**} = 1 - \frac{\theta^{**}}{r_i} = \frac{r_i}{r_i + r_j}. \tag{13}$$

Therefore, everything else remaining constant, p_i^{**} increases in r_i.

Also notice that (12) can be rewritten as follows:

$$\frac{1}{r_i} + \frac{1}{r_j} < \frac{1}{r_k}.$$

That inequality will be satisfied if r_k is sufficiently small. Hence, if s_k provides a relatively small amount of resources, then c_k prefers not to fight at all. The results of this section can be used to test the following hypothesis empirically:

H1: If a sponsor commits a small amount of resources, then the corresponding combatant does not fight at all.

5 Period 1: Resource allocation stage

Having fully characterized the subgame equilibrium choices of effort levels by the combatants, in this section we determine the equilibrium values of r_i, for $i = 1, 2, 3$. In particular, we are interested in identifying subgame perfect Nash equilibria in which each sponsor selects a pure strategy in the first period. We show that three types of equilibria in pure strategies are possible. In the spirit of subgame perfection, each equilibrium depends on the outcome in period 2. The major results in this section are summarized in Table A.2 in the Appendix.

5.1 Type 1 equilibrium: All three combatants compete actively in period 2

The payoff function of sponsor s_i $(i = 1, 2)$ is given by (3) and of s_3 is given by (4). The equilibrium levels of resource allocation are described in the proposition below, with the following notation: $\Delta = 1 + \sqrt{1 + 8(1 - \beta)}$.

Proposition 3 *Consider an equilibrium in which all three combatants participate in period* 2*. In period* 1 *of such an equilibrium,* s_i *(*$i = 1, 2$*) provides resources of*

$$r_1^* = r_2^* = r^* = \frac{V}{t}\frac{2\Delta^2}{(8 + \Delta)^2} \tag{14}$$

and s_3 *provides resources of*

$$r_3^* = \frac{V}{t}\frac{8\Delta}{(8 + \Delta)^2}. \tag{15}$$

The subsequent effort levels of the combatants in period 2 *are given by Proposition 1.*

Proof. See the Appendix. ■

In this equilibrium, the winning probabilities are given by

$$p_i^* = \frac{\Delta}{8+\Delta} \text{ for } i = 1, 2,$$

and

$$p_3^* = \frac{8-\Delta}{8+\Delta} = 1 - 2p_i^*;\ i = 1, 2.$$

The aggregate effective effort is therefore given by

$$\theta^* = \frac{256\Delta}{32+4\Delta}\frac{V}{t}.$$

When $\beta = 0$, then $\Delta = 4$, in which case $p_1^* = p_2^* = p_3^* = \frac{1}{3}$. Furthermore, since $\frac{\partial\Delta}{\partial\beta} < 0$, it follows that

$$\frac{\partial p_i^*}{\partial\beta} < 0 \text{ for } i = 1, 2 \text{ and } \frac{\partial p_3^*}{\partial\beta} > 0. \tag{16}$$

Therefore, the larger is the spillover effect, the smaller are the winning probabilities of sponsors 1 and 2, and the larger is the winning probability of s_3. When $\beta = 1$, then $\Delta = 2$, in which case $p_1^* = p_2^* = 0.2$ and $p_3^* = 0.6$.

Also $\frac{\partial\theta^*}{\partial\beta} < 0$, that is, the aggregate effective effort declines in the spillover effect. The aggregate effective effort captures the extent of damage from the proxy war. This result shows that the spillover effect serves to reduce the extent of damage from war. When the spillover effect increases, then c_1 and c_2 reduce their efforts while c_3 increases its effort. However, the result indicates that the increase in c_3's effort remains less than the aggregate reduction in its rivals' efforts.

Below, we list two hypotheses that follow from the discussion above and can be validated empirically:

H2: Everything else remaining constant, if multiple sponsors participate in the proxy war as in the Type 1 equilibrium, the winning probability of the most hated sponsor increases in β.

H3: Everything else remaining constant, if multiple sponsors participate in the proxy war as in the Type 1 equilibrium, the intensity of the war (captured by θ) declines in β.

Let us examine conditions under which the equilibrium is valid. In the type 1 equilibrium, all three combatants compete actively in period 2. requiring (5) to hold, which in turn requires $\Delta \leq 8$. Since the maximum possible value of Δ is 4, the condition is satisfied. Therefore, if the combatants are provided the equilibrium level of resources, then each of them prefers to participate in the contest. However, that alone does not guarantee that the type 1 equilibrium occurs, because it is possible that a sponsor might find it preferable not to participate in the contest. Below, we examine the equilibrium payoffs of each sponsor and find conditions under which each of them prefers to participate in the contest.

5.1.1 Payoffs of sponsors

The equilibrium payoffs of sponsors - derived using (3), (4) and Proposition 3 - are presented in the corollary below:

Corollary 2 *In the type* 1 *equilibrium, the payoffs of* s_1, s_2 *and* s_3 *are:*

$$W_i^* = p_i^* \left\{ 1 + \beta - 4p_i^{*3} \frac{V}{t} \right\} V \text{ } for \text{ } i = 1, 2$$

and

$$W_3^* = \left\{ 1 - 2p_i^* - p_i^{*2} \left(1 - p_i^*\right)^2 \frac{V}{t} \right\} V.$$

Let us first consider the participation decision of s_i ($i = 1, 2$). If the sponsor does not participate in the contest (and the others do), then its expected payoff will be

$$\frac{p_j^*}{p_j^* + p_3^*}\beta V;\ j = 1, 2,\ j \neq i. \tag{17}$$

In the above expression, $\frac{p_j^*}{p_j^*+p_3^*}$ is the conditional probability that s_j will win the contest if s_i pulls out and in that case s_i will enjoy the spillover effect.[15] Now, using the facts that $p_j^* + p_3^* = 1 - p_i^*$ and $p_j^* = p_i^*$, we can rewrite the outside option of s_i as $\frac{p_i^*}{1-p_i^*}\beta V$ for $i = 1, 2$. The participation constraint of s_i is therefore

$$W_i^* \geq \frac{p_i^*}{1 - p_i^*}\beta V \text{ for } i = 1, 2. \tag{18}$$

Note that the values of the outside options for s_1 and s_2 increase in V and β. The participation constraint of s_3 is simply $W_3^* \geq 0$. The conditions under which these participation constraints are satisfied are shown in the left panel of Figure 1. Notice that the participation constraint is satisfied when the *ex post* value of winning V is relatively small or if the spillover effect β is relatively large.

