March, 2006

Bias in What We Truly Want to Measure in the Most Common Test of Uncovered Interest Parity and a Suggestion for an Unbiased Alternative

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March 4th, 2006

Background

The idea comes from Froot and Thaler's 1990 Journal of Economic Perspectives survey paper, "Anomalies: Foreign Exchange".

- The main idea is essentially that 2 assets with equal default risk, denominated in different currencies, should have the same expected *real* return – if investors are rational and risk neutral.

If not it is said there is a forward discount bias, and uncovered interest parity does not hold.

- A test for forward discount bias, very frequently performed in the literature (75 documented tests) involves running a regression on the following equation:

$$\Delta S_{t+k}^{F/H} = \alpha + \beta(i_t^H - i_t^F) + \eta_{t+k} \qquad (1)$$

where;

η_{t+k} is a 0 mean noise term.

$$\Delta S_{t+k}^{F/H} = \log(S_t^{F/H}) - \log(S_{t+k}^{F/H}) = \log\left(\frac{S_t^{F/H}}{S_{t+k}^{F/H}}\right)$$

= The *continuously* *compounded* *net* depreciation in the foreign currency.

$(i_t^H - i_t^F)$ = The difference between the *continuously compounded* *net* *nominal* interest rate in the foreign country and in the home country.

- For example;

Continuously Compounded Nominal Net Interest Rate on a Grade B bond:

Bolivia: 50% USA: 10%

Given this, Froot and Thaler assert that uncovered interest parity implies that on average the continuous net depreciation in the Bolivian currency will be 40%.

Thus, they assert (and imply that a large literature does also), that uncovered interest parity implies that the regression equation:

$$\Delta S_{t+k}^{F/H} = \alpha + \beta(i_t^H - i_t^F) + \eta_{t+k}$$

will have an $\alpha = 0$ and a $\beta = 1$.

- In fact, 75 published regressions have been run on this equation and not one has found a $\beta \geq 1$. The average β from these regressions is -0.88, with only "a few" showing positive β's.

→ This is quite a surprising finding in that it means that the country with the higher nominal interest rate – on bonds with the same default risk – doesn't have the countervailing penalty of a currency depreciation, on average.

Instead,In fact, you get the higher nominal rate *plus* , on average, a currency *appreciation*.

→ So, in the example above, the Bolivian grade B bond would have a 40 percentage point higher nominal interest rate over US grade B bonds, *and* its currency on average would appreciate 40 x .88 = 35.2%.

→ **So, there is a question of how this can persist if markets are efficient.**

• **Explanations in the literature**

1) Peso Problem

2) Risk Aversion – The hypothesis is that when there is a large expected appreciation in a currency, there is a lot of volatility around that expected appreciation. Thus, although the currency may on average appreciate, there is a substantial probability of an extreme depreciation (like a hyper-inflation), and therefore an extreme loss to the investor.

Furthermore, this risk may be substantially undiversifiable.

3) Actions by Central Banks

Issue:

• An aside from my paper idea, but something I would really like to clear up;

• Froot and Thayler state several times that they think the evidence in the literature points to risk aversion, as well as risk aversion and the peso problem combined, not having much power in explaining the highly negative β. For example, on page 190 they state:

"Indeed, the conclusion we draw from the tests completed so far is that there is no positive evidence that the forward discount's bias is due to risk (as opposed to expectational errors)."

and they go on to state:

"Taken as a whole the evidence suggests that explanations which allow for the possibility of market inefficiency should be seriously investigated."

• But, the question arises, if risk is not an issue, and the high nominal interest rate bonds pay so much of a higher real rate on average, why wouldn't investors jump on this, and force those high nominal rates down?

And, Froot and Thayler do actually address this. They state on page 189:

"Whether or not there really is money to be made based on the apparent inefficiency of foreign exchange markets, it is worth emphasizing that the risk return trade off for a single currency is not very attractive. The annualized standard error of the regression estimates of equation (1) is about 36%. This implies that a strategy that generates expected profits of \$1 comes with a standard deviation of profits of \$15...With transactions costs the risk-return trade off becomes even less favorable. Although much of the risk in these strategies may be diversifiable in principle, more complex diversified strategies may be much more costly, unreliable, or difficult to execute."

