Modeling of concrete cracking due to corrosion process of reinforcement bars

Tullio Monetta, University of Napoli
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Antonio Bossio, Tullio Monetta, Francesco Bellucci, Gian Piero Lignola, Andrea Prota

1. Introduction

Reinforced Concrete (RC) is one of the most durable construction materials. However, the presence of soluble chlorides from deicing salts (15 million tons of de-icing salts are used each year in the United States of America and 4 to 5 million tons in Canada [1]) or marine exposure and the loss of alkalinity due to the carbonation of the concrete could destroy the passive film protecting the steel inducing corrosion of reinforcement [2,3]. Corrosion of steel reinforcement represents the major concern of degradation of RC structures. The corrosion process leads to the following coupled effects: (i) longitudinal cracking of concrete cover due to the expansion of corrosion products [4–9], (ii) steel cross section reduction and, (iii) the variation of bond at the steel–concrete interface [10,11]. As a result of these effects, service life and load-bearing capacity of RC elements are considerably reduced [12–14].

In U.S.A., for example, during the 1996, 2 billions of dollars has been spent to repair damages to highway bridges and the cost is increasing at a rate of 500 million of dollars per year [15]. According to the World Corrosion Organization (WCO) the annual cost of corrosion worldwide is over 3% of the world’s Gross Domestic Product (GDP) [16].

Nowadays, prevention and detection of deterioration of RC infrastructures is one of the greatest challenges. Various non-destructive quantitative techniques based on electrochemical methods [17–19] can be used to detect the corrosion at an early stage in order to predict residual life, and suggesting the most suitable repair technique to be used [20–24].

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Abstract

The reinforcement corrosion in Reinforced Concrete (RC) is a major reason of degradation for structures and infrastructures throughout the world leading to their premature deterioration before design life was attained. The effects of corrosion of reinforcement are: (i) the reduction of the cross section of the bars, and (ii) the development of corrosion products leading to the appearance of cracks in the concrete cover and subsequent cover spalling. Due to their intrinsic complex nature, these issues require an interdisciplinary approach involving both material science and structural design knowledge also in terms of International and National codes that implemented the concept of durability and service life of structures.

In this paper preliminary FEM analyses were performed in order to simulate pitting corrosion or general corrosion aimed to demonstrate the possibility to extend the results obtained for a cylindrical specimen, reinforced by a single bar, to more complex RC members in terms of geometry and reinforcement. Furthermore, a mechanical analytical model to evaluate the stresses in the concrete surrounding the reinforcement bars is proposed. In addition, a sophisticated model is presented to evaluate the non-linear development of stresses inside concrete and crack propagation when reinforcement bars start to corrode. The relationships between the cracking development (mechanical) and the reduction of the steel section (electrochemical) are provided. Finally, numerical findings reported in this paper were compared to experimental results available in the literature and satisfactory agreement was found.

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The aim of this paper is to propose an analytical model for predicting the amount of oxide leading to concrete cracking in both general and localized attack. The proposed model can represent a good support also to design durable new buildings. In fact, once geometrical and the main parameters of materials are known, the model allows to estimate the appearance of first visible crack on concrete surface. This is the appropriate stage to restore concrete before the penetration of a large amount of aggressive agents onto the reinforcement through forming cracks.

The proposed model accounts for the main parameters involved in the overall degradation process namely: concrete strength class, concrete cover, size and type of aggregates, exposure class of concrete and bar diameter. In addition, volumetric expansion factor of oxide, Young’s moduli and Poisson coefficients of steel, oxide, and concrete, and creep effect were also accounted for. Preliminary FE simulations were carried out in order to evaluate the possibility to correlate results for cylindrical specimens, like those usually adopted in laboratory tests, to real RC cross sections. Once the effectiveness of cylindrical models to estimate the behavior of real members was verified, two mechanical analytical models are proposed. The former simulates the effects of corrosion yielding to the initiation of the crack where concrete stress-strain behavior is assumed to be linear. The latter simulates the propagation of the crack where concrete stress-strain behavior is mainly non-linear, until it reaches the external surface of concrete cover. Both models allow discussing the influence of main parameters on cracking process based on the reduction of the bar cross section due to corrosion.

It is well known that the timing of cracking process is influenced by many parameters. However, the evaluation of time dependent process is out of the scope of the present paper. Hence parametric analyses were performed on creep effect while evolution of corrosion penetration was simulated starting from a non-corroded situation up to a value of corrosion penetration yielding to full concrete cover cracking.

### 3. Mechanical parameters affecting oxide and concrete behavior

Numerical modeling was limited to preliminary analyses to establish the differences between pitting corrosion and general corrosion, being these the two representative forms of corrosion processes affecting reinforcement bar. Subsequently, FE analyses aimed to correlate the results obtained on cylindrical laboratory specimens to real RC members have been performed.

Analytical and numerical models discussed in the subsequent part of this paper require the definition of mechanical parameters for oxide and concrete. As far as the oxides are concerned, their formation, structure and composition have been summarized in a simple volumetric expansion factor, n. However, the oxide layer has to be considered with an elastic behavior since from the mechanical point of view it affects the behavior of concrete.

#### 3.1. Constitutive modeling of oxide layer

Since the oxide layer transfers normal and shear stresses between the steel surface and the surrounding concrete, model parameters characterizing deformability and shear behavior of corrosion products need to be defined. A survey of literature data on the elastic modulus of oxide can be found in a paper of Chernin and Val [6] summarizing the findings of Samsonov et al. [29], Yoshioka and Yonezawa [30], Suda et al. [31] and Ouglova et al. [27]. According to Samsonov [29], the modulus of elasticity of iron oxide crystals (Fe₂O₃ or Fe₃O₄) is in the range from 215 to 350 GPa. However, it must point out that these values cannot be used directly for corrosion products because of their granular nature, which results in a significant reduction of their stiffness. Yoshioka and Yonezawa [48] measured expansive strains in steel plates subjected to galvanostatic corrosion and confined by concrete. Based on the latter data, Suda et al. [31] estimated that the Young’s modulus of corrosion products, Eₜ, was in the range from 0.1 to 0.5 GPa. Ouglova et al. [27] using a number of techniques found that as the degree of compaction increased, the modulus of elasticity of oxide increased nonlinearly from 1 ÷ 2 GPa up to several hundreds of GPa for pure oxide crystals (showing the typical
behavior of granular materials). However, according to Chernin and Val [6], the so called thick-walled uniform cylinder (TWUC) model significantly overestimates the internal pressure causing the concrete cover cracking, Pcr, for high concrete cover to bar diameter, dMAX, ratios due to its inability to account for nonlinear behavior of concrete after cracking. Based on their considerations, the error in evaluating the oxide expansion at cover cracking due to neglecting the deformability of the oxide layer should be less than 10%. Due to an existing debate on such value, the elastic modulus of oxide in this paper is parametrically discussed ranging from a minimum value of 130 MPa [32] up to 210,000 MPa i.e., the value of steel Table 1 summarizes literature’s results.