Another question is the relationship between the equilibrium payoffs of the sponsors and the spillover effect β. It is interesting to note that this relationship need not be monotonic. To see that, consider the right panel of Figure 1, which illustrates the equilibrium payoffs of the sponsors when $\frac{V}{t} = 2$. Notice that W_i^* first increases with β and then decreases. The intuition is as follows. When β increases, then s_i gains from the spillover effect. However, the problem is that s_j also gains from the spillover effect, which induces s_j to reduce its expenditure. Such a reduction now hurts s_i. As long as the former effect is the stronger one, an increase in β increases W_i^*. However, when the latter effect is the stronger one, an increase in β reduces W_i^*.

[15] For details, see the Appendix.

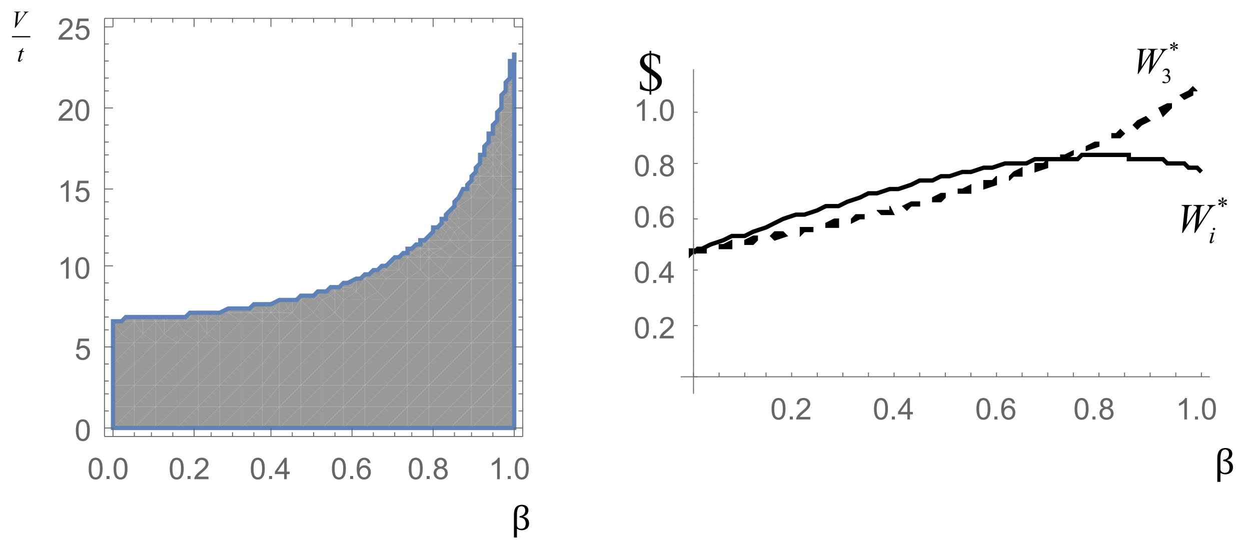

Figure 1: (a) The shaded region in the left panel shows the combinations of β and $\frac{V}{t}$ for which the type 1 equilibrium exists. In this case, the participation constraint of each sponsor is satisfied. (b) The right panel shows the payoffs of sponsor i ($i = 1, 2$) and sponsor 3 in the type 1 equilibrium for $\frac{V}{t} = 2$. The payoff function of sponsor i is non-monotonic in β.

5.1.2 Payoffs of combatants

The equilibrium payoffs of the combatants can be derived by plugging the values of r_i from Proposition 3 into the payoff function of the combatants, given by (2). It can be shown that in the type 1 equilibrium, the payoffs of the combatants are as follows:

$$U_i = \begin{cases} p_i^* \left[1 - p_i^{*2}\left(1 - p_i^*\right)\frac{V}{t}\right] & \text{if} \quad i = 1, 2, \\ p_3^{*2} & \text{if} \quad i = 3. \end{cases}$$

If a combatant does not participate in the contest, then its payoff is 0. Therefore, in equilibrium, every combatant must earn at least 0. To check if a combatant earns at least 0 in equilibrium, consider the payoff function of c_i given in (2). Notice that $U_i > 0$ as long as $p_i^* > x_i^*$. Since $p_i^* = \frac{r_i^* x_i^*}{\theta^*}$, this exporession ultimately reduces to a requirement that (5) is satisfied (which we have argued always is true, since $\Delta \leq 4$).

5.2 Type 2 equilibrium: Either c_1 or c_2 drops out (and others stay) in period 2

Let us now consider the equilibrium in period 1 that induces only c_i ($i = 1$ or 2) to drop out in period 2. The equilibrium is presented in the proposition below.

Proposition 4 *Consider the equilibrium in which only c_i $(i = 1, 2)$ drops out in period 2. In period 1 of such an equilibrium, s_j $(j = 1, 2;\ j \neq i)$ and s_3 provide resources of*

$$r_j^{**} = r_3^{**} = \sqrt{\frac{1}{8}\frac{V}{t}} \quad \textit{for } j = 1, 2,\ j \neq i.$$

The subsequent effort levels of the combatants in period 2 are given by Proposition 2.

Proof. See the Appendix. ■

Using (12), it can be shown that an equilibrium exists only if the following condition holds:

$$\frac{V}{t} \geq 32, \tag{19}$$

implying that when the *ex post* value of winning is sufficiently large, then s_1 and s_2 coordinate their actions such that only one of them fights. However, such coordination is not possible when the *ex post* value of winning is small. Therefore, we have the following empirically testable hypothesis:

H4: Everything else remaining constant, coordination is more likely if the value of winning V is higher.

5.2.1 Payoffs of sponsors

The equilibrium payoffs of sponsors in the type 2 equilibrium are presented in the following corollary.

Corollary 3 *Suppose that in the type* 2 *equilibrium,* s_i *drops out and* s_j *participates in the contest* $(i, j = 1, 2; i \neq j)$*. In that case, the payoffs of the sponsors are:*

$$W_i^{**} = \frac{1}{2}\beta V$$

and

$$W_j^{**} = W_3^{**} = \frac{3}{8}V.$$

In equilibrium, the payoffs of the participants is independent of β, while the payoff of the non-participant increases in β. Furthermore, we have the curious result that $W_i^{**} > W_j^{**}$ for $\beta > \frac{3}{4}$ – revealing that strong incentives to stay out of the contest arise when β is sufficiently large, since s_i realizes a larger payoff from dropping out than s_j realizes from participating.