→ I don't understand this. At one point they're saying that risk aversion is not a very significant factor and that markets may well be inefficient, but then they say the expected return from trading on this phenomenon looks like it's not worth the risk? especially including transactions costs, which makes the market look efficient and rational, not inefficient and irrational?

→ It seems like what they're saying is not internally consistent?

→ How can they say that risk is not a substantial explanation for this phenomena, and then say that the reason why investors don't bid/buy the phenomena away is that the excess expected returns are too risky? So, I'm unclear on this.

End of aside; now to some ideas I had.

My Ideas

Froot and Thaler, and the authors of the 75 tests of the $\beta = 1$ hypothesis, start with the following assumptions

i) Investors are rational

ii) Investors are risk neutral

I will attempt to show that given these assumptions their test, analysis, and conclusions are significantly flawed, and I will give suggestions for a test of uncovered interest rate parity that avoids these flaws.

- Lets start by comparing 2 investment plans.

→ **Plan 1:** An investor invests 1 home currency unit in a k period home bond at time t (today).

Under this plan at time t + k the investor gets $1+\mathrm{i}_t^H$, where i_t^H is a *simple* net interest rate. This payoff, in home currency (assuming there's no default risk) is certain. It is nonstochastic.

→ **Plan 2:** The investor takes his 1 home currency unit, converts it to foreign currency units at the time t (current) spot rate, $S_t^{F/H}$, then invests those foreign currency units in a k period foreign bond (that has the same default risk as the home bond in plan 1), and then at the end of the k periods he cashes out his foreign bond and converts his foreign currency units back to home currency units at whatever the spot exchange rate ends up being at time t+k, that is at $\widehat{S}_{t+k}^{H/F}$. There is a hat on this variable because it is stochastic. At time t, the present, the investor does not know for certain what it is going to be.

Under this plan at time t + k the investor gets $S_t^{F/H}(1+\mathrm{i}_t^F)S_{t+k}^{H/F} + \eta_{t+k}$, where i_t^F is a *simple* net interest rate and η_{t+k} is a 0 mean noise term (perhaps N[0, σ^2]).

→ Now, why is this the investors payoff. Like F&T, I assume that investors can make unbiased forecasts, otherwise any phenomena can be explained by just saying that investors are consistently poor forecasters. They may *actually* be poor forecasters, but it is still worthwhile to see if the data is consistent with them being good (close to) unbiased forecasters, as well as being rational and risk neutral (or behaving as though they are risk neutral because the "risks" turn out to be (close to) diversifiable).

→ Note how the payoff is broken into 2 parts: $S_t^{F/H}(1+\mathrm{i}_t^F)S_{t+k}^{H/F}$, which is what the payoff <u>*actually*</u> will be at time t + k (note there's no hat on the $S_{t+k}^{H/F}$ variable.) , plus a 0 mean error term, η_{t+k} . This reflects our assumption that investors are able to forecast future payoffs unbiasedly.

- When investors are deciding between investment plans 1 and 2, they will try to forecast what the total payoff will be for both plans at the end date, and again we assume that they are able to forecast unbiasedly. The investors then, being rational and risk neutral, will choose whichever plan they think has the higher expected end date <u>*payoff*</u>. Thus, in equilibrium it must be the case that;

$$\left[1+\mathrm{i}_t^H\right] = \left[S_t^{F/H}(1+\mathrm{i}_t^F)\ S_{t+k}^{H/F}\right] + \ \eta_{t+k} \quad (*)$$

That is to say <u>*in repeated sampling*</u>, asymptotically, (*) should hold; or, more precisely, it should be the case that:

$$\lim_{T\to\infty}\left[\frac{\sum\limits_{t=1}^{T}\left(1+\mathrm{i}_t^H\right)}{\sum\limits_{t=1}^{T}\left(\left[S_t^{F/H}(1+\mathrm{i}_t^F)\ S_{t+k}^{H/F}\right] + \ \eta_{t+k}\right)}\right] = 1$$

and in a random sample of "large" size, T, we should find that

$\sum\limits_{t=1}^{T}\left(1+i_t^H\right) = \sum\limits_{t=1}^{T}\left(\left[S_t^{F/H}(1+\mathrm{i}_t^F)\ S_{t+k}^{H/F}\right] + \ \eta_{t+k}\right)$ within a reasonable degree of statistical error.