3.2. Constitutive modeling of concrete

The concrete behavior around a corroding reinforcing bar is dominated by tensile cracking at a low level of (radial) compressive stresses. Thus, constitutive modeling of concrete can be based on an elasto-cracking (softening) model in tension, while concrete between cracks can be modeled as an isotropic linear elastic material in compression.

One of the most important concrete parameter to consider herein is the fracture energy, Gt, because of its effect on ultimate strain, εcu. The concrete behavior in tension is characterized by fracture energy, Gt, and by the maximum aggregate size, dMAX, according to the Model Code 2010 [33]. The area Gt/lc under the linear softening stress–strain curve, related to fracture energy, Gt, is evaluated according to Gt = 73·fct 16 2.17 (Eq. 5.1-9 [33]). In addition, following the Model Code, concrete is assumed elastic in compression due to the reduced stress levels, while it is fully nonlinear in tension.

Namely, a bilinear curve is assumed for concrete having an elastic branch up to peak tensile strain, εcu, followed by a softening linear branch to zero stress at ultimate strain, εcu. In Fig. 1, the elastic branch is a simplified version of the original curve, passing through 0.9·fct and fct, proposed in [33], and the two curves are almost overlapping. To move from crack opening, wc, to ultimate strain, εcu, a characteristic length, l, (e.g. for concrete it is assumed equal to 3 · dMAX), was assumed, being, dMAX, the maximum aggregate dimension.

Elastic modulus of concrete, Ec, is affected by aggregate types and sustained or long-term loads. According to table 5.1-6 [33] and compared to quartzite aggregates, the elastic modulus of concrete can be increased by 20% (Basalt) or decreased by 30% (Sandstone) only by changing the aggregate type. The value for the modulus of elasticity at concrete age of 28 days, Ecu, can be estimated according to Equation 5.1-20 [33] reported as Eq. 1 in the following:

\[ E_{cu} = E_c \cdot \alpha_c \left( \frac{f_{ck} + \Delta f}{10} \right)^{\frac{2}{3}}. \]

In Eq. (1) \( \alpha_c \) is the factor reported in table 5.1-6 [33] and \( \Delta f \) is equal to 8 MPa (5.1.4 [33]). In Table 2 the values of concrete maximum tensile strain, εcu; of compressive strength, fcm; of fracture energy, Gt, and the width of the crack when the concrete has reached the value of the ultimate tensile strain, εcu, depending on the strength class of concrete, are reported. By an inspection of Fig. 1 and data reported in Table 2, it is clearly shown that the maximum dimension of aggregates influences the tensile behavior of concrete owing to its dependence on the value of the ultimate tensile strain, εcu, (hence of aggregate dimension) and from the value of the elastic modulus of concrete, Ec (hence of aggregate type).

Furthermore, an additional parameter affecting the elastic modulus of concrete is the creep. In fact, it reduces the elastic modulus of concrete. The elastic modulus of concrete due to creep has been considered equal to Ec,eq = Ec / (1 + ϕ) where parameter ϕ has been considered variable from 0 to 3, being its time-dependency out of the scope of the present paper.

4. Preliminary numerical modeling

In order to understand the general behavior of corrosion process in the cases of pitting and general corrosion and if it is possible to extend results obtained on cylindrical specimens to real RC members, some preliminary numerical modeling was carried out.
4.1. FEM analysis of pitting and general corrosion

FEM analyses have been performed by means of commercial code TNO DIANA v9.4, in order to consider the differences between the pitting and the general corrosion [5]. General corrosion and localized corrosion were simulated considering a single bar embedded in a cylinder of concrete. A wide range of typical values for reinforcement diameters and concrete covers were considered. Namely, reinforcement bar diameters, $2 \cdot R_0$, are 10 mm (stirrups), 16 mm and 20 mm, while concrete covers, $c_c$, investigated were 10 mm, 30 mm and 50 mm.

In order to extend experimental results (usually carried out by means of cylindrical specimens reinforced by a single bar [34, 35]) to real structures, cylindrical models are the main focus of the proposed FEM analyses providing some simplifications. In the FEM analysis carried out in this paper, pitting corrosion and general corrosion were simulated as described in the following.

It is well known that pitting is a localized corrosion phenomenon, therefore it is simulated via a radial unitary pressure applied locally on the interface between reinforcement bar and concrete, at the mid-span plane of the concrete cylinder. The length of the pitting zone, $w_p$, was assumed parametrically equal to 0.2 mm and 2 mm. On the other hand, general corrosion affects all bar surface thus it was modeled as a radial unitary pressure applied along the whole length of the concrete cylinder.

Meshes for both pitting and general corrosion analyses are made of more than 2500 CQ16A elements, eight-node isoparametric axisymmetric solid ring elements with quadrilateral cross-section, based on quadratic interpolation and Gauss integration.

Results of FEM analyses are reported in Figs. 2 and 3. Fig. 2 shows the circumferential stress inside the thickness of the concrete cover and $R$ represents the position vector along the cover from bar surface. Namely, $R_0$ represents initial bar radius, the net cover is equal to the difference of $R_0 - R_4$ and $\sigma_{\theta}$ represents the tensile stress of concrete. Fig. 2a compares FEM results for pitting and general corrosion simulations. Applying the same radial pressure along the reinforcing bar groove, pitting corrosion presents a highly localized behavior; conversely, general corrosion shows a wider spreading of tensile stresses $\sigma_{\theta}$ through the concrete cover. Fig. 2b shows the effect of bar diameter when localized corrosion and general corrosion occur. As can be seen from the results reported in Fig. 2b when pitting is the form of corrosion, the circumferential tensile stress is not affected by the bar diameter. Conversely, the lower is the bar radius, $R_0$, the lower is the circumferential tensile stress in concrete when general corrosion is observed. However, to have a sound comparison of the two forms of corrosion, a further evaluation adopts different values of radial pressure in order to provide the same radial displacement.