5.2.2 Payoffs of combatants

The equilibrium payoffs of the combatants can be derived by plugging the results from Proposition 4 into the payoff function of the combatants, given by (2). Suppose that sponsor s_i decides not to participate in the contest. In the absence of resources from s_i, the corresponding combatant c_i likewise will not participate in the contest. It can be shown that in the type 2 equilibrium, the payoffs of the combatants are:

$$U_k = \begin{cases} \frac{1}{4} & \text{if} \quad k = j, 3, \\ 0 & \text{if} \quad k = i. \end{cases}$$

Notice that the payoffs of the combatants are independent of β in the type 2 equilibrium.

5.3 Type 3 equilibrium: c_3 drops out (and others stay) in period 2

Let us now consider the equilibrium in period 1 that induces only c_3 to drop out in period 2. The equilibrium is presented in the proposition below.

Proposition 5 *Consider the equilibrium in which only c_3 drops out in period 2. In period 1 of such an equilibrium, s_i $(i = 1, 2)$ provides resources of:*

$$r_i^{***} = r^{***} = \sqrt{\frac{(1-\beta)}{8} \frac{V}{t}} \text{ for } i = 1, 2.$$

The subsequent effort levels of the combatants in period 2 are given by Proposition 2.

Proof. See the Appendix. ∎

When $\beta = 0$, the resources spent by each participant in equilibrium are $\sqrt{\frac{1}{8}\frac{V}{t}}$, which is the same as the expenditure in the type 2 equilibrium. Consequently, when $\beta = 0$ the type 2 equilibrium and the type 3 equilibrium are the same. However, meaningful differences can be found between them in the presence of spillover effects.

Let us first check the validity of the type 3 equilibrium. It was discussed above in Proposition 2 that c_3 drops out if the following condition holds: $r_1 + r_2 \leq r_1 r_2$. Using the equilibrium values of r_1 and r_2 from Proposition 5, it follows that the following condition must hold for the type 3 equilibrium to exist:

$$\frac{V}{t}(1-\beta) \geq 32. \tag{20}$$

Note that (20) is a more restrictive condition than (19). When β is sufficiently large, then (20) is violated. In such a case, the type 3 equilibrium cannot exist.

5.3.1 Payoffs of sponsors

Below, we present the payoffs of the sponsors in the type 3 equilibrium.

Corollary 4 *In the type* 3 *equilibrium, the payoffs of* s_1*,* s_2 *and* s_3 *are as follows:*

$$W_1^{***} = W_2^{***} = \left(\frac{3+5\beta}{8}\right) V$$

and

$$W_3^{***} = 0.$$

We also can compare the aggregate payoffs of s_1 and s_2 when both type 2 and type 3 equilibria exist. Notice that $\left(\frac{3+5\beta}{8}\right) V > \max\left\{\frac{1}{2}\beta V,\ \frac{3}{8}V\right\}$. Hence, both s_1 and s_2 are better off in the type 3 equilibrium than in the type 2 equilibrium. In contrast, s_3 is worse off in the type 3 equilibrium.

5.3.2 Payoffs of combatants

The equilibrium payoffs of the combatants can be derived by plugging the results from Proposition 5 into the payoff function of the combatants, given by (2). It can be shown that in the type

3 equilibrium, the payoffs of the combatants are:

$$U_i = \begin{cases} \frac{1}{4} & \text{if} \quad i = 1, 2, \\ 0 & \text{if} \quad i = 3. \end{cases}$$

Notice that the payoff of c_1 and c_2 are independent of β in the type 3 equilibrium.

We also compare the payoffs of c_1 and c_2 in the type 2 and type 3 equilibria. To make a meaningful comparison, we restrict ourselves to the case in which (20) holds. The payoff of c_i ($i = 1, 2$) is $\frac{1}{4}$ in the type 3 equilibrium and is either $\frac{1}{4}$ or 0 in the type 2 equilibrium. Hence, if (19) holds, then c_1 and c_2 both weakly prefer the type 3 equilibrium over the type 2 equilibrium. It is easy to see that c_3 prefers the type 2 equilibrium over the type 3 equilibrium.

5.4 Alliance formation

In this paper, we have allowed for implicit coordination between s_1 and s_2 (in the Type 2 equilibrium), but have not considered a formal alliance between them. Below, we discuss the impact of such an alliance. This analysis is an adaptation of Sanchez-Pages (2007) to our context.

For this analysis, we introduce a new stage: period 0 in which $s_1$and s_2 decide whether or not to form an alliance. If they decide to form an alliance, then in period 1 the alliance and s_3 simultaneously and independently choose their resource expenditures and in period 2 the corresponding combatants choose their efforts. It is assumed that if s_1 and s_2 decide to form an alliance, then they can commit to it. One way to ensure such commitment is to impose a large penalty if a sponsor chooses to leave the alliance after its formation. If s_1 and s_2 decide not to form the alliance, the rest of the game has the same structure as in the previous sections. In that case, each sponsor chooses his resource expenditure in period 1 and each combatant chooses his effort in period 2 .

Let us examine the equilibrium if an alliance is formed in period 0. Let W_A denote the alliance's aggregate payoff. It is given by

$$W_A = p_A (1+\beta) V - tr_A^2,$$

where p_A is the probability of the alliance winning the contest and r_A is the total amount of resources that the sponsors provide. It follows from (13) that $p_A = \frac{r_A}{r_A + r_3}$ and $p_3 = \frac{r_3}{r_A + r_3}$.

The alliance maximizes W_A with respect to r_A. In equilibrium, the probabilities of winning for the alliance and s_3, respectively, are $\hat{p}_A = \frac{\sqrt{1+\beta}}{1+\sqrt{1+\beta}}$ and $\hat{p}_3 = \frac{1}{1+\sqrt{1+\beta}}$. It can be shown that the equilibrium payoff of the alliance is

$$\hat{W}_A = (1+\beta)\, \hat{p}_A \,(1 - 0.5\hat{p}_3)\, V.$$

Let us now examine whether an alliance is viable. An alliance will not be viable if either s_1 or s_2 are better off going it alone. Suppose the equilibrium is of type 1 if s_1 and s_2 do not form an alliance. It follows from the left panel of Figure 1 that in this case $\frac{V}{t}$ must be sufficiently small (that is, must belong to the shaded area of that diagram). Also, the outside option of s_i $(i = 1, 2)$ is W_i^* (given in Corollary 2). In this case, an alliance is viable only if $\hat{W}_A \geq 2W_i^*$.[16] In the left panel of Figure 2, we compare $\hat{W}_A$ and $2W_i^*$ for $\frac{V}{t} = 3$. Notice that in this example, an alliance is not always viable if the outside option of each sponsor is given by the type 1 equilibrium.