Thus, the assumptions imply a model of;

$$\left[1+\mathrm{i}_t^H\right] = \left[S_t^{F/H}(1+\mathrm{i}_t^F)\ S_{t+k}^{H/F}\right] + \ \eta_{t+k} \qquad (3)$$

And this is intuitively appealing, after all the goal is to test "uncovered interest parity", and that's what (3) is; the interest for the 2 methods is in parity – $E[LHS] = E[RHS]$, but uncovered – there's 0 mean randomness, η_{t+k} *.*

Now, doing some algebra on (3);

$$\Longrightarrow \frac{1+\mathrm{i}_t^H}{1+\mathrm{i}_t^F} = \frac{S_t^{F/H}}{S_{t+k}^{F/H}} + \frac{\eta_{t+k}}{1+\mathrm{i}_t^F} \qquad ; \text{ note that } \frac{1}{S_{t+k}^{F/H}} = S_{t+k}^{H/F} \quad . \qquad (4)$$

Next, take the log of both sides of (4);

$$\Longrightarrow \underbrace{\log\left(S_t^{F/H}\right) - \log\left(S_{t+k}^{F/H}\right)}_{\equiv \Delta S_{t+k}^{F/H}} \;=\; \log\left[\frac{1+\mathrm{i}_t^H}{1+\mathrm{i}_t^F} - \frac{\eta_{t+k}}{1+\mathrm{i}_t^F}\right] =$$

$$\log\left[(1+\mathrm{i}_t^F)^{-1}\right] + \log\left[1+\mathrm{i}_t^H - \eta_{t+k}\right] \qquad (5)$$

Now, keep in mind that $\log\left(1+\mathrm{i}^{simple}\right) = \mathrm{i}^{continuously\ coumpounded}$, and note that i_t^H in equation (6) below (with the italic looking i) is a *continuously compounded* net interest rate, as opposed to i_t^H in equation (5) above (with the very straight i) , which is a *simple* net interest rate.

$$(5) \Longrightarrow \Delta S_{t+k}^{F/H} = -i_t^F + \log\left[1+\mathrm{i}_t^H - \eta_{t+k}\right] \qquad (6)$$

$$\Longrightarrow \Delta S_{t+k}^{F/H} = \log\left[\exp\left(i_t^H\right) - \eta_{t+k}\right] - i_t^F \qquad (7)$$

• So, $\Delta S_{t+k}^{F/H}$ is *not* a linear function of i_t^H and i_t^F as Froot and Thaler, and the authors of the 75 other studies, assume when they test the specification:

$$\Delta S_{t+k}^{F/H} \;=\; \alpha + \beta(i_t^H - i_t^F) + \eta_{t+k}$$

In fact, $\Delta S_{t+k}^{F/H}$ *is instead a highly non-linear function of* i_t^H *and* i_t^F *, so there may be significant specification error in these tests of uncovered interest parity.*

An alternative test of uncovered interest parity that does not have specification error

My idea is to just do a regression on (3),

$$\left[1+\mathrm{i}_t^H\right]=\left[S_t^{F/H}(1+\mathrm{i}_t^F)\ S_{t+k}^{H/F}\right]+\ \eta_{t+k}$$

which truly is a linear specification. For convenience (3) could be rewritten as;

$$P_{n,t+k}^H=\alpha+\beta P_{n,t+k}^F+\ \eta_{n,t+k}\quad, \qquad (8)$$

where;
$P_{n,t+k}^H=\left[1+\mathrm{i}_{n,t}^H\right]$; the ex-post payoff for fixed income security n, n = 1...N.

$P_{n,t+k}^F=S_t^{F/H}(1+\mathrm{i}_{n,t}^F)\ S_{t+k}^{H/F}$; the ex-post payoff for the foreign counterpart to fixed income security n (which has equal

default risk – perhaps equal credit rating).

And, uncovered interest parity implies that $\alpha=0$ and $\beta=1$.

- The data would thus be a panel, and the best panel data techniques would be searched for to utilize here, and maybe there's even some Bayesian panel data techniques I could use.

- Note: One cannot say that the direction of causation in (8) is RHS $\rightarrow$ LHS , or that it's LHS $\rightarrow$ RHS ; it looks clearly to be RHS $\leftrightarrow$ LHS , and so an econometric technique which takes this into account would have to be used.