Fig. 3 shows the correlation between the ratio of radial tension, $\sigma_{\theta}$, (in case of general corrosion), and radial tension, $\sigma_{\theta,\text{pitting}}$, (in case of pitting corrosion) and the concrete cover relative position, $\frac{R_0 - R_4}{R_0}$ (where 0 is close to the bar and 1 is at the external surface of concrete cylinder) for cylindrical models.

The first clear outcome of the analyses is that maximum tensile stress, $\sigma_{\theta}$, in the circumferential direction is attained close to the steel bar. Then the crack develops in the radial direction until it reaches the external surface of concrete, in both the case of pitting corrosion and general corrosion. However, the main difference is in the tensile stress level that is higher close to the bar in the case of pitting. Conversely, the volume of concrete involved by relevant tensile stresses is much lower. Due to the observed huge difference in behavior the two forms of corrosion provide quite different stress states. Thus, a specific analytical model for pitting will not be provided, but its effects have been discussed by means of FEM. In the following, the general corrosion will be the only form of corrosion analyzed accounting for the cylindrical symmetry conditions. It is worth of mention that FEM analyses better fit the general corrosion process.

4.2. FEM analysis of cylindrical geometry

The main aim of the following 3D FEM modeling is the comparison between the cylindrical specimen model (usually used for laboratory tests) and the real RC members. In the following analyses three strength classes for concrete, three diameters for reinforcement bars and two values for concrete cover will be considered. Both 3D and 2D (plain strain) analyses (Fig. 4) were performed to evaluate the influence of...
the modeling dimensions. The 3D model is uniform and symmetrical along the longitudinal axis of the column, so it is possible to scan it in 2D plain-strain, considering a single cross-section. Fully comparable results were found, thus 2D plain strain analyses were performed thereafter. In such analyses, an increasing pressure was applied to let the tensile principal stresses attain the tensile strength of concrete.

It is worth noted that the response of 3D FEM is axisymmetric so the stress is uniform in the circumferential direction.

Further analyses have been performed using three strength classes for concrete according to Model Code 2010 [33]: C20/25, C25/30 and C30/37. Furthermore, two other variables were considered: three bar diameters, $2 \cdot R_0$, namely 12 mm, 16 mm and 20 mm and two concrete cover values, $c_c$, equal to 20 mm and 30 mm, respectively. Concrete cover dimension was chosen considering the exposure classes for concrete defined by UNI EN 206-1 [36] assuming reinforcement corrosion due to carbonation (XC1) and sea water chlorides (XS1) whose high level of chloride concentration leads to general corrosion. The outcomes reported in Table 3 clearly show that this dependency is almost weak. In fact, the higher is the Young’s modulus of concrete, $E_c$, (depending on strength class of concrete) the higher is the tensile strength, $f_{ctm}$. The tensile strength increases of about 30% moving from strength class of concrete C20/25 to C30/37, while the increase in terms of internal displacement is about 19% and the increase in terms of internal pressure is 23%, being both almost independent on the bar diameter and concrete cover.

The influence of bar diameter is relevant on internal displacement (extending the bar diameter of about 70%, i.e. from 12 mm to 20 mm, the same increment of about 70% is found in terms of displacement), while the variation is almost negligible in terms of inner pressure, leading to a pressure reduction of about 10%. A further interesting result appears from data reported in Table 3 in which the influence of concrete cover is negligible on internal displacement (increasing the concrete cover of 50%, the increment of displacement is about 1%), while concrete cover variations lead to a pressure increment ranging between 5% and 10%.

### 4.3. FEM analysis of real member geometry

Real RC members have been numerically simulated as described below. Fig. 5a shows an RC beam, where reinforcement bars were arranged to fulfill the indications of Eurocode 2 (see section 8.2) [37]. Namely, the clear distance between two bars, $i_c$, and between horizontal layers of bars should not be less than one bar diameter. Fig. 5b shows the deformed shape of a quarter of the beam after the expansion exerted by the oxide formed on the corroded bar. It is worth noted that the corrosion is simulated as a pressure induced by the oxide on concrete circular groove, in place of the bar. Furthermore, it is clearly shown that there is a higher displacement outwards the cross section due to the constraint exerted by the concrete core to the expansion of oxide and, consequently, of concrete.

The expansion of the oxide layer due to corrosion leads to circumferential tensile stresses around the bars, similarly to the case of cylindrical specimens. To provide a comparison, and to evaluate the differences between the tensile stress predictions for the cylindrical model, and the effective stress in a real RC member (once bar diameter, $2 \cdot R_0$, concrete cover, $c_c$, and clear distance, $i_c$, are given), the same level of pressure used to perform the cylindrical model yielding to the tensile strength of concrete was applied to circular grooves (i.e. bars) of the RC beam. Pressure load was preferred to displacement load in order to...
prevent forcing the shape of the expanded corroded bar. However, the scatter in terms of principal tensile stress predictions between the two load schemes (displacement vs. pressure) was always lower than about ±10%. For this reason, the pressure load is preferred to the direct displacement load.

The effect of clear distance values, ic, the same concrete cover values used before, cc, concrete properties and steel bars adopted for parametric analyses were replicated in this case. The outcomes of the subsequent 54 analyses are summarized in Table 4. Principal tensile stresses are evaluated according to the same values of inner pressure adopted in the cylindrical modeling. In brackets are reported the percentage difference with the cylindrical model.

Fig. 6 shows a contour plot of the principal tensile stresses for four combinations of analyzed parameters for a given strength class of concrete (even if results are clearly independent on concrete strength, see Table 4). Results obtained indicate that the maximum tensile stresses are always in the direction of the layer of bars, while in the direction of the concrete cover the stresses are usually smaller than about 20%. In addition, this difference is further reduced in the case of larger clear distances, ic.