Now suppose that $\frac{V}{t} \geq 32$. In that case, the type 2 equilibrium is the only equilbrium when $\frac{V}{t}(1-\beta) < 32$ and is one of two possible equilibria when $\frac{V}{t}(1-\beta) \geq 32$. For the type 2 equilibrium, s_1 and s_2 coordinate their actions with one of them staying out of the contest. It fol-

[16]Following Sanchez-Pages (2007), we do not explicitly discuss the division of the alliance's payoff among its individual constituents. However, one way to divide the aggregate payoff is to pay each constituent an amount equal to the outside option plus a fraction of the leftover surplus.

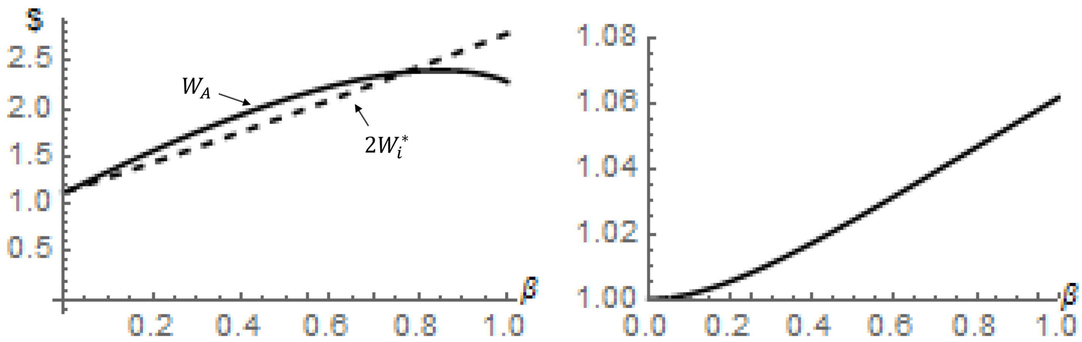

Figure 2: (a) The left panel compares the payoff of the alliance with the aggregate payoff of sponsors 1 and 2 in the type 1 equilibrium for $V = 3$ and $t = 1$. It follows from the plot that the payoff can go either way. (b) The right panel is the ratio of the payoff of the alliance to the aggregate payoff of sponsors 1 and 2 in the type 2 equilibrium. The payoff is higher for an alliance.

lows from Corollary 3 that under such implicit coordination, the aggregate payoff of s_1 and s_2 is $W_1^{**} + W_2^{**} = \frac{3+4\beta}{8}V$. The right panel of Figure 2 shows the plot of $\frac{\hat{W}_A}{W_1^{**}+W_2^{**}}$.[17] It follows that an alliance is viable in this case for all values of β.

Finally, suppose that $\frac{V}{t}(1-\beta) \geq 32$. In this case, a possible equilibrium is the type 3 one in which s_3 stays out of the contest. Using the same method as above, it can be shown that an alliance is never viable for all feasible values of β.

6 Concluding remarks

This model presented herein shows that a civil war fought by proxies might evolve in several different ways depending on the context of the war and the strategies followed by the belligerents. The context is captured in the model by parameters such as V (a sponsor's *ex post* value of winning), t (a scale parameter of the cost of sponsoring a proxy), and β (the magnitude of a spillover effect). This model illustrates how the likely outcome of a proxy war can depend critically upon the values of all of those different parameters.

In this paper, we consider only pure strategy equilibria. We identify conditions under which each type of equilibrium arises. We find several striking results. First, we show that an increase in the spillover effect can affect the payoffs of the sponsors non-monotonically. Second, we show that situations exist in which a sponsor who stays out of the contest is better off than a sponsor who participates. Third, we identify conditions under which sponsors might want to form an alliance. Fourth, we show that more resources from sponsors can induce their proxy combatants to exert more effort if their chances of winning are sufficiently low (below 50% in our model). However,

[17]We show this ratio since it is independent of V and t.

if that is not the case, combatants have a tendency to free-ride on their sponsors by exerting less effort. In the Type 1 equilibrium, the chance of winning can be greater than 50% only for combatant 3 (that is, only ISIS in the Syrian civil war). Therefore, only sponsor 3 can suffer from free-riding in that equilibrium.

The analysis adds to our understanding of proxy wars by characterizing the different kinds of possible equilibria. It shows that such wars can evolve in many non-obvious ways. The theoretical predictions can be tested. In our discussion of the various equilibria, we mentioned four hypotheses that can be confronted with data. Given our discussion of alliance formation, a fifth hypothesis is that for small values of $\frac{V}{t}$, s_1 and s_2 are unlikely to form an alliance if the spillover effect β is large. Our list is not exhaustive and we leave it to future researchers to identify other interesting hypotheses based on the paper at hand.

References

Anderton, C.H., & Carter, J.R. (2009). *Principles of conflict economics.* New York, NY: Cambridge University Press.

Arce, D. (2010). Terrorist spectaculars: Backlash attacks and the focus of intelligence. *Journal of Conflict Resolution*, 54, 354-373.

Azam, J-P. (2002). Looting and conflict between ethnoregional groups: Lessons for state formation in Africa. *Journal of Conflict Resolution*, 46, 131-153.

Baliga, S., & Sjostrom, T. (2013). Bargaining and war: A review of some formal models. *Korean Economic Review,* 29, 235-266.

Barros, C., & Proença, I. (2005). Mixed logit estimation of radical Islamic terrorism in Europe and North America: A comparative study. *Journal of Conflict Resolution,* 49, 298-314.

Beviá C., & Corchón, L.C. (2010). Peace agreements without commitment. *Games and Economic Behavior,* 68, 469-487.

Beviá, C., & Corchón, L.C. (2013). Endogenous strength in conflicts. *International Journal of Industrial Organization,* 31, 297-306.

Blimes, R. J. (2006). The indirect effect of ethnic heterogeneity on the likelihood of civil war onset. *Journal of Conflict Resolution,* 50, 536-547.

Blomberg, B. S., & Hess, G.D. (2002). The temporal links between conflict and economic activity. *Journal of Conflict Resolution,* 46, 74-90.