It is clearly shown that increasing the clear distance, ic, (Fig. 6a vs. Fig. 6b) bar interaction and tensile stresses reduce while stress difference increases as reported in Table 4. Furthermore, by increasing the concrete cover (Fig. 6a vs. Fig. 6c) bar interaction and tensile stresses are almost similar whereas by increasing bar diameter (Fig. 6b vs. Fig. 6d), bar interaction and tensile stresses increase (and stress difference, too; Table 4).

Fig. 7a shows the percentage of the stress increment between the tensile stresses provided by cylindrical model and real RC beam simulations at identical values of parameters, but neglecting the clear distance values, ic.

To reduce the remarkable effect of the clear distance, ic, (leading to stress increments up to 47%) the analysis was performed considering a concrete cover value for the cylindrical model equal to the minimum between concrete cover, cc, (equal to R4 − R0), and half of clear distance, ic.

This means that two virtual circular crowns of concrete are assumed around each bar and they are not intersecting, but, at least, they result tangent to each other. The pressure exerted by the cylindrical model (under these assumptions) was applied again to the previously analyzed 54 cases, and the stress increment (see Fig. 7b) was found to be dramatically reduced and always smaller than about 10%.

Results discussed so far suggest that it is possible to operate with a single bar cylindrical model to simulate the behavior of RC real members in terms of initiation of cracking. However, a similar quantitative extension of the non-linear fracture mechanics process requires further research efforts.

Table 4

<table>
<thead>
<tr>
<th>Strength class of concrete</th>
<th>fct [MPa]</th>
<th>Ec [GPa]</th>
<th>Bar diameter 2 ⋅ R0 [mm]</th>
<th>Concrete cover cc [mm]</th>
<th>Maximum principal tensile stress [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ic = 20 mm</td>
<td>ic = 30 mm</td>
<td>ic = 40 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C20/25</td>
<td>2.265</td>
<td>30.2</td>
<td>12</td>
<td>20</td>
<td>2.622 (+16%)</td>
</tr>
<tr>
<td>C25/30</td>
<td>2.558</td>
<td>31.4</td>
<td>30</td>
<td>20</td>
<td>2.961 (+16%)</td>
</tr>
<tr>
<td>C30/37</td>
<td>2.942</td>
<td>33.0</td>
<td>16</td>
<td>20</td>
<td>3.406 (+16%)</td>
</tr>
<tr>
<td>C20/25</td>
<td>2.265</td>
<td>30.2</td>
<td>30</td>
<td>20</td>
<td>2.699 (+19%)</td>
</tr>
<tr>
<td>C25/30</td>
<td>2.558</td>
<td>31.4</td>
<td>30</td>
<td>20</td>
<td>3.048 (+19%)</td>
</tr>
<tr>
<td>C30/37</td>
<td>2.942</td>
<td>33.0</td>
<td>16</td>
<td>20</td>
<td>3.504 (+19%)</td>
</tr>
<tr>
<td>C20/25</td>
<td>2.265</td>
<td>30.2</td>
<td>16</td>
<td>20</td>
<td>2.870 (+27%)</td>
</tr>
<tr>
<td>C25/30</td>
<td>2.558</td>
<td>31.4</td>
<td>30</td>
<td>20</td>
<td>3.241 (+27%)</td>
</tr>
<tr>
<td>C30/37</td>
<td>2.942</td>
<td>33.0</td>
<td>16</td>
<td>20</td>
<td>3.727 (+27%)</td>
</tr>
<tr>
<td>C20/25</td>
<td>2.265</td>
<td>30.2</td>
<td>30</td>
<td>20</td>
<td>3.293 (+30%)</td>
</tr>
<tr>
<td>C25/30</td>
<td>2.558</td>
<td>31.4</td>
<td>30</td>
<td>20</td>
<td>3.324 (+30%)</td>
</tr>
<tr>
<td>C30/37</td>
<td>2.942</td>
<td>33.0</td>
<td>16</td>
<td>20</td>
<td>3.822 (+30%)</td>
</tr>
<tr>
<td>C20/25</td>
<td>2.265</td>
<td>30.2</td>
<td>20</td>
<td>20</td>
<td>3.037 (+34%)</td>
</tr>
<tr>
<td>C25/30</td>
<td>2.558</td>
<td>31.4</td>
<td>20</td>
<td>20</td>
<td>3.430 (+34%)</td>
</tr>
<tr>
<td>C30/37</td>
<td>2.942</td>
<td>33.0</td>
<td>20</td>
<td>20</td>
<td>3.944 (+34%)</td>
</tr>
<tr>
<td>C20/25</td>
<td>2.265</td>
<td>30.2</td>
<td>30</td>
<td>20</td>
<td>3.338 (+47%)</td>
</tr>
<tr>
<td>C25/30</td>
<td>2.558</td>
<td>31.4</td>
<td>30</td>
<td>20</td>
<td>3.770 (+47%)</td>
</tr>
<tr>
<td>C30/37</td>
<td>2.942</td>
<td>33.0</td>
<td>30</td>
<td>20</td>
<td>4.335 (+47%)</td>
</tr>
</tbody>
</table>

Fig. 5. a) Typical beam considered in the analyses; b) deformed shape of a quarter of beam with bar radius R0 = 12 mm, strength class of concrete C20/25, cc = ic = 20 mm.
A further remarkable result of FEM analysis is that the crack initiates close to the steel bar as indicated by tensile stresses in the circumferential direction in agreement with experimental data reported in the literature [38]. This preliminary numerical study showed that the stress induced by the expansion of corrosion products can be simulated both by a pressure or a displacement. With regard to the proposed analytical models, the use of pressure exerted by corrosion products allows also to consider the effects of deformability of the oxide layer, while the displacements are used as a compatibility condition.

This part of the work has underlined the possibility to compare cylindrical specimen simplified models with the behavior of real RC members in terms of concrete cracking. The average underestimation of the proposed method is almost negligible only if, in cylindrical specimen model, concrete cover is assumed equal to the minimum value between concrete cover, $c_c$, and half the distance between two bars, $i_c$, is adopted. Maximum underestimation of the model was found equal to about 10% thus still comparable, for instance, to the usual uncertainty on concrete tensile strength.

5. Analytical modeling

Once the suitability of cylindrical specimens to approximate stress state in real member geometry was verified, the main goal of this work is to provide analytical formulations easily implementable.