Blomberg, S.B., Hess, G.D., & Orphanides, A. (2004). The macroeconomic consequences of terrorism. *Journal of Monetary Economics,* 51, 1007-1032.

Boudreau, J. W., Rentschler, L. & Sanders, S. (2017). Stag hunt contests and alliance formation. Working Paper

Brito, D., & Intriligator, M. (1985). Conflict, war and redistribution. *American Political Science Review,* 79, 943-957.

Brown, J. (2011). Quitters never win: The (adverse) incentive effects of competing with superstars. *Journal of Political Economy,* 119, 982-1013.

Bueno de Mesquita, B., Morrow, J.D., Siverson R., & Smith, A. (1999). An institutional explanation of the democratic peace. *American Political Science Review,* 93, 791-808.

Carey, S., Colaresi, M.P., & Mitchell, N.J. (2015). Governments, informal links to militias, and accountability. *Journal of Conflict Resolution,* 59, 850-876.

Cederman, L-E., & Gleditsch, K.S. (2009). Introduction to special issue on "Disaggregating

Civil War". *Journal of Conflict Resolution,* 53, 487-495.

Choudhury, S.M., & Sheremeta, R.M. (2011). Multiple equilibria in Tullock contests. *Economics Letters,* 112, 216-219.

Collier, P., & Duponchel, M. (2013). The economic legacy of civil war. *Journal of Conflict Resolution,* 57, 65-88.

Collier, P., & Hoeffler, A. (2002). On the incidence of civil war in Africa. *Journal of Conflict Resolution* 46, 13-28.

Collier, P., & Sambanis, N. (2002). Understanding civil war. *Journal of Conflict Resolution*, 46, 3-12.

Corchón, L. C. (2007). The theory of contests: A survey. *Review of Economic Design*, 11, 69-100.

Corchón, L. C., & Yildizparlak, A. (2013). Give peace a chance: The effect of ownership and asymmetric Information on peace. *Journal of Economic Behavior & Organization*, 92, 116-126.

Corchon, L. C., & Serena, M. (2018). Contest Theory: A survey. In L.C. Corchon & M.A. Marini (Ed.s) *Handbook of Game Theory and Industrial Organization* (Vol II, 125-146). Northampton, MA: Edward Elgar Publishing.

Dahm, M., & Porteiro, N. (2008). Biased contests. *Public Choice*, 136, 55-67.

Dixit, A. (1987). Strategic behavior in contests. *American Economic Review*, 77, 891-898.

Dodd, V. (2016). Isis planning 'enormous and spectacular attacks', anti-terror chief warns, Accessed on April 20 2017 https://www.theguardian.com/uk-news/2016/mar/07/isis-planning-enormous-and-spectacular-attacks-uk-counter-terrorism-chief-warns.

Enders, W., and Sandler, T. (2000). Is transnational terrorism becoming more threatening? A time-series investigation. *Journal of Conflict Resolution*, 44, 307-332.

Enders, W., & Sandler, T. (2012). *The political economy of terrorism.* New York, NY: Cambridge University Press.

Faria, J.R., Silva, E., & Arce, A. (2017). Intertemporal versus spatial externalities in counterterror policy games. *Dynamic Games and Applications*, 7, 402-421.

Fearon J. D., & Laitin, D. D. (2003). Ethnicity, insurgency, and civil war. *American Political Science Review*, 97, 75–90.

Fey, M. & Ramsay, K. (2007). Mutual optimism and war. *American Journal of Political Science*, 51, 738-54.

Filson, D., & Werner, S. (2002). A bargaining model of war and peace: Anticipating the onset, duration, and outcome of war. *American Journal of Political Science*, 46, 819-38.

Gaibulloev, K., & Sandler, T. (2009). The impact of terrorism and conflicts on growth in Asia. *Economics & Politics*, 21, 359-383.

Gause, F. G. (2014). ISIS and the new Middle East cold war. *Brookings Institution*, Washington D.C., Accessed on January 18 2017 https://www.brookings.edu/blog/markaz/2014/08/25/isis-and-the-new-middle-east-cold-war/.

Genicot, G., & Skaperdas, S. (2002). Investing in conflict management. *Journal of Conflict Resolution*, 46, 154-170.

Gleditsch, K. S., & Beardsley, K. (2004). Nosy neighbors: Third-party actors in Central American conflicts. *Journal of Conflict Resolution*, 48, 379-402.

Gleditsch, K. S., Salehyan, I. & Schultz, K. (2008). Fighting at home, fighting abroad: How civil wars lead to international disputes. *Journal of Conflict Resolution*, 52, 479-506.

Hinnebusch, R. (2014). Syria, in E. Lust (Ed.) *The Middle East* (764-788). Washington: CQ Press.

Hirshleifer, J. (1989). Conflict and rent-seeking success functions: Ratio vs. difference models of relative success. *Public Choice*, 63, 101-112.

Hoffman, B. (2006). *Inside terrorism.* New York, NY: Columbia University Press.

Hwang, S. (2012). Technology of military conflict, military spending, and war. *Journal of Public Economics*, 96, 226–236.

Jackson, M., & Morelli, M. (2007). Political bias and war. *American Economic Review*, 97, 1353-1373.

Katz, E., Nitzan, S., & Rosenberg, J. (1990). Rent-Seeking for pure public goods. *Public Choice*, 65, 49-60.

Klose, B. & Kovenock, D. (2015). The all-pay auction with complete information and identity-dependent externalities. *Economic Theory*, 59, 1-19.

Konrad, K. (2012) Dynamic contests and the discouragement effect. *Revue d'Economie Politique*, 122, 233-256

Levy, A., & Faria, J.R. (2007). Ramsey in dual-population lands: Internal conflict and utility-maximizing consumption. *Defence and Peace Economics*, 18, 339-352.

Linster, B. G. (1993). A generalized model of rent-seeking behavior. *Public Choice*, 77, 421-435.

Moldovanu, B. & Sela, A., (2001). The optimal allocation of prizes in contests. *American Economic Review*, 91, 542-558

Moller, M. (2012). Incentives versus competitive balance. *Economics Letters*, 117, 505-508.

Murdoch J. C., & Sandler, T. (2002a). Civil wars and economic Growth: A regional comparison.

Defence and Peace Economics, 13, 451–64.

Murdoch J. C., & Sandler, T. (2002b). Economic growth, civil wars, and spatial spillovers. *Journal of Conflict Resolution*, 46, 91–110.

Oliveira, A.R., Faria, J.R. & Silva, E. (2018). Transnational terrorism: Externalities and coalition formation, *Journal of Conflict Resolution*, 62, 496-528.

Otto, S. (2017). The grass is always greener? Armed group side switching in civil wars. *Journal of Conflict Resolution*, forthcoming.

Phares, W. (2005). *Future jihad.* New York, NY: St. Martin's Griffin.

Reynal-Querol, M. (2002). Ethnicity, political systems, and civil wars. *Journal of Conflict Resolution*, 46, 29-54.

Rogin, J. (2014). America's allies are funding ISIS. *Daily Beast*, Accessed on January 18 2017 http://www.thedailybeast.com/articles/2014/06/14/america-s-allies-are-funding-isis.html.

Russett, B. & Oneal, J. (2001). *Triangulating peace: Democracy, interdependence, and international organizations.* New York, NY: Norton

Sambanis, N. (2001). Do ethnic and non-ethnic civil wars have the same causes? A theoretical and empirical inquiry (Part 1). *Journal of Conflict Resolution*, 45, 259-82.

Salehyan, I. (2010). The delegation of war to rebel organizations. *Journal of Conflict Resolution*, 54, 3, 493-515.

Salehyan, I., Gleditsch, K.S. & Cunningham, D.E. (2011). Explaining external support for insurgent groups. *International Organization*, 65, 709-744.

Sanchez-Pages, S. (2007). Endogenous coalition formation in contests. *Review of Economic Design,* 11, 139-163.

Schelling, T. (1966). *Arms and influence.* New Haven, CT: Yale University Press.

Sela, A. (2011). Best-of-three all-pay auctions. *Economics Letters*, 112, 67-70.

Sørli, M. E., Gleditsch, N.P. & Strand, H. (2005). Why is there so much conflict in the Middle East? *Journal of Conflict Resolution*, 49, 141-165.

Skaperdas, S. (1996). Contest success functions. *Economic Theory*, 7, 283-290.

Staniland, P. (2012). Between a rock and a hard place: Insurgent fratricide, ethnic defection, and the rise of pro-state paramilitaries. *Journal of Conflict Resolution*, 56, 16-40.

Staniland, P. (2015). Militias, ideology and the state. *Journal of Conflict Resolution*, 59, 770-793.

Tavares, J. (2004). The open society assesses its enemies: Shocks, disasters and terrorist attacks. *Journal of Monetary Economics*, 51, 1039-1070.

Tir, J. & Jasinski, M. (2008). Domestic-level diversionary theory of war: Targeting ethnic minorities. *Journal of Conflict Resolution* 52, 641-664.

Tullock, G. (1980). Efficient rent-seeking. In J.M. Buchanan, R.D. Tollison and G. Tullock (Eds.), *Toward a theory of the rent-seeking society* (97-112). College Station, TX: Texas A&M University Press.

Wood, G. (2015). What ISIS really wants?, *Atlantic*, Accessed on April 20 2017 http://www.theatlantic.com/magazine/archive/2015/03/what-isis-really-wants/384980/?utm_source=SFFBants.

Zollner, B. (2009). *The Muslim Brotherhood: Hasan Al-Hudaybi and ideology.* London: Palgrave.

Zorthian, J. (2015). Who's fighting who in Syria. *Time*, Accessed on January 18 2017 http://time.com/4059856/syria-civil-war-explainer/.

Electronic Appendix

Notation	Description
s_i	Sponsor i
c_i	Combatant i
r_i	Amount of resources provided by s_i to c_i
t	Scale parameter that determines the cost to a sponsor of providing resources
x_i	Effort of c_i in the contest
p_i	Probability that s_i and c_i together win the contest
θ	Aggregate effective effort in the contest. This captures the intensity of combat
V	*Ex post* value of winning of the sponsor
U_i	Payoff of c_i
W_i	Payoff of s_i
β	Positive spillover to s_j $(j = 1, 2; j \neq i)$ if s_i $(i = 1, 2)$ wins
$E_{x,r}$	Elasticity of combat effort with respect to the amount of resources
Δ	$1 + \sqrt{1 + 8(1 - \beta)}$
A	Alliance between s_1 and s_2

Table A.1: List of Notations

A Proof of Proposition 1

In the combat subgame, each combatant solves the following problem:

$$\max_{x_i} \quad U_i = \tfrac{r_i x_i}{\theta} - x_i$$

for $i = 1, 2, 3$. It follows from the above that

$$\frac{\partial U_i}{\partial x_i} = \frac{\theta r_i - r_i x_i \frac{\partial \theta}{\partial x_i}}{\theta^2} - 1 \text{ for } i = 1, 2, 3.$$

Since

$$\frac{\partial \theta}{\partial x_i} = r_i \text{ for } i = 1, 2, 3,$$

therefore,

$$\frac{\partial U_i}{\partial x_i} = \frac{r_i \sum_{j \neq i} r_j x_j}{\theta^2} - 1.$$

Also,

$$\frac{\partial^2 U_i}{\partial x_i^2} = -2 \frac{r_i^2 \sum_{j \neq i} r_j x_j}{\theta^3} < 0,$$

that is, U_i is a concave function of x_i. At the optimum, $\frac{\partial U_i}{\partial x_i} = 0$. Hence, at the optimum,

$$r_i \sum_{j \neq i} r_j x_j = \theta^2.$$

For convenience, we re-write the first order condition as follows:

$$r_i (\theta - r_i x_i) = \theta^2. \tag{21}$$

It follows from (21) that

$$r_i x_i = \theta \left(1 - \frac{\theta}{r_i}\right). \tag{22}$$

By substituting (22) into (1), we obtain the following:

$$\begin{aligned}\theta &= \sum_{i=1}^{3} r_i x_i \\ &= \theta \sum_{j\neq i} \left(1 - \frac{\theta}{r_i}\right).\end{aligned}$$

Hence,

$$1 = \sum_{j\neq i} \left(1 - \frac{\theta}{r_i}\right) = 3 - \theta \sum_{i=1}^{3} \frac{1}{r_i},$$

that is,

$$\theta = \frac{2}{\sum_{i=1}^{3} \frac{1}{r_i}}. \tag{23}$$

The above expression yields the equilibrium value of θ given in (7). The equilibrium value is denoted by θ^*.

The equilibrium effort level of x_i is therefore given by (22) and (23).