In order to develop in details the phases of the process of cracking initiation and propagation, two analytical models will be discussed. The former is aimed to simulate initiation of crack opening due to corrosion process. The latter simulates crack propagation until the crack reaches the external surface of concrete. It is assumed, as common in smeared approaches, that only one crack generates, being virtually under radial symmetry, the opening of a single crack equal to the sum of the openings of potential multiple cracks. However, the non-linear fracture mechanics requires further investigations to be extended in general to non-axisymmetric conditions. A set of parameters has been considered (e.g. concrete cover, concrete strength class, type and size of aggregates, bar diameter, etc.) including some time-dependent parameters. Their time-dependent law is influenced by the environmental conditions and the properties of the materials used to build the structures. Even if this time dependency is out of the scope of the paper, parametrical studies have been performed to evaluate its effects.
5.1. Initiation of cracking

The analytical model proposed in this paper is based on a non-linear system of equations ensuring the radial equilibrium of the bar–oxide–concrete system as well as the compatibility of displacements at the interfaces between the layers made of dissimilar materials. In this model, an outer layer (bar) of radius, \( R_0 \), an intermediate layer (oxide) of radius, \( R_2 \), and an outer layer (concrete) of radius, \( R_s \) (Fig. 8a) are introduced. The aim of the model is to define the internal pressure due to oxide expansion leading to concrete cover cracking and to calculate the corresponding cross section reduction of reinforcement bar [39–43].

The general equations of continuity, equilibrium and compatibility are:

\[
\begin{align*}
R_1 &= R_0 - x \\
R_2 &= R_1 + x + y \\
\pi \cdot [(R_0 + y)^2 - (R_0 - x)^2] &= n \cdot \pi \cdot [R_0^2 - (R_0 - x)^2] \\
S_{\text{concrete}} &= S_{\text{oxide}} - y \\
S_{\text{steel}} &= S_{\text{oxide}} \\
\end{align*}
\]

where, \( R_1 \) represents the reduced bar radius due to steel corrosion; \( R_0 \) is the initial bar radius; \( x \) is the corrosion penetration; \( R_2 \) is the outer radius of oxide layer; \( y \) is the oxide displacement; \( n \) is the volumetric expansion factor for oxide; \( S_{\text{concrete}} \) is the internal concrete displacement; \( S_{\text{oxide}} \) is the external oxide displacement and \( S_{\text{steel}} \) is the steel displacement. Eq. (2) indicates that the radius of the bar, \( R_0 [\text{mm}] \), is reduced by the quantity of steel consumed, \( x [\mu \text{m}] \), and it is worth, \( R_1 [\text{mm}] \). Eq. (3) represents the outer radius, \( R_2 [\text{mm}] \) of the oxide layer having a total thickness \((x + y) [\text{mm}]\), referring to the oxide inner radius, \( R_1 [\text{mm}] \). Eq. (4) represents the continuity equation where the volume of formed oxide \( \pi \cdot [(R_0 + y)^2 - (R_0 - x)^2] \) is equal to \( n \) times the volume of bar consumed \( \pi \cdot [R_0^2 - (R_0 - x)^2] \). It is trivial to observe that at the beginning of the process \( R_0 = R_1 = R_2 \).

The coefficient, \( n \), is the volumetric expansion factor of the oxide. It can be reportedly taken between 2 and 6 in agreement with the range \((1.7 \div 6.2)\) currently reported in the scientific literature [13,44] on the basis of oxide formed [3]. Eq. (5) represents the compatibility of displacements between the oxide and the concrete layer, that, displaced by the quantity, \( y \), (due to the produced oxide), reacts with an inward pressure, \( q_2 \), (related to the displacement \( S_{\text{concrete}} \) evaluated in the crown of radii \( R_2, R_s \) and loaded by internal pressure). The oxide layer (crown of radii \( R_1 \) and \( R_2 \)) is loaded by an internal contact pressure with steel, \( q_1 \), and an external contact pressure with concrete, \( q_2 \). The external displacement of oxide layer is \( S_{\text{oxide}} \) while the internal displacement is \( S_{\text{concrete}} \). In Eq. (6), \( S_{\text{oxide}} \) is equal to the steel bar displacement, of radius \( R_1 \), induced by the contact internal pressure with the oxide layer, \( q_1 \). The following displacement equations are evaluated according to Lignola et al. [45]:

\[
\begin{align*}
S_{\text{steel}} &= \frac{q_2 R_1 (v_s + 1) - (1 - 2v_s)}{E_s} \\
S_{\text{oxide}} &= \frac{(1 + v_o) R_1}{E_o} \left( 1 - 2v_o \right) \left( \frac{q_2 R_1^2 - q_1 R_1^2}{R_2^2 - R_1^2} \right) + \frac{1}{R_1^2} \left( \frac{q_2 - q_1}{R_2^2 - R_1^2} \right) \frac{R_2^2 - R_1^2}{R_2^2 - R_1^2} \\
S_{\text{concrete}} &= \frac{q_2 R_2 (v_s + 1) R_2^2 (2v_s - 1) - R_2^2}{E_c (R_2^2 - R_1^2)} \\
S_{\text{oxide}} &= \frac{(1 + v_o) R_2}{E_o} \left( 1 - 2v_o \right) \left( \frac{q_2 R_2^2 - q_1 R_2^2}{R_2^2 - R_1^2} \right) + \frac{1}{R_2^2} \left( \frac{q_2 - q_1}{R_2^2 - R_1^2} \right) \frac{R_2^2 - R_1^2}{R_2^2 - R_1^2} \\
\end{align*}
\]

where \( v_o, v_s \), and \( v_c \) represent the Poisson coefficients and \( E_o, E_s \) and \( E_c \) are the Young’s elastic moduli, for steel, oxide and concrete, respectively. All Poisson coefficients were assumed equal to the value of concrete (0.2) in order to neglect the effects of differential expansion. The circumferential tension, \( \sigma_0 \), at the oxide–concrete interface can be evaluated according to:

\[
\sigma_0 = \frac{q_2 (R_2^2 + R_1^2)}{R_2^4 - R_1^4}.
\]

When \( \sigma_0 \) reaches the concrete tensile strength, \( f_{ct} \), it causes the initiation of concrete cracking.