B Proof of Corollary 1

(a) We first determine the value of $\frac{\partial}{\partial r_i}(r_i x_i^*)$. It follows from (22) that

$$r_i^2 x_i^* = (r_i - \theta^*)\,\theta^*.$$

Differentiating both sides with respect to r_i and using (9), we obtain the following:

$$\begin{aligned}2r_i x_i + r_i^2 \frac{\partial x_i^*}{\partial r_i} &= \left(1 - \frac{\partial \theta^*}{\partial r_i}\right)\theta^* + (r_i - \theta^*)\,\frac{\partial \theta^*}{\partial r_i} \\ &= \theta^* + (r_i - 2\theta^*)\,\frac{\partial \theta^*}{\partial r_i}. \\ &= \theta^* + \frac{1}{2r_i^2}(r_i - 2\theta^*)\,\theta^{*2}.\end{aligned}$$

Now substitute the expression for x_i from (6) in the left hand side of the above expression to obtain the following:

$$2\frac{(r_i - \theta^*)\,\theta^*}{r_i} + r_i^2 \frac{\partial x_i^*}{\partial r_i} = \theta^* + \frac{1}{2r_i^2}\,(r_i - 2\theta^*)\,\theta^{*2}.$$

Finally, we substitute (10) in the above expression to obtain the following:

$$\frac{r_i^2}{\theta^*}\frac{\partial x_i^*}{\partial r_i} = p_i^*\,(1 - 2p_i^*)\,.$$

The left hand side of the above expression can be written in the following form:

$$p_i^* E_{x,r}.$$

Hence, we have the following result:

$$E_{x,r} = (1 - 2p_i^*)\,.$$

C Proof of Proposition 2

First consider combatants c_i and c_j. Following the same steps as in the proof of Proposition 1, the first order conditions can be shown to be the following:

$$\frac{\partial U_i}{\partial x_i} = \frac{r_i r_j x_j}{\theta^2} - 1 \;= 0 \tag{24}$$

and

$$\frac{\partial U_j}{\partial x_j} = \frac{r_i r_j x_i}{\theta^2} - 1 \;= 0.$$

Therefore, in equilibrium,

$$x_i = x_j = x.$$

Hence, the aggregate effective effort is given by

$$\theta = r_i x_i + r_j x_j = (r_i + r_j)\,x. \tag{25}$$

It follows from (24) and (25) that

$$r_i r_j x = (r_i + r_j)^2 x^2,$$

that is,

$$x_i^{**} = x_j^{**} = \frac{r_i r_j}{(r_i + r_j)^2}$$

and

$$\theta^{**} = \frac{r_i r_j}{r_i + r_j}.$$

We also need to check that

$$\frac{\partial U_k}{\partial x_k}|_{x_k=0} < 0.$$

It can be shown that when $x_i = x_i^{**}$ and $x_j = x_j^{**}$, then

$$\frac{\partial U_k}{\partial x_k}|_{x_k=0} = \frac{r_k}{\theta^{**}} - 1.$$

Hence,

$$\frac{\partial U_k}{\partial x_k}|_{x_k=0} < 0 \text{ if } r_k < \theta^{**} = \frac{r_i r_j}{r_i + r_j}.$$

Hence, the result follows.

D Proof of Proposition 3

First consider sponsors s_1 and s_2. The payoff function of these sponsors is given by (3). By differentiating this expression with respect to r_i, we obtain the following:

$$\frac{\partial W_i}{\partial r_i} = \left(\frac{\partial p_i^*}{\partial r_i} + \beta \frac{\partial p_j^*}{\partial r_i} \right) V - 2tr_i \text{ for } i = 1, 2;\ i \neq j.$$

In the above expression, the values of $\frac{\partial p_i^*}{\partial r_i}$ and $\frac{\partial p_j^*}{\partial r_i}$ can be obtained from (9) and (11). Substituting these values into the expression for $\frac{\partial W_i}{\partial r_i}$, we obtain the following:

$$\begin{aligned}\frac{\partial W_i}{\partial r_i} &= \frac{r_j + (1-\beta) r_3}{r_i r_j + r_3 (r_i + r_j)} \theta^* V - 2 t r_i \\ &= 2 \left\{ \frac{r_j + (1-\beta) r_3}{[r_i r_j + r_3 (r_i + r_j)]^2} r_j r_3 V - t \right\} r_i.\end{aligned}$$

In an interior solution, $\frac{\partial W_i}{\partial r_i} = 0$, that is,

$$\frac{r_j + (1-\beta) r_3}{[r_i r_j + r_3 (r_i + r_j)]^2} r_j r_3 V = t \text{ for } i = 1, 2;\ i \neq j. \tag{26}$$

For this to be a valid solution, the second order condition must hold. Let us therefore check the second order conditions. It can be shown that

$$\begin{aligned}\frac{\partial^2 W_i}{\partial r_i^2} &= -4 \frac{r_j + (1-\beta) r_3}{[r_i r_j + r_3 (r_i + r_j)]^3} r_i r_j r_3 (r_j + r_3) V \\ &\quad + 2 \left\{ \frac{r_j + (1-\beta) r_3}{[r_i r_j + r_3 (r_i + r_j)]^2} r_j r_3 V - t \right\}.\end{aligned}$$

In an interior solution, the second term above must be 0. Hence in an interior solution,

$$\frac{\partial^2 W_i}{\partial r_i^2} = -4 \frac{r_j + (1-\beta) r_3}{[r_i r_j + r_3 (r_i + r_j)]^3} r_i r_j r_3 (r_j + r_3) V < 0$$

if

$$r_i > 0,\ r_j > 0 \text{ and } r_3 > 0.$$

Therefore, the second order condition is satisfied if all three sponsors spend a positive amount.

Sponsor s_3's payoff function is given by (4). By applying a similar argument, the first order condition in this case can be shown to be the following:

$$\frac{(r_i + r_j) r_i r_j}{[r_i r_j + r_3 (r_i + r_j)]^2} V = t. \tag{27}$$

Next, notice from (26) that

$$r_i = r_j = r.$$

Hence, the first order conditions (26) and (27) can be written as follows:

$$\frac{r + (1-\beta) r_3}{r^2 (r + 2r_3)^2} r r_3 V = t$$

and

$$\frac{2r^3}{r^2 (r + 2r_3)^2} V = t. \tag{28}$$

The above two conditions can be combined to imply the following:

$$2r^2 - r_3 r - (1-\beta) r_3^2 = 0.$$

Hence,

$$r(r_3) = \frac{1}{4} r_3 \left\{ 1 \pm \sqrt{1 + 8(1-\beta)} \right\}$$

are two candidate solutions. Among these two solutions,

$$\frac{1}{4} r_3 \left\{ 1 - \sqrt{1 + 8(1-\beta)} \right\}$$

is negative and is not a valid value for r.