By using non-linear Eqs. (2) to (11) it is possible to solve the system of equations to find the bar reduction, \( x \), and the percentage of section loss defined as \( 1 - \left( \frac{R_0}{R_s} \right)^2 \) at initiation of cracking for given values of \( \sigma_0 \), \( R_0, R_s, n \), Poisson coefficients and Young’s moduli.

---

Fig. 8. Geometric scheme for analytical modeling of crack: a) initiation, b) propagation.
5.2. Propagation of Cracking

In order to simulate crack propagation, concrete layer (internal radius $R_2$, external radius $R_4$) has been split into two layers: cracked concrete of internal radius $R_2$ and external radius $R_3$, and uncracked concrete of internal radius $R_3$ and external radius $R_4$. Hence a new continuity equation needs to be defined.

The applicability of this model starts when the circumferential tension, $\sigma_\theta$, at the inner concrete radius, $R_2$, location is equal to the tensile strength value, $f_{\text{ctm}}$, (i.e. since crack initiation). A fourth layer of internal radius $R_2$ and external radius $R_3$, made of cracked concrete, is then defined (Fig. 8b). Referring to the crack initiation, i.e. at the beginning of the propagation, $R_3 = R_2$. It is possible to define the equilibrium equations for the four layers (bar, oxide, cracked concrete and not cracked concrete) and the compatibility equations for the three interfaces between the layers. The Eqs. (2), (3), (4), (5), and (6) are still valid, (even if, in Eq. (5) $S_{2\text{cracked concrete}}$ replaces $S_{2\text{concrete}}$, where $S_{2\text{cracked concrete}}$ is the internal displacement of cracked concrete and $S_{2\text{concrete}}$ is the external displacement of cracked concrete and internal displacement of not cracked concrete, due to continuity) and the following two equations are need to be added:

\[
q_2 \cdot R_2 = q_3 \cdot R_3 + \int_{R_2}^{R_3} \sigma_\theta(R) \, dR
\]  

(12)

\[
S_{\text{concrete}} - S_{2\text{cracked concrete}} = \frac{(q_2 + q_3) \cdot (R_3 - R_2)}{2 \cdot E_c}.
\]  

(13)

Solving the integral in Eq. (12), according to the assumptions on concrete model in tension, as discussed in Section 3.2, the following is obtained:

\[
q_2 \cdot R_2 = q_3 \cdot R_3 + \sigma_\theta^\text{cu}(R) \cdot \left[ -R_2 + R_3 + \frac{(R_2 - R_3) \cdot (\epsilon_{\text{ct}} \cdot R_2 + S_{2\text{concrete}})}{2 \cdot (\epsilon_{\text{ct}} - \epsilon_{\text{cu}}) \cdot R_2} \right]
\]  

(12b)

where $\sigma_\theta^\text{cu}(R)$ is the circumferential stress at position $R$, accounting for post peak non-linear behaviour of concrete in tension; $\epsilon_{\text{ct}}$ is the maximum tensile strain and $\epsilon_{\text{cu}}$ is the ultimate tensile strain. Eqs. (12) and (12b), after solving the integral, represent the equilibrium of the cracked concrete compressed radially along the inner ($R_3$) and outer ($R_4$) circumferences by the pressures $q_2$ and $q_3$ and circumferentially by the stresses of the cracked concrete in softening (whose resultant is represented by the integral). Eq. (13) represents the radial deformability of cracked concrete due to the pressures $q_2$ and $q_3$. Note that cracked concrete is not modeled as a continuous crown but as a series of struts delineated by radial cracks. The following displacement equations are evaluated according to Lignola et al. [45] for oxide and not cracked concrete crowns:

\[
S_{2\text{oxide}} = \frac{(1 + \nu_o)R_2}{E_o} \left( 1 - 2\nu_o \right) \left( q_1 R_2^2 - q_1 R_1^2 \right) \frac{1}{R_2^2 - R_1^2} + \frac{1}{R_2^2} (q_2 - q_1) R_2^2 R_1^2 
\]  

(14)

\[
S_{2\text{concrete}} = \frac{(1 + \nu_c)R_2}{E_c} \left( 1 - 2\nu_c \right) \left( q_1 R_2^2 - q_1 R_3^2 \right) \frac{1}{R_2^2 - R_3^2} + \frac{1}{R_2^2} (q_2 - q_1) R_2^2 R_3^2 
\]  

(15)

The circumferential stress, $\sigma_\theta$, at the interface between the crown of cracked concrete and not cracked concrete (i.e. at $R_3$) must always be equal to the maximum tensile stress of concrete. The position of the crack tip, $R_3$, where tensile strength is attained, is given by solving Eq. (16):

\[
\sigma_\theta = \frac{q_3 (R_3^2 + R_2^2)}{R_2 - R_3} = f_{\text{ct}}
\]  

(16)

The increase of tensile circumferential strains generates increments of cracked concrete radius $R_3$, to the point of full propagation of cracking at which $R_3 = R_4$ when the crack front reaches the outer surface of the concrete cover.

Given the values of $R_0$, $R_4$, $n$, Poisson coefficients and Young’s moduli, by using the nonlinear set of Eqs. (2) to (16) it is possible to find values of bar reduction, $x$, and the percentage of section loss for a given propagation of crack, $R_3$. The crack propagation is 0% and 100% when $R_3 = R_2$ and $R_3 = R_4$, respectively. In this way, by increasing independently the bar reduction, $x$, it is possible to check the propagation of cracked concrete front, $R_3$ (providing a relationship between $x$ and $R_3$).

6. Parametric analyses

In this paragraph results obtained by solving analytical-parametric Eqs. 2 to 16 are discussed.

Fig. 9a shows the evolution of circumferential stresses in concrete, $\sigma_\theta$, related to the bar reduction, $x$, at crack initiation. In this example $R_0 = 8$ mm, concrete strength class C30/37 ($E_c = 29,738$ MPa), oxide Young’s modulus $E_o = 130$ MPa (according to Carè et al. [32]) and variability of oxide expansion factor, $n$, are considered. Maximum dimension, $d_{\text{max}}$, of quartzite type aggregate is assumed equal to $32$ mm.