Hence, in equilibrium,

$$\begin{aligned} r(r_3) &= \frac{1}{4} r_3 \left\{ 1 + \sqrt{1 + 8(1-\beta)} \right\} \\ &= \frac{1}{4} \Delta r_3. \end{aligned} \tag{29}$$

By substituting (29) into (28), we obtain (15). Finally, we obtain (14) from (29) and (15).

E Proof of (17)

Suppose s_1 chooses to deviate and not participate in the contest (and let the others play their equilibrium strategies). In that case, s_1's payoff will be βV if s_2 wins the contest and will be 0 otherwise. The probability that s_2 will win the contest can be derived by setting $r_1 = 0$ in the contest success function and is given by

$$\frac{r_2^* x_2^*}{r_2^* x_2^* + r_3^* x_3^*}.$$

Let us now divide both the numerator and the denominator of the above expression by

$$r_1^* x_1^* + r_2^* x_2^* + r_3^* x_3^*$$

to obtain the following:

$$\begin{aligned} & \frac{\frac{r_2^* x_2^*}{r_1^* x_1^* + r_2^* x_2^* + r_3^* x_3^*}}{\frac{r_2^* x_2^* + r_3^* x_3^*}{r_1^* x_1^* + r_2^* x_2^* + r_3^* x_3^*}} \\ = & \frac{p_2^*}{p_2^* + p_3^*}. \end{aligned}$$

Hence we obtain the result.

F Proof of Proposition 4

The payoff function of s_j $(i = 1, 2;\ j \neq i)$ is given by

$$W_j = p_j^{**} V - t r_j^2$$

where it follows from (13) that

$$p_j^{**} = \frac{r_j}{r_j + r_3}.$$

Hence, s_j solves the following first order condition:

$$\frac{\partial W_j}{\partial r_j} = \frac{\partial p_j^{**}}{\partial r_j} V - 2tr_j = 0.$$

Since

$$\frac{\partial p_j^{**}}{\partial r_j} = \frac{r_3}{(r_j + r_3)^2},$$

therefore, the first order condition for s_j can be re-written as follows:

$$r_j V = 2tr_3 (r_j + r_3)^2. \tag{30}$$

Similarly, it can be shown that s_3 solves the following first order condition:

$$r_3 V = 2tr_j (r_j + r_3)^2.$$

It follows from the above two first order conditions that

$$r_3 = r_j = r. \tag{31}$$

By substituting (31) into (30), we obtain the following:

$$V = 8tr^2.$$

Hence the result follows.

G Proof of Proposition 5

The payoff function of s_i $(i = 1, 2)$ is given by

$$W_i = \left(p_i^{**} + \beta p_j^{**}\right) V - tr_i^2 \text{ for } i = 1, 2;\ i \neq j,$$

where p_i^{**} and p_j^{**} is given by (13). Hence, the first order condition is as follows:

$$\frac{\partial W_i}{\partial r_i} = \left(\frac{\partial p_i^{**}}{\partial r_i} + \beta \frac{\partial p_j^{**}}{\partial r_i} \right) V - 2tr_i.$$

Using (13) it follows that

$$\frac{\partial p_i^{**}}{\partial r_i} = \frac{r_j}{(r_i + r_j)^2} = -\frac{\partial p_j^{**}}{\partial r_i} \ .$$

Hence the first order condition can be re-written as follows:

$$\frac{\partial W_i}{\partial r_i} = \frac{(1-\beta)\, r_j}{(r_i + r_j)^2} V - 2tr_i \tag{32}$$

In an interior solution, $\frac{\partial W_i}{\partial r_i} = 0$ and $\frac{\partial W_j}{\partial r_j} = 0$. Therefore, in an interior solution, the following equalities must hold:

$$(1-\beta)\, r_j V = 2tr_i \,(r_i + r_j)^2$$

and

$$(1-\beta)\, r_i V = 2tr_j \,(r_i + r_j)^2 \,.$$

It follows from the above two conditions that in an interior solution, we must have

$$r_1 = r_2 = r. \tag{33}$$

Using (32) and (33), it can be shown that

$$r^{**} = \sqrt{\frac{(1-\beta)}{8} \frac{V}{t}}.$$

		Equilibrium		
		Type 1	Type 2	Type 3
Quantity of	$r_i \ (i=1,2)$	$\frac{V}{t}\frac{2\Delta^2}{(8+\Delta)^2}$	$\sqrt{\frac{1}{8}\frac{V}{t}}$ or 0	$\sqrt{\frac{(1-\beta)}{8}\frac{V}{t}}$
Resources	r_3	$\frac{V}{t}\frac{8\Delta}{(8+\Delta)^2}$	$\sqrt{\frac{1}{8}\frac{V}{t}}$	0
Winning	$p_i \ (i=1,2)$	$\frac{\Delta}{8+\Delta}$	$\frac{1}{2}$ or 0	$\frac{1}{2}$
Probability	p_3	$\frac{8-\Delta}{8+\Delta}$	$\frac{1}{2}$	0
Payoff	$W_i \ (i=1,2)$	$p_i^* \left\{1+\beta-4p_i^{*3}\frac{V}{t}\right\} V$	$\frac{3}{8}V$ or $\frac{1}{2}\beta V$	$\left(\frac{3+5\beta}{8}\right) V$
of Sponsors	W_3	$\left\{1-2p_i^*-p_i^{*2}\left(1-p_i^*\right)^2\frac{V}{t}\right\} V$	$\frac{3}{8}V$	0
Payoff	$U_i \ (i=1,2)$	$p_i^*\left[1-p_i^{*2}\left(1-p_i^*\right)\frac{V}{t}\right]$	$\frac{1}{4}$ or 0	$\frac{1}{4}$
of Combatants	U_3	p_3^{*2}	$\frac{1}{4}$	0
Aggregate Effort	θ	$\frac{256\Delta}{32+4\Delta}\frac{V}{t}$	$\frac{1}{2}\sqrt{\frac{1}{8}\frac{V}{t}}$	$\frac{1}{2}\sqrt{\frac{(1-\beta)}{8}\frac{V}{t}}$

Table A.2: Summary of all Equilibria