In this model the value of $E_o$ is changed in a wide range from about 130 MPa (proposed by Carè et al. [32]) up to a very stiff value equal to...
Young’s modulus of steel (E_s = 210,000 MPa) due to the not existing agreement in the scientific literature on the value of E_o [10,46,47]. It is worth to mention that other works consider some oxide penetrating into concrete pores and cracks too [46, 48, 49]. However, since the amount of oxide penetrating is still under debate, and operating on safe side, authors decided not to consider any volume of oxide penetrating pores or cracks [50].

Fig. 9b shows results of concrete cover (c_c = R_4 - R_0) and the strength class of concrete variability. Fig. 9b clearly shows that in terms of initiation of crack, from a mechanical point of view, greater dependence is on the strength class of concrete, while the thickness of the concrete cover has less impact, similarly to the Young’s modulus of oxide, E_o.

Fig. 10a shows the propagation of cracked concrete front, R_3, (R_3 increases from R_2 to R_4) related to bar reduction, x, and the percentage of section loss for the same case examined in Fig. 9a. It is observed that cracked concrete front, R_3, starts from a value of 8.0010^3 mm (corresponding to the initiation of cracking with a bar reduction x = 0.6885785 μm) and asymptotically reaches the value R_4 (e.g. with a bar reduction x = 6.48911 μm, in the case of n = 2.5, strength class of concrete C30/37, bar radius R_0 = 8 mm and concrete cover c_c = R_4 - R_0 = 35 mm).

Fig. 10b shows the variability of Young’s modulus of oxide, E_o, when crack propagation matches concrete cover. In terms of crack propagation, greater dependence is on the concrete cover dimensions, while the class of concrete has less effect. This result suggests that, contrary to the crack initiation model, the crucial parameter is the concrete cover.

6.1. Initiation of cracking: plots and discussion

Fig. 11a shows corrosion penetration related to bar radius, R_0, for various types of aggregates. It shows that concrete made of basalt type aggregates needs less corrosion penetration to reach the stress values for crack initiation. Fig. 11b shows that the percentage of section loss is almost independent on the bar radius, R_0.

Fig. 12a shows that the lower is the expansion factor of oxide, n, the higher is the corrosion penetration, x, leading to crack initiation, but the percentage of section loss (Fig. 12b) is independent on bar radius, R_0.

To account for creep effects, the Young’s modulus of concrete is reduced by means of the factor \( \frac{1}{1 + \phi} \). When a structure is exposed to
low aggressive agents, it could happen that creep phenomenon starts before the crack initiation. Therefore it is important to evaluate the corrosion penetration, $x$, (or section loss) and creep effect. Fig. 14 considers section loss and bar penetration related to the creep factor, $\phi$, for various types of aggregates.

6.2. Propagation of cracking: plots and discussion

Fig. 15 shows the section loss at full propagation of crack front up to the external concrete surface related to bar radius, $R_0$, according to values of Young’s modulus of oxide, $E_o$. As can be seen from this figure,
the section loss has a negligible dependency on Young’s modulus. The lack of dependency on Young’s modulus of oxide, $E_o$, could be attributed to the thickness of the oxide layer because of its lowest thickness. Due to both the uncertainty on Young’s modulus of oxide, $E_o$, value and to the latter finding, this result might lead to not consider the Young’s modulus of oxide, $E_o$, as a critical variable.

Fig. 16 shows bar section loss related to bar radius, $R_o$, according to volumetric expansion factor of oxide, $n$, in case of concrete cover $c_c = R_4 - R_0 = 35$ mm. As can be seen from this figure the lower is the volumetric expansion factor of oxide, $n$, the higher is the percentage of bar section loss leading to crack propagation until $R_3$ is equal to $R_4$. Fig. 17 shows that the lower is the bar radius, $R_o$, the higher is the percentage of bar section loss leading to complete crack propagation. It is worth to observe that from a structural point of view it could be better to use larger bar diameters because they ensure a long-lasting contribution to structural behavior.

Fig. 18 shows bar section loss related to bar radius, $R_o$, according to maximum size of aggregates. The lower is $d_{\text{MAX}}$, the higher is the bar section loss. This result can be attributed to the less brittle behavior of concrete in tension with smaller aggregate dimensions.

Fig. 19 shows bar section loss related to bar radius, $R_o$, according to various types of aggregate. Results obtained indicate that concrete made of basalt type aggregates needs less corrosion penetration (e.g. bar loss) to reach complete concrete cover cracking than concrete made of sandstone type aggregates.

Fig. 20 shows the bar section loss and corrosion penetration, $x$, related to the creep factor, $\phi$, for various types of aggregates. The higher is
the creep factor, $\phi$, the higher is corrosion penetration, $x$, leading to complete concrete cover cracking. It is worth to observe that the longer is the time to corrosion initiation the higher is the potential creep effect. However, this kind of assessment is out of the scope of this paper.

Analytical results presented above were compared with those of Du et al. [51] obtained via finite element analyses in order to validate the proposed model. Du et al. [51] performed FEM analyses based on the geometric parameters used by Clark and Saifullah experiments [52]. Figs. 21 and 22 show results from Du et al. [51] compared to those obtained from the model proposed in this paper in case of complete concrete cover cracking and in case of initiation of cracking, respectively. As can be seen from this figure results are almost overlapping.

In addition, our model is further tested by comparing data reported by Konishi et al. [53] who studied the expansive behavior of corrosion products, both experimentally and analytically. Their specimens had a concrete cover $cc = R_0^4$, deformation factor of corrosion products were $0.0026 \div 0.0031$ mm. Based on their experimental results, Konishi et al. [53] reported that the expansive amount of corrosion needed to induce surface cracks was $0.0036$ mm. Indeed, by using the same value of the ratio $n = 2.5$ and the strength class of concrete is C30/37. Fig. 24a shows the results for corrosion penetration, $x$, as a function of crack propagation where the calculated expansive deformation was only $0.0018$ mm.

A further test of the model presented in this paper can be envisaged by using the data of Tsukamoto et al. [54]. These authors simulated experimentally the expansive behavior of corrosion products. In such FEM analysis the expansive behavior of corrosion products was introduced by means of a volumetric strain. Based on their numerical outcomes it is reported that when the corrosion cracks appeared on the specimen surface, both the internal pressure at the bar surface and the expansion coefficient of corrosion products were about $10.0 \text{N/mm}^2$ and $2.0 \div 2.2$, respectively. The expansive coefficient of $2.0 \div 2.2$ of the corrosion products corresponds to a radial expansion of $0.0026 \div 0.0031$ mm. By using data reported in [54] into the model presented in this paper, a radial expansion at complete cracking of about $0.0035 \div 0.0045$ mm is obtained and the corresponding radial pressure is in the range $9.0 \div 13.2 \text{N/mm}^2$. As can be seen both data are in reasonable agreement with literature results.

### 6.3. Different models for cracked concrete layer

In Eq. 12 the integral of $\sigma_0(R)$ represents the softening contribution in the case of quasi-brittle concrete in tension. In the case of brittle concrete the integral is taken equal to zero to simulate the brittle drop of tensile capacity after peak. To account for the cracking of concrete in tension the integral reported in Eq. 12 must be replaced by the following Eq. (17):

$$
\alpha_{0b} \cdot (R_2 - R_1) \cdot \left( \frac{2 \cdot \varepsilon_y \cdot R_2 \cdot R_1}{2 \cdot R_2 \cdot R_3 \cdot (\varepsilon_{ct} - \varepsilon_y)} \right). 
$$

(Rigid cracked concrete means that there is no radial relative displacement of the two circumferences bounding the cracked concrete layer, i.e. $S_{\text{oxide}}$ is equal to the inner cracked concrete displacement given by $S_{\text{oxide}} = -y$. Therefore the following Eq. 13b replaces Eq. 13 (valid for deformable concrete).

$$
S_{\text{concrete}} = S_{\text{oxide}} - y. 
$$

where $S_{\text{concrete}}$ is related to the geometric and elastic parameters according to Eq. 18:

$$
S_{\text{concrete}} = \frac{(1 + \nu) \cdot R_3}{E_c} \cdot \left( 1 - 2 \cdot \nu \cdot \left( \frac{-q_3 \cdot R_3^2}{R_2^2 - R_3^2} \right) \right) \cdot \frac{1}{R_3^2} 
$$

Fig. 23 shows the corrosion penetration, $x$, as function of crack propagation, $R_b$, for various concrete cover thicknesses in tension. It is remarkable to observe that the quasi-brittle models are almost similar, even if, for values of $R_b$ above 20, the corrosion penetration, $x$, is higher for the rigid behavior of concrete. Results for brittle models differ from the quasi-brittle ones exhibiting half values of corrosion penetration. Results reported in Fig. 23 were obtained by using the following parameters: bar diameter $R_0 = 8$ mm, concrete cover $cc = R_0^4$, strength class of concrete C30/37 and expansion factor of oxide, $n = 2.5$.

Table 5 summarizes the results obtained for the various models with concrete cover, $cc = R_0 - R_0 = 30$ mm, bar radius, $R_0 = 8$ mm, volumetric expansion factor of oxide, $n = 2.5$ and strength class of concrete C30/37.

Fig. 24a shows the results for corrosion penetration, $x$, as a function of the bar radius, $R_b$, for the models investigated with concrete cover, $cc = R_4 - R_0 = 35$ mm, bar radius, $R_0 = 8$ mm, oxide expansion factor, $n = 2.5$ and the strength class of concrete is C30/37. Fig. 24b shows corrosion penetration, $x$, and the percentage of bar section loss, related to the concrete cover $cc = R_4 - R_0$. Deformable quasi-brittle model behaves in both cases differently from other models. In fact the value of corrosion penetration is about double than those exhibited by other models.

![Fig. 21. Radial expansion, y, vs. $c_{/2R_0}$ ratio, compared to [51] at complete concrete cover cracking.](image)

![Fig. 22. Radial expansion, y, vs. $c_{/2R_0}$ ratio, compared to [51] at initiation of cracking.](image)
The aim of the work is to correlate bar corrosion penetration leading to concrete cover cracking initiation as a function of both geometric and structural parameters of materials. FEM analyses were performed to evaluate the behavior of both localized and general corrosion processes.

The first outcomes of the analyses are: (i) the maximum tensile stress, \( \sigma_{th} \), is attained close to the bar in the circumferential direction, and (ii) the crack develops in the radial direction until it reaches the external surface of concrete. These findings were observed for both localized corrosion and general corrosion even if, in case of pitting, tensile stress level is higher close to the bar. Conversely, the volume of concrete involved by tensile stresses is much lower. In the light of the conclusions outlined above, only the general corrosion process was addressed in the final part of this paper.

In order to check the possibility to compare the behavior of cylindrical simplified models to real RC members in terms of concrete cracking, a methodology was shown to assess corrosion of steel reinforcing bars in concrete. It has been noticed that the average underestimation of the proposed method is almost negligible if, in cylindrical models, a value of concrete cover is assumed equal to the minimum between concrete cover, \( c_c = R_4 - R_0 \), and the half distance between two consecutive bars, \( i_c/2 \). Maximum underestimation of the model was found equal to about 10% thus still comparable to usual uncertainty on concrete tensile strength.

Two analytical models were presented in order to evaluate the corrosion penetration, \( x \), leading to concrete cover cracking. The proposed model suggests that, oxide expansion factor, \( n \), bar radius, \( R_0 \) and concrete radius, \( R_4 \), are the most relevant parameters as far as cracking initiation is concerned. From a mechanical point of view, the dependency on concrete cover \( c_c = R_4 - R_0 \) is reduced even if, from an electrochemical point of view, a greater concrete cover slows down the penetration of aggressive agents and the consequent corrosion process taking place at the bar interface. However, in terms of crack propagation, results obtained indicate that propagation depends still on the same parameters, but with a major dependency from the concrete cover. In addition, oxide penetration increases slightly as elastic modulus of concrete, \( E_o \), decreases. Parameters as type or dimensions of aggregates, and creep effects, also influence the crack initiation and propagation and all contribution can be simulated by means of the proposed models.

Main outcomes of the parametric analyses by means of the proposed analytical models refer to deformable quasi-brittle behavior of concrete. However, other concrete behaviors in tension were also addressed in this paper. In the crack propagation model, the cracked concrete crown has been modeled as rigid or deformable, with a brittle or quasi-brittle cracking behavior. Quasi-brittle behavior leads to higher corrosion penetration compared to a brittle behavior, considering rigid or deformable concrete assumptions. Finally, results of the analytical models were in good agreement with experimental findings reported in the literature.

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